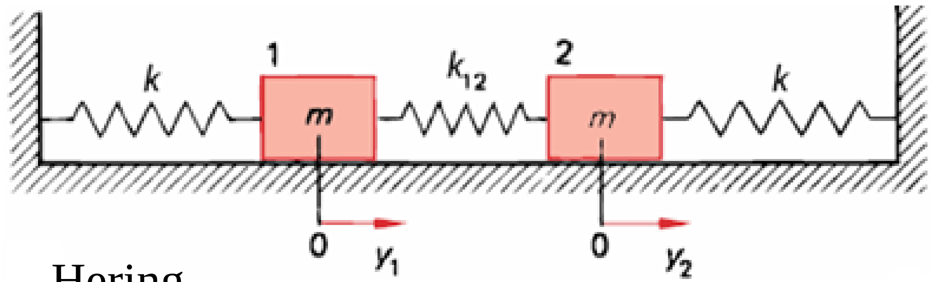
gekoppeltes Schwingungssystem



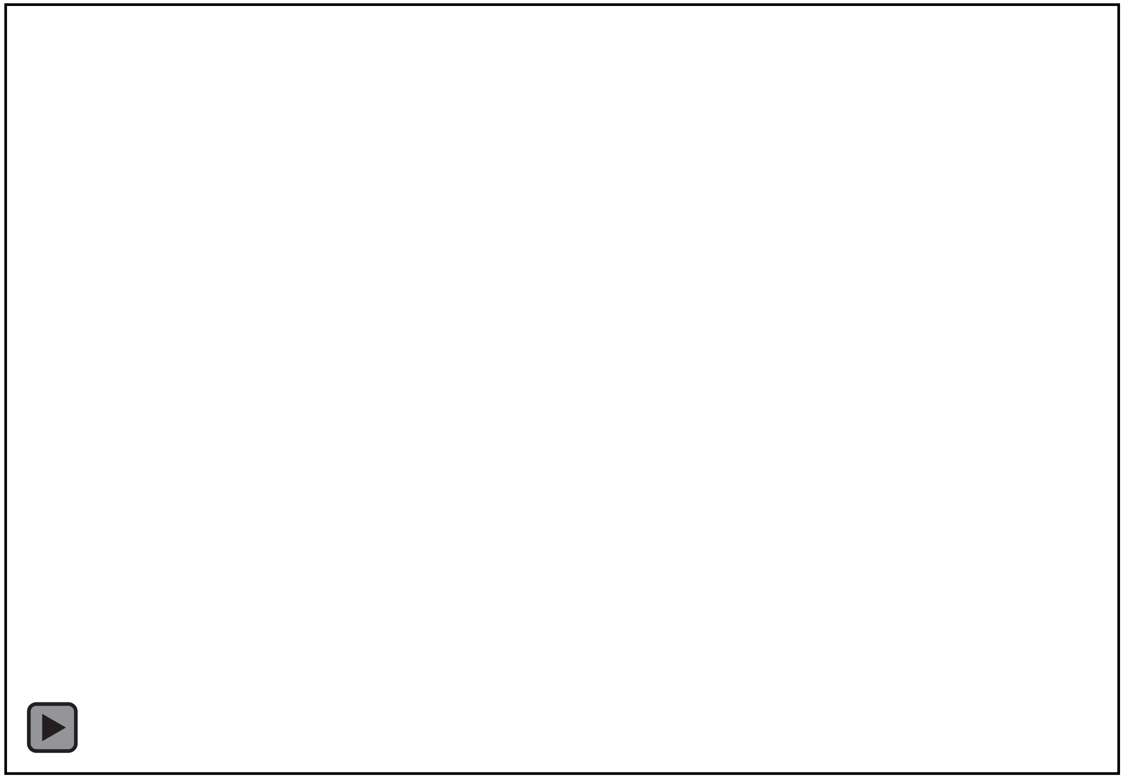
Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$

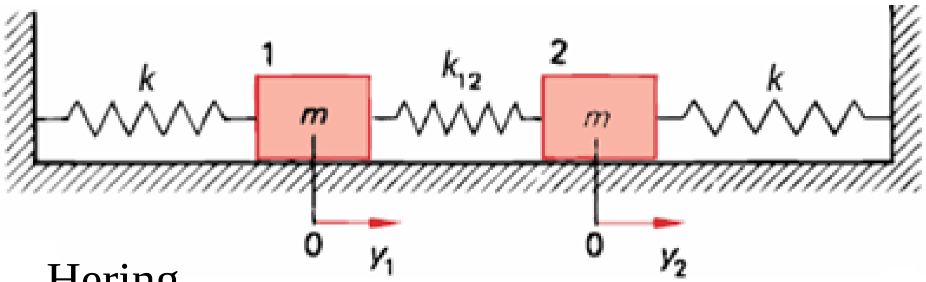
Eigenmoden: harmonische Bewegung
Alle Teile schwingen mit der gleichen Frequenz

zwei Eigenmoden



https://www.youtube.com/watch?v=8RV3gXm6j2I

gekoppeltes Schwingungssystem



Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$

Numerisches Lösen von Differentialgleichungen 6. Ordnung

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ -x-0.3*(y-x)

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ -y-0.3*(x-y)

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ 0

Anfangsbedingungen:

$t_0 =$ 0

$x(t_0) =$ 0

$v_x(t_0) =$ 1

$y(t_0) =$ 0

$v_y(t_0) =$ 1

$z(t_0) =$ 0

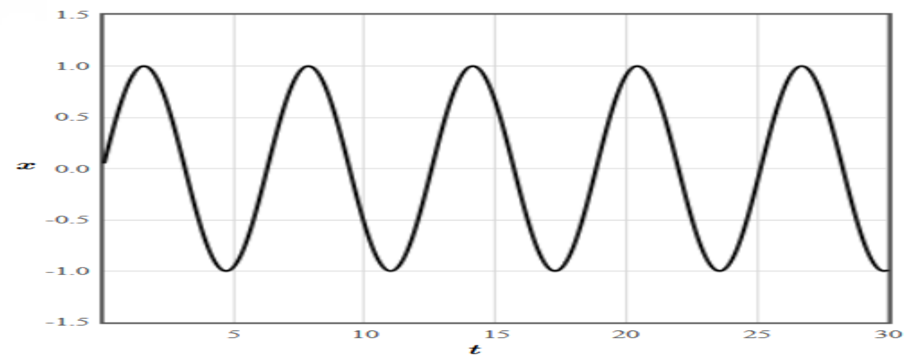
$v_z(t_0) =$ 0

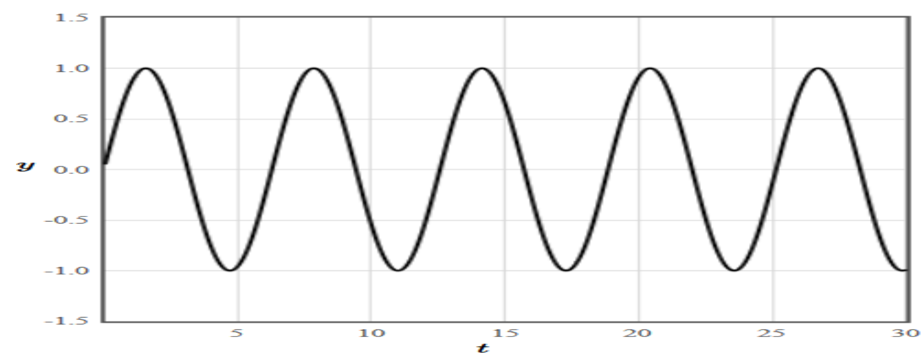
$\Delta t =$ 0.05

N_{steps} 600

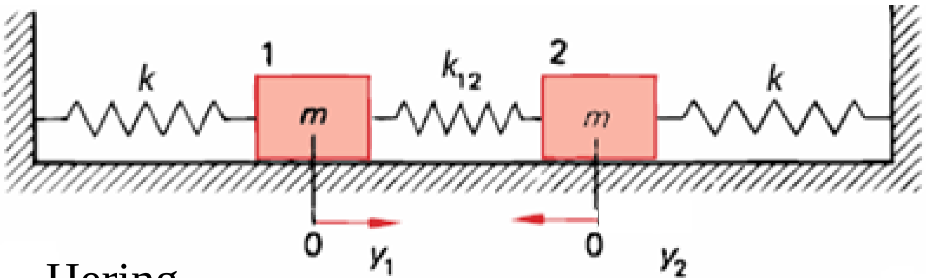
Graphische Darstellung: x vs. t

Absenden





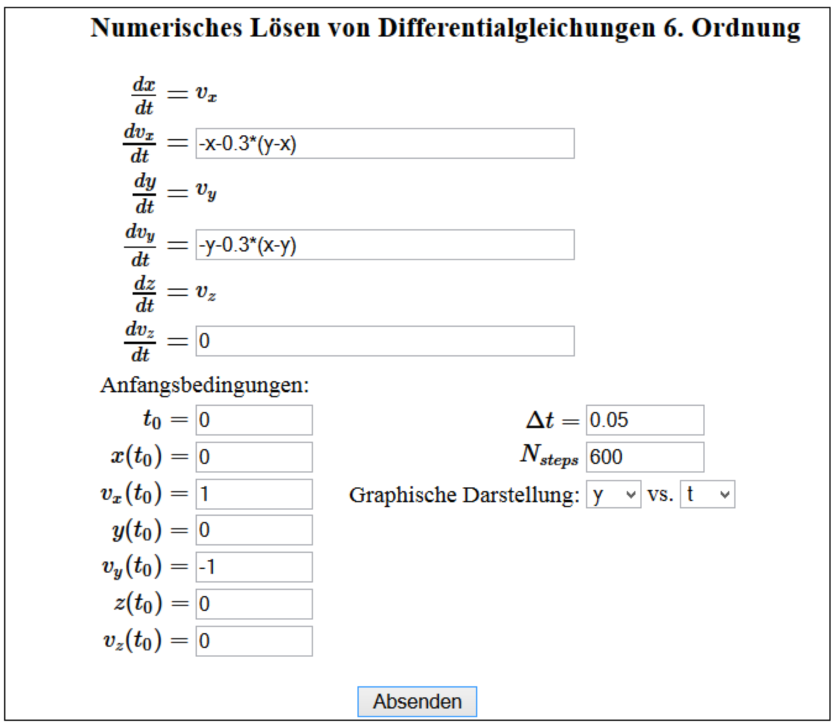
gekoppeltes Schwingungssystem

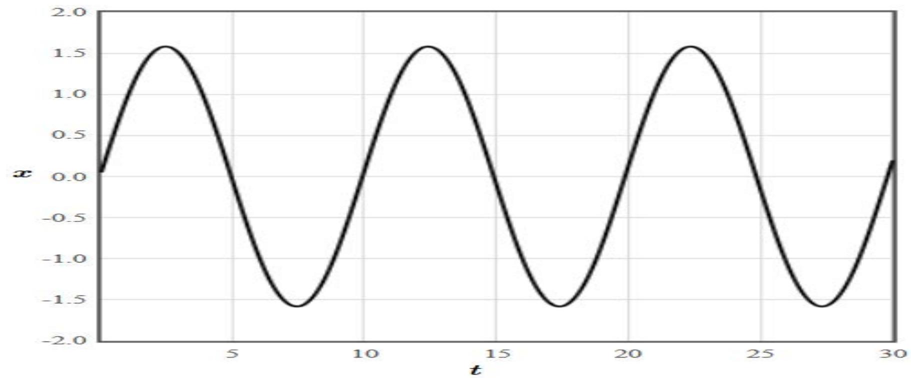


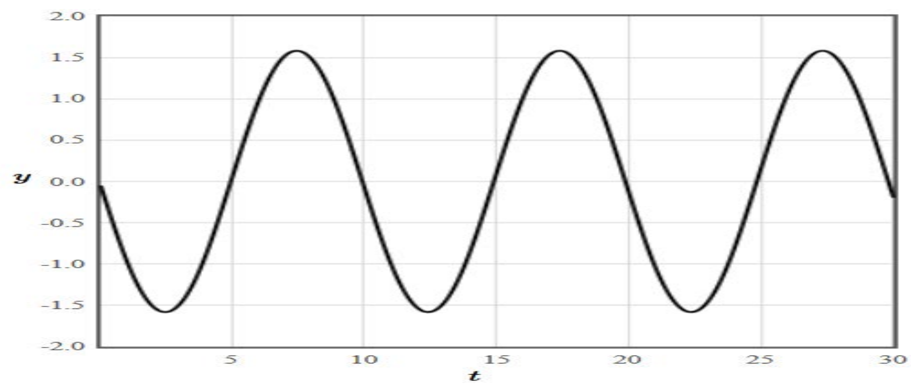
Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

