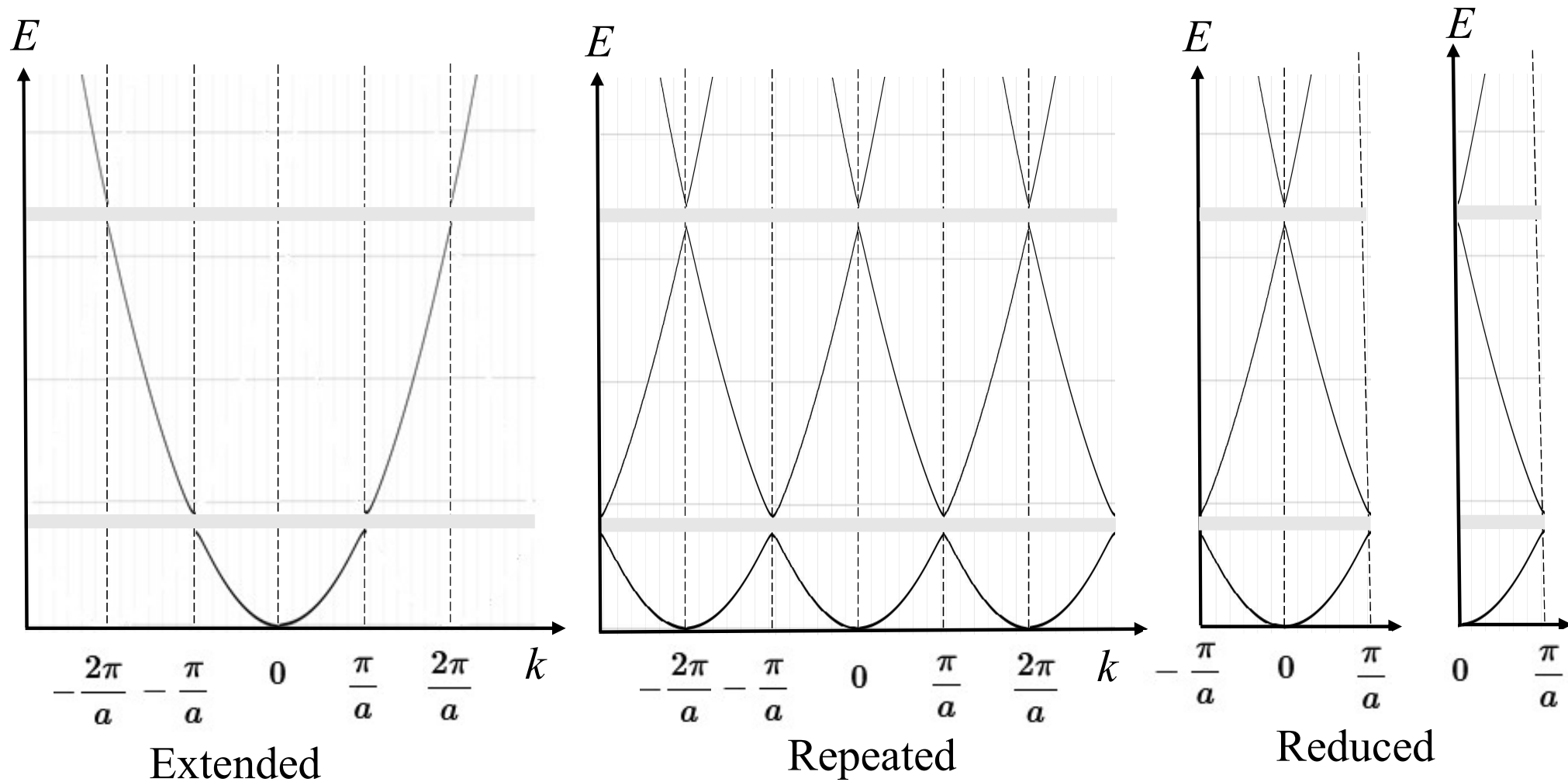


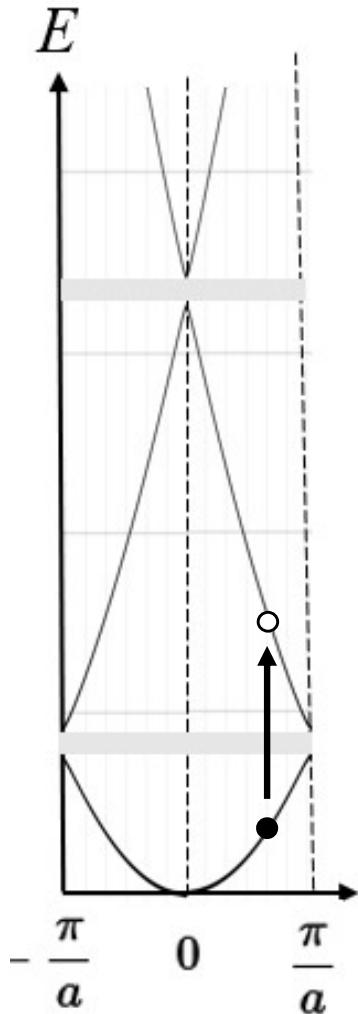
Electron Band Structure

Empty lattice approximation



$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \sum_{\vec{G}} C_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} = e^{i\vec{k}\cdot\vec{r}} \underbrace{e^{i\vec{G}_0\cdot\vec{r}} e^{-i\vec{G}_0\cdot\vec{r}}}_1 \sum_{\vec{G}} C_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} = e^{i(\vec{k}-\vec{G}_0)\cdot\vec{r}} \sum_{\vec{G}} C_{\vec{G}} e^{i(\vec{G}+\vec{G}_0)\cdot\vec{r}}$$

Conservation of momentum

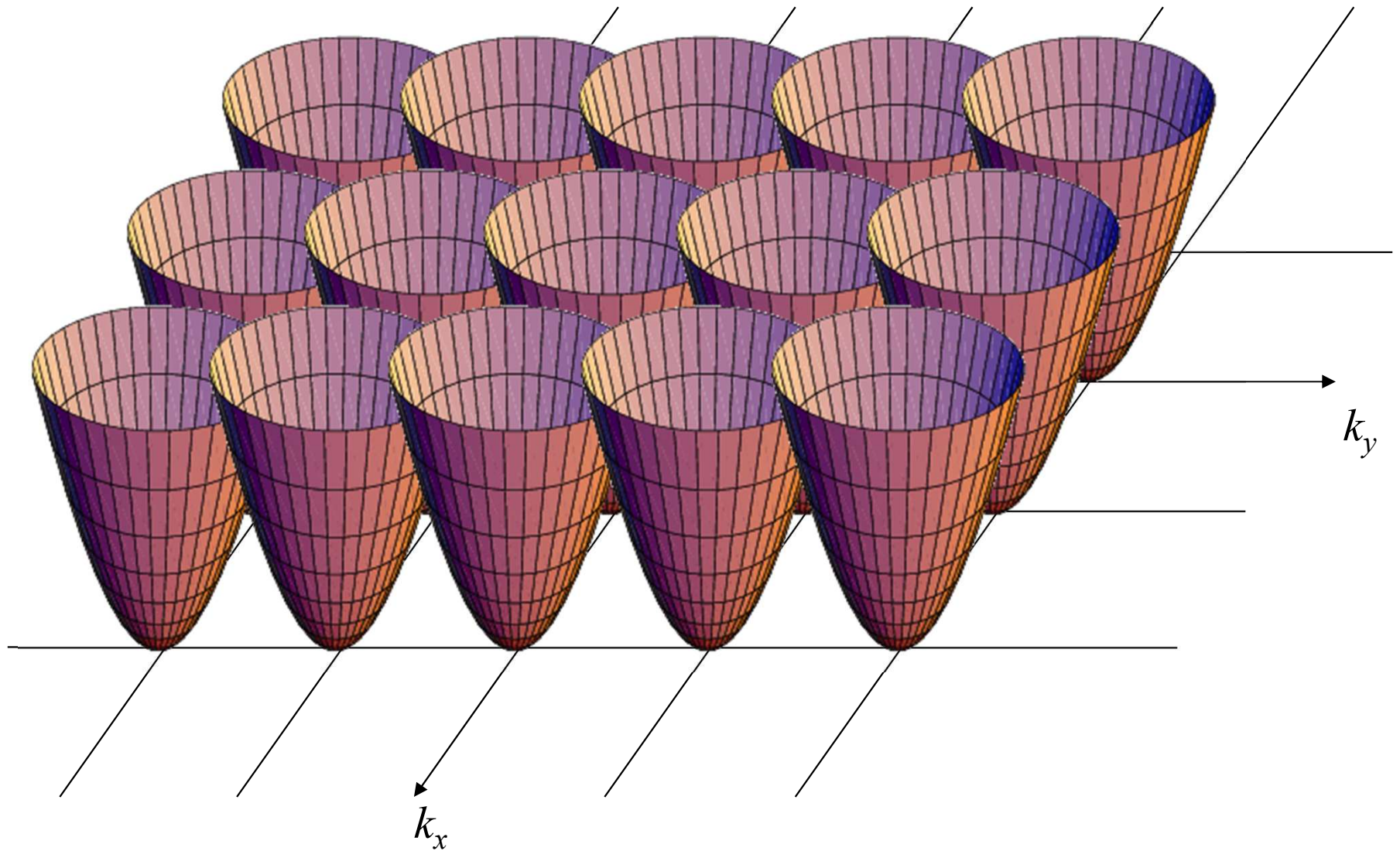


When an electron absorbs a photon and makes a transition from an occupied state to an empty state, energy and momentum are conserved.

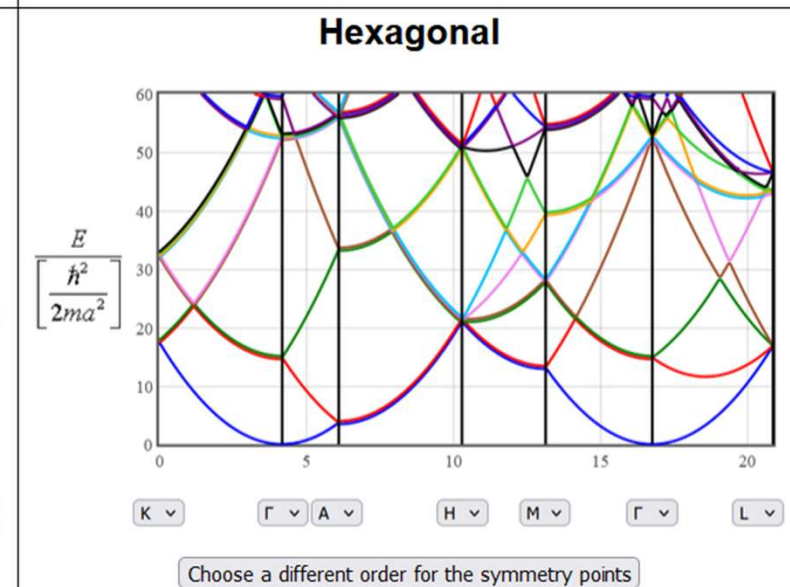
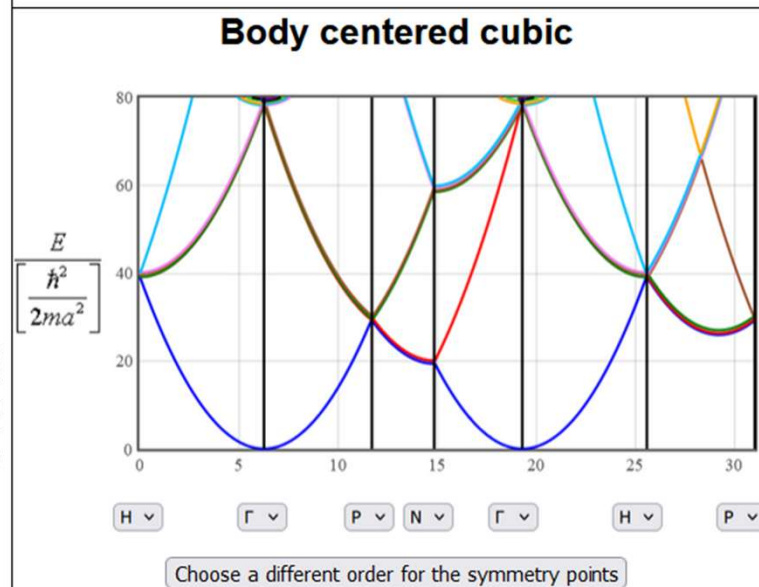
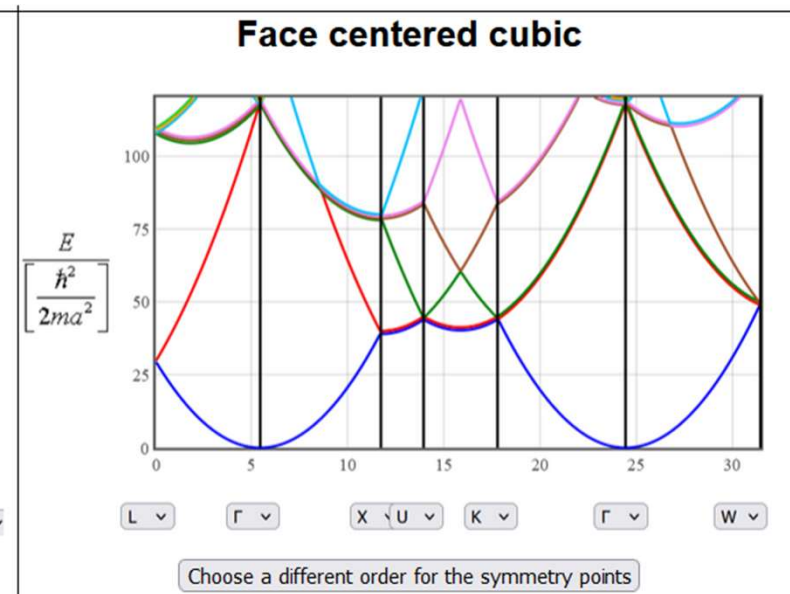
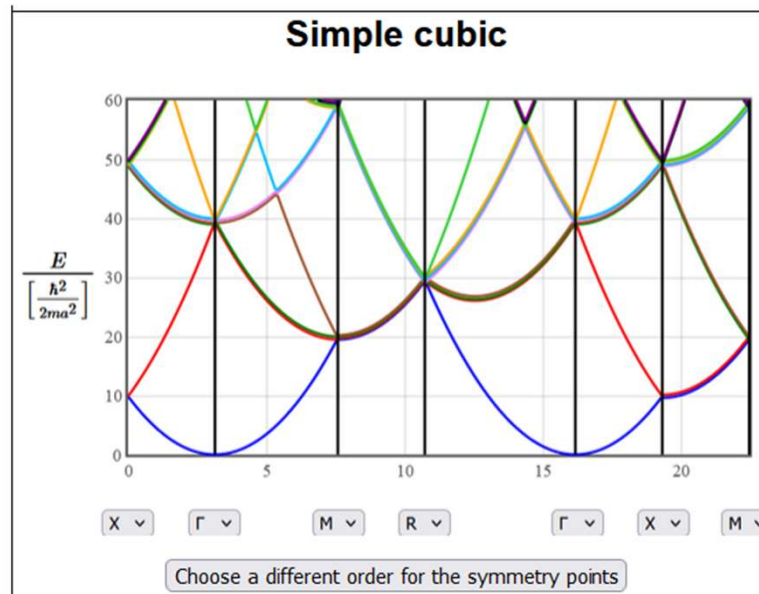
$$\hbar \vec{k}_{\text{photon}} + \hbar \vec{k}_{\text{initial}} = \hbar \vec{k}_{\text{final}} + \hbar \vec{G}$$

$$\vec{k}_{\text{initial}} = \vec{k}_{\text{final}} + \vec{G}$$

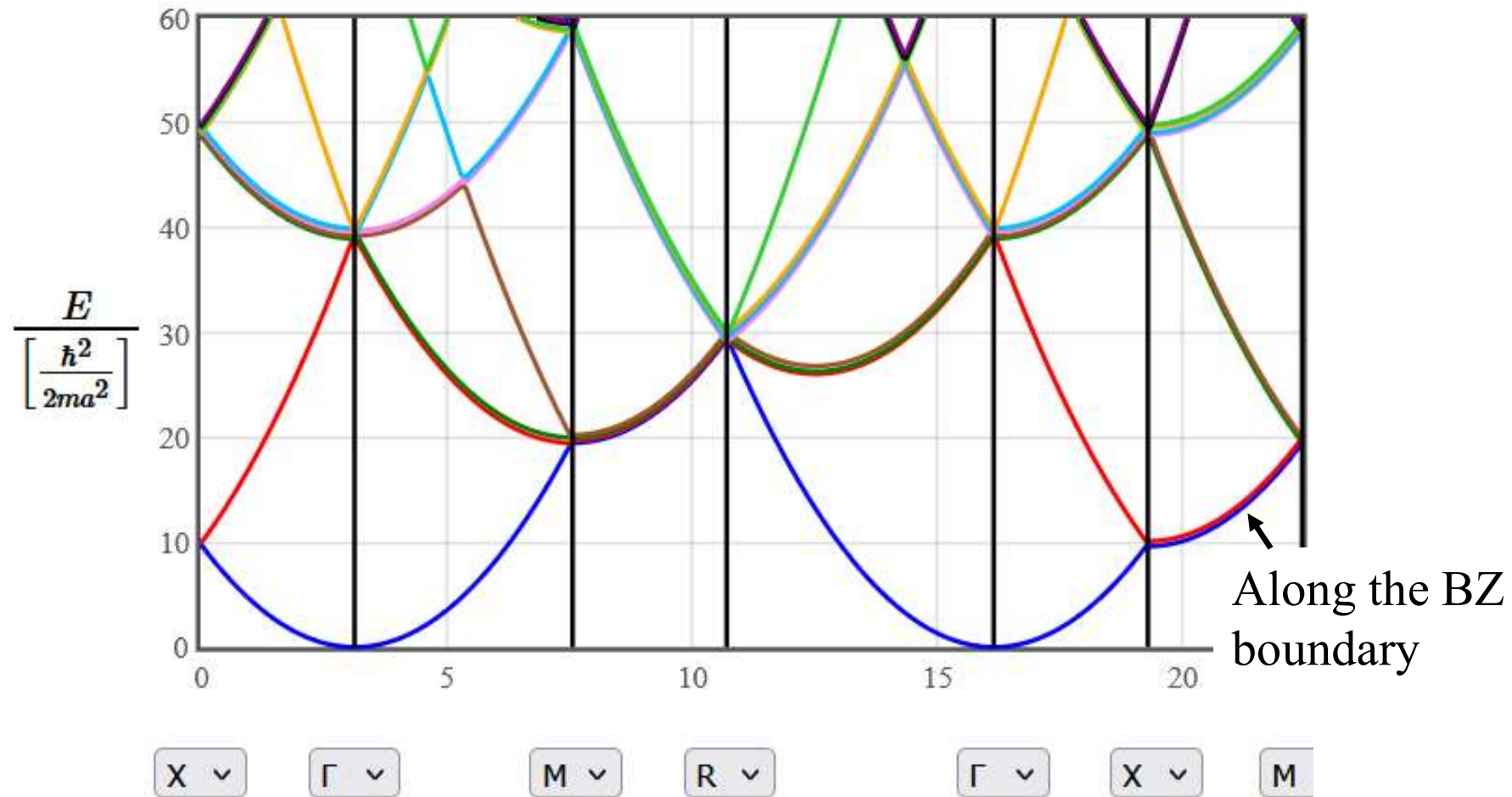
Empty lattice approximation



Empty lattice approximation



Empty lattice approximation



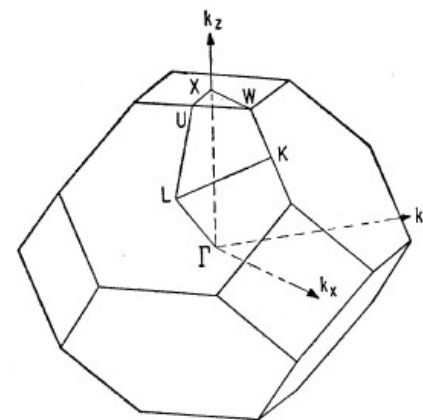
There are as many k states per band as there are primitive unit cells in the crystal.

There are 2 electrons states (spin) per k state.

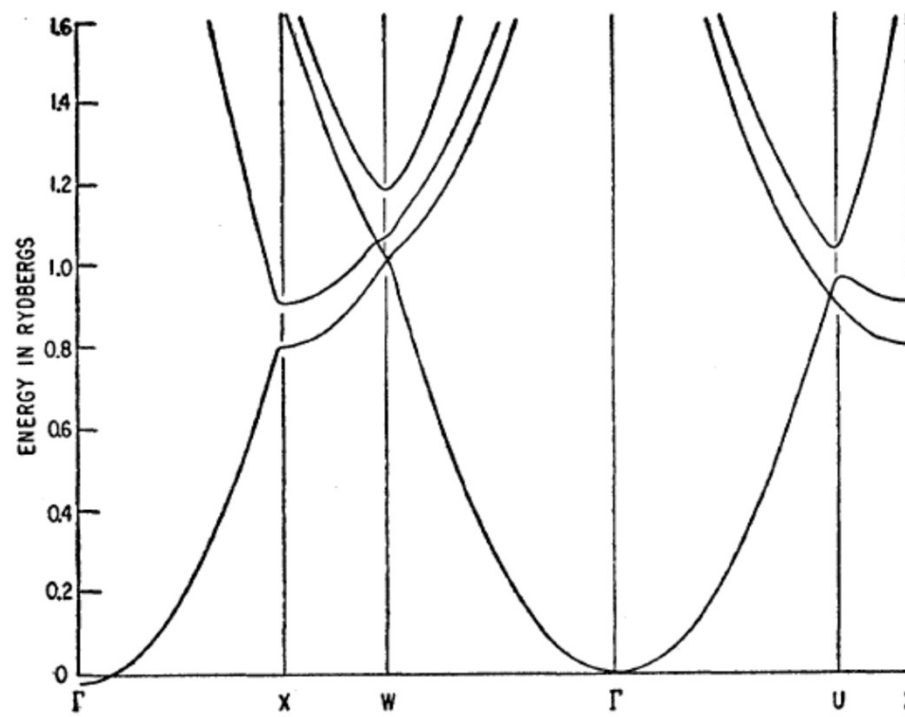
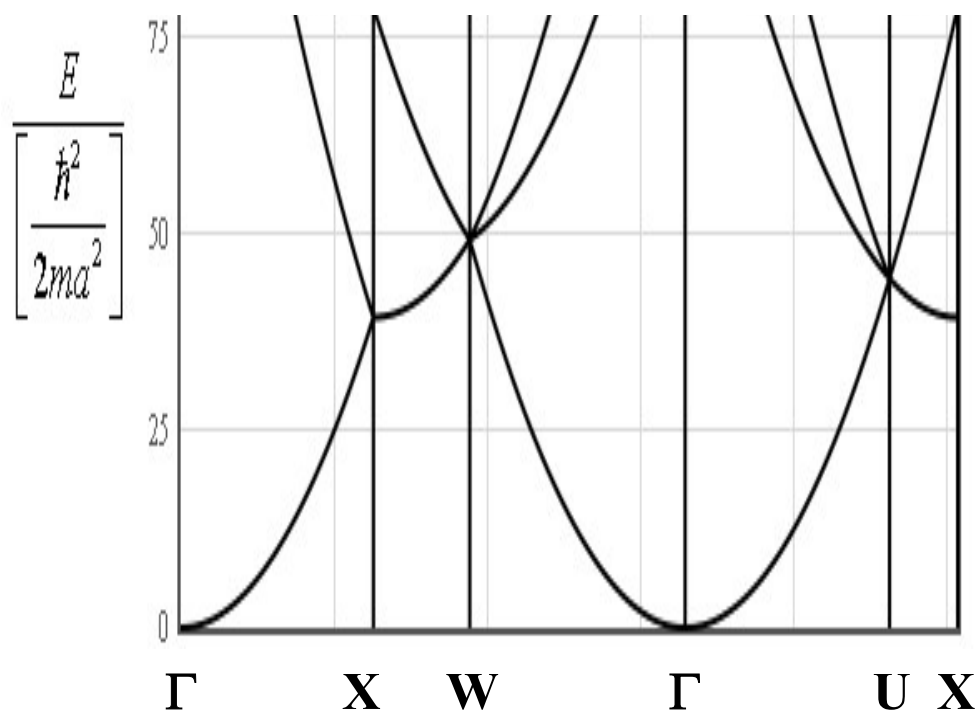
Band Structure of Aluminum

WALTER A. HARRISON

General Electric Research Laboratory, Schenectady, New York

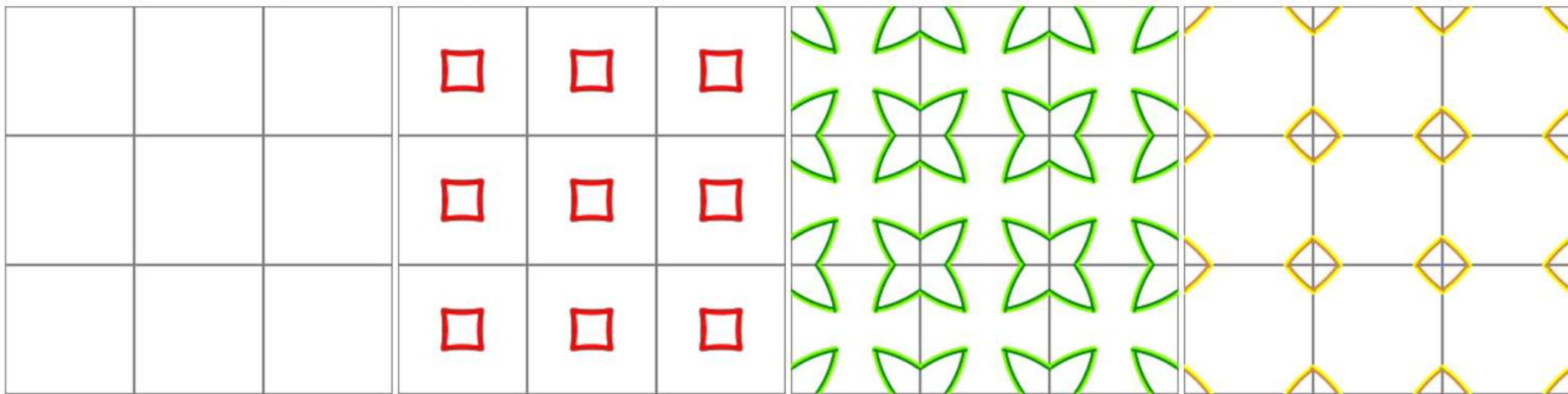
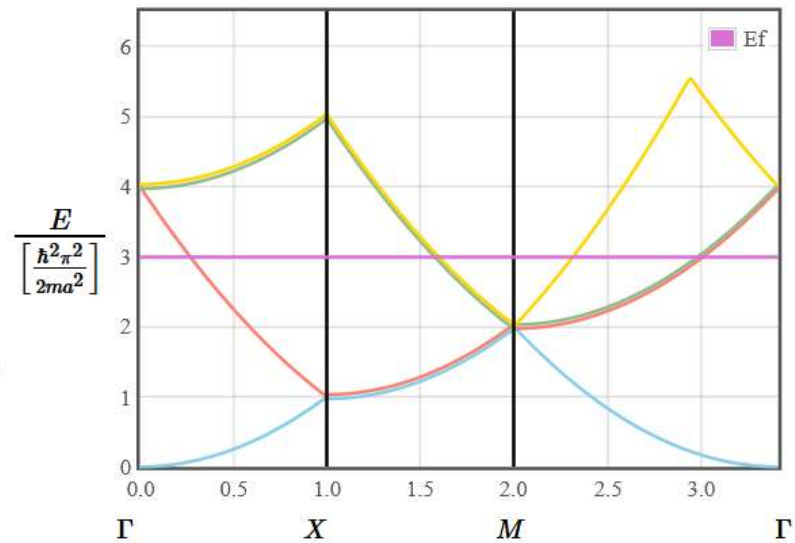
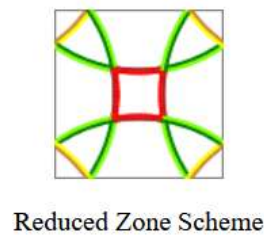
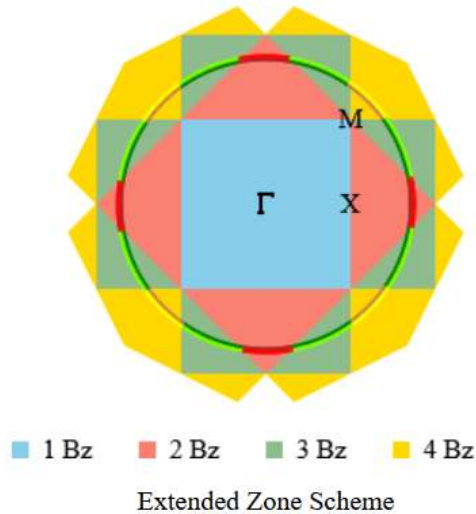


empty lattice approximation



Fermi surface of a two-dimensional square lattice

$n = 4.7$ $\frac{k_F a}{\pi} = 1.730$ $\frac{E_F}{\left[\frac{\hbar^2 \pi^2}{2ma^2}\right]} = 2.992$ $\frac{E_F}{\left[\frac{\hbar^2 \pi^2}{2ma^2}\right]}$ electrons/unit cell



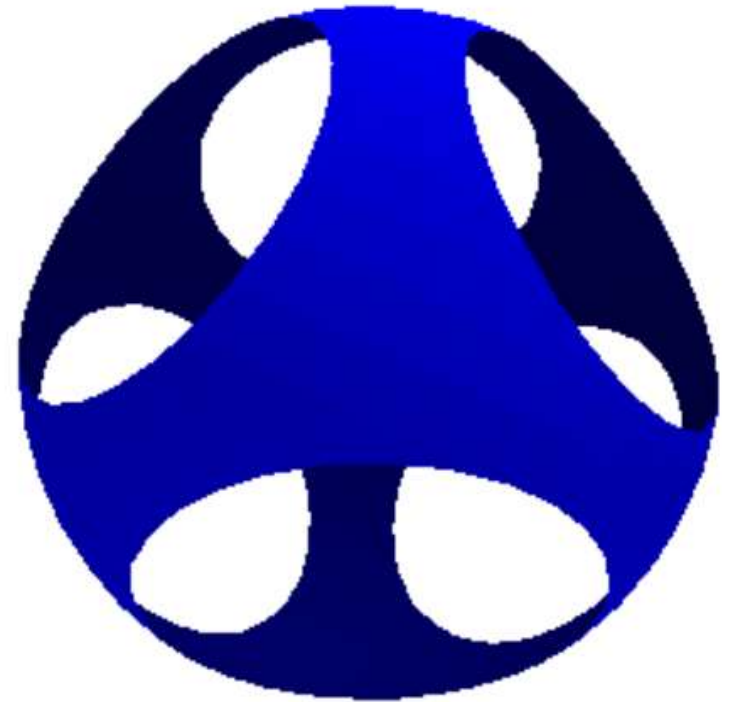
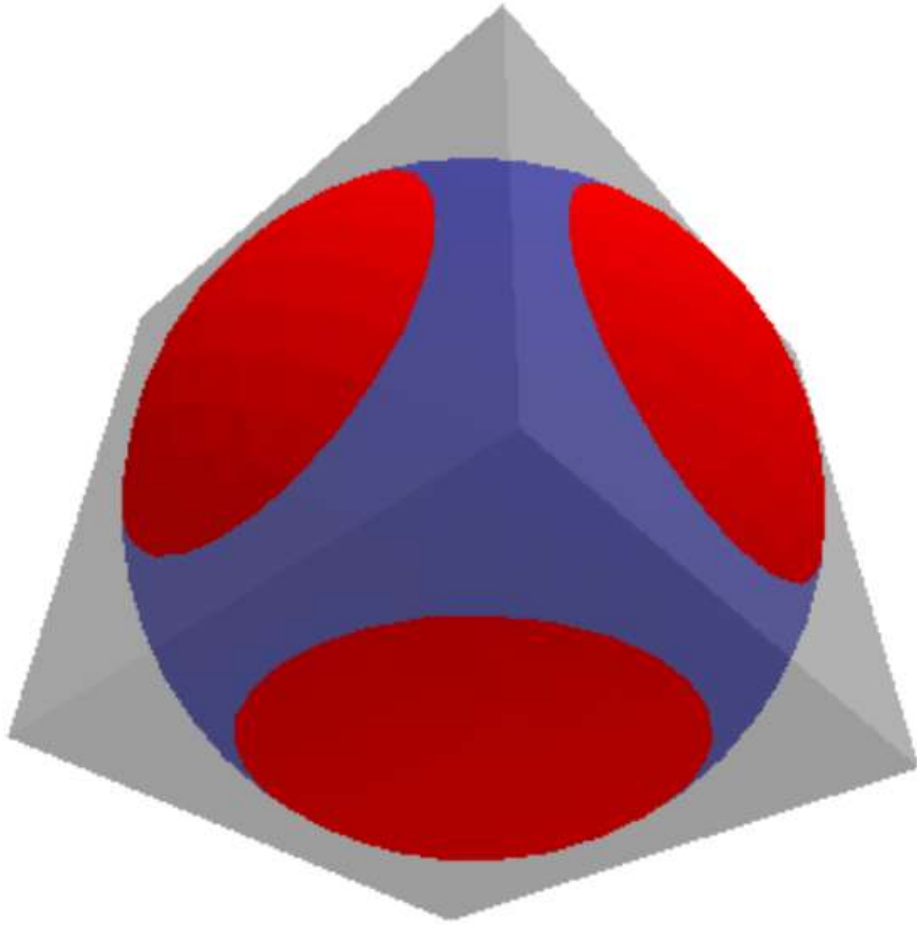
1st Brillouin Zone
Repeated Zone Scheme

2nd Brillouin Zone
Repeated Zone Scheme

3rd Brillouin Zone
Repeated Zone Scheme

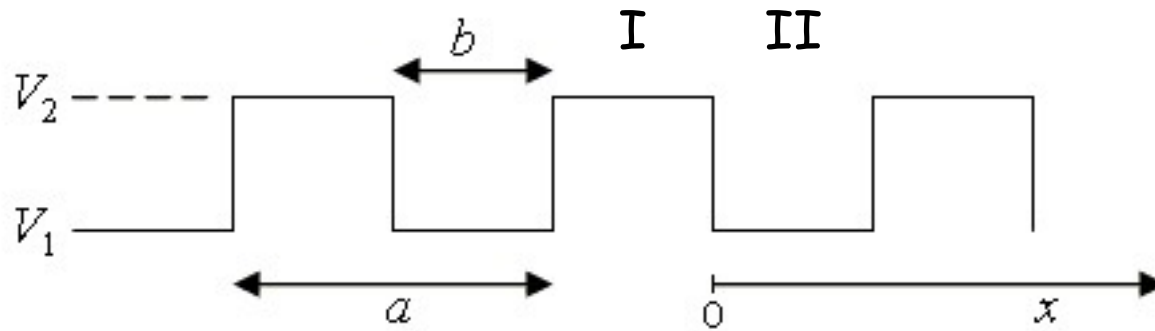
4th Brillouin Zone
Repeated Zone Scheme

Fermi Surfaces



https://lampz.tugraz.at/~hadley/ss1/book/bands/3d_fermisurface/index.php

A separable potential

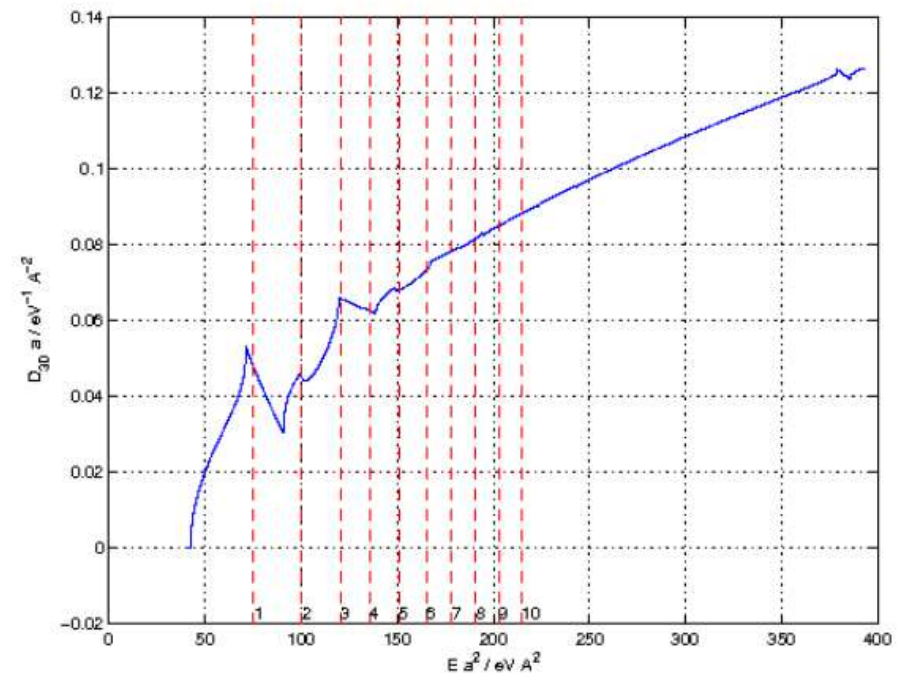
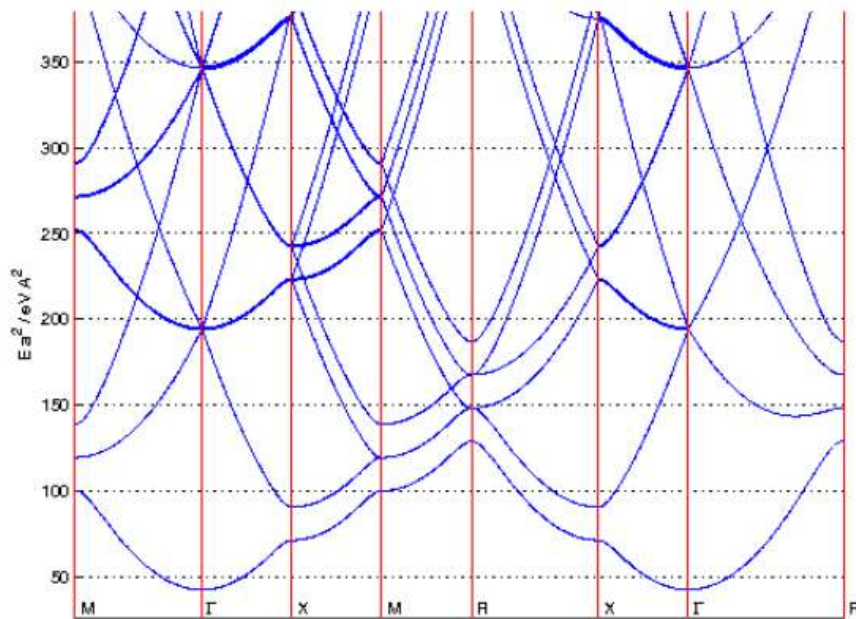
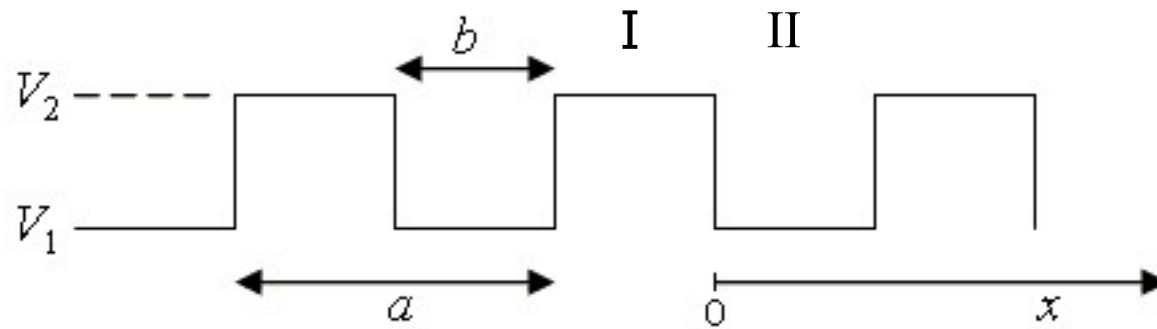


$$-\frac{\hbar^2}{2m}\nabla^2\Psi + (V(x) + V(y) + V(z))\Psi = E\Psi$$

Ψ is the product of the solutions to the Kronig-Penney model.

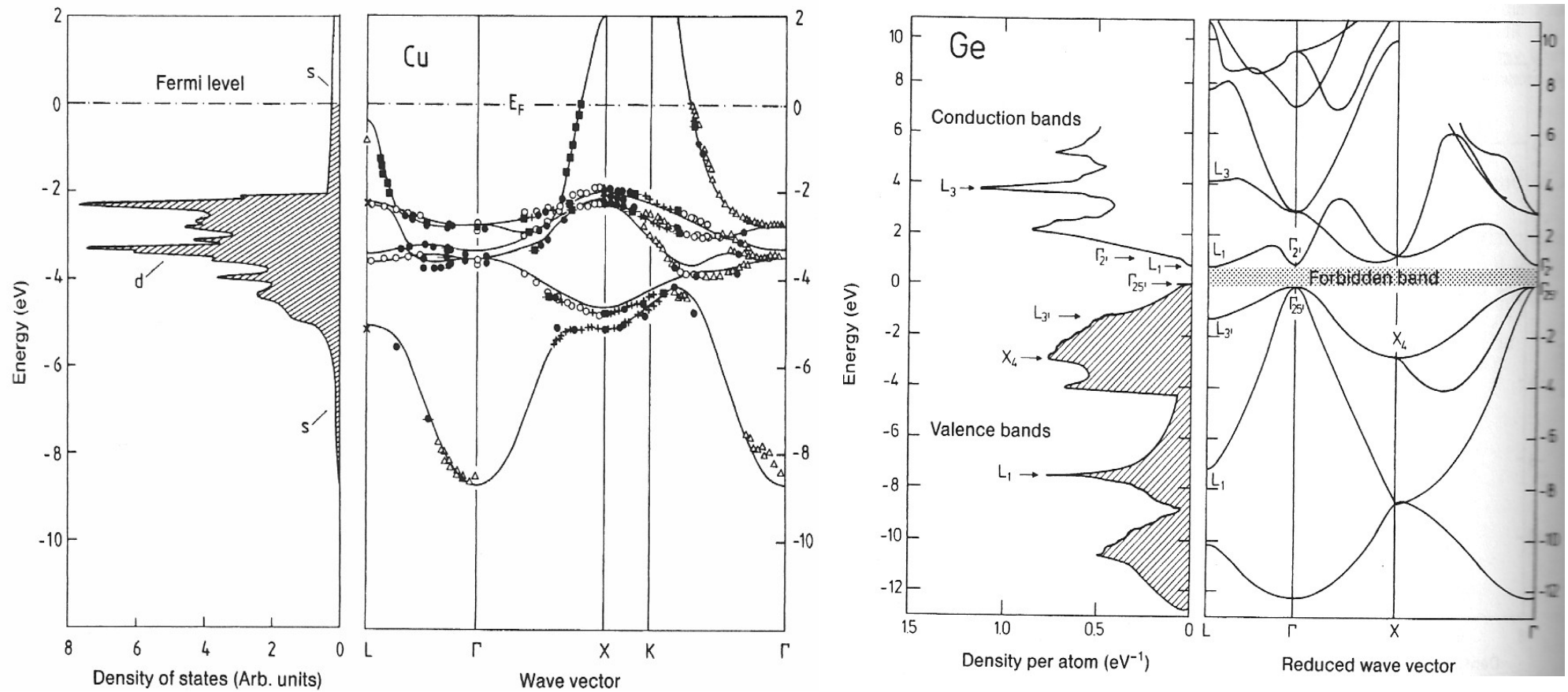
$$\Psi(x, y, z) = \psi_{KP}(x)\psi_{KP}(y)\psi_{KP}(z)$$

A separable potential



<http://lampx.tugraz.at/~hadley/ss1/separablecrystals/thermo.html>

Metals, semiconductors, and insulators

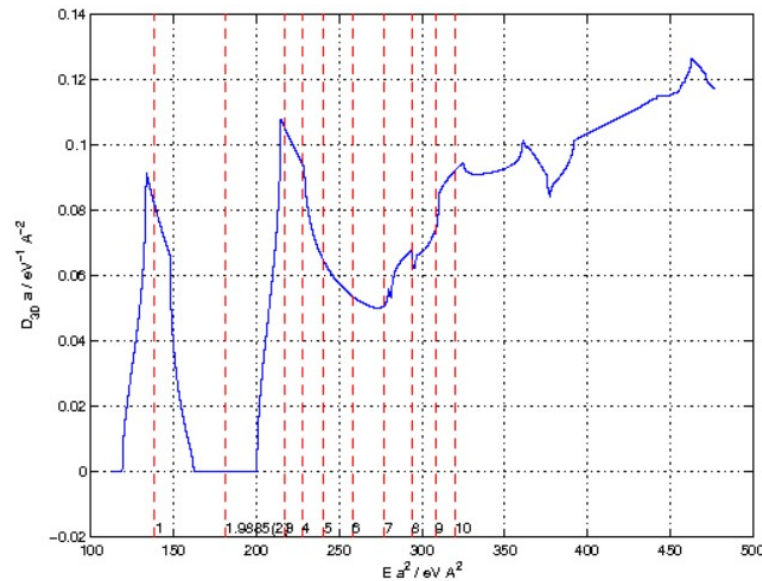


Insulators: band gap > 3 eV

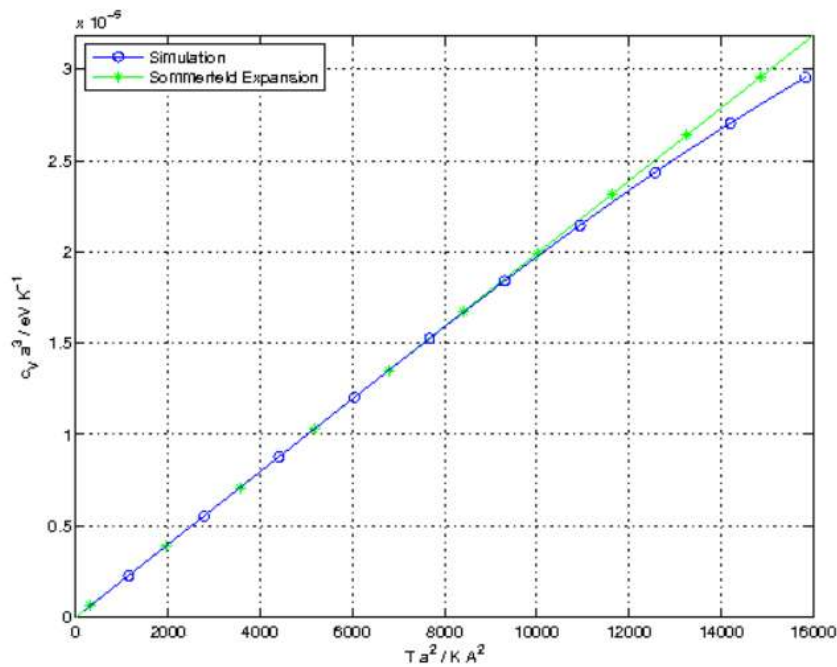
From Ibach & Lueth

The thermodynamic properties depend dramatically on the number of electrons in the primitive unit cell

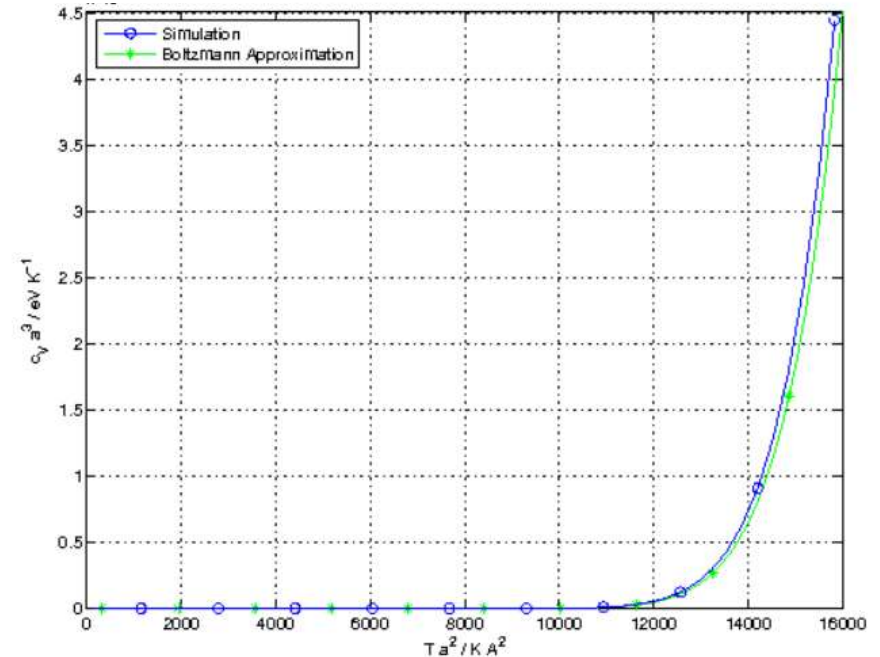
Metal
 $n = 1$



Insulator
 $n = 2$



Specific heat c_v



Specific heat c_v

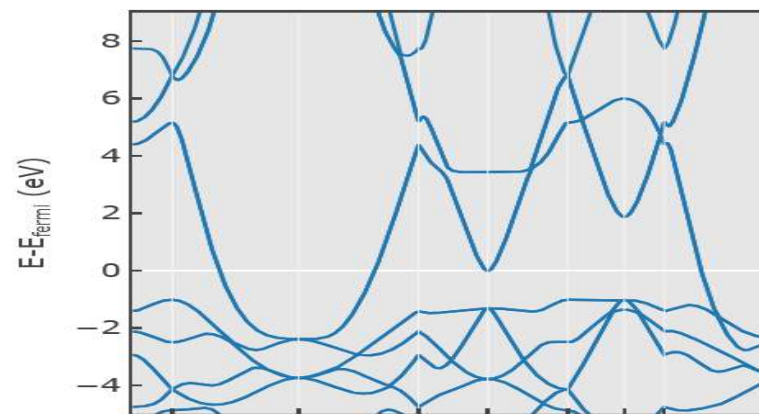
Band structure calculations

Package ↕	License† ↕	Language ↕	MPI ↕	OpenMP ↕	GPU ↕	I/O libraries ↕	Parallel I/O ↕
ABINIT	Free, GPL	Fortran	Yes	Yes	Yes, CUDA	Yes, HDF5, NetCDF	Yes, Fortran and HDF5
ACES ^[1]	Free, GPL	Fortran, C++	Yes	No	Yes	Unknown	Unknown
ADF, Amsterdam Modeling Suite	Commercial	Fortran	Unknown	Unknown	Yes, CUDA	Yes, HDF5, custom	Unknown
AMPAC	Academic	Unknown	Unknown	Unknown	No	Unknown	Unknown
Atomistix ToolKit (QuantumATK)	Commercial	C++, Python	Yes	Yes	Yes, CUDA	Yes, HDF5, NetCDF	Yes, HDF5
BigDFT	Free, GPL	Fortran	Yes	Yes	Yes	Yes, HDF5, NetCDF	Yes, HDF5, NetCDF
CADPAC	Academic	Fortran	Unknown	Unknown	No	Unknown	Unknown
CASINO (QMC)	Academic	Fortran 2003	Yes	Yes	Yes, OpenACC	No	No
CASTEP	Academic, commercial	Fortran 95, Fortran 2003	Yes	Yes	No	Unknown	Unknown

Quantum Espresso
VASP
FHI-AIMS
ABINIT



The Materials Project



Band structure calculations

