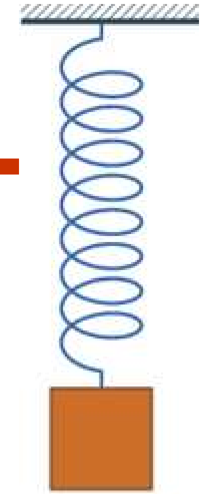
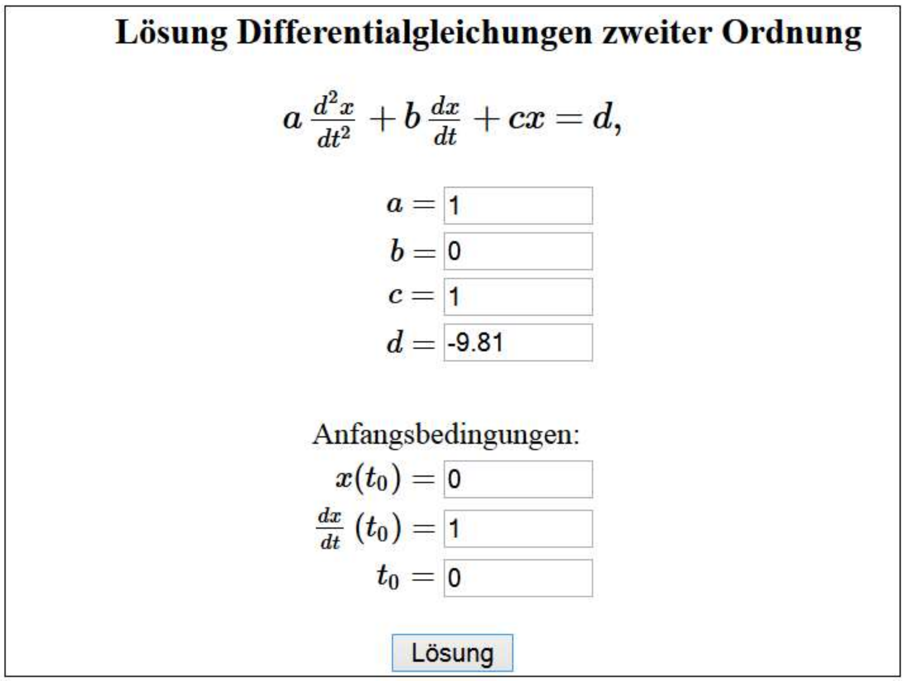
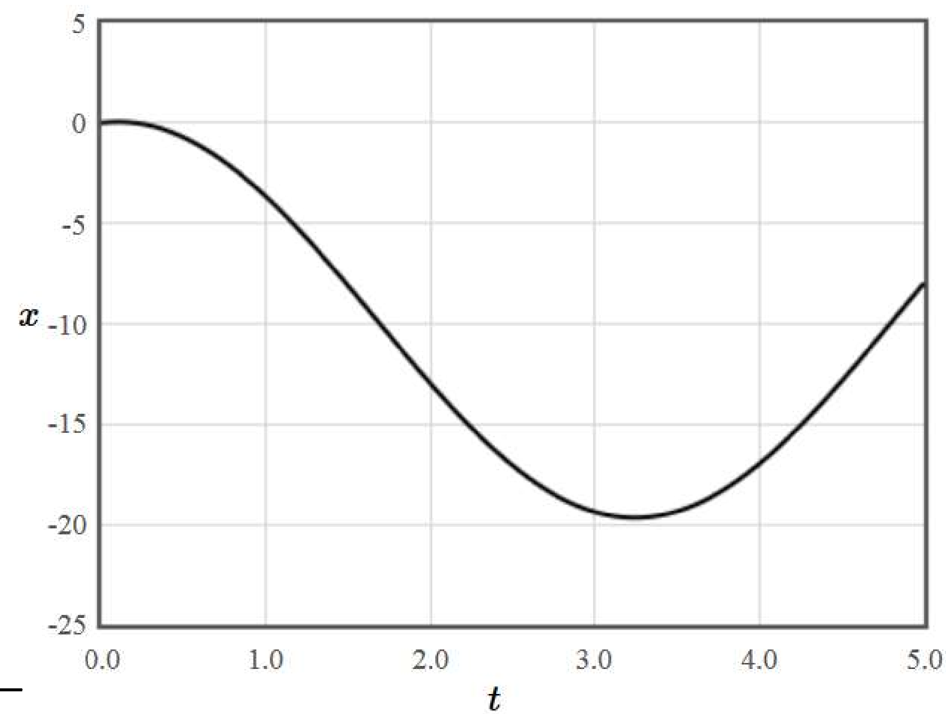
Freie Schwingung



