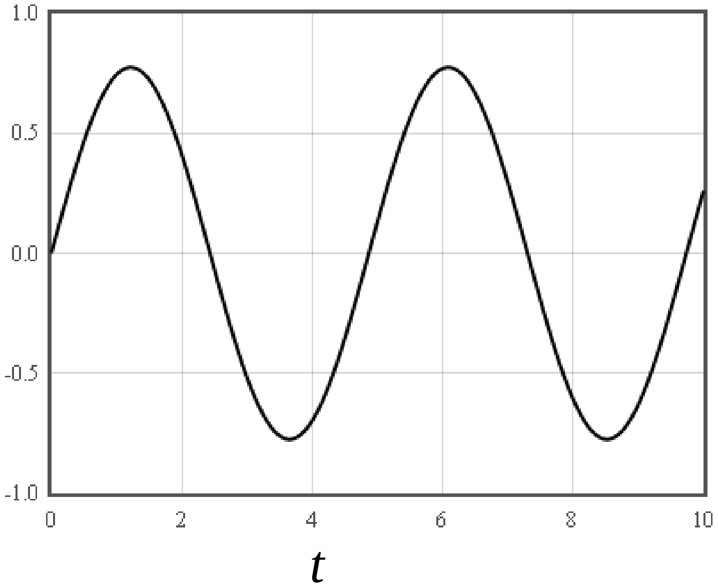
Harmonische Schwingung

$$f = \frac{1}{T}$$



sinusförmig = harmonisch

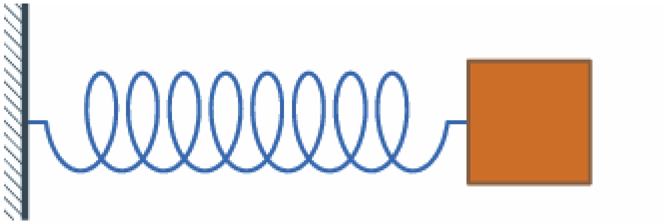
$$\sin(\omega t + \theta)$$

Radiant

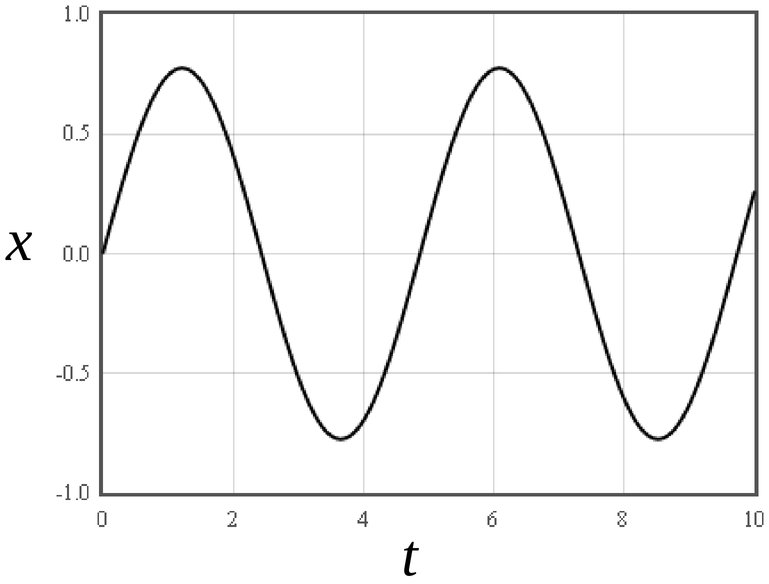
Kreisfrequenz $$\omega = 2\pi f = \frac{2\pi}{T}$$ rad/s

