Potentielle energie

Konservative Kraft

Konservative Kraft: Arbeit entlang eines beliebigen Weges ist nur vom Anfangs- und Endpunkt abhängig.

$$\oint_C \vec{F}_{konservative} \cdot d\vec{r} = 0$$

konservative Kräfte: Schwerkraft, Coulombkraft, elastische Kraft

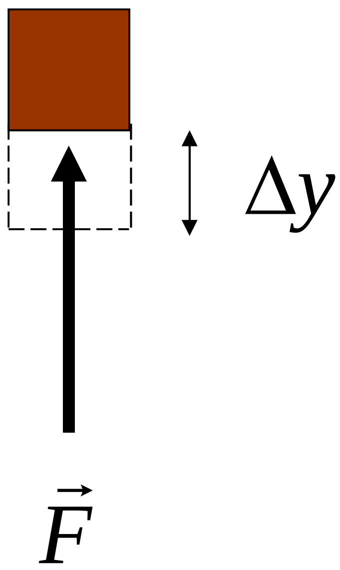
nicht konservative Kräfte: Reibungskräfte, dissipative Kräfte

$$\oint_C \vec{F} \cdot d\vec{r} \geq 0$$

konservative Kraft → Potentielle energie

$$\Delta E_{pot}(x, y, z) = -W$$

Hubarbeit gegen Gewichtskraft

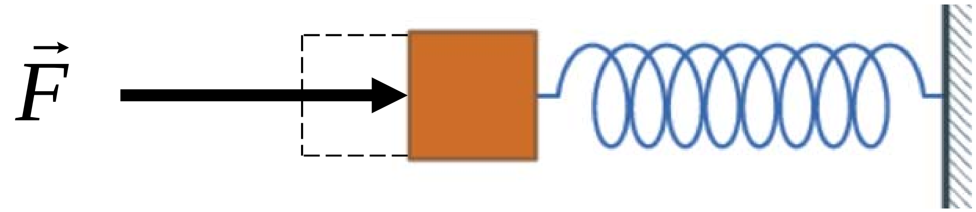


$$\vec{F} = -mg\hat{z}$$

$$\Delta E_{pot} = -W$$

Potentielle energie: $$\Delta E_{pot}(x, y, z) = mg\Delta y$$

Feder



Hookesches Gesetz: $F(x) = -kx$

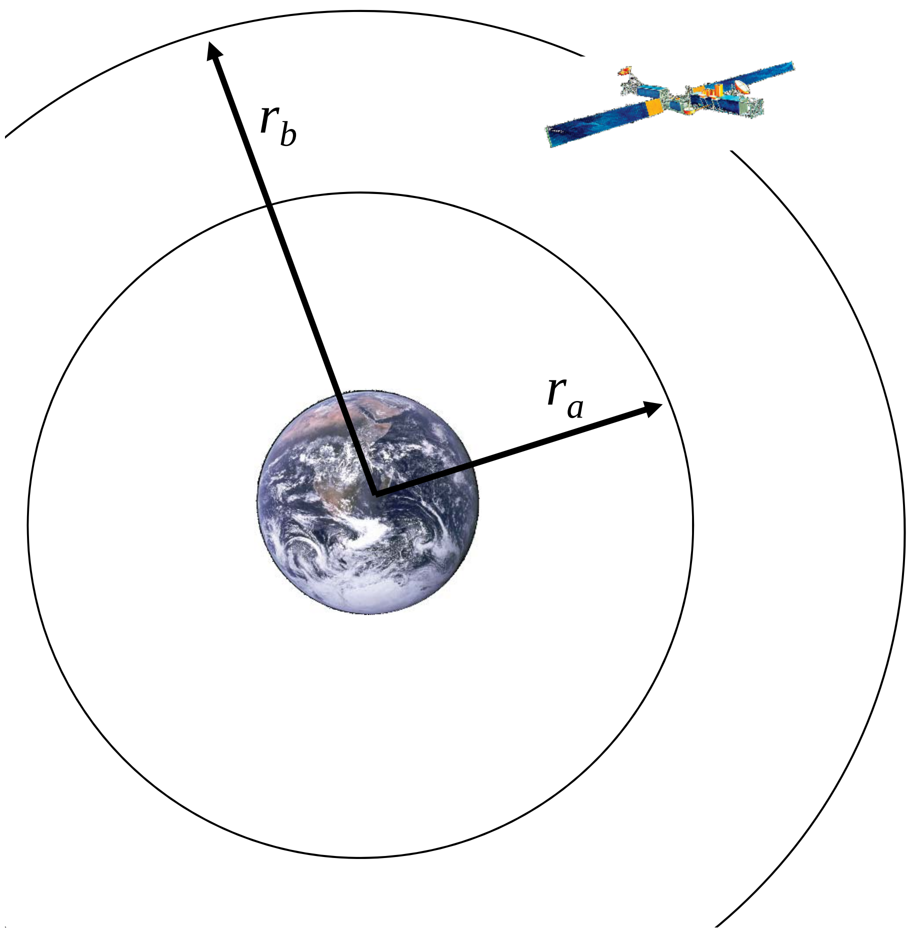
$$W = -\int_0^{x_e} kx\,dx = -\tfrac{1}{2} k x_e^2 \quad [\mathrm{J}]$$