$$m\frac{d^2x}{dt^2}+kx=-mg$$

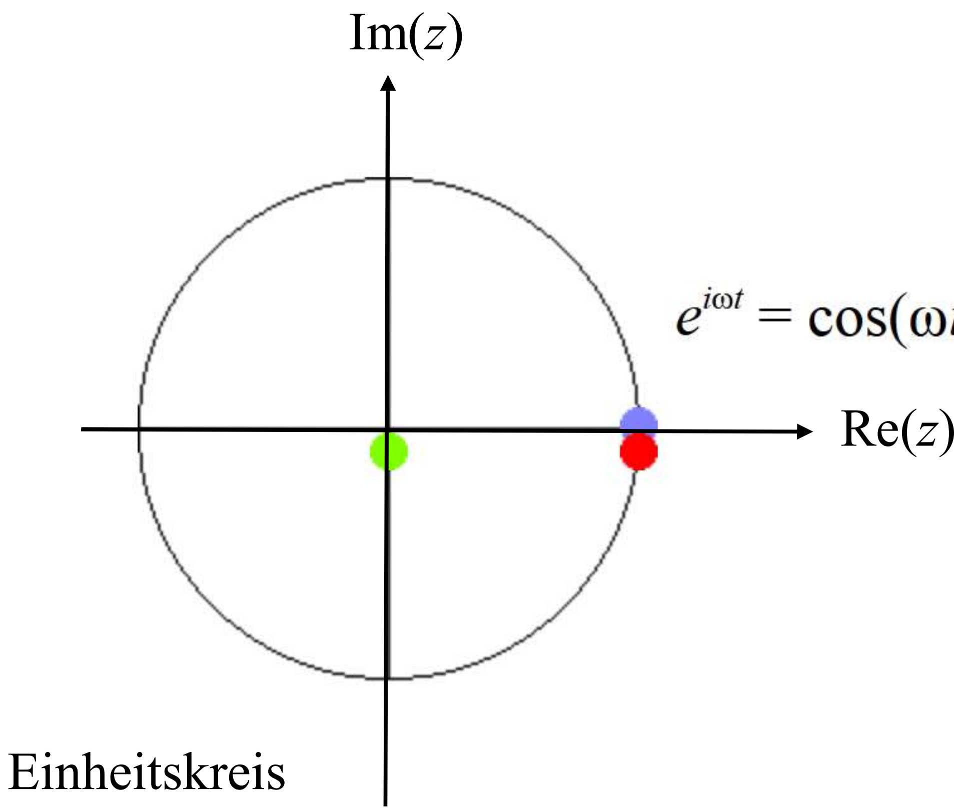




$$\omega_0=\sqrt{\frac{k}{m}}$$

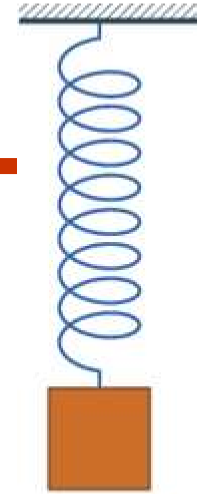
Euler'sche Formel $e^{i\theta} = \cos\theta + i\sin\theta$