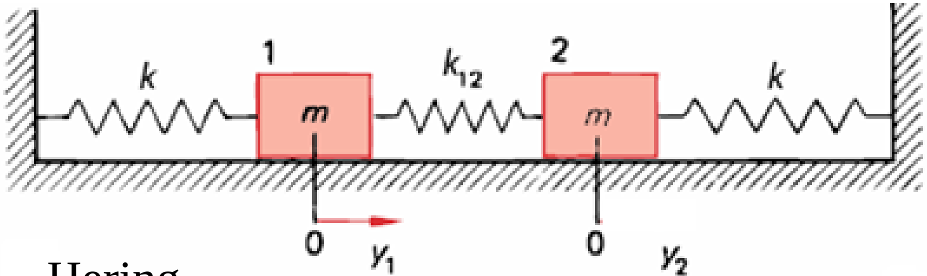
$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$







gekoppeltes Schwingungssystem



Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$

Numerisches Lösen von Differentialgleichungen 6. Ordnung

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ (-x+0.1*(y-x))

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ (-y+0.1*(x-y))

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ 0

Anfangsbedingungen:

$t_0 =$ 0

$x(t_0) =$ 0

$v_x(t_0) =$ 1

$y(t_0) =$ 0

$v_y(t_0) =$ 0

$z(t_0) =$ 0

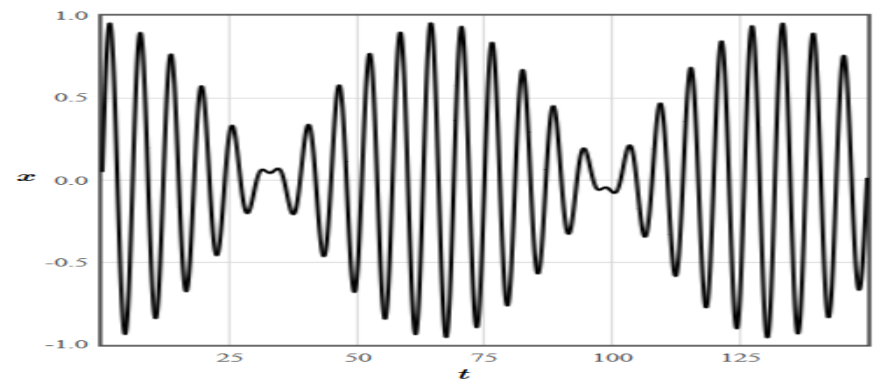
$v_z(t_0) =$ 0

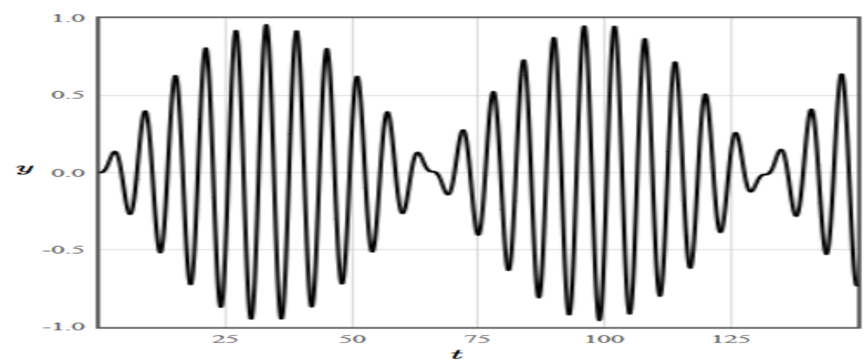
$\Delta t =$ 0.05

N_{steps} 3000

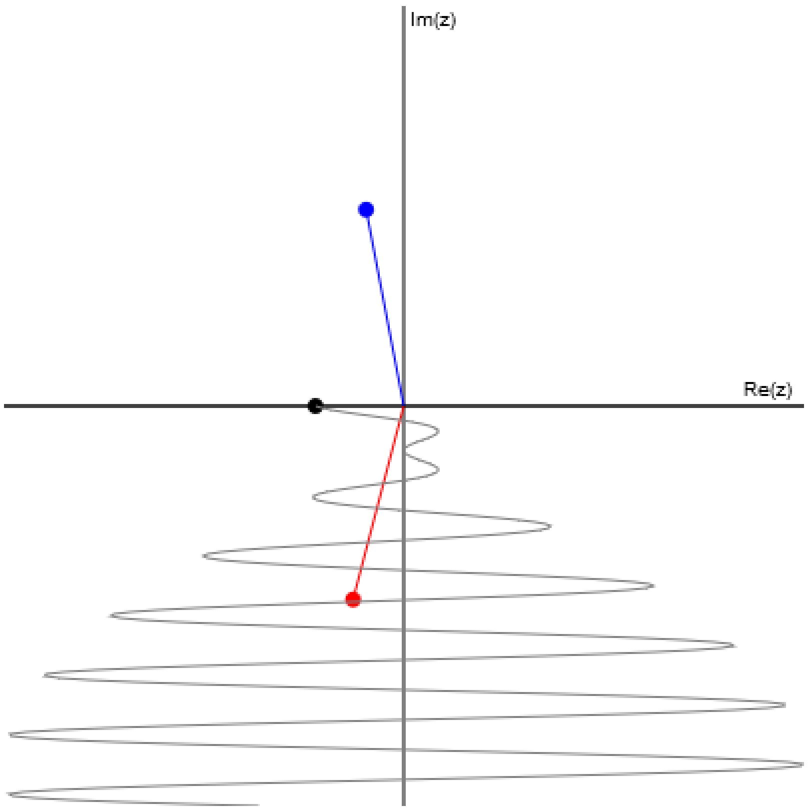
Graphische Darstellung: y vs. t

Absenden

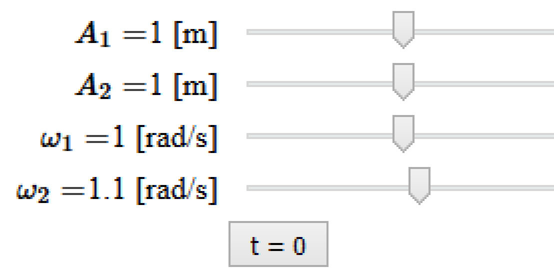


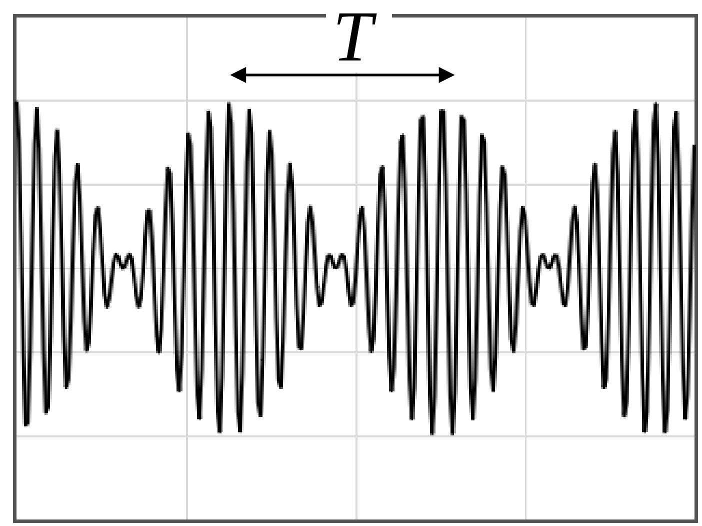


Schwebung