Freie Schwingung

$$m\frac{d^2x}{dt^2} = -kx$$

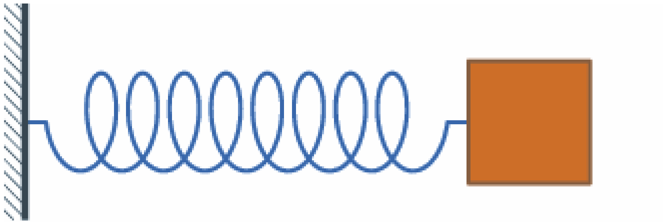


Lösung: $x(t) = C_1 \sin\left(\omega_0 t\right) + C_2 \cos\left(\omega_0 t\right)$



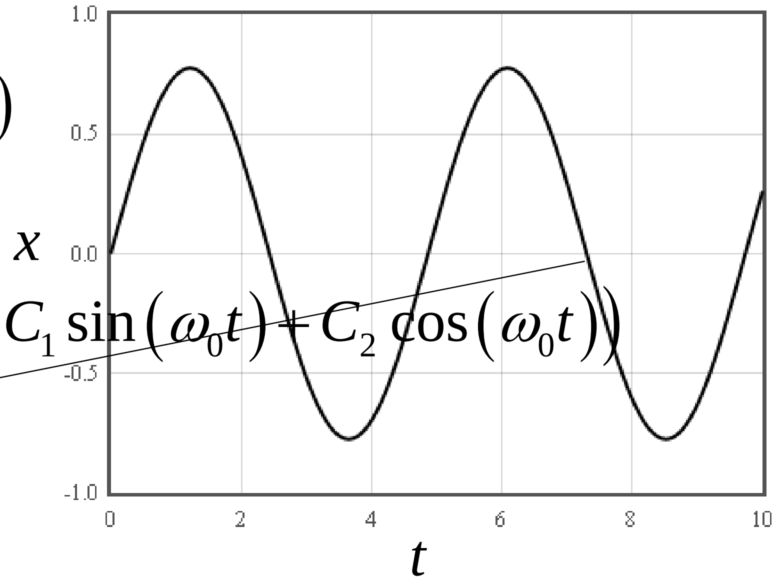
$$\omega_0 = ?$$

Freie Schwingung



$$m\frac{d^2x}{dt^2} = -kx$$

$$x(t) = C_1 \sin(\omega_0 t) + C_2 \cos(\omega_0 t)$$



$$\frac{d^2x}{dt^2} = -\omega_0^2 C_1 \sin(\omega_0 t) - \omega_0^2 C_2 \cos(\omega_0 t)$$

$$-m\omega_0^2 \left(C_1 \sin(\omega_0 t) + C_2 \cos(\omega_0 t)\right) = -k\left(C_1 \sin(\omega_0 t) + C_2 \cos(\omega_0 t)\right)$$