Potentielle energie: $\Delta E_{pot} = \frac{kx^2}{2}$

Gravitation

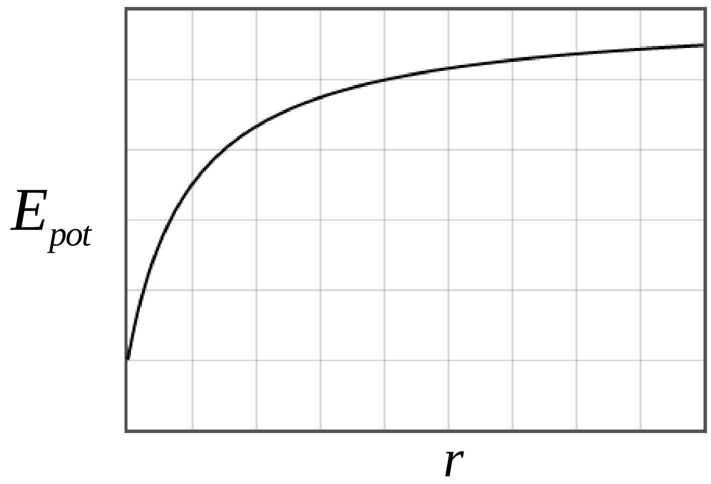
$$\Delta E_{pot}(\vec{r}) = -W$$

$$-W = -\int_{r_a}^{r_b} \frac{-Gm_1 m_2}{r^2} dr = \frac{-Gm_1 m_2}{r_b} - \frac{-Gm_1 m_2}{r_a}$$



üblicherweise $E_{pot}(r_a = \infty) = 0$

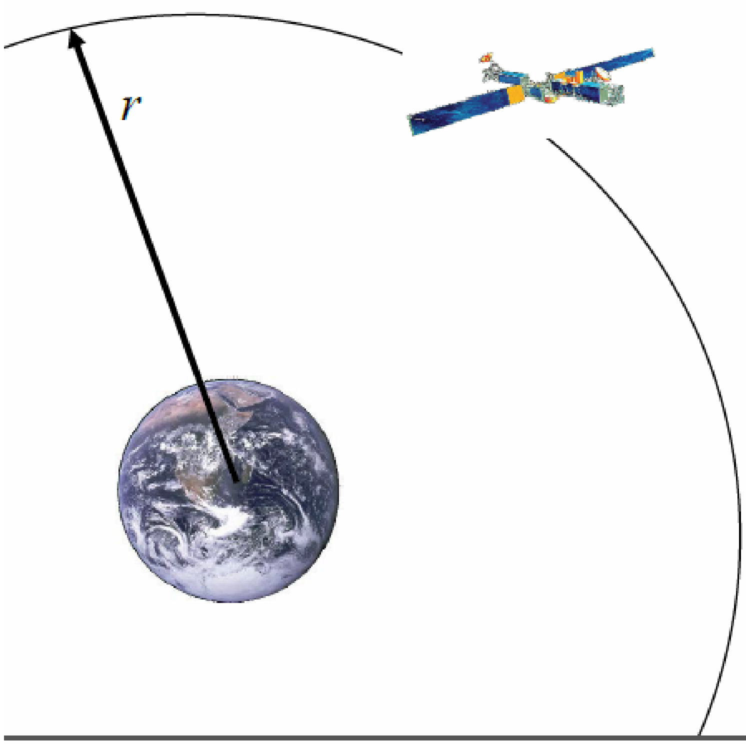
$$E_{pot}(\vec{r}) = \frac{-Gm_1 m_2}{|\vec{r}|}$$