$$x = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t).$$

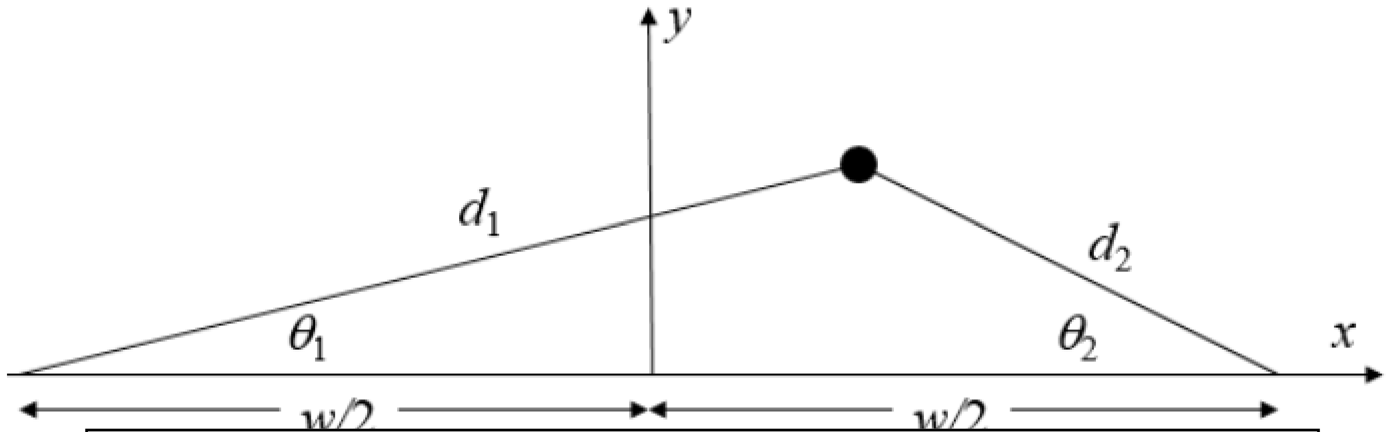


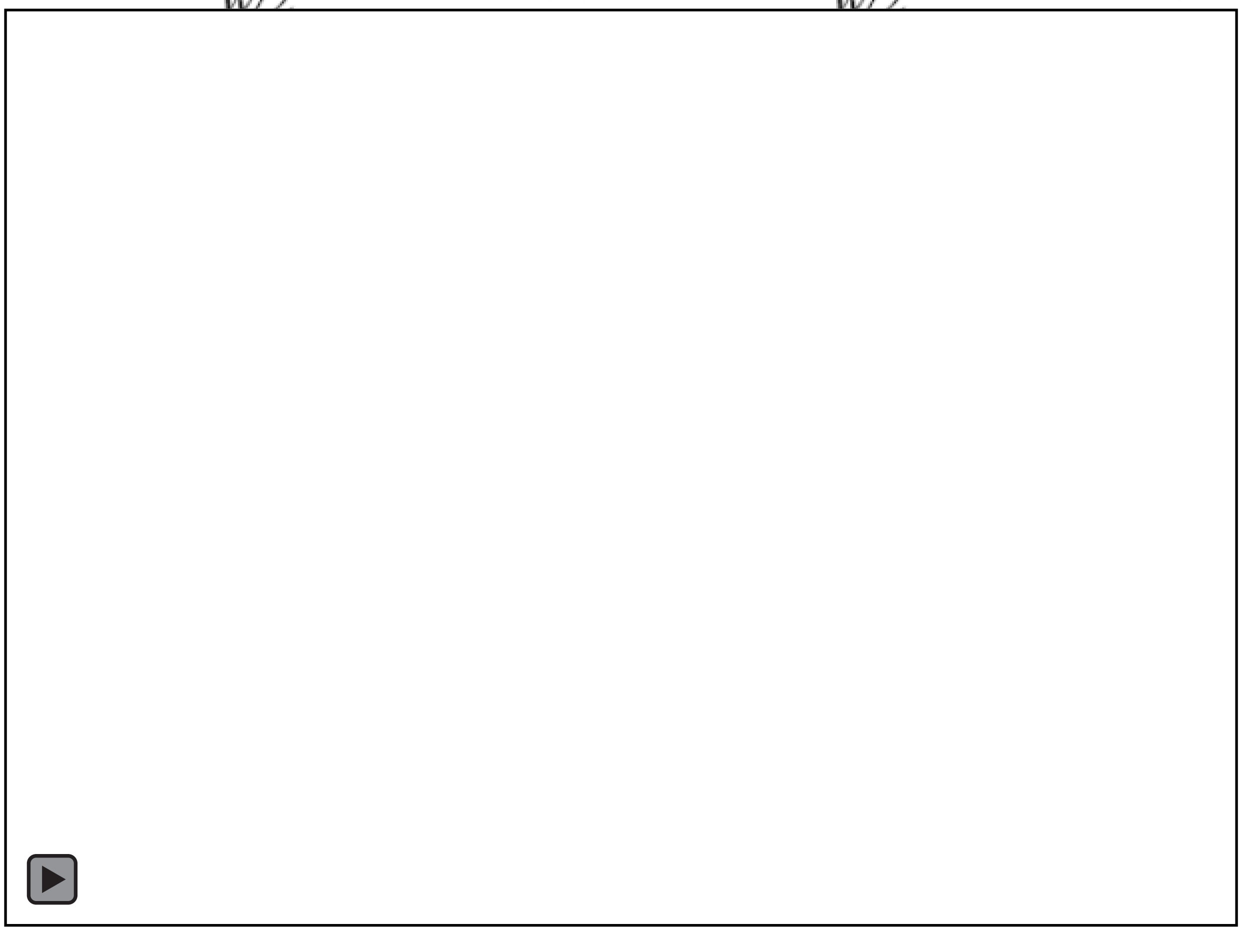


$$\omega_1 T - \omega_2 T = 2\pi$$

$$T = \frac{2\pi}{\omega_1 - \omega_2}$$

http://lamp.tu-graz.ac.at/~hadley/physikm/apps/resonance/schwebung.de.php

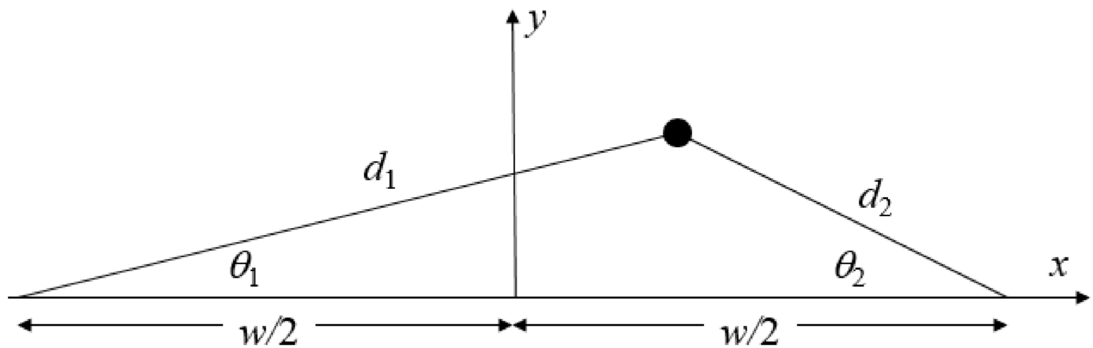




The Open University
https://www.youtube.com/watch?v=VQ5xRhNlQiY

A mass on a frictionless table connected to two springs

A mass m on a frictionless horizonal table is connected to the left and to the right by two springs each with a spring constant k. The length of the unstretched springs is l and the width of the table is $w > 2l$.



The equilibrium position for the mass is in the middle of the table $(x = 0, y = 0)$ where the forces exerted by the springs cancel each other out. If the mass is moved away from its equilibrium position then the force on on the mass is,