$$i^2 = -1 = \left(j^2\right)$$



$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$b^2 < 4km$ Schwingfall



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = -mg$$

$m = 1$ [kg] - +

$b = 0.05$ [kg/s] - +

$k = 0.9$ [N/m] - +

Lösung Differentialgleichung zweiter Ordnung

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = d,$$

$a =$ 1

$b =$ 0.05

$c =$ 0.9

$d =$ 0

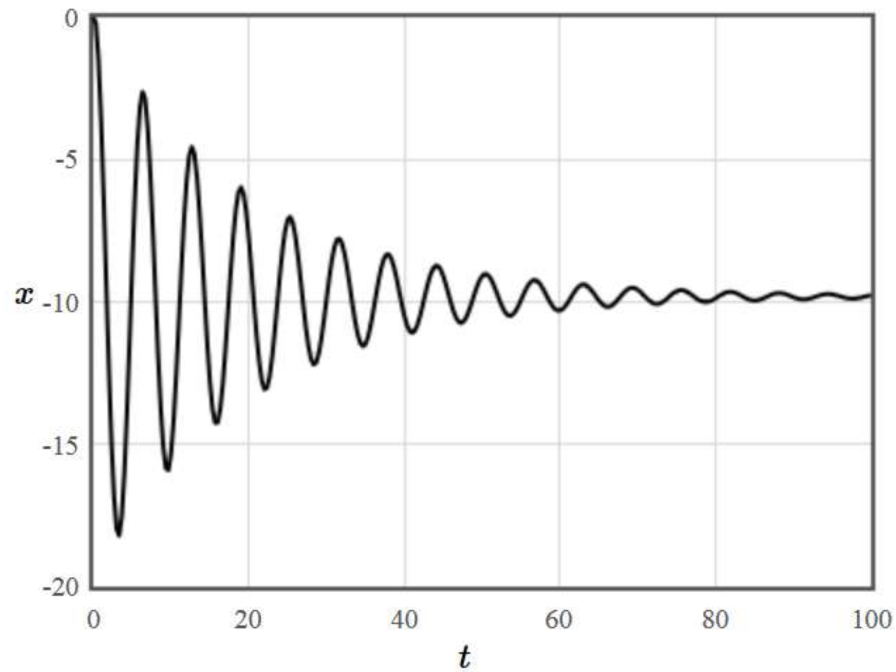
Anfangsbedingungen:

$x(t_0) =$ 0.02

$\frac{dx}{dt}(t_0) =$ 0

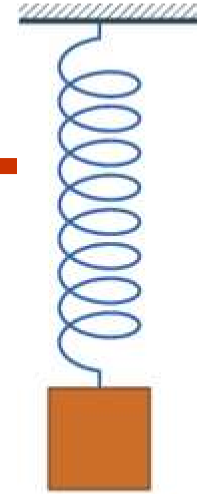
$t_0 =$ 0

submit



$$\tau = \frac{2m}{b} \qquad \omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$b^2 = 4km$ aperiodischer Grenzfall



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = -mg$$

$m = 1$ [kg] - +

$b = 0.05$ [kg/s] - +

$k = 0.9$ [N/m] - +

Lösung Differentialgleichungen zweiter Ordnung

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = d,$$

$a =$ 1

$b =$ 2

$c =$ 1

$d =$ -9.81

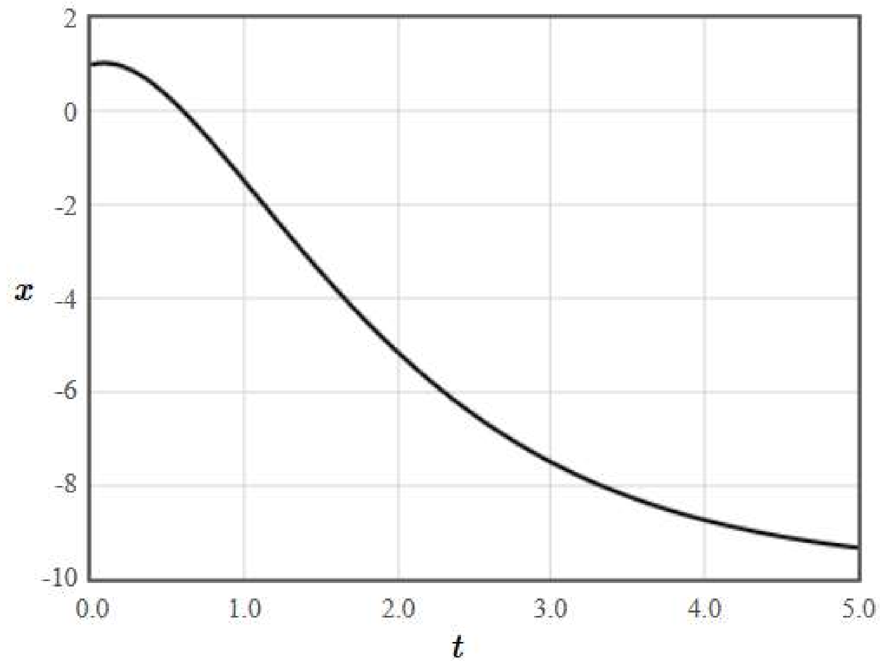
Anfangsbedingungen:

$x(t_0) =$ 1

$\frac{dx}{dt}(t_0) =$ 1

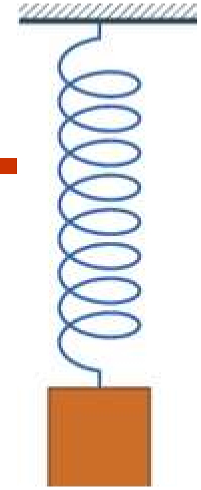
$t_0 =$ 0

Lösung

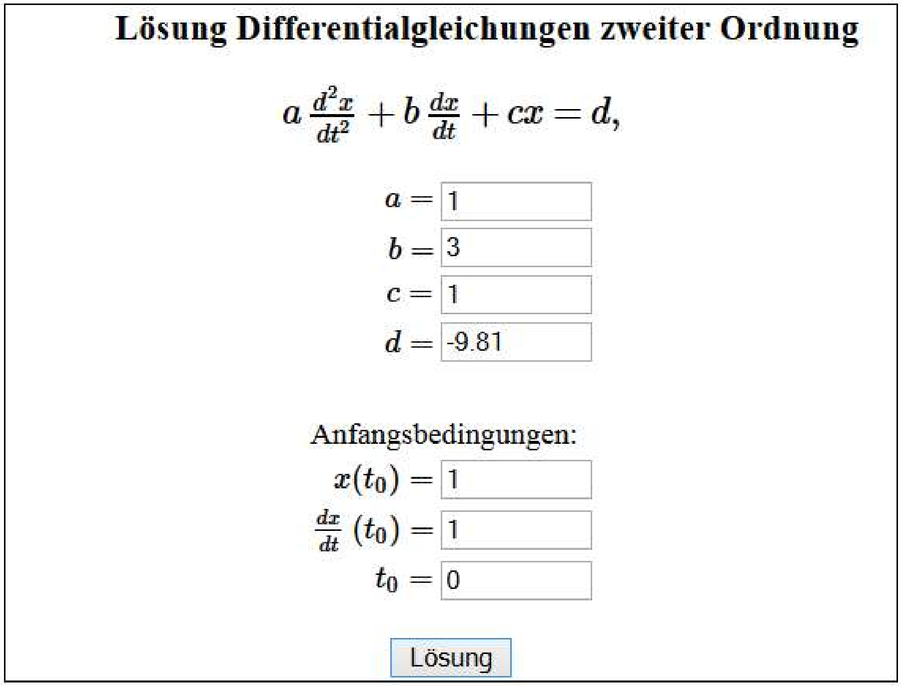


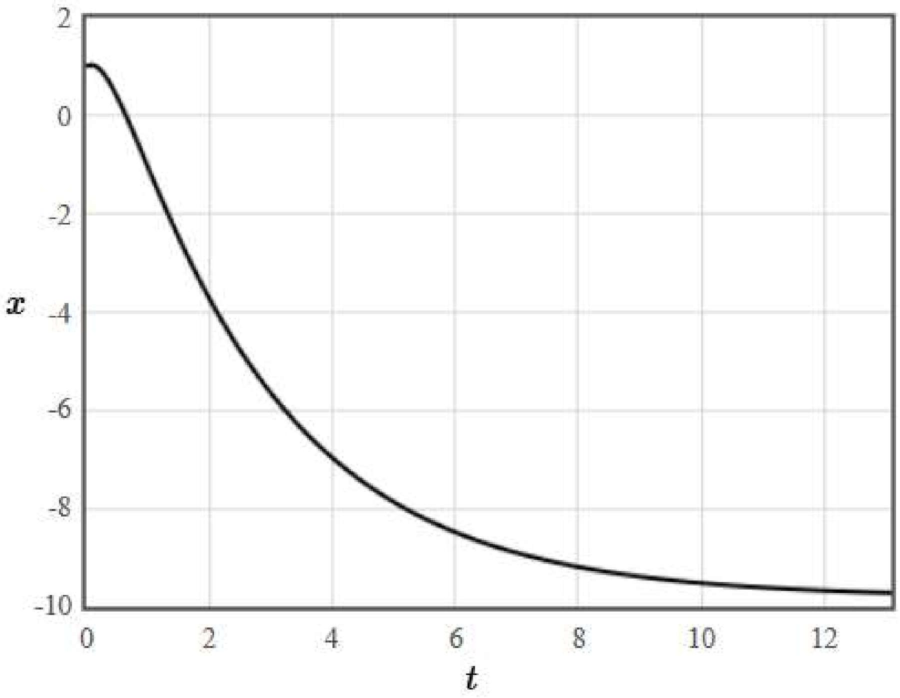
$$\tau = \frac{2m}{b}$$

