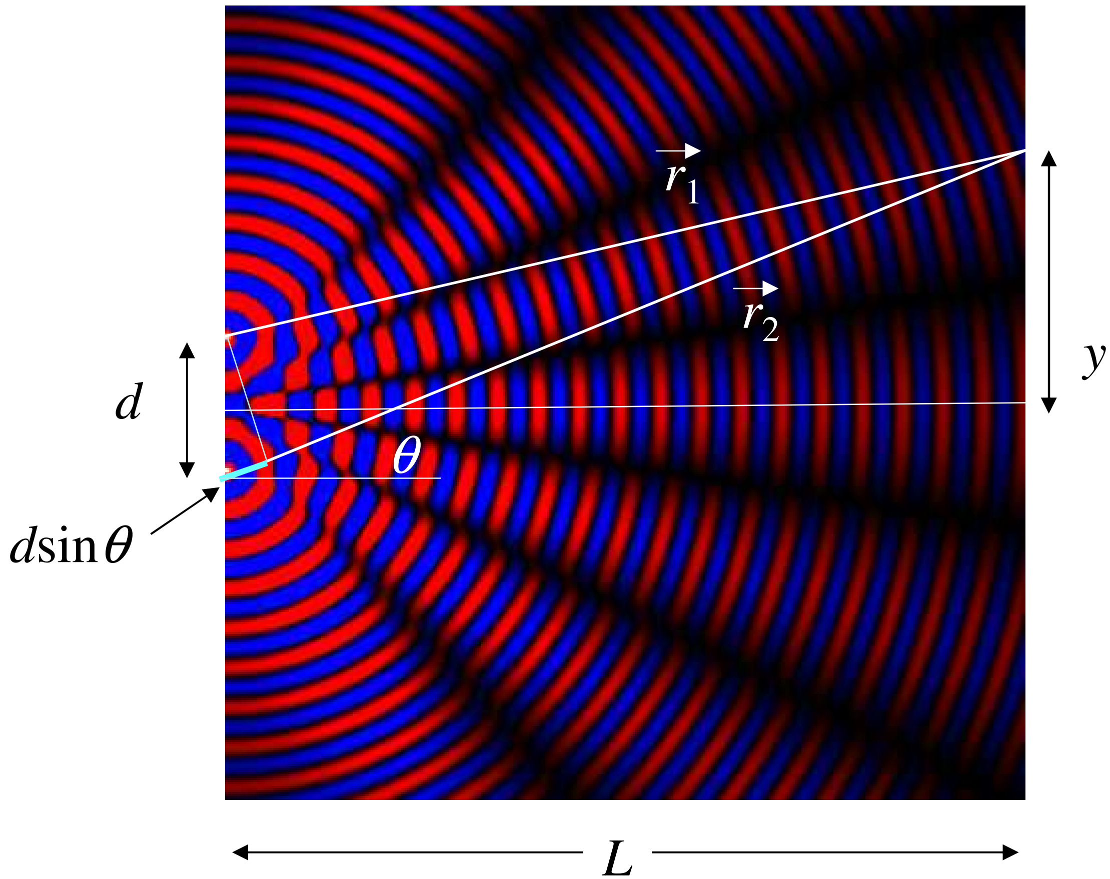
23. Wellen

14. Jan. 2019

Fernfeld

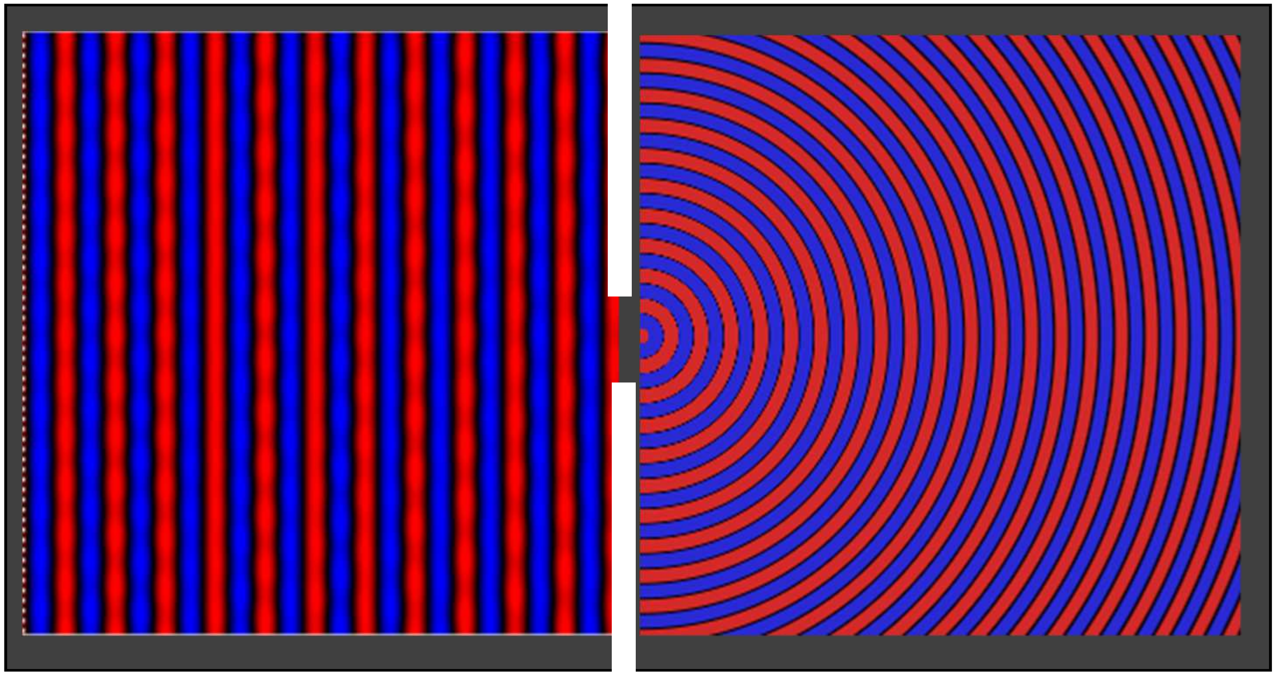


$$\theta \approx \frac{y}{L}$$

Konstruktive Interferenz: $\left|\vec{r}_2\right| - \left|\vec{r}_1\right| = n\lambda \approx d\sin\theta \approx \frac{yd}{L}$

Destruktive Interferenz: $\left|\vec{r}_2\right| - \left|\vec{r}_1\right| = \left(n + \frac{1}{2}\right)\lambda \approx d\sin\theta \approx \frac{yd}{L}$