$$\vec{F} = k\left[(d_2 - l)\cos\theta_2 - (d_1 - l)\cos\theta_1\right]\hat{x} - k\left[(d_2 - l)\sin\theta_2 + (d_1 - l)\sin\theta_1\right]\hat{y}.$$

Here $d_1 = \sqrt{(w/2 + x)^2 + y^2}$, $d_2 = \sqrt{(w/2 - x)^2 + y^2}$, $\theta_1 = \text{atan}\left(\frac{y}{w/2+x}\right)$, and $\theta_2 = \text{atan}\left(\frac{y}{w/2-x}\right)$.

The form below will calculate the trajectory of the mass when it is released from postion (x_0, y_0) at rest.

$m =$ 0.4 kg $k =$ 0.2 N/m $l =$ 0.15 m $w =$ 0.8 m $x_0 =$ 0.15 m $y_0 =$ -0.07 m

Load these equations into the form below

Numerical 6th order differential equation solver

$\frac{dx}{dt} = v_x$

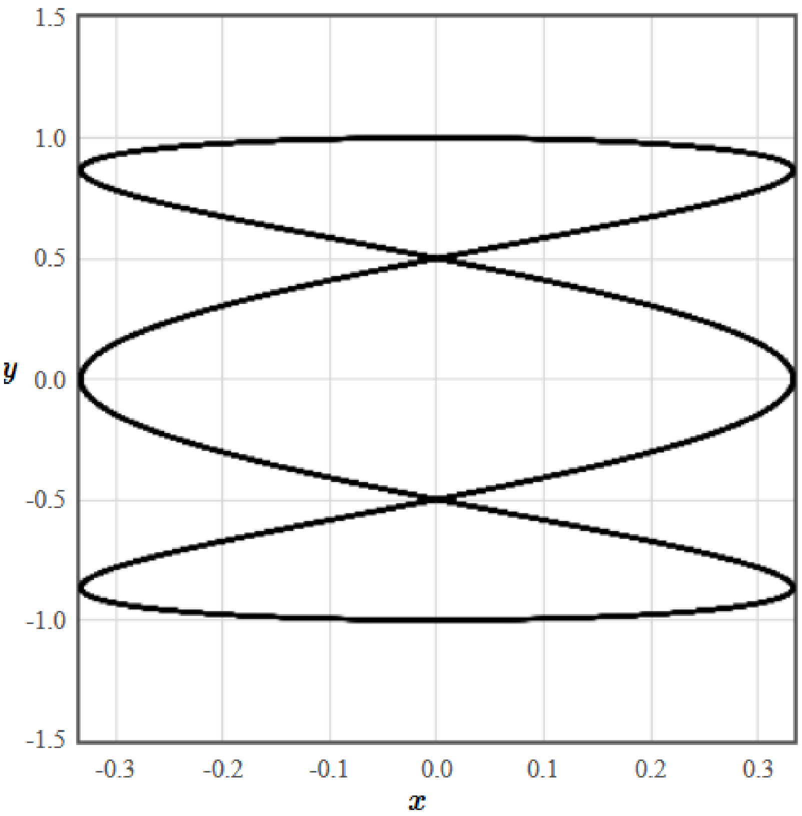
$\frac{dv_x}{dt} =$

```
(0.2*(sqrt((0.4-x)*(0.4-x)+y*y)-
0.15)*cos(atan(y/(0.4-x)))-0.2*(sqrt((0.4+x)*
(0.4+x)+y*y)-0.15)*cos(atan(y/(0.4+x))))/(0.4)
```