Satellitenbahnen

$$\vec{F} = \frac{-G m_{erde} m_{sat}}{r^2} \hat{r}$$

$$\frac{dv_x}{dt} = \frac{-G m_{erde} m_{sat} x}{m_{sat} \left(x^2 + y^2 + z^2\right)^{3/2}}$$



Numerical 6th order differential equation solver

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ -x*6.6726E-11*5.97219E24/pow(x*x+y*y+z*z,3/2)

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ -y*6.6726E-11*5.97219E24/pow(x*x+y*y+z*z,3/2)

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ -z*6.6726E-11*5.97219E24/pow(x*x+y*y+z*z,3/2)

Initial conditions:

$t_0 =$ 0

$x(t_0) =$ 0

$v_x(t_0) =$ 7900

$y(t_0) =$ 6371000

$v_y(t_0) =$ 0

$z(t_0) =$ 0

$v_z(t_0) =$ 0

$\Delta t =$ 60

N_{steps} 1500

Plot: y vs. x

konservative Kraft → Potentielle energie

$$E_{pot}(x,y,z) = -W$$

	Kraft	Potentielle energie
Schwerkraft	$\vec{F} = -mg\ \hat{y}$	$E_{pot}(x,y,z) = mgy$
Feder	$\vec{F} = -kx\ \hat{x}$	$E_{pot}(x,y,z) = \frac{kx^2}{2}$
Gravitation	$\vec{F} = -\frac{Gm_1m_2}{r^2}\ \hat{r}$	$E_{pot}(x,y,z) = -\frac{Gm_1m_2}{r}$
Coulomb	$\vec{F} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}\ \hat{r}$	$E_{pot}(x,y,z) = \frac{q_1q_2}{4\pi\varepsilon_0 r}$

Energieerhaltungssatz

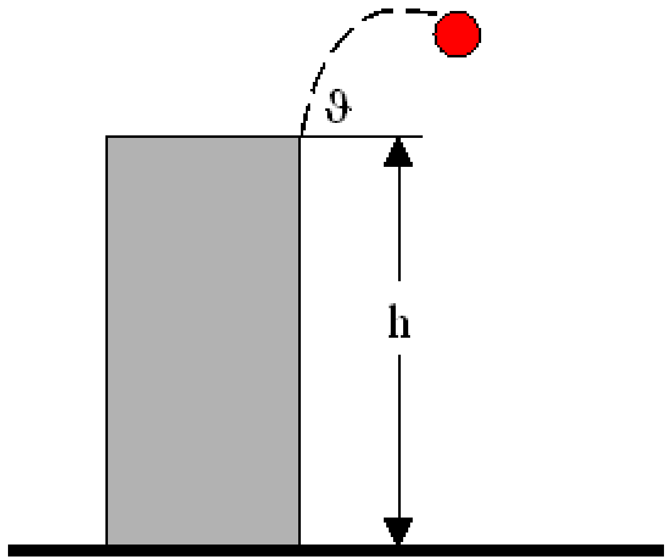
$$\Delta E = 0$$

$$E_{kin,nachher} - E_{kin,vorher} + W = 0$$

konservative Kräfte:

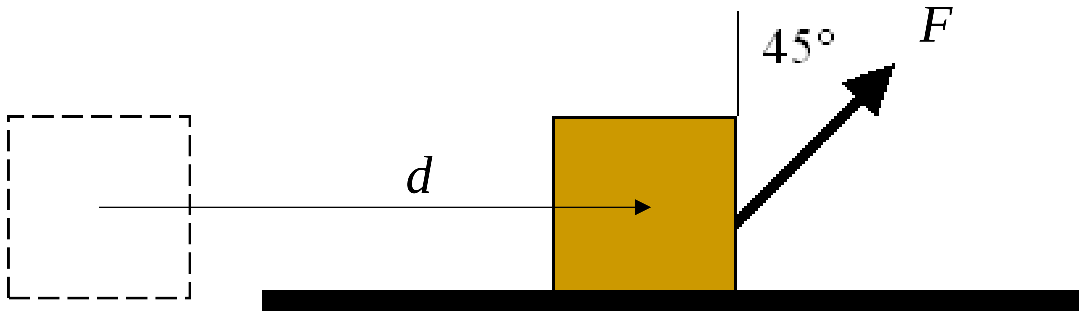
$$E_{pot,nachher} - E_{pot,vorher} + E_{kin,nachher} - E_{kin,vorher} = 0$$

Energieerhaltungssatz



$$|\vec{v}(y=0)|\,?$$

Energieerhaltungssatz



$$E_{therm} \,?$$

Potentielle energie $\rightarrow$ Kraft

$$\vec{F} = -\nabla E_{pot}(x, y, z)$$

Gradient: $$\nabla E_{pot} = \frac{\partial E_{pot}}{\partial x}\hat{x} + \frac{\partial E_{pot}}{\partial y}\hat{y} + \frac{\partial E_{pot}}{\partial z}\hat{z}$$