$b^2 > 4km$ Kriechfall



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = -mg$$





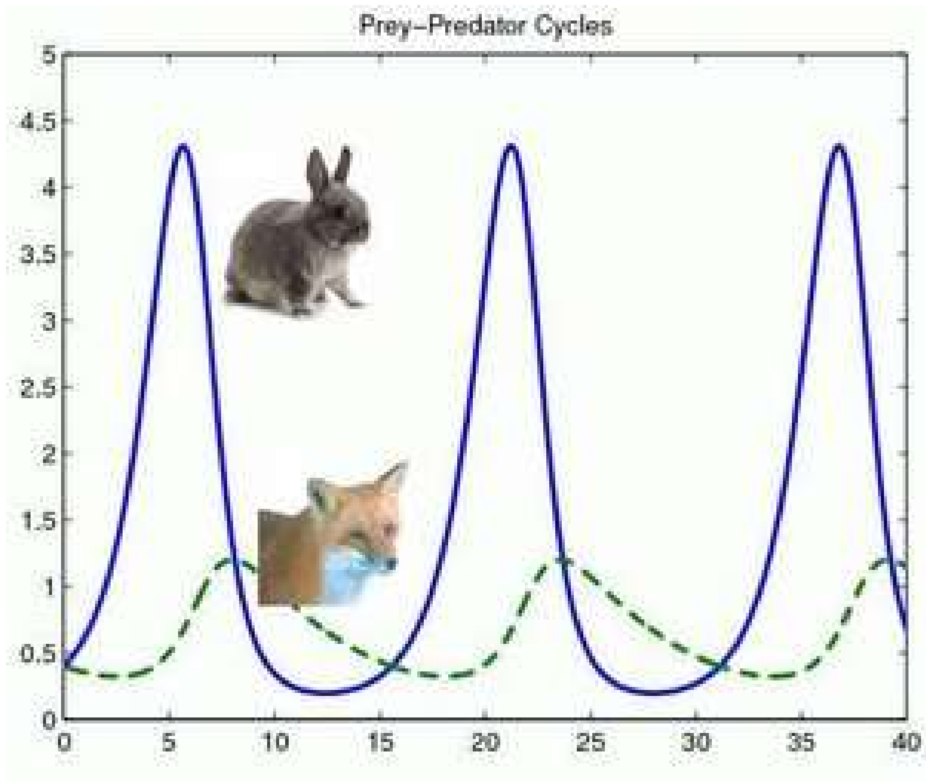
$$\tau_1 = \frac{-1}{\lambda_1} = \frac{2m}{b + \sqrt{b^2 - 4km}}$$