Überlagerung harmonischer Schwingungen mit ganzzahligem Frequenzverhältnis, die senkrecht zueinander schwingen (Lissajous-Figuren)

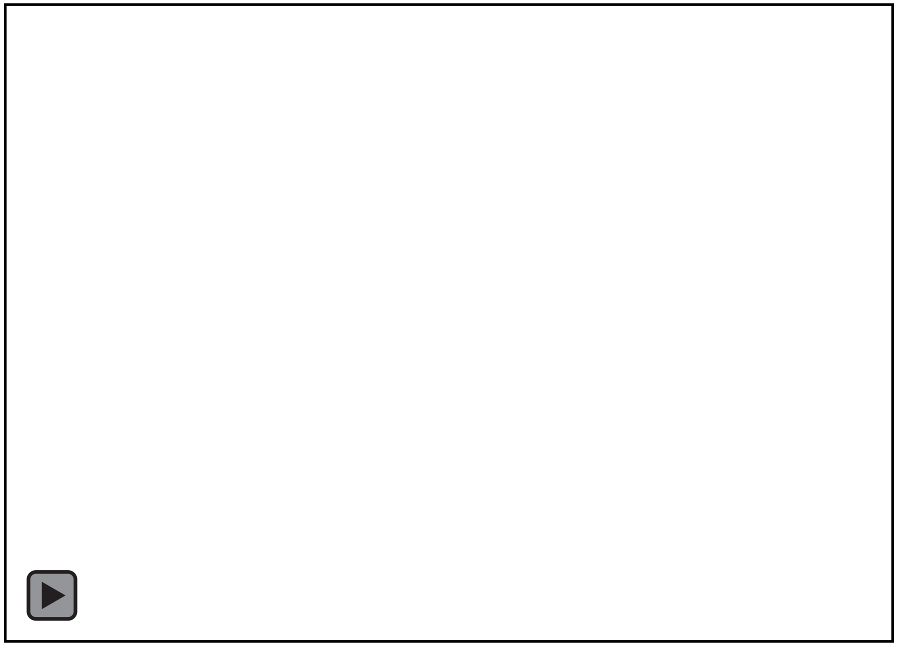
$$M\frac{d^2x}{dt^2} = -k_2 x$$

$$M\frac{d^2y}{dt^2} = -k_2 y$$

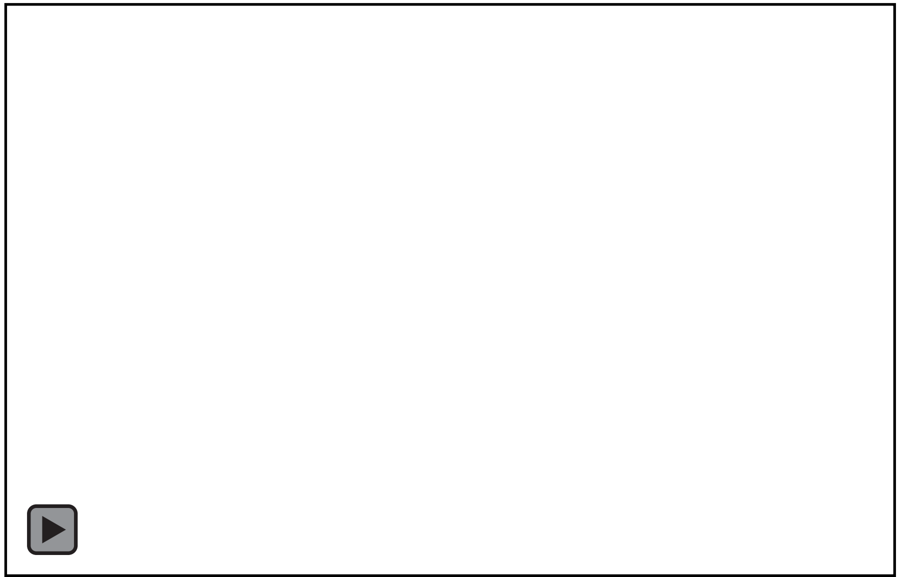


3 Eigenmoden

jede Bewegung kann als eine Superposition von Eigenmoden geschrieben werden.



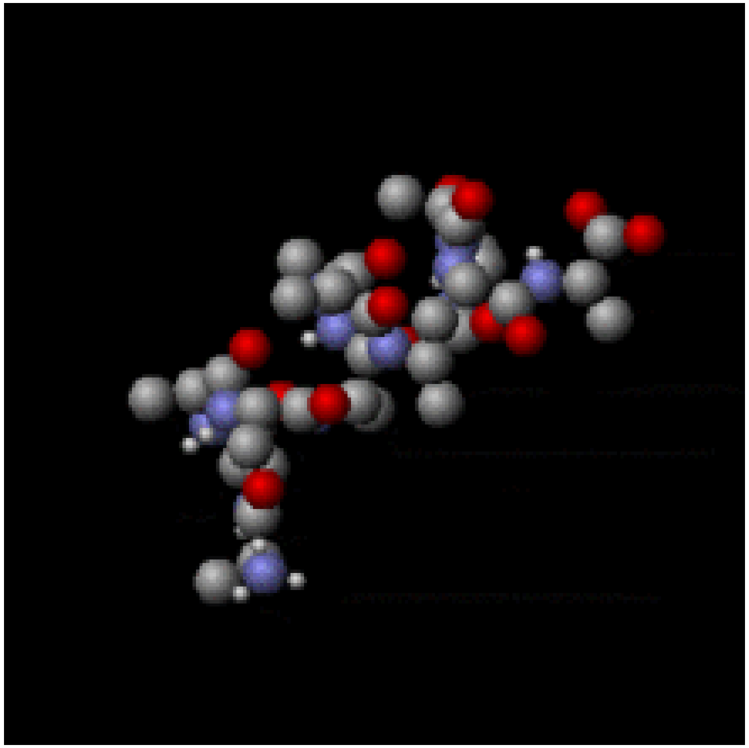
Eigenmoden:
harmonische Bewegung
Alle Teile schwingen mit der gleichen Frequenz



Open University
https://www.youtube.com/watch?v=jhGhSXAqXmU

Eigenmoden

jede Bewegung kann als eine Superposition von Eigenmoden geschrieben werden.