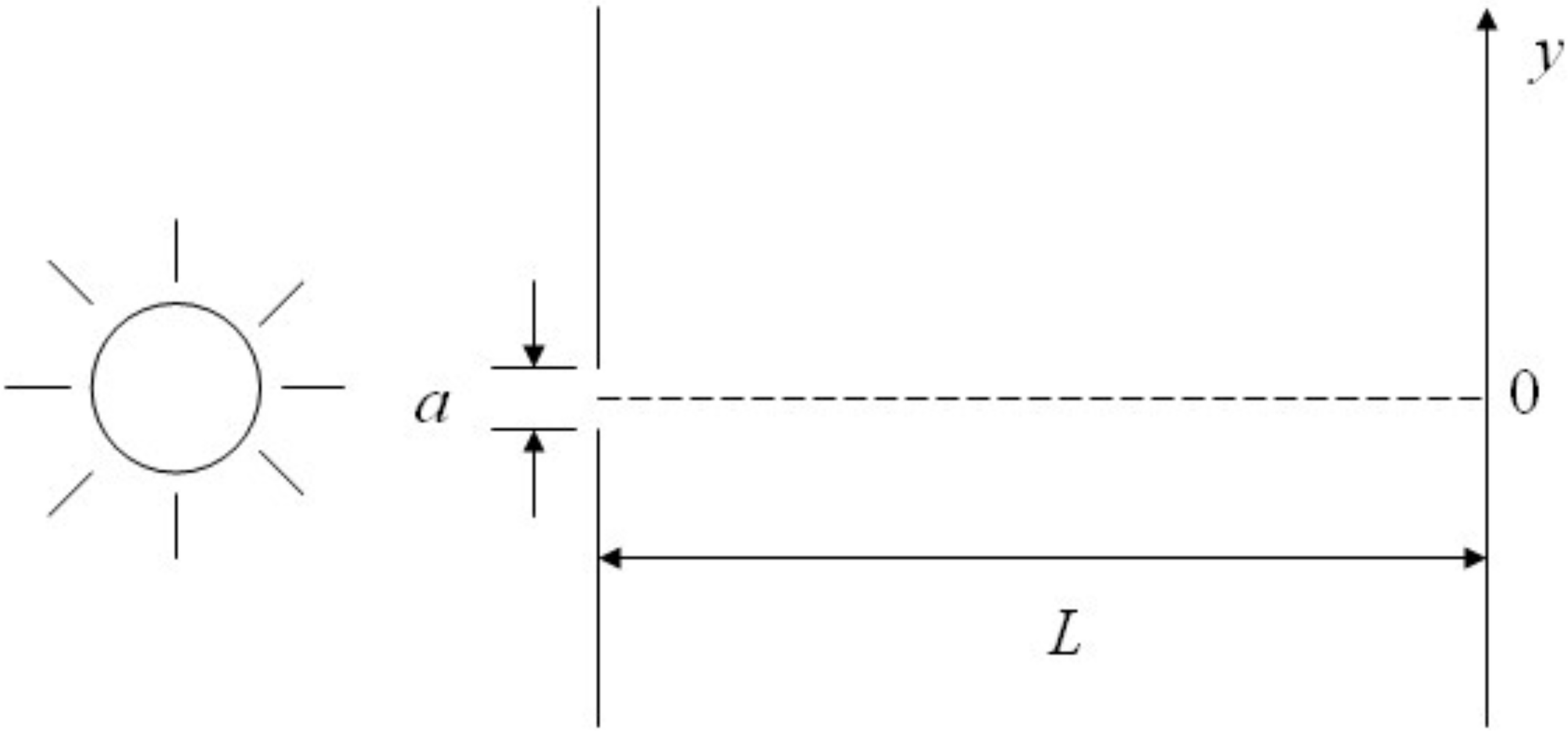
Elementarwelle



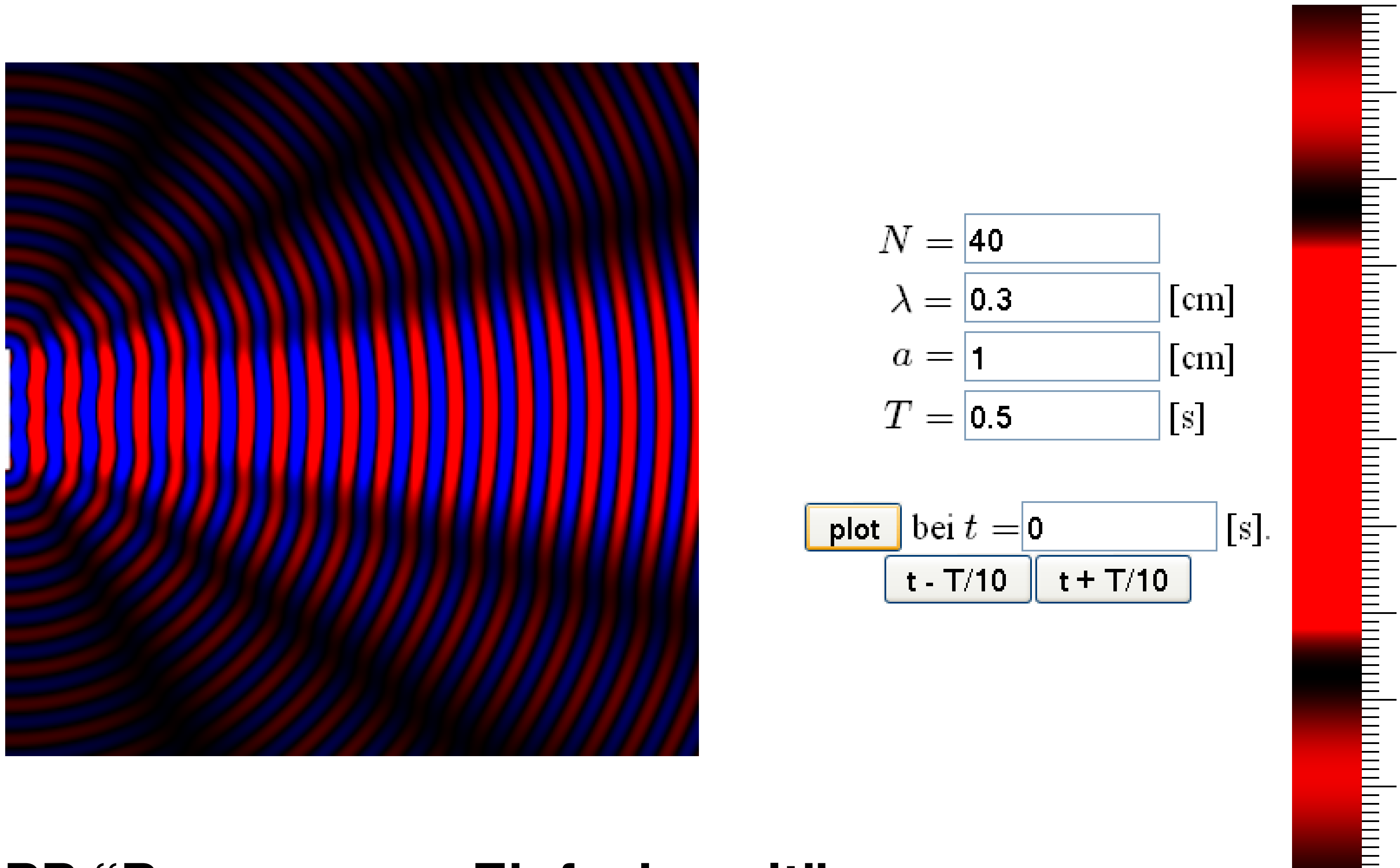
“Huygenssches Prinzip: ebene Welle”

“Huygenssches Prinzip” (N = 1)