$$m\omega_0^2 = k$$

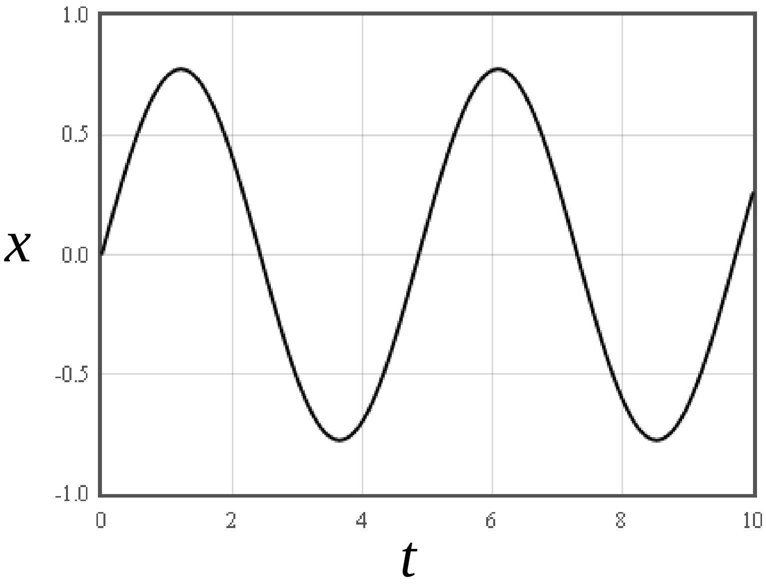
$$\omega_0 = \sqrt{\frac{k}{m}}$$

Anfangsbedingungen

$$x(t=0)=0 \qquad \frac{dx}{dt}(t=0)=1$$

$$x(t)=C_1\sin\left(\omega_0 t\right)+C_2\cos\left(\omega_0 t\right)$$

$$\frac{dx}{dt}=\omega_0 C_1\cos\left(\omega_0 t\right)-\omega_0 C_2\sin\left(\omega_0 t\right)$$



$$x(t=0)=0=C_2$$

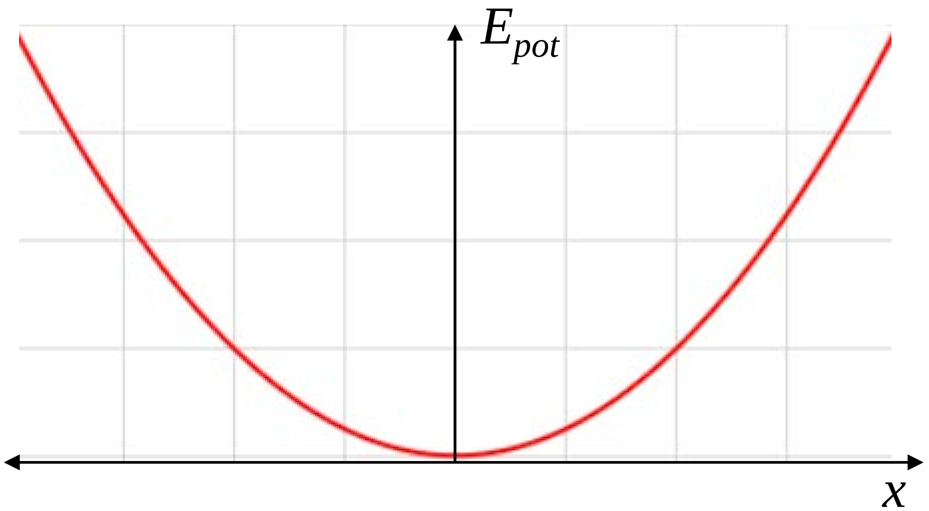
$$C_2=0$$

$$\frac{dx}{dt}(t=0)=1=\omega_0 C_1$$

$$C_1=\frac{1}{\omega_0}$$

andere Schwingungssysteme

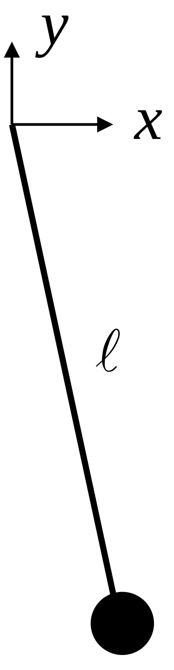
E_{pot} hat ein Minimum bei dem Gleichgewichtspunkt



$$F_x = -\frac{dU}{dx}$$

Pendel

$$x^2 + y^2 = \ell^2$$



$$E_{pot} = -mgy = -mg\sqrt{\ell^2 - x^2}$$

$$F_x = -\frac{\partial E_{pot}}{\partial x} = \frac{1}{2}\frac{mg\left(-2x\right)}{\sqrt{\ell^2 - x^2}}$$