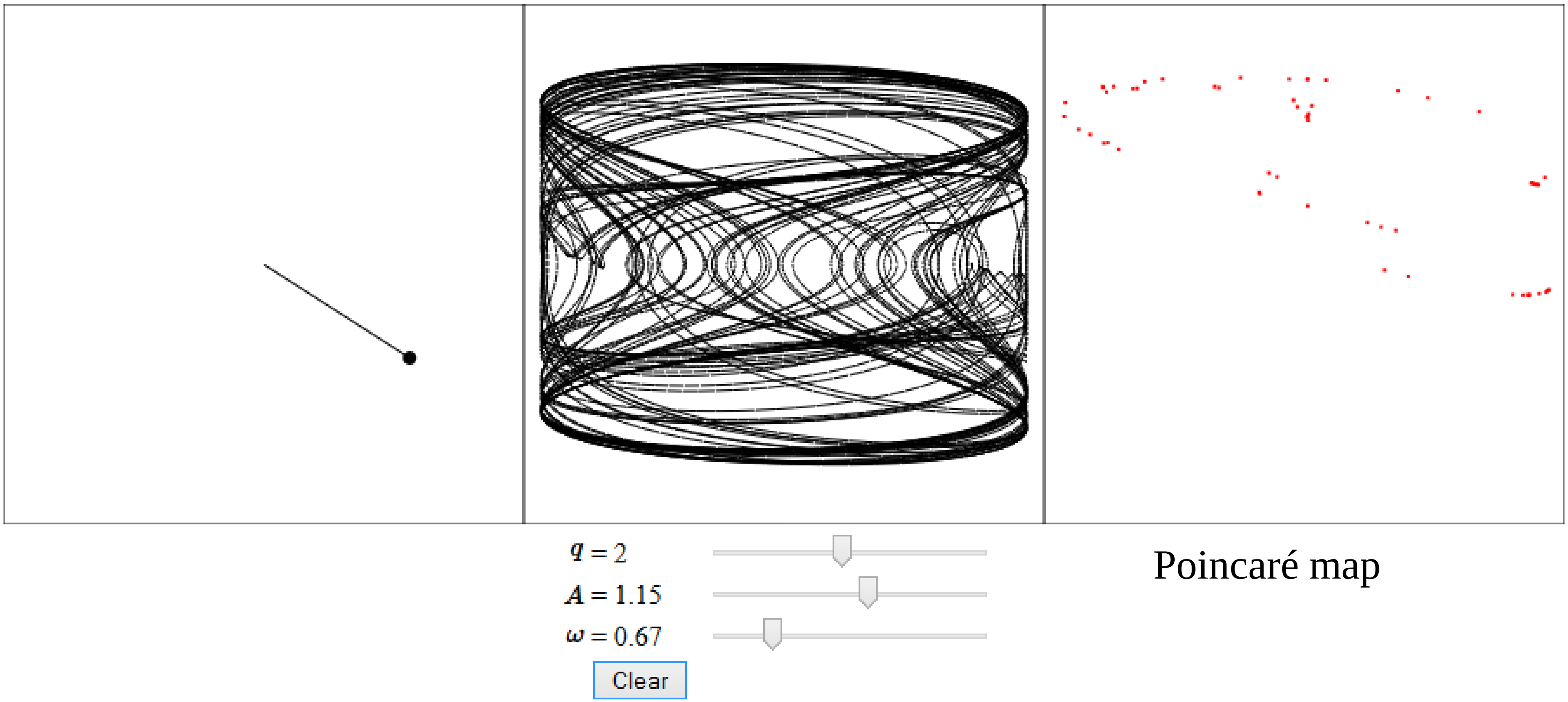
3 Translationsfreiheitsgrade
3 Rotationsfreiheitsgrade

3N - 6 Eigenmoden
(Schwingunen)

http://de.wikipedia.org/wiki/Freiheitsgrad

The driven pendulum

$$\frac{d^2\theta}{dt^2} + \frac{1}{q}\frac{d\theta}{dt} + \sin(\theta) = A\cos(\omega t)$$