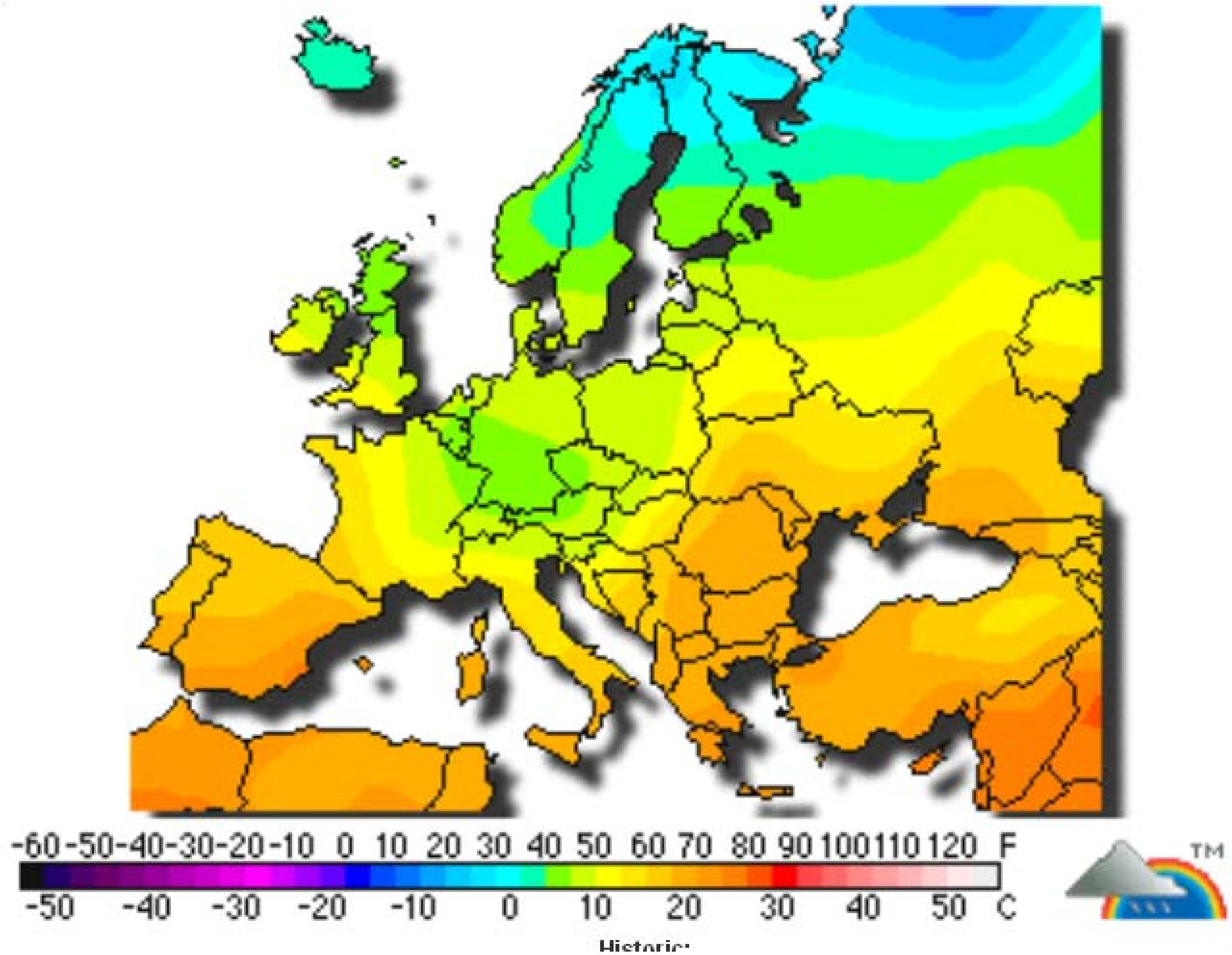
Partielle Ableitung

Potentielle energie → konservative Kraft

$$\vec{F} = -\nabla E_{pot}(x,y,z)$$

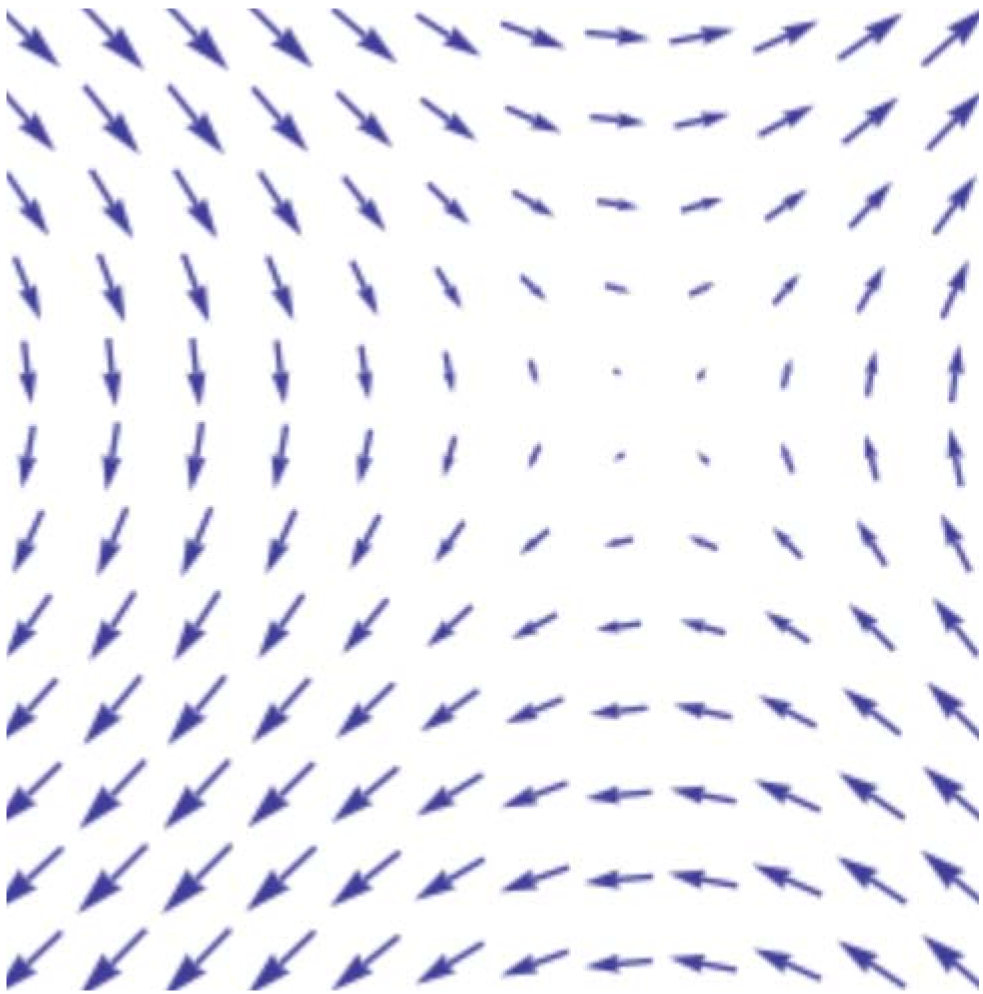
	Potentielle energie	Kraft
Schwerkraft	$E_{pot}(x,y,z) = mgy$	$\vec{F} = -mg\ \hat{y}$
Feder	$E_{pot}(x,y,z) = \frac{kx^2}{2}$	$\vec{F} = -kx\ \hat{x}$
Gravitation	$E_{pot}(x,y,z) = -\frac{Gm_1m_2}{r}$	$\vec{F} = -\frac{Gm_1m_2}{r^2}\ \hat{r}$
Coulomb	$E_{pot}(x,y,z) = \frac{q_1q_2}{4\pi\varepsilon_0 r}$	$\vec{F} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}\ \hat{r}$