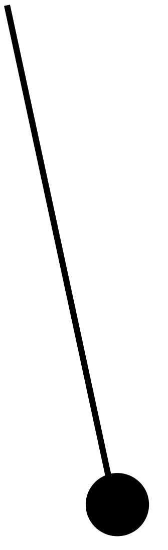
für $\quad x \ll y \approx \ell$

$$F_x \approx -\frac{mgx}{\ell}$$

Pendel

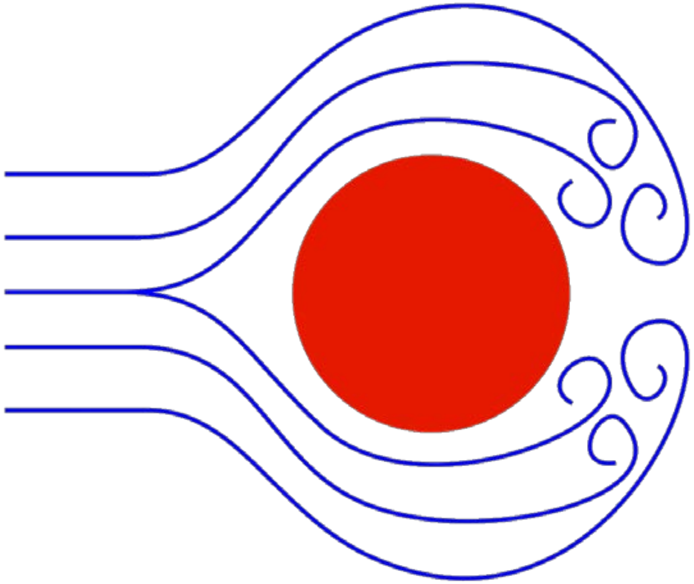
$$m\frac{d^2x}{dt^2} \approx -\frac{mg}{\ell}x$$

$$\omega_0 = \sqrt{\frac{g}{\ell}}$$



Reibungskraft (Strömungswiderstand)

$$\vec{F} = -a\vec{v} - b\vec{v}\left|\vec{v}\right| - c\vec{v}\left|\vec{v}\right|^2 + \cdots$$



$$F_x \approx -a\frac{dx}{dt}$$

http://en.wikipedia.org/wiki/Drag_%28physics%29

Schwingungssysteme

für kleine Amplitudenschwankungen und niedrigen Geschwindigkeiten die Differentialgleichung für eine schwingende Systeme ist

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = d$$

Lineare gewöhnliche Differentialgleichung zweiter Ordnung

Lineare gewöhnliche Differentialgleichung

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

Lineare Differentialgleichung zweiter Ordnung

Nichtlineare Differentialgleichung zweiter Ordnung

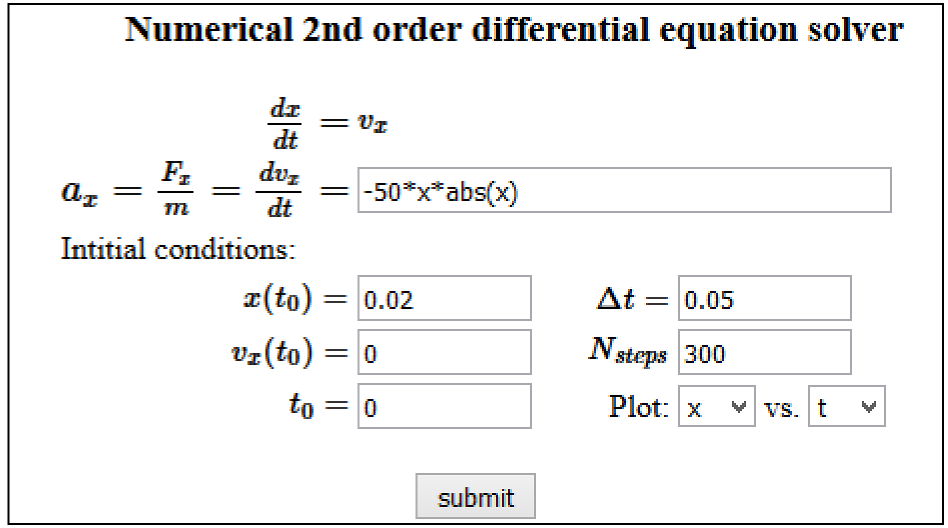
$$m\left(\frac{d^2x}{dt^2}\right)^2+b\frac{dx}{dt}+kx=0$$

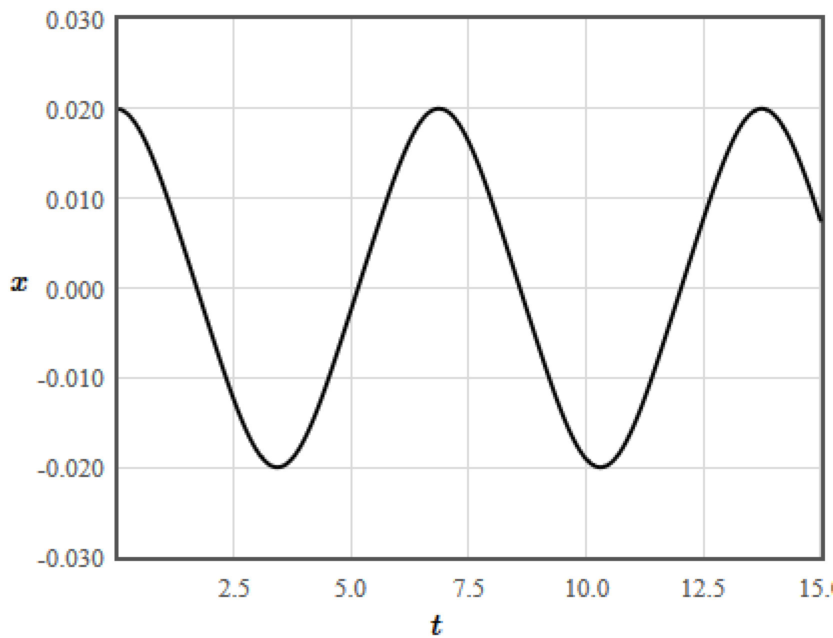
$$m\frac{d^2x}{dt^2}+b\left(\frac{dx}{dt}\right)^2+kx=0$$

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx^3=0$$

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

$$a_x=-\frac{b}{m}v_x-\frac{k}{m}x$$





Differentialgleichungen zweiter Ordnung

Lösung Differentialgleichungen zweiter Ordnung

$$a\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + cx = d,$$

$a =$ 1

$b =$ 3

$c =$ 1

$d =$ 0

Anfangsbedingungen:

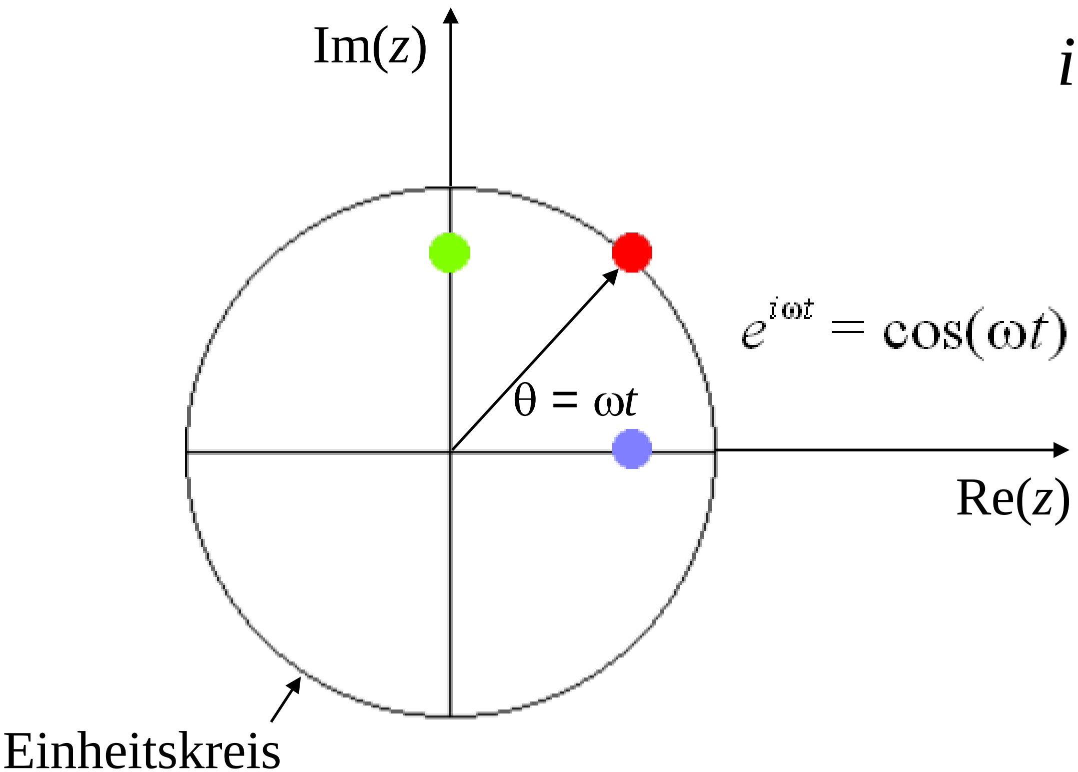
$x(t_0) =$ 1

$\frac{dx}{dt}\,(t_0) =$ 0

$t_0 =$ 0

Lösung

Euler'sche Formel $e^{i\theta} = \cos\theta + i\sin\theta$



$$i^2 = -1 = \left(j^2\right)$$

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

ω = Kreisfrequenz

$$\left|e^{i\theta}\right| = \sqrt{e^{-i\theta}e^{i\theta}} = \sqrt{e^{0}} = 1 = \sqrt{\left(\cos\theta - i\sin\theta\right)\left(\cos\theta + i\sin\theta\right)} = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

Lineare gewöhnliche Differentialgleichung

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

Lösung: $$x=Ce^{\lambda t}$$

Konstant

$$m\lambda^2Ce^{\lambda t}+b\lambda Ce^{\lambda t}+kCe^{\lambda t}=0$$

$$m\lambda^2+b\lambda+k=0$$

Lineare gewöhnliche Differentialgleichung

$$m\lambda^2 + b\lambda + k = 0$$

$$\lambda_1 = \frac{-b + \sqrt{b^2 - 4mk}}{2m} \qquad \lambda_2 = \frac{-b - \sqrt{b^2 - 4mk}}{2m}$$

Lösung: $$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Anfangsbedingungen:

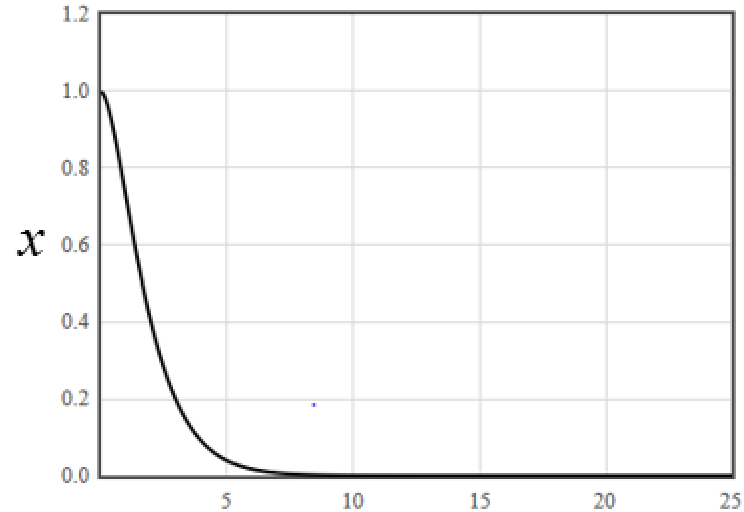
$$x(0) = C_1 + C_2$$

$$\frac{dx}{dt}(0) = \lambda_1 C_1 + \lambda_2 C_2$$

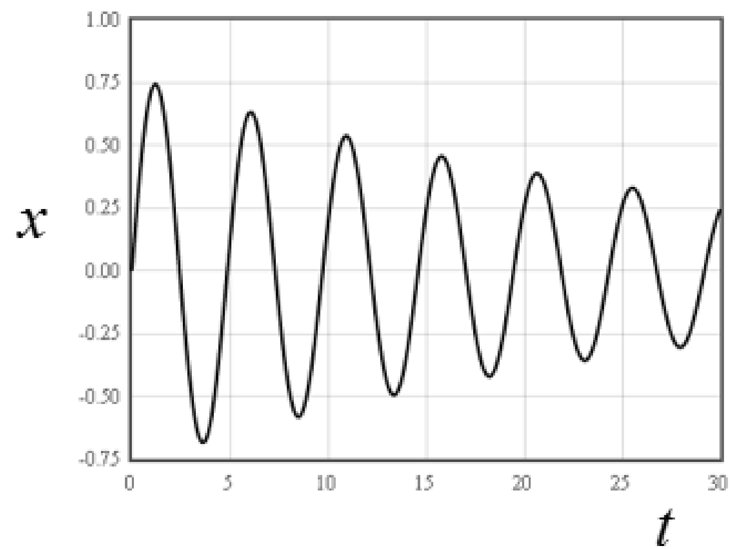
$$C_1 = \frac{\lambda_2 x(0) - \frac{dx}{dt}(0)}{\lambda_2 - \lambda_1} \qquad C_2 = \frac{\lambda_1 x(0) - \frac{dx}{dt}(0)}{\lambda_1 - \lambda_2}$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

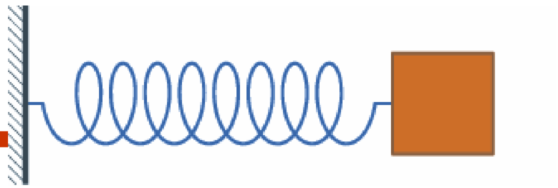
$b^2 > 4km$ Kriechfall



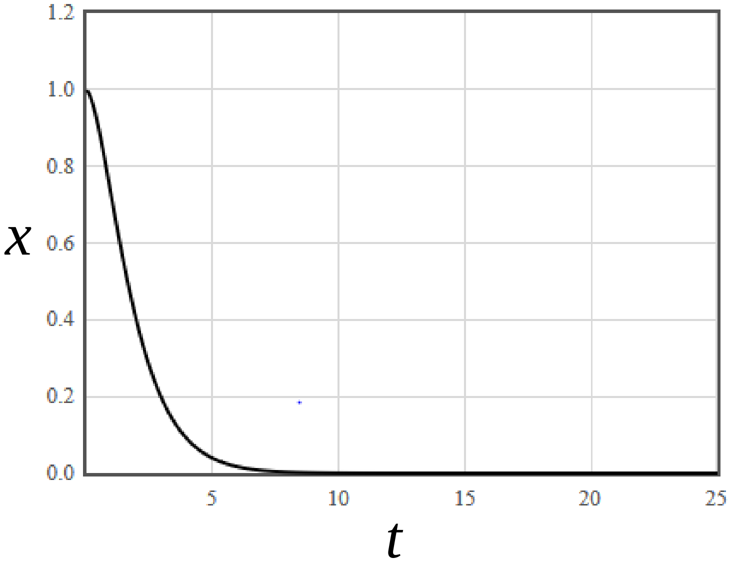
$b^2 < 4km$ Schwingfall



$b^2 > 4km$ Kriechfall



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$



Lösung: $$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$$x(t) = C_1 \exp(-t/\tau_1) + C_2 \exp(-t/\tau_2)$$

$$\tau_1 = \frac{-1}{\lambda_1} = \frac{2m}{b + \sqrt{b^2 - 4km}} \qquad \tau_2 = \frac{-1}{\lambda_2} = \frac{2m}{b - \sqrt{b^2 - 4km}}$$