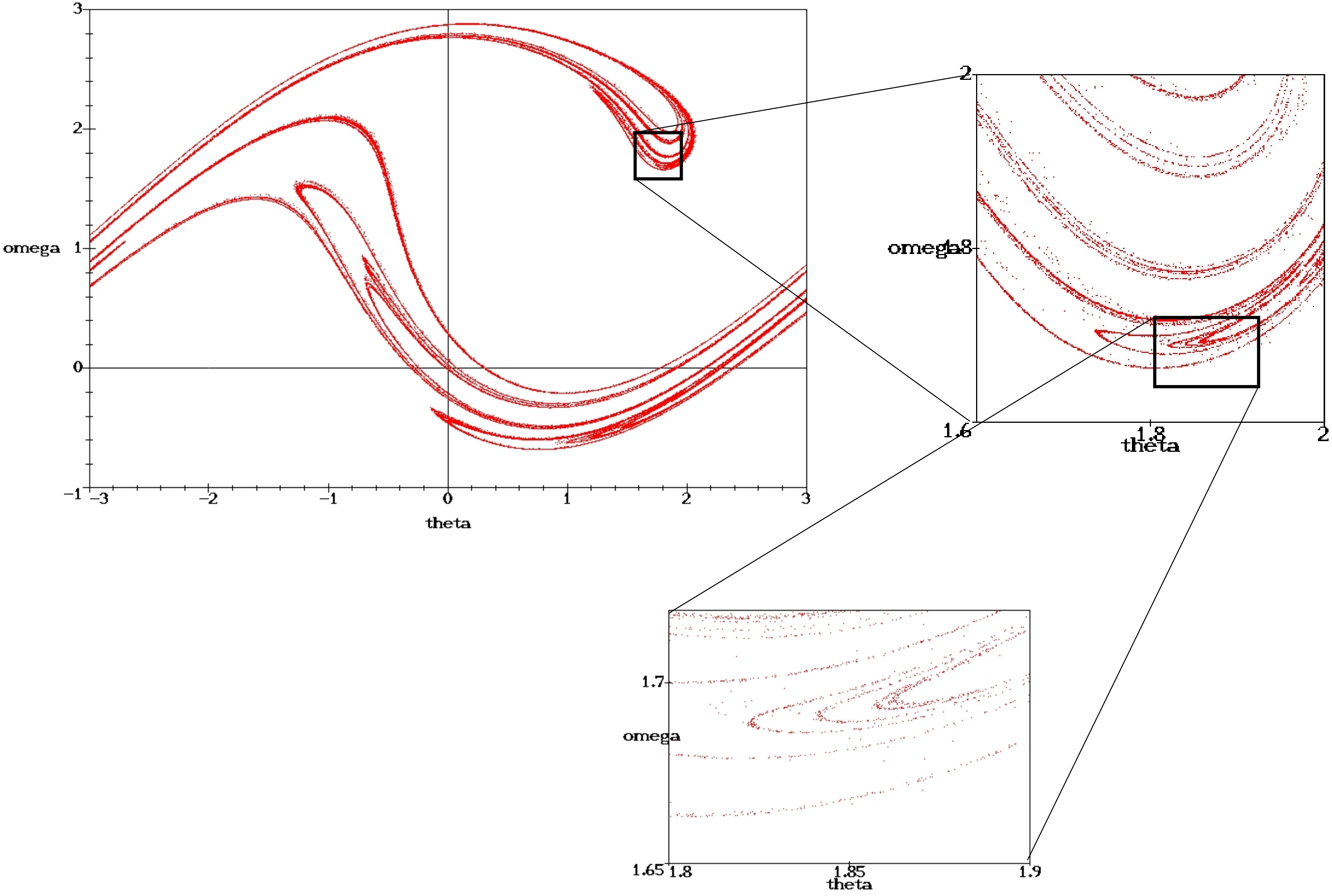


A = 0, 0.6, 1.04, 1.08, 1.1, 1.15

Symmetriebrechung → 1.04; Periodenverdopplung → 1.08; Chaos → 1.15

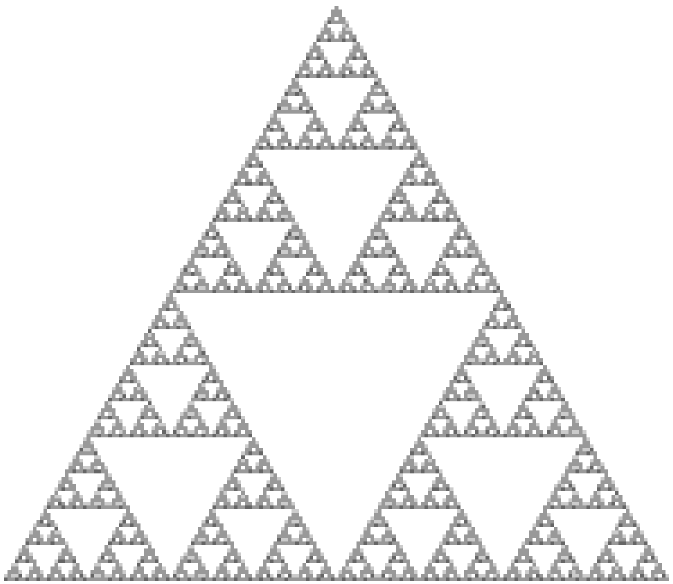
http://lamp.tu-graz.ac.at/~hadley/physikm/apps/numerical_integration/pendulum_ani.html

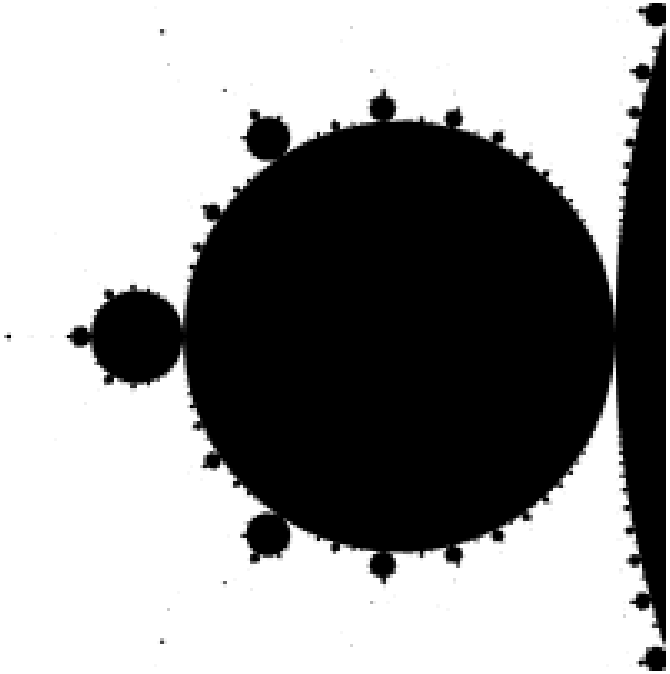
Seltsamer Attraktor



http://www.thphys.uni-heidelberg.de/~gasenzer/index.php?n1=teaching&n2=chaos

Selbstähnlichkeit





http://en.wikipedia.org/wiki/Self-similarity

Bonus Frage 18.12.2015

Nicht lineare Feder

Ein Gewicht der Masse 4 kg wird an einer nicht linearen Feder angebracht. Die Federkraft ist $F_{\text{feder}} = -4x|x|$ N. x ist hierbei die Auslenkung von der Feder in Meter. Das Gewicht schwingt horizontal auf einem Tisch mit einer Reibungskraft, die entgegen der Richtung der Geschwindigkeit des Gewichts zeigt, $F_{fric} = -0.3 \frac{dx}{dt}$ N.

Bei $t = 0$ s: $x = 0$ m, $\frac{dx}{dt} = 2$ m/s.

Wo ist das Gewicht zum Zeitpunkt t =10 s?

$x =$ ______ [m]

Dieses Problem muß numerisch gelöst werden.