$$\tau_2 = \frac{-1}{\lambda_2} = \frac{2m}{b - \sqrt{b^2 - 4km}}$$

Räuber-Beute-Gleichungen

$$\frac{dx}{dt} = (b - py)\,x$$

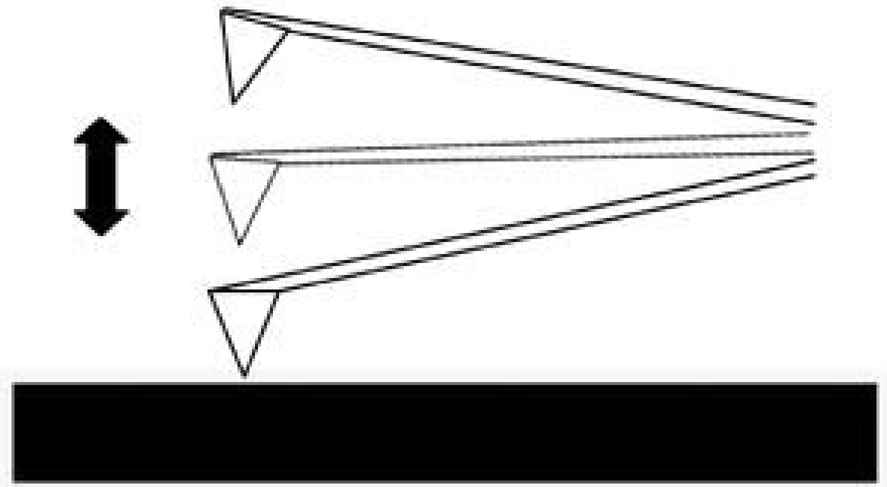
$$\frac{dy}{dt} = (ry - d)\,y$$

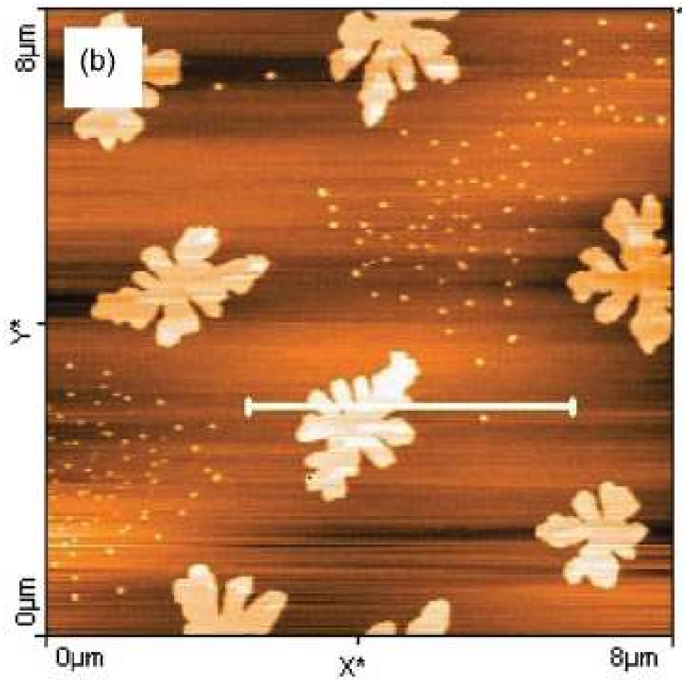


http://www.scholarpedia.org/article/Predator-prey_model

Rasterkraftmikroskop

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

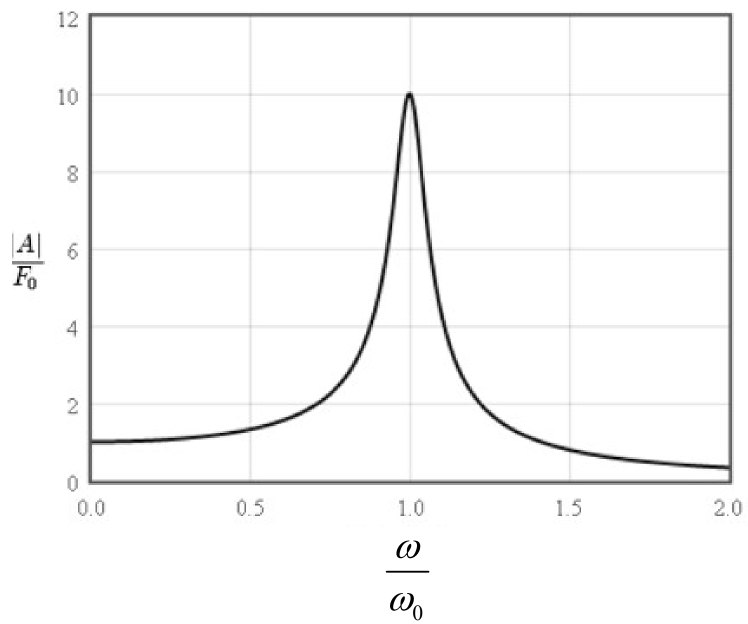




L. Tumbek, C. Gleichweit, K. Zojer, and A. Winkler
Phys. Rev. B 86, 085402 (2012)

Resonaz

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx + F_0\cos(\omega t)$$