Einfachspalt



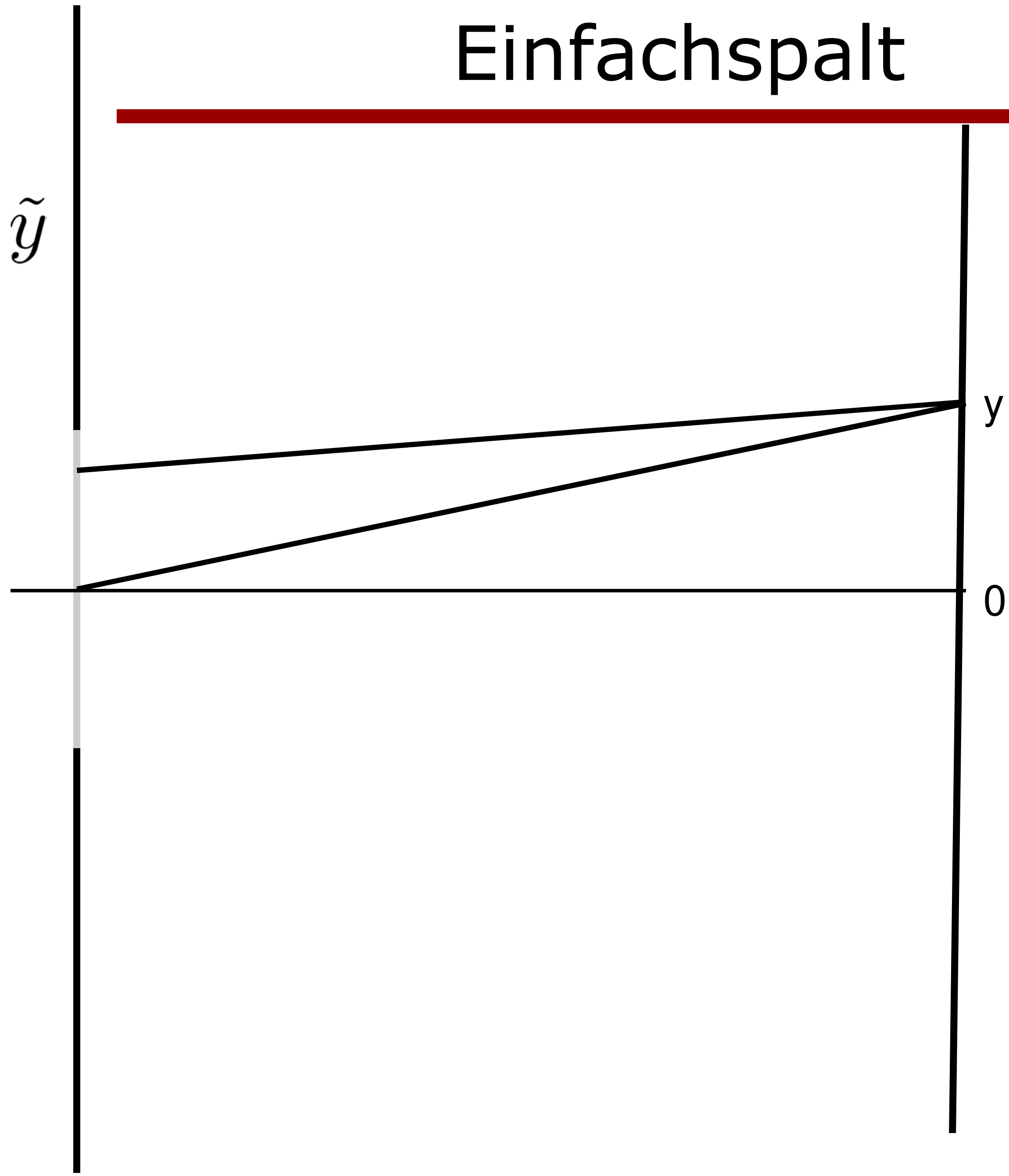
Einfachspalt



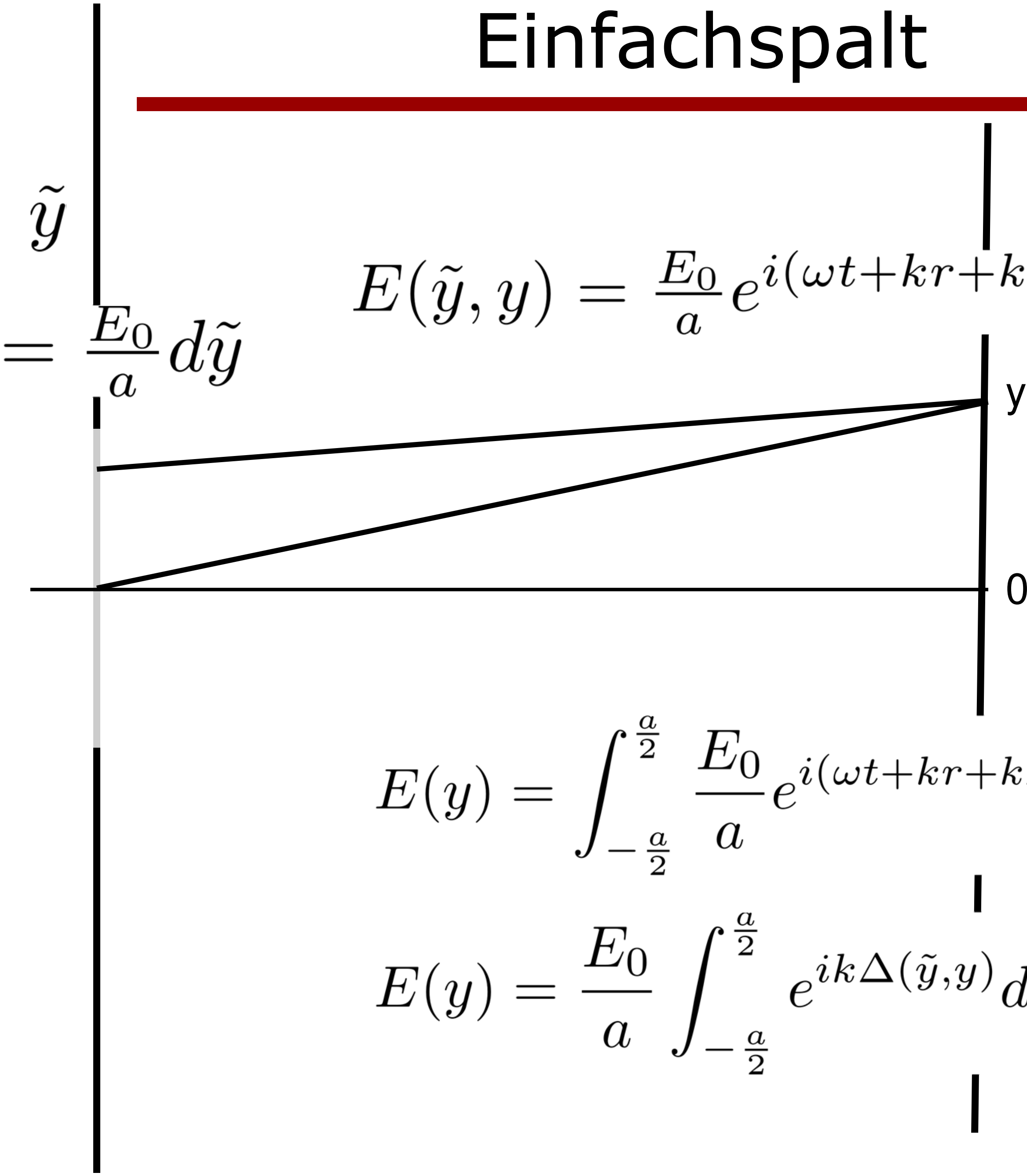
APP “Beugung am Einfachspalt”