Skalarfeld



Potentielle energie ist ein Skalarfeld

Vektorfeld



Elektrisches Feld, Magnetfeld

Gradient

Der Druck P ist in einem bestimmten Gebiet im Raum durch die folgende Funktion bestimmt:

$$P = 7x^3y^{-9}z^6$$

Berechnen Sie den Gradienten des Drucks!

$\nabla P =$ ______ $\hat{x}$ + ______ $\hat{y}$ + ______ $\hat{z}$ [K/m]

Lösung

$$\nabla P = (21x^2y^{-9}z^6)\hat{x} + (-63x^3y^{-10}z^6)\hat{y} + (42x^3y^{-9}z^5)\hat{z}.$$

Coulombsches Gesetz

$$E_{pot}(x,y,z) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

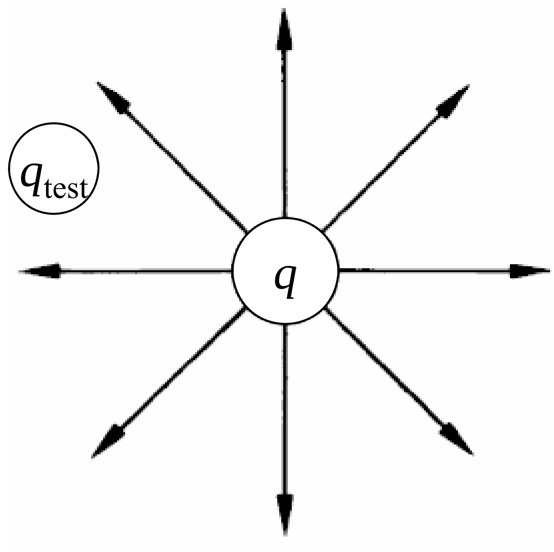
Skalarfeld

$$\vec{F} = -\nabla E_{pot}$$

Vektorfeld

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\ \hat{r}$$

Elektrisches Feld



$$\vec{F} = \frac{q_{test} q}{4\pi\varepsilon_0 r^2}\hat{r}$$

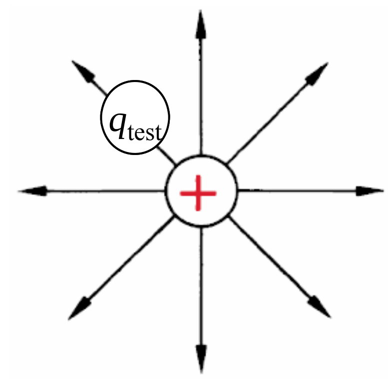
$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Vektorfeld

$$\vec{F} = q_{test}\vec{E}$$

Elektrostatische Potential

Elektrisches Feld $$\vec{E}(\vec{r}) = \frac{\vec{F}}{q_{test}} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$



Elektrostatische Potential $$\varphi(\vec{r}) = \frac{E_{pot}}{q_{test}} = \frac{q}{4\pi\varepsilon_0 r}$$

$$\vec{F} = -\nabla E_{pot}$$

$$E_{pot} = -\int \vec{F} \cdot d\vec{r}$$

$$\vec{E} = -\nabla \varphi$$

$$\varphi = -\int \vec{E} \cdot d\vec{r}$$