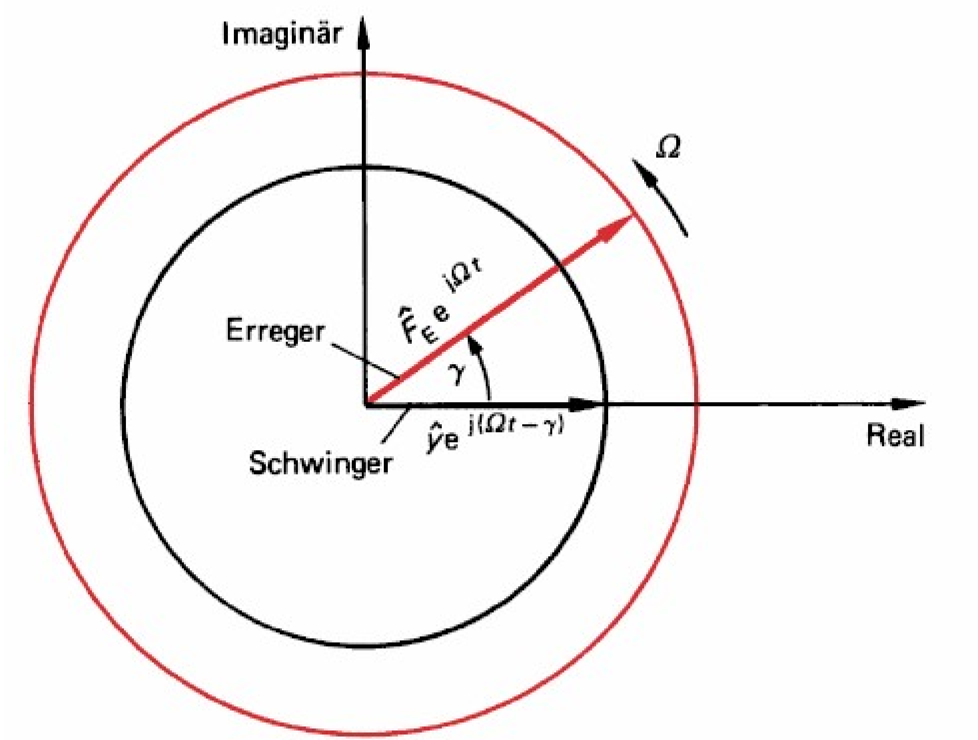
Resonaz

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

$$\Downarrow$$

Komplexe Ebene: $$m\frac{d^2z}{dt^2} + b\frac{dz}{dt} + kz = F_0\exp(i\omega t)$$

Lösung: $$z = A\exp(i\omega t)$$



Erreger: $F_0 \exp(i\omega t)$

Schwinger: $z = A\exp(i\omega t - \theta)$

Resonaz

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F_0\exp\left(i\omega t\right)$$

$$z=x_1+ix_2$$

$$m\frac{d^2x_1}{dt^2}+b\frac{dx_1}{dt}+kx_1=F_0\cos\left(\omega t\right) \qquad m\frac{d^2x_2}{dt^2}+b\frac{dx_2}{dt}+kx_2=F_0\sin\left(\omega t\right)$$