Spalt ~ Wellenquellen in linearer Anordnung

Einfachspalt



Einfachspalt



$$E = \frac{E_0}{a} d\tilde{y}$$

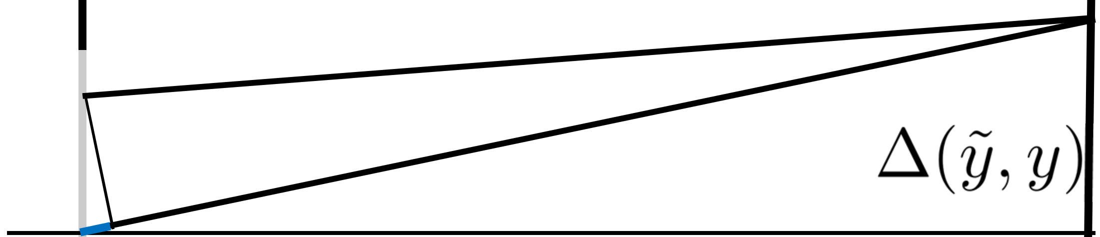
$$E(\tilde{y}, y) = \frac{E_0}{a} e^{i(\omega t + kr + k\Delta(\tilde{y}, y))} d\tilde{y}$$

$$E(y) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{E_0}{a} e^{i(\omega t + kr + k\Delta(\tilde{y}, y))} d\tilde{y}$$

$$E(y) = \frac{E_0}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{ik\Delta(\tilde{y}, y)} d\tilde{y} \cdot e^{i(\omega t + kr)}$$

Einfachspalt

$$E(y) = \frac{E_0}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{ik\Delta(\tilde{y},y)} d\tilde{y} \cdot e^{i(\omega t + kr)}$$



$$\Delta(\tilde{y}, y) = \tilde{y} \sin\theta(y) \approx \tilde{y} \frac{y}{L}$$

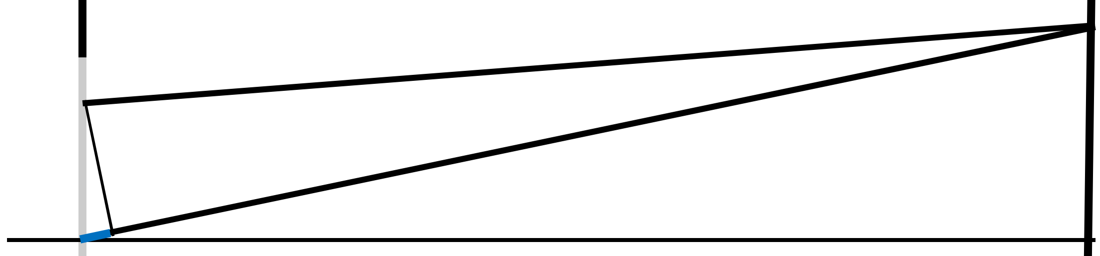
$$\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{ik\Delta(\tilde{y},y)} d\tilde{y} = \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{i\gamma\tilde{y}} d\tilde{y} = \frac{1}{i\gamma}\left(e^{i\gamma\frac{a}{2}} - e^{-i\gamma\frac{a}{2}}\right) = \frac{2}{\gamma} \sin\frac{a\gamma}{2}$$

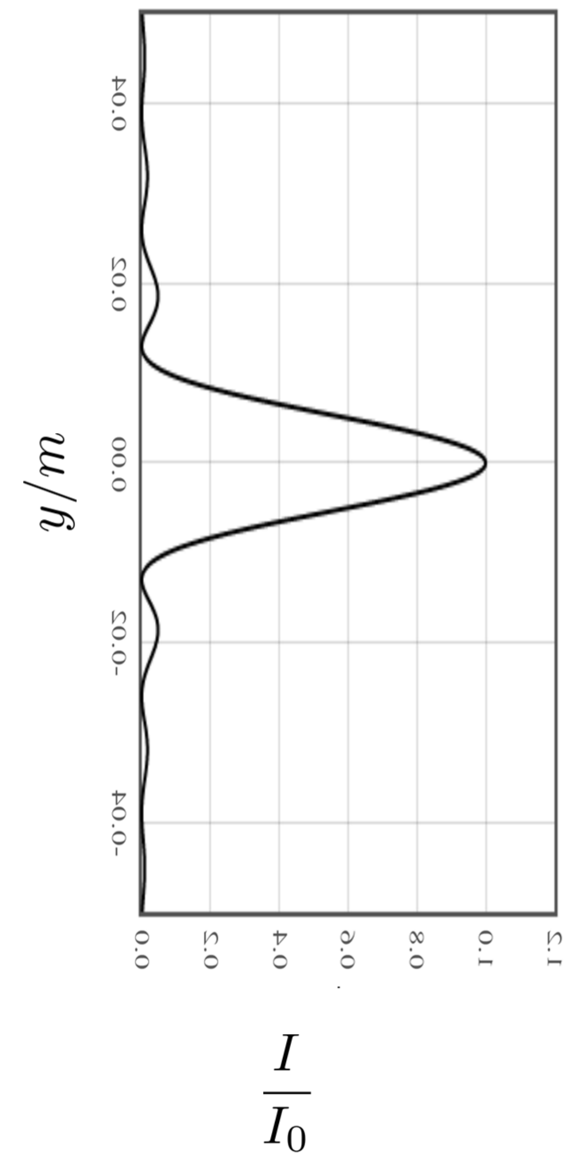
$$\gamma = \frac{2\pi}{\lambda} \sin\theta \approx \frac{2\pi}{\lambda} \frac{y}{L}$$

$$E(y) = \frac{2E_0}{a\gamma} \sin\frac{a\gamma}{2} \cdot e^{i(\omega t + kr)}$$

Einfachspalt

$$E(y) = \frac{2E_0}{a\gamma} \sin \frac{a\gamma}{2} \cdot e^{i(\omega t + kr)}$$

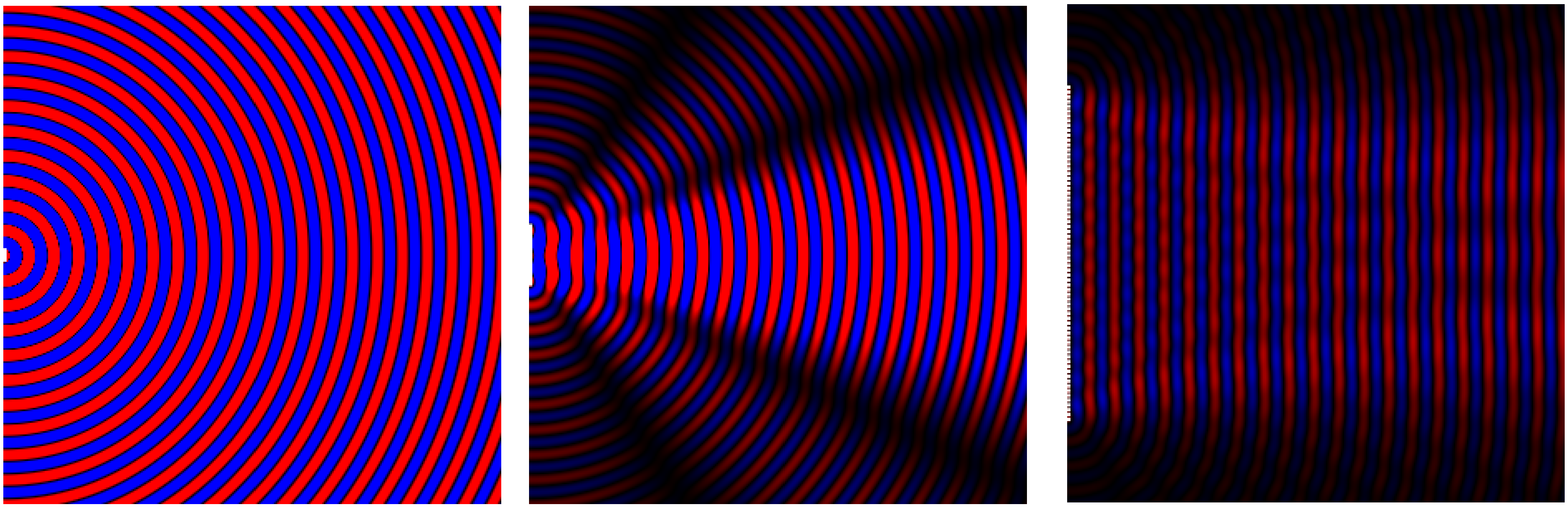




$$I(y) = E^*(y)E(y) = \frac{E_0^2}{(\frac{a}{2}\gamma)^2} \sin^2 \frac{a\gamma}{2}$$

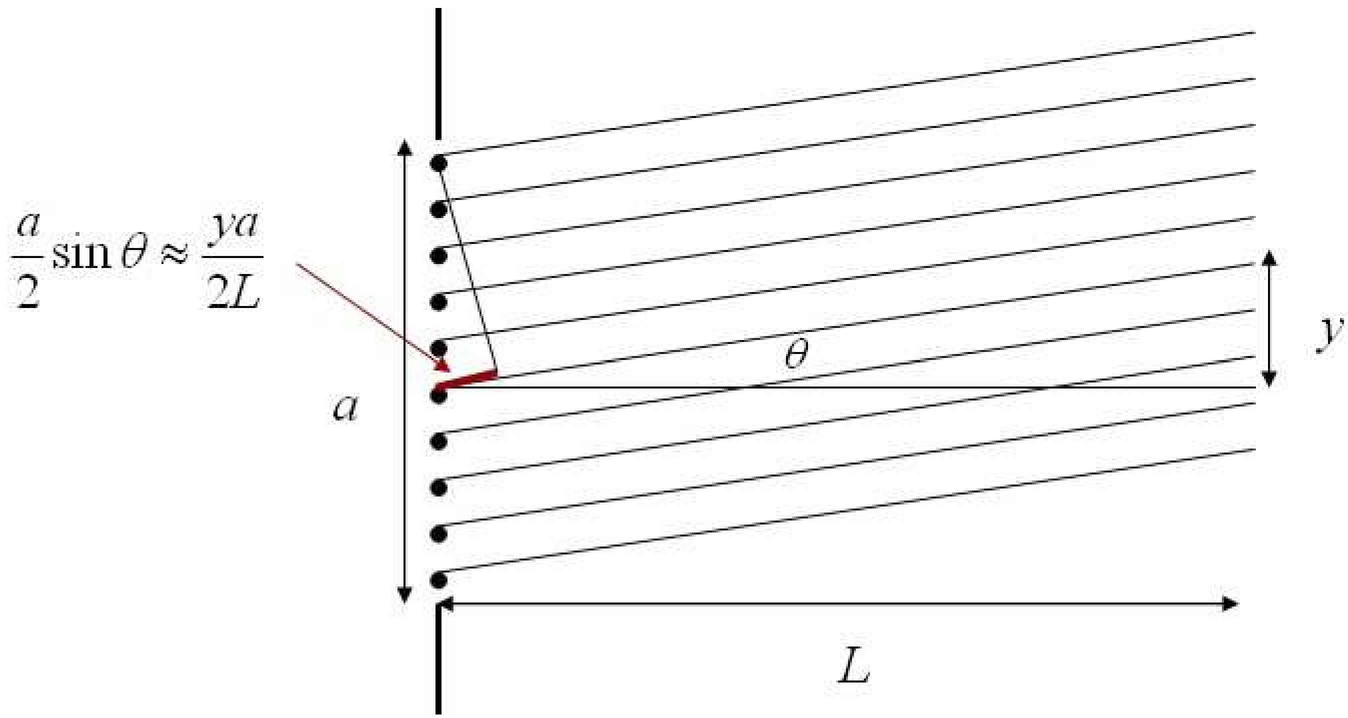
$$\gamma = \frac{2\pi}{\lambda} \sin\theta \approx \frac{2\pi}{\lambda} \frac{y}{L}$$

Beugung

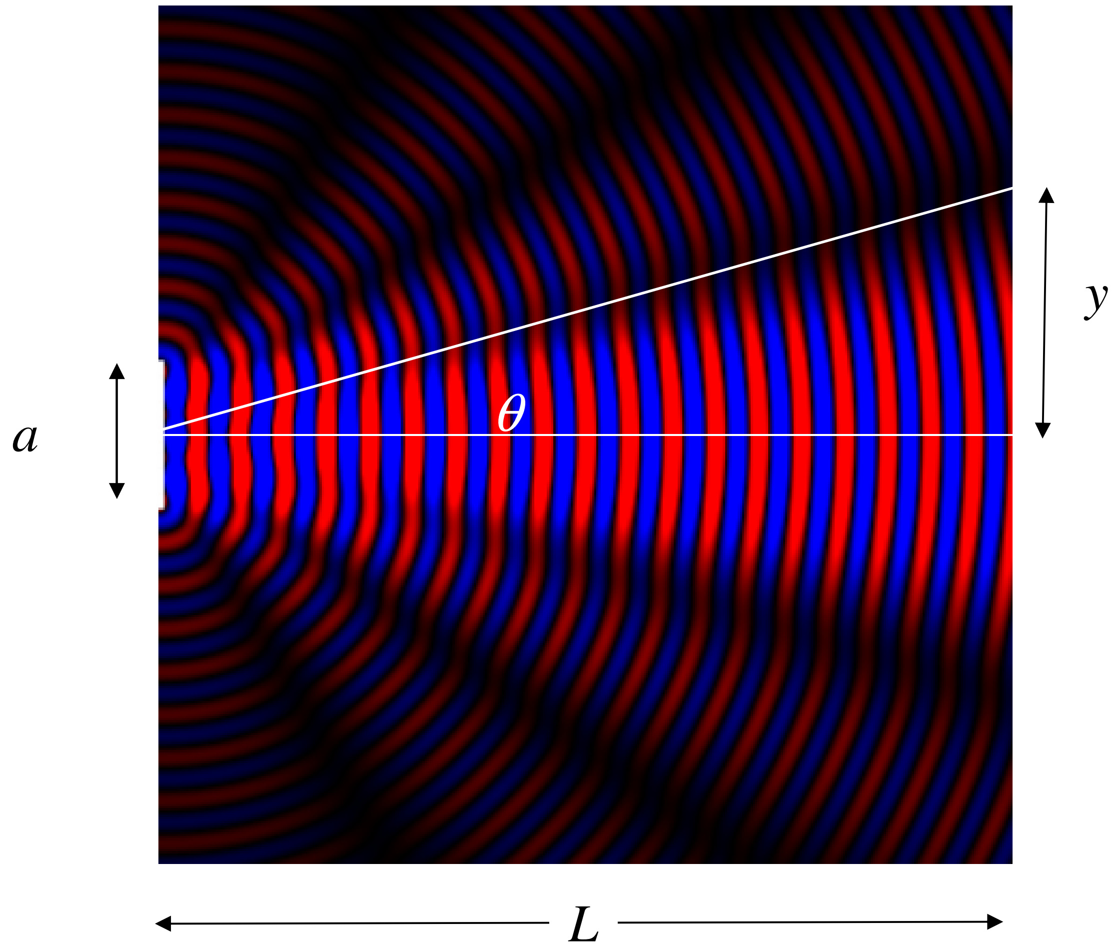


Spaltbreite gegenüber Wellenlänge ?

Einfachspalt



Einfachspalt Beugung

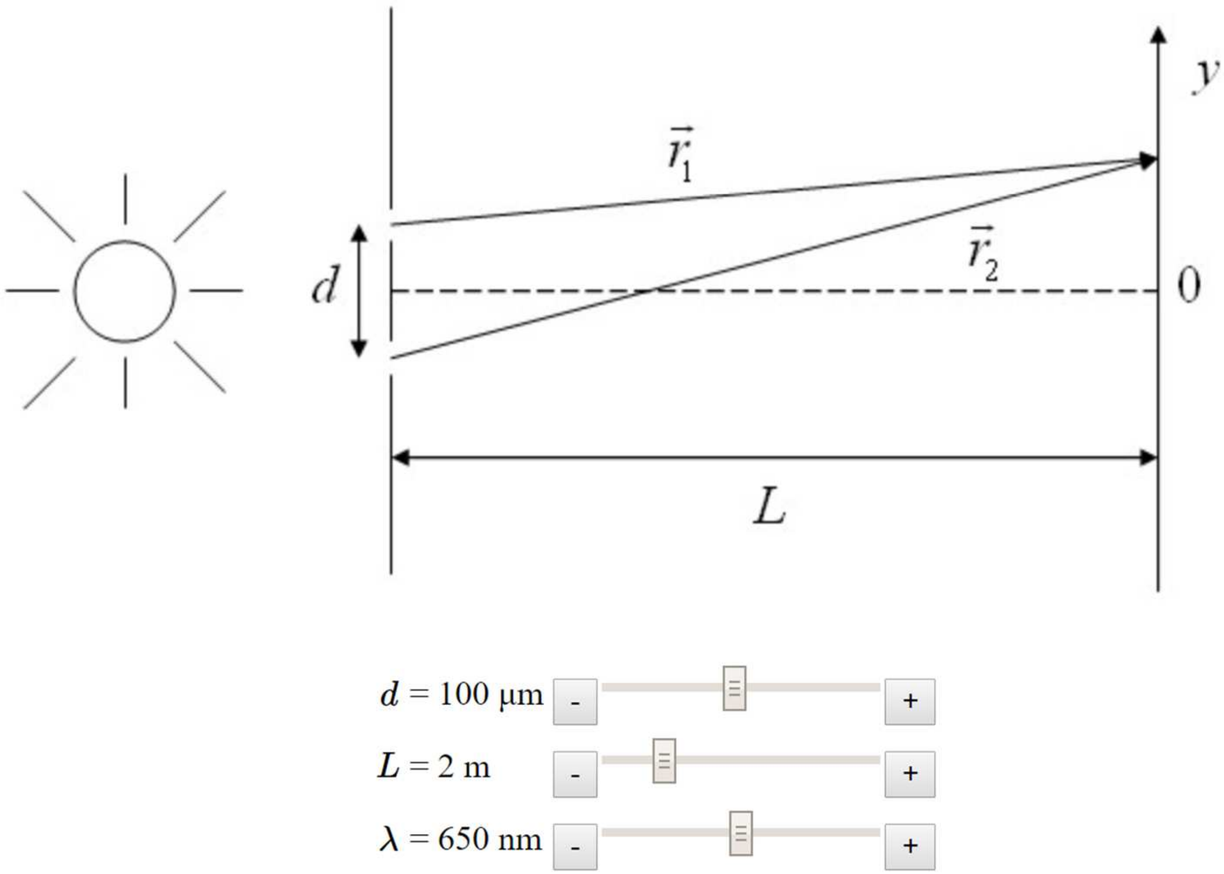


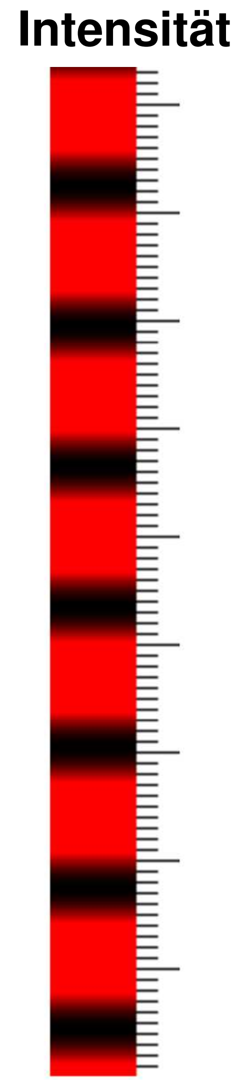
$$\theta \approx \frac{\lambda}{a}$$

Destruktive Interferenz: $|\vec{r}_2| - |\vec{r}_1| = \frac{\lambda}{2} \approx \frac{a}{2}\sin\theta \approx \frac{ya}{2L}$

(Fernfeld)

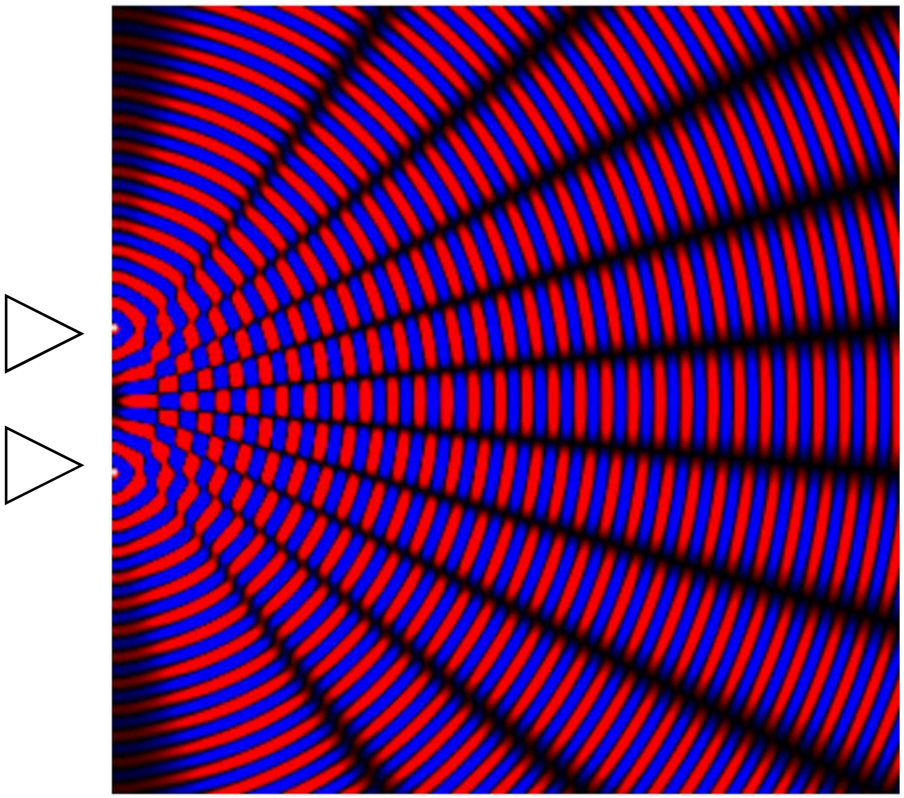
Interferenz am Doppelspalt



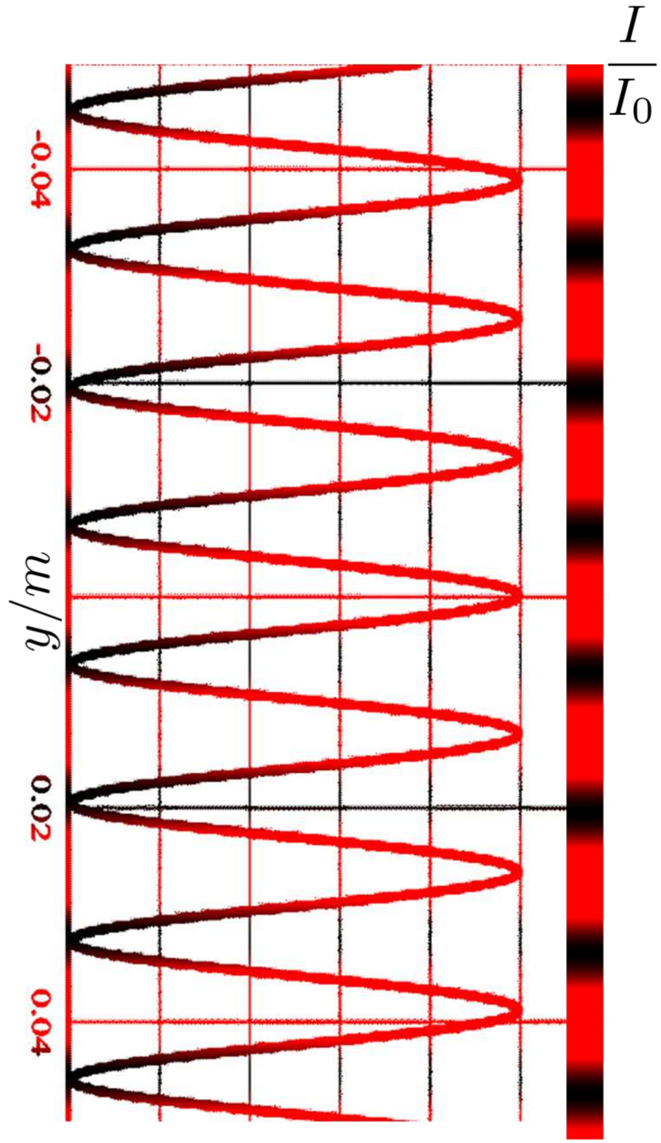


Interferenz am Doppelspalt

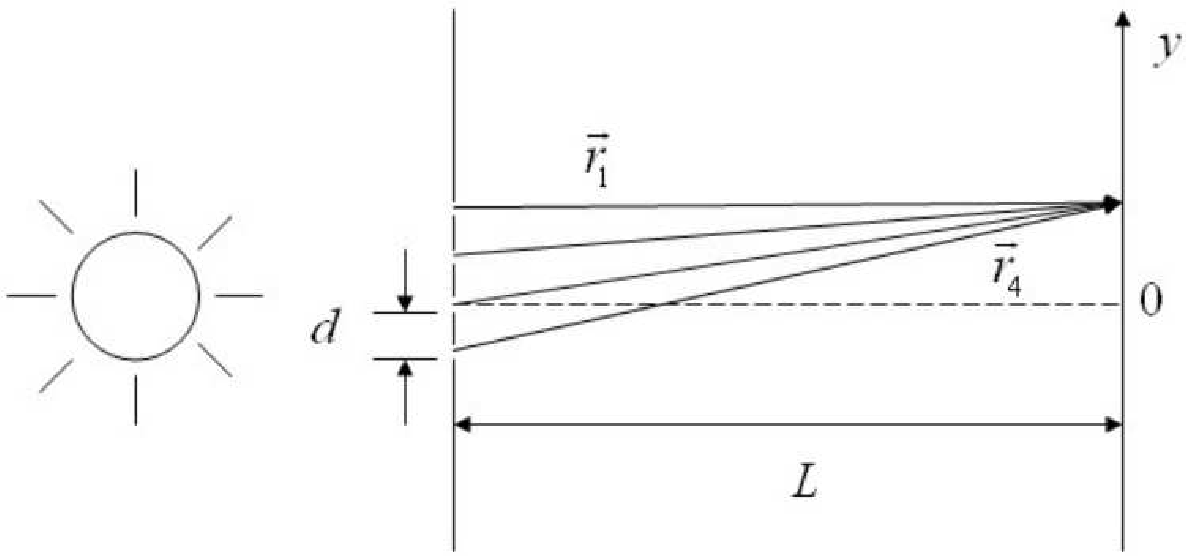
Amplitude

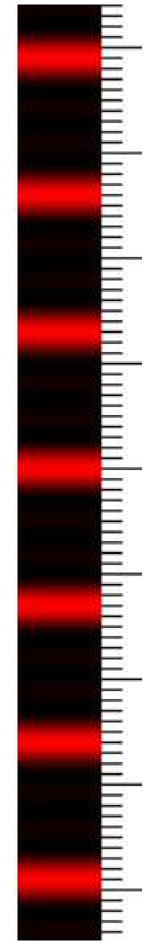


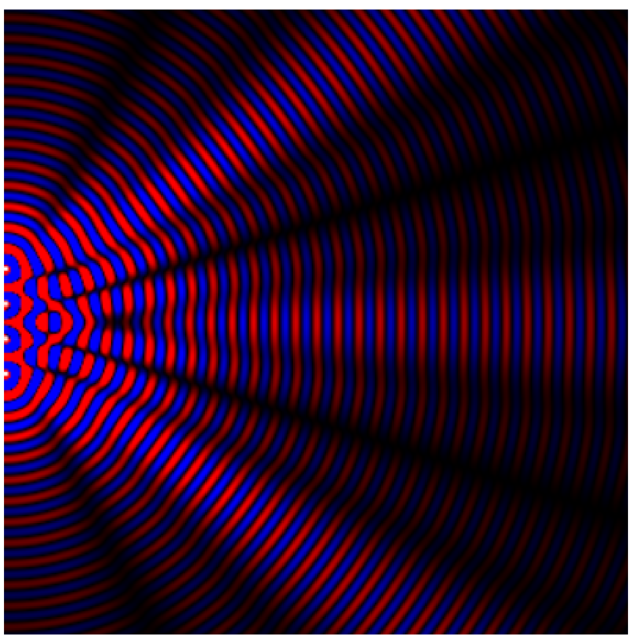
Intensität

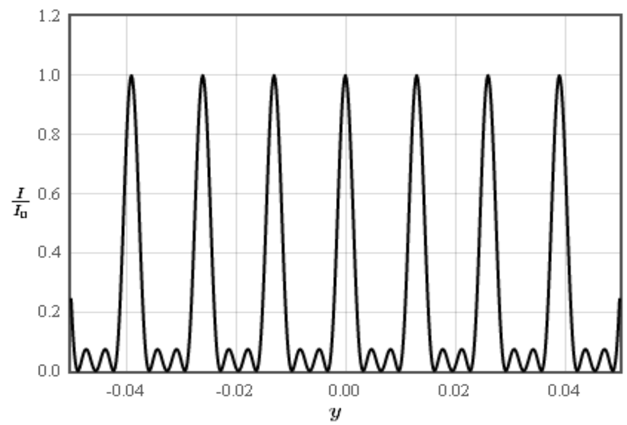


Interferenz am *N*-fach Spalt



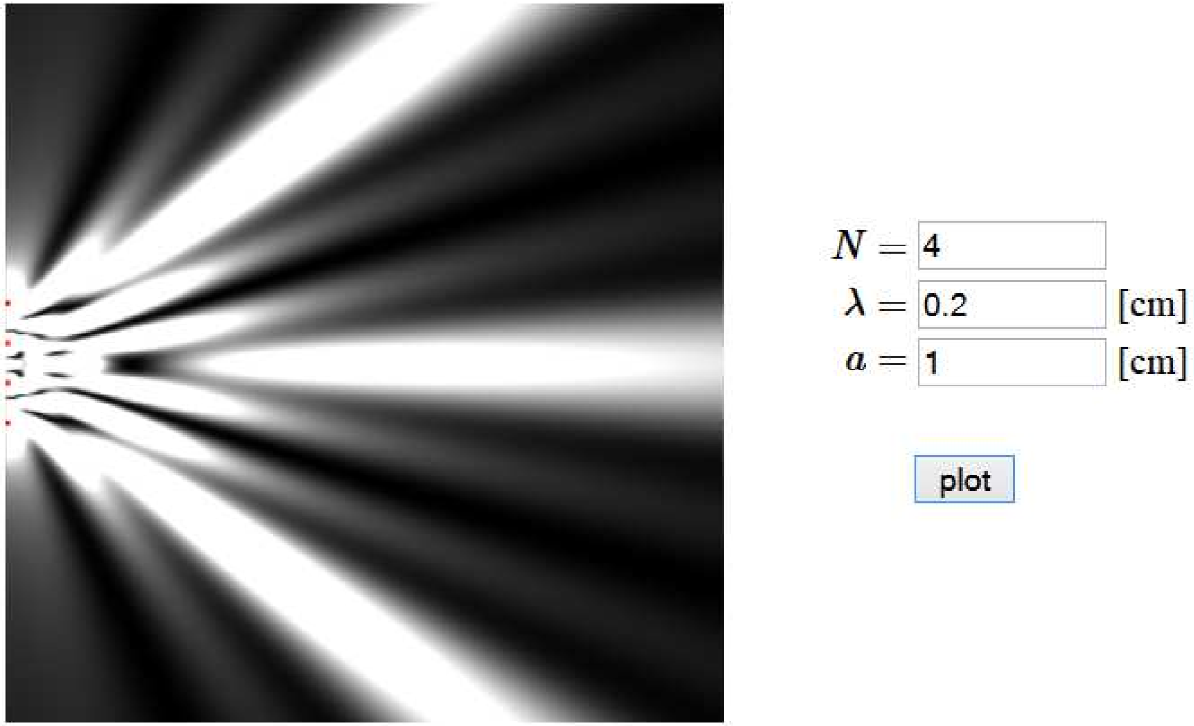






Konstruktive Interferenz: $\left|\vec{r}_2\right| - \left|\vec{r}_1\right| = n\lambda \approx d \sin\theta \approx \frac{yd}{L}$
(Fernfeld)

Intensität vieler interferierender Punktquellen



$$\sum_{j=1}^{N} \frac{A_j}{\sqrt{|\vec{r}-\vec{r}_j|}} e^{i(k|\vec{r}-\vec{r}_j|-\omega t+\phi_j)} = \left(\sum_{j=1}^{N} \frac{A_j}{\sqrt{|\vec{r}-\vec{r}_j|}} e^{i(k|\vec{r}-\vec{r}_j|+\phi_j)} \right) e^{-i\omega t}.$$

http://lamp.tu-graz.ac.at/~hadley/physikm/apps/huygensintensity.de.php

Wellenamplitude

Zwei Punktquellen emittieren Oberflächenwellen, die miteinander interferieren. Die Amplituden der von den Quellen emittierten Wellen sind gegeben durch:

$$z_1 = A_1 \cos(k|\vec{r} - \vec{r}_1| - \omega t)/\sqrt{|\vec{r} - \vec{r}_1|} \text{ cm}$$

$$z_2 = A_2 \cos(k|\vec{r} - \vec{r}_2| - \omega t)/\sqrt{|\vec{r} - \vec{r}_2|} \text{ cm}$$

Hier ist $|\vec{r} - \vec{r}_1|$ der Abstand von Quelle 1 und $|\vec{r} - \vec{r}_2|$ der Abstand von Quelle 2. Die Wellenzahl k hängt mit der Wellenlänge λ zusammen, $k = \frac{2\pi}{\lambda}$ und die Kreisfrequenz ω mit der Periodendauer T, $\omega = \frac{2\pi}{T}$.

Punktquelle 1 emittiert Wellen von der Position $\vec{r}_1 = 2\hat{x} + 9\hat{y}$ (/cm) mit der Amplitude $A_1 = 0.5$ cm$^{3/2}$.

Punktquelle 2 emittiert Wellen von der Position $\vec{r}_2 = 7\hat{x} + 2\hat{y}$ (/cm) mit der Amplitude $A_2 = 0.6$ cm$^{3/2}$.

Die Ausbreitungsgeschwindigkeit der Wellen ist $c = 49$ cm/s und die Kreisfrequenz der Schwingungen ist $\omega = 9$ rad/s.

Das Interferenzmuster dieser Wellen ruft vertikale einfach harmonische Schwingungen an jedem Punkt hervor. Wie groß ist die Amplitude dieser Oszillationen an der Position $\vec{r} = -7\hat{x} + 7\hat{y}$ cm?

$z_{\text{amplitude}} =$ ______ cm Lösung

Die Wellenzahl ist $k = \frac{\omega}{c}$ =0.129 cm^{-1}.

Die durch Punktquelle 1 verursachten Schwingungen können durch einen in der komplexen Ebene rotierenden Phasor beschrieben werden:

$$\frac{A_1}{\sqrt{|\vec{r}-\vec{r}_1|}}\, e^{i(k|\vec{r}-\vec{r}_1|-\omega t)} = 0.0697(\cos(1.30 - 8t) + i\sin(1.30 - 8t)).$$

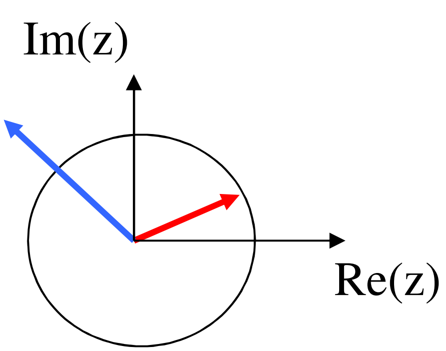
Die durch Punktquelle 2 verursachten Schwingungen können ebenso durch einen Phasor in der komplexen Ebene beschrieben werden:

$$\frac{A_2}{\sqrt{|\vec{r}-\vec{r}_2|}}\, e^{i(k|\vec{r}-\vec{r}