## Microelectronics and Micromechanics

## 1 Mikro- und Nanotechnologien in der Mikroelektronik

### 1.1 Grundprozesse

Ausgangspunkt ist der Siliziumwafer (früher Ge, heute auch GaAs). Ist ein Einkristall mit sehr geringer Defektdichte und hochrein (typisch einige Defekte/cm ${ }^{2}$ ). Herstellung meist nach dem Czochralskiverfahren (Ziehen aus der Schmelze).
Zur Zeit typische Waferdurchmesser $10 \mathrm{~cm}\left(4\right.$ ") bis 20 cm ( $8^{\prime \prime}$ ). Länge des Einkristalls bis 2 m ; Dicke der Wafer $\approx 0.5 \mathrm{~mm}$. (Fa. Wacker Chemie/Burghausen, Bayern).
Auf dem Wafer werden in vielfacher Anzahl die IC's (integrated circuits) hergestellt (chips, dies), typische Größen: $5 \times 6 \mathrm{~mm}^{2}$ (Logikbausteine), $4 \times 10 \mathrm{~mm}^{2}$ (Speicher). Pro chip mehrere Millionen Bausteine (Transistoren, Kondensatoren). Dimensionen im $\mu \mathrm{m}$ und sub- $\mu \mathrm{m}$ Bereich.


## Jack Kilby's first integrated circuit 1958

## Moore's law

The complexity for minimum component costs has increased at a rate of roughly a factor of two per year. Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will not remain nearly constant for at least 10 years.
G. Moore, 1965

Microprocessor Transistor Counts 1971-2011 \& Moore's Law



## Microelectronics



Silicon chips are used in computers, mobile telephones, and microcontrollers.

1 trillion transistors are produced simultaneously on a wafer.

Gate length ~ 15 nm
$1 \mu$ processor $\sim 1$ billion transistors ~ 100 Euros
1 transistor for $10^{-5}$ cents.
$10^{5}$ transistors $\sim 1$ cent $\rightarrow$ packaging is the major cost for simple circuits

100 euro/cm ${ }^{2}$


## International Technology Roadmap for Semiconductors

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## ITRS 2009 Edition

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http://www.itrs.net/reports.html

International Technology Roadmap for Semiconductors

Table PIDS2a High-performance (HP) Logic Technology Requirements - TCAD

| Year of Production | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 | 2026 | 2027 | 2028 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  <br>  | "16/14" |  | "11/10" |  | "8/7" |  | "6/5" |  | "4/3" |  | "3/2.5" |  | "2/1.5" |  | "1/0.75" |  |
|  | 40 | 32 | 32 | 28.3 | 25.3 | 22.5 | 20.0 | 17.9 | 15.9 | 14.2 | 12.6 | 11.3 | 10.0 | 8.9 | 8 | 7.1 |
|  | 20 | 18 | 16.7 | 15.2 | 13.9 | 12.7 | 11.6 | 10.6 | 9.7 | 8.8 | 8.0 | 7.3 | 6.7 | 6.1 | 5.6 | 5.1 |
|  | 16.0 | 14.4 | 13.4 | 12.2 | 11.1 | 10.2 | 9.3 | 8.5 | 7.8 | 7.0 | 6.4 | 5.8 | 5.4 | 4.9 | 4.5 | 4.1 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| BulkiSOI\|MG | 0.86 | 0.85 | 0.83 | 0.81 | 0.80 | 0.78 | 0.77 | 0.75 | 0.74 | 0.72 | 0.71 | 0.69 | 0.68 | 0.66 | 0.65 | 0.64 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bulk/SOlMG (nm) | 0.80 | 0.77 | 0.73 | 0.70 | 0.67 | 0.64 | 0.61 | 0.59 | 0.56 | 0.54 | 0.51 | 0.49 | 0.47 | 0.45 | 0.43 | 0.41 |
| Dielectric constant (K) of gate dielectrics | 12.5 | 13.0 | 13.5 | 14.0 | 14.5 | 15.0 | 15.5 | 16.0 | 16.5 | 17.0 | 17.5 | 18.0 | 18.5 | 19.0 | 19.5 | 20.0 |
| Physical gate oxide thickness (nm) | 2.56 | 2.57 | 2.53 | 2.51 | 2.49 | 2.46 | 2.42 | 2.42 | 2.37 | 2.35 | 2.29 | 2.26 | 2.23 | 2.19 | 2.15 | 2.10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bulk | 6.0 | 7.0 | 7.7 | 8.4 | 9.0 |  |  |  |  |  |  |  |  |  |  |  |
| SOllMG | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Bordy 7hekhess/imi/5] |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| SOI |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| MG | 6.4 | 5.8 | 5.3 | 4.9 | 4.4 | 4.1 | 3.7 | 3.4 | 3.1 | 2.8 | 2.6 | 2.3 | 2.1 | 2.0 | 1.8 | 1.6 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| SOI |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bulk'SOIIMG | 1.10 | 1.07 | 1.03 | 1.00 | 0.97 | 0.94 | 0.91 | 0.89 | 0.86 | 0.84 | 0.81 | 0.79 | 0.77 | 0.75 | 0.73 | 0.71 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| BulkiSOIIMG | 0.502 | 0.465 | 0.448 | 0.420 | 0.396 | 0.373 | 0.352 | 0.329 | 0.311 | 0.289 | 0.273 | 0.255 | 0.240 | 0.225 | 0.212 | 0.198 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bulk | 400 | 400 | 400 | 400 | 400 |  |  |  |  |  |  |  |  |  |  |  |
| SOI |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| MG | 250 | 250 | 250 | 250 | 250 | 250 | 200 | 200 | 200 | 200 | 200 | 150 | 150 | 150 | 150 | 150 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bulk'SOIIMG | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bulk | 1,348 | 1,355 | 1,340 | 1,295 | 1,267 |  |  |  |  |  |  |  |  |  |  |  |
| SOI |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| MG | 1670 | 1,680 | 1,700 | 1,660 | 1,660 | 1,610 | 1,600 | 1,480 | 1,450 | 1,350 | 1,330 | 1,170 | 1,100 | 1,030 | 970 | 900 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bulk | 0.306 | 0.327 | 0.334 | 0.357 | 0.378 |  |  |  |  |  |  |  |  |  |  |  |
| SOI |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| MG | 0.219 | 0.225 | 0.231 | 0.239 | 0.264 | 0.266 | 0.265 | 0.276 | 0.295 | 0.303 | 0.306 | 0.319 | 0.334 | 0.340 | 0.354 | 0.364 |

http://www.itrs.net/reports.html

## Micromechanics



Figure 1.2 Micromirror made of silicon, 1 mm in diameter, is supported by torsion bars $1.2 \mu \mathrm{~m}$ wide and $4 \mu \mathrm{~m}$ thick (detail figure). Reproduced from Greywall et al. (2003), Copyright 2003, by permission of IEEE

## Micromechanics


http://en.wikipedia.org/wiki/Cantilever\#mediavie wer/File:AFM_\%28used\%29_cantilever_in_Scan ning_Electron_Microscope,_magnification_1000 x.GIF


Figure 1.13 Silicon microneedle, about $100 \mu \mathrm{~m}$. Reproduced from Griss and Stemme (2003), Copyright 2003, by permission of IEEE

## Micromechanics



Figure 29.20 Mechanical gears made in multiple layer polysilicon process. Courtesy Sandia National Laboratories

## III <br> On Newsstands Now

Issue 8.04 | Apr 2000

## Why the future doesn't need us.

## Pg 1 of $11 \gg$

Print, email, or fax this article for free.

Our most powerful 21st-century technologies - robotics, genetic engineering, and nanotech - are threatening to make humans an endangered species.

By Bill Joy

From the moment I became involved in the creation of new technologies, their ethical dimensions have concerned me, but it was only in the autumn of 1998 that I became anxiously aware of how great are the dangers facing us in the 21st century. I can date the onset of my unease to the day I met Ray Kurzweil, the deservedly famous inventor of the first reading machine for the blind and many other amazing things.

Ray and I were both speakers at George Gilder's Telecosm conference, and I encountered him by chance in the bar of the hotel after both our sessions were over. I was sitting with John Searle, a Berkeley philosopher who studies consciousness. While we were talking, Ray approached and a conversation began, the subject of which haunts me to this day.

## http://www.if.tugraz.at/memm.html

### 513.160 Microelectronics and Micromechanics

Home

## Outline

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Microfabrication is a collection of production methods that are used to create very many electrical or mechanical components simultaneously. These methods have allowed us to produce microcontrollers and computers that affect virtually all aspects of our lives. Microcontrollers are found in household appliances such as coffee makers, vacuum cleaners, dishwashers, heating systems, and televisions. Mobile telephones and the internet have transformed how we communicate. Computers are essential for transportation systems, weather prediction, science, medicine, education, industrial design, banking, and retail sales. Even though computers are so important to modern life, there are relatively few people who understand how they are made.This course describes microelectronic and micromechanical devices and how they are fabricated. We will concentrate on silicon devices produced by optical lithography.

Technische Universität Graz


Electronic Materials


## ITIS

## Books



> International Technology Roadmap for Semiconductors

Principles of
Semiconductor Devices
by
Bart Van Zeghbroeck
e-book

### 513.160 Microelectronics and Micromechanics

## Home

Outline
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## Outline

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- Chemical vapor deposition
- Atomic layer deposition
- Laser ablation
- Thermal evaporation
- Sputtering


## Evaluation

## July 2 <br> Bring your phone (or a laptop).

## Examination

1 Contribution to improve the course
Oral exam

## Silicon

- Important semiconducting material
- 2nd most common element on earths crust (rocks, sand, glass, concrete)
- Often doped with other elements
- Oxide $\mathrm{SiO}_{2}$ is a good insulator

silicon crystal $=$ diamond crystal structure


## Silicon

e first transistor was made of polycrystalline germaum in 1947. Electron mobility in germanium is higher an in silicon, and germanium was readily available. owever, silicon, with its larger band gap, was favoured cause of smaller leakage currents. Initially there was consensus whether single crystalline or polycrystalline aterial was better, but the rapid development of sine crystal silicon growth in the 1950's soon dominated e market. The real breakthrough came when the beneial role of silicon dioxide was recognized: it provided ssivation of semiconductor surfaces, and it resulted in aproved transistor reliability. When it was further noed that the $\mathrm{SiO}_{2}$ layer could act as a diffusion mask d as isolation for integrated metallization, the way was en for the invention of the integrated circuit.
multicrystalline silicon! As short forms, c-Si is used for crystalline silicon and a-Si for amorphous silicon, while polycrystalline silicon is known simply as poly. In the solar cell industry crystalline silicon is sometimes called X-Si.

### 4.1 Silicon Material Properties

Silicon material properties are an excellent compromise between performance and stability. An energy gap of 1.12 eV makes silicon devices less prone to thermal noise than germanium devices with a 0.67 eV gap. Silicon is transparent in the infrared (above $1.1 \mu \mathrm{~m}$ wavelength) which means that it can used as an optical material at $1.55 \mu \mathrm{~m}$ telecom wavelength applications.

## Silicon

Table 4.1 Properties of silicon at 300 K

## Structural and mechanical

Atomic weight
Atoms, total $\left(\mathrm{cm}^{-3}\right)$
Crystal structure
Lattice constant $(\AA)$
Density ( $\mathrm{g} / \mathrm{cm}^{3}$ )
Density of surface atoms $\left(\mathrm{cm}^{-2}\right)$

Young's modulus (GPa)
Yield strength (GPa)
Fracture strain
Poisson ratio, $v$
28.09
$4.995 \times 10^{22}$
diamond (FCC)
5.43

Knoop hardness ( $\mathrm{kg} / \mathrm{mm}^{2}$ )
2.33
(100) $6.78 \times 10^{14}$
(110) $9.59 \times 10^{14}$
(111) $7.83 \times 10^{14}$

190
(111) crystal orientation

7
$4 \%$
0.27

Electrical
Energy gap (eV)
Intrinsic carrier concentration $\left(\mathrm{cm}^{-3}\right)$
Intrinsic resistivity (ohm-cm)
Dielectric constant
Intrinsic Debye length (nm)
Mobility (drift) ( $\mathrm{cm}^{2} / \mathrm{V}-\mathrm{s}$ )
Temperature coeff. of resistivitv $\left(\mathrm{K}^{-1}\right)$
1.12
$1.38 \times 10^{10}$
$2.3 \times 10^{5}$
11.8

24
1500 (electrons)
475 (holes)
0.0017

## Silicon

| Thermal |  |  |
| :--- | :--- | :--- |
| Coefficient of thermal expansion $\left({ }^{\circ} \mathrm{C}^{-1}\right)$ |  | $2.6 \times 10^{-6}$ |
| Melting point $\left({ }^{\circ} \mathrm{C}\right)$ | 1421 |  |
| Specific heat $(\mathrm{J} / \mathrm{kg}-\mathrm{K})$ |  | 700 |
| Thermal conductivity $(\mathrm{W} / \mathrm{m}-\mathrm{K})$ |  | 150 |
| Thermal diffusivity | $0.8 \mathrm{~cm}^{2} / \mathrm{s}$ |  |
| Optical |  |  |
| Index of refraction | 3.42 | $\lambda=632 \mathrm{~nm}$ |
|  | 3.48 | $\lambda=1550 \mathrm{~nm}$ |
| Energy gap wavelength | $1.1 \mu \mathrm{~m}^{2}$ | $($ transparent at larger wavelengths $)$ |
| Absorption | $>10^{6} \mathrm{~cm}^{-1}$ | $\lambda=200-360 \mathrm{~nm}$ |
|  | $10^{5} \mathrm{~cm}^{-1}$ | $\lambda=420 \mathrm{~nm}$ |
|  | $10^{4} \mathrm{~cm}^{-1}$ | $\lambda=550 \mathrm{~nm}$ |
|  | $10^{3} \mathrm{~cm}^{-1}$ | $\lambda=800 \mathrm{~nm}$ |
|  | $<0.01 \mathrm{~cm}^{-1}$ | $\lambda=1550 \mathrm{~nm}$ |

Franssila

## Semionoductors

## on NSM

## Semiconductors

$n, k$ database
InGaAsP
Equivalents

| Si | - Silicon | Ge | - Germanium |
| :--- | :--- | :--- | :--- |
| GaP | - Gallium Phosphide | GaAs | - Gallium Arsenide |
| InAs | - Indium Arsenide | C | - Diamond |
| GaSb | - Gallium Antimonide | InSb | - Indium Antimonide |
| InP | - Indium Phosphide | $\mathrm{GaAs}_{1-\mathrm{x}} \mathrm{Sb}_{\mathrm{x}}$ | - Gallium Arsenide Antimonide |
| $\mathrm{Al}_{\mathrm{X}} \mathrm{Ga}_{1-\mathrm{x}} \mathrm{As}$ | - Aluminium Gallium Arsenide |  |  |
| AlN | - Aluminium Nitride | InN | - Indium Nitride |
| BN | - Boron Nitride | GaN | - Gallium Nitride |

## We are going to add new data for:

$\mathrm{Gax}_{\mathrm{x}} \mathrm{In}_{1-\mathrm{x}} \mathrm{As}_{\mathrm{y}} \mathrm{Sb}_{1-\mathrm{y}} \quad$ - Gallium Indium Arsenide Antimonid
$\operatorname{Gax}_{\mathrm{x}} \mathrm{In}_{1-\mathrm{x}} \mathrm{A}$
InAs $1_{1-x} \mathrm{Sb}_{\mathrm{x}}$
$\mathrm{Sil}_{1-\mathrm{x}} \mathrm{Ge}_{\mathrm{x}}$

- Gallium Indium Arsenide
- Indium Arsenide Antimonide
- Silicon Germanium
$\mathrm{Gax}_{\mathrm{x}} \mathrm{In}_{1-\mathrm{x}} \mathrm{P}$
$\mathrm{Gax}_{\mathrm{x}} \mathrm{In}_{1-\mathrm{x}} \mathrm{Sb}$
$\mathrm{Gax}_{\mathrm{x}} \mathrm{In}_{1-\mathrm{x}} \mathrm{A}_{\mathrm{y}} \mathrm{P}_{1-\mathrm{y}}$ SiC
- Gallium Indium Phosphide
- Gallium Indium Antimonide
- Gallium Indium Arsenide Phosphide
- Silicon Carbide


Temperature Dependences
Temperature dependence of the energy gap

$$
\mathrm{E}_{\mathrm{g}}=1.17-4.73 \cdot 10^{-4} \cdot \mathrm{~T}^{2} /(\mathrm{T}+636)(\mathrm{eV})
$$

where T is temperature in degrees K .

| Energy gap | 1.12 eV |
| :--- | :--- |
| Energy separation ( $\mathrm{E}_{\Gamma \mathrm{L}}$ ) | 4.2 eV |
| Energy spin-orbital splitting | 0.044 eV |
| Intrinsic carrier concentration | $1 \cdot 10^{10} \mathrm{~cm}^{-3}$ |
| Intrinsic resistivity | $3.2 \cdot 10^{5} \Omega \cdot \mathrm{~cm}$ |
| Effective conduction band density of states | $3.2 \cdot 10^{19} \mathrm{~cm}^{-3}$ |
| Effective valence band density of states | $1.8 \cdot 10^{19} \mathrm{~cm}^{-3}$ |

## Temperature dependence of the direct band gap $\mathrm{E}_{\Gamma 2}$

$$
\mathrm{E}_{\Gamma 2}=4.34-3.91 \cdot 10^{-4} \cdot \mathrm{~T}^{2} /(\mathrm{T}+125)(\mathrm{eV})
$$

## Intrinsic carrier concentration

$$
n_{i}=\left(N_{c}-N_{v}\right)^{1 / 2} \cdot \exp \left(-E g /\left(2 k_{B} T\right]\right)
$$

## Effective density of states in the conduction band

$$
\mathrm{N}_{\mathrm{c}}=4.82 \cdot 10^{15} \cdot \mathrm{M} \cdot\left(\mathrm{~m}_{\mathrm{c}} / \mathrm{m}_{\mathrm{o}}\right)^{3 / 2} \cdot \mathrm{~T}^{3 / 2}=4.82 \cdot 10^{15} \cdot \mathrm{M} \cdot\left(\mathrm{~m}_{\mathrm{cd}} / \mathrm{m}_{\mathrm{o}}\right)^{3 / 2} \cdot \mathrm{~T}^{3 / 2}\left(\mathrm{~cm}^{-3}\right),
$$

or

$$
\mathrm{N}_{\mathrm{c}}=6.2 \cdot 10^{15} \cdot \mathrm{~T}^{3 / 2}\left(\mathrm{~cm}^{-3}\right),
$$

$\mathrm{M}=6$ is the number of equivalent valleys in the conduction band.
$\mathrm{m}_{\mathrm{c}}=0.36 \mathrm{~m}_{\mathrm{o}}$ is the effective mass of the density of states in one valley of conduction $\mathrm{m}_{\mathrm{cd}}=1.18 \mathrm{~m}_{0}$ is the effective mass of the density of states.

## Effective density of states in the valence band

$$
\mathrm{N}_{\mathrm{v}}=3.5 \cdot 10^{15} \cdot \mathrm{~T}^{3 / 2}\left(\mathrm{~cm}^{-3}\right)
$$



The temperature dependence of the intrinsi (Shur [19907).

Fermi level versus temperature for differen (Grove [19677).

## diamond

c Si Ge


Number: 227
-Primitive Vectors:

$$
\begin{aligned}
& \vec{a}_{1}=\frac{a}{2} \hat{y}+\frac{a}{2} \hat{z} \\
& \vec{a}_{2}=\frac{a}{2} \hat{x}+\frac{a}{2} \hat{z} \\
& \vec{a}_{3}=\frac{a}{2} \hat{x}+\frac{a}{2} \hat{y} \quad a=0.543 \mathrm{~nm}
\end{aligned}
$$

Active


$$
\vec{B}_{1}=(0,0,0)
$$

-Basis Vectors:

$$
\vec{B}_{2}=(1 / 4,1 / 4,1 / 4)
$$

Point group: m3m $\left(\mathrm{O}_{\mathrm{h}}\right) 6$ 2-fold rotations, 4 3-fold rotations, 3 4-fold rotations, 9 mirror planes, inversion

## Silicon surfaces



## Miller indices: Crystal planes


( ) specific plane
\{ \} family of equivalent planes

MOSFETs are made on <100> wafers


A plane with the intercepts
$1 / h, 1 / k, 1 / l$ is the ( $h, k, l$ ) plane.
always use integers for $h, k, l$

(100)

(101)

(110)

(010)

## Strain

A distortion of a material is described by the strain matrix

$$
\begin{aligned}
& x^{\prime}=\left(1+\varepsilon_{x x}\right) \hat{x}+\varepsilon_{x y} \hat{y}+\varepsilon_{x z} \hat{z} \\
& y^{\prime}=\varepsilon_{y x} \hat{x}+\left(1+\varepsilon_{y y}\right) \hat{y}+\varepsilon_{y z} \hat{z} \\
& z^{\prime}=\varepsilon_{z x} \hat{x}+\varepsilon_{z y} \hat{y}+\left(1+\varepsilon_{z z}\right) \hat{z}
\end{aligned}
$$

## Stress

9 forces describe the stress $X x, X y, X z, Y x, Y y, Y z, Z x, Z y, Z z$

$$
\begin{aligned}
& \text { LreSS } \\
& \text {-direction to the } \\
& \text { stress tensor: } \sigma=\left[\begin{array}{lll}
\frac{X_{x}}{A_{x}} & \frac{X_{y}}{A_{y}} & \frac{X_{z}}{A_{z}} \\
\frac{Y_{x}}{A_{x}} & \frac{Y_{y}}{A_{y}} & \frac{Y_{z}}{A_{z}} \\
\frac{Z_{x}}{A_{x}} & \frac{Z_{y}}{A_{y}} & \frac{Z_{z}}{A_{z}}
\end{array}\right]
\end{aligned}
$$

$X x$ is a force applied in the $x$-direction to the plane normal to $x$
$X y$ is a sheer force applied in the $x$-direction to the plane normal to $y$

Stress is force $/ \mathrm{m}^{2}$

## Stress and Strain

$$
\varepsilon_{i j}=s_{i j k l} \sigma_{k l}
$$

The stress - strain relationship is described by a rank 4 stiffness tensor. The inverse of the stiffness tensor is the compliance tensor.

$$
\sigma_{i j}=c_{i j k l} \varepsilon_{k l}
$$

Einstein convention: sum over repeated indices.

$$
\begin{aligned}
& \varepsilon_{x x}=s_{x x x x} \sigma_{x x}+s_{x x x y} \sigma_{x y}+s_{x x x z} \sigma_{x z}+s_{x x y x} \sigma_{y x}+s_{x x y y} \sigma_{y y} \\
& +s_{x x y z} \sigma_{y z}+s_{x x z x} \sigma_{z x}+s_{x x y y} \sigma_{z y}+s_{x x z z} \sigma_{z z}
\end{aligned}
$$

## Mechanical properties

The stress-strain relation for silicon is described by the tensor equation,

$$
\sigma_{i j}=c_{i j k l} \epsilon_{k l},
$$

where the elements of the compliance tensor are,

$$
\begin{aligned}
& c_{11}=165.7 \mathrm{GPa}=c_{x x x x}=c_{y y y y}=c_{z z z z}, \\
& c_{12}=63.9 \mathrm{GPa}=c_{x x y y}=c_{x y z z}=c_{z z y y}=c_{y y x x}=c_{y y z z}=c_{z z x x}, \\
& c_{44}=79.6 \mathrm{GPa}=c_{x y x y}=c_{x z x z}=c_{y z y z}=c_{y x y x}=c_{z x z x}=c_{z y z y}, \\
& \text { all other components } c_{i j k l}=0 .
\end{aligned}
$$

Yield strength: 7 GPa
Fracture strain: 4\%
Possion's ratio: 0.27
When an electric field is applied to a silicon crystal, the resulting strain can be expressed as a Taylor series,

$$
\epsilon_{i j}=d_{i j k} E_{k}+Q_{i j k l} E_{k} E_{l}+\cdots,
$$

where $\epsilon_{i j}$ is the strain, $E_{k}$ is the electric field, $d_{i j k}$ is the reciprocal piezoelectric tensor, and $Q_{i j k l}$ is the electrostriction tensor. Silicon has inversion symmetry in its point group so the reciprocal piezoelectric tensor is zero by symmetry and the leading order term is electrostriction.