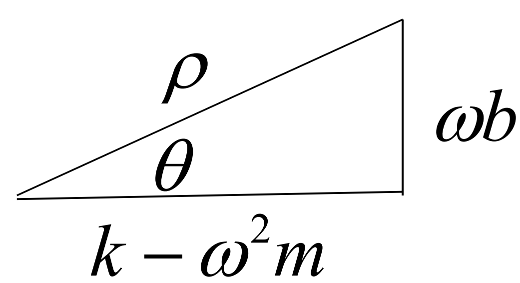
Resonaz

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F_0\exp(i\omega t)$$

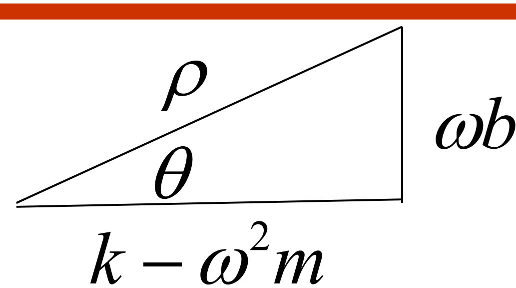
$$z=A\exp(i\omega t)$$

$$\left(-\omega^2 m+i\omega b+k\right)A=\rho e^{i\theta}A=F_0$$

$$-\omega^2 m+i\omega b+k=\rho e^{i\theta}=\rho\cos\theta+i\rho\sin\theta$$



Resonaz



$$\rho=\sqrt{\left(k-m\omega^2\right)^2+\omega^2 b^2} \qquad \tan\theta=\frac{\omega b}{k-m\omega^2}$$

$$\rho e^{i\theta}A=F_0 \quad \Rightarrow \quad A=\frac{F_0}{\rho}e^{-i\theta}$$

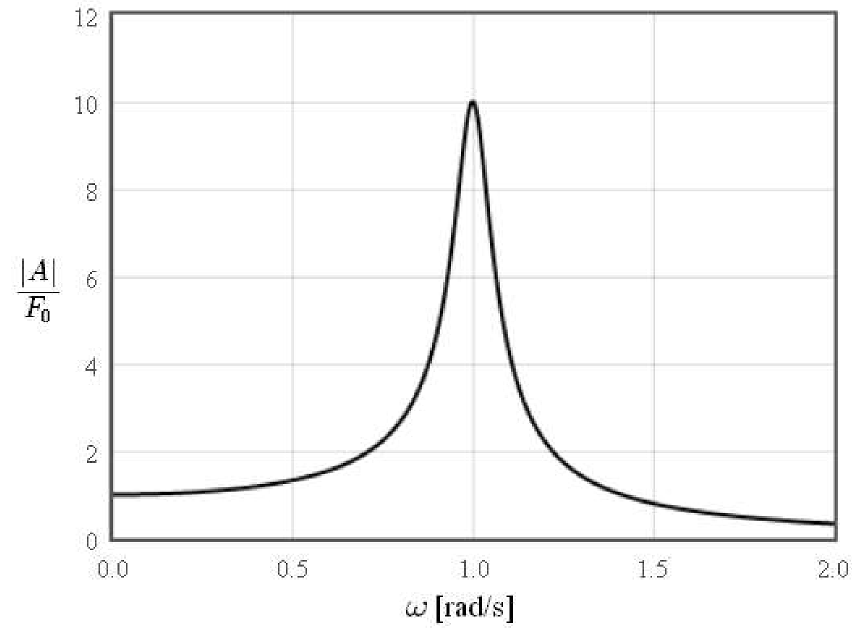
$$z=Ae^{i\omega t}=\frac{F_0}{\rho}e^{i\omega t}e^{-i\theta}=\frac{F_0}{\rho}e^{i(\omega t-\theta)}$$

$$x_1=\mathrm{Re}(z)=\frac{F_0}{\rho}\cos(\omega t-\theta)$$

$m =$ 1 [kg] $b =$ 0.1 [N s/m] $k =$ 1 [N/m] Plot

$Q = \frac{\sqrt{mk}}{b} =$ 10

$$\frac{|A|}{F_0} = \frac{1}{\sqrt{\left(k - m\omega^2\right)^2 + \omega^2 b^2}}$$



$$A = \frac{F_0}{\rho} e^{-i\theta}$$

$$\tan\theta = \frac{\omega b}{k - m\omega^2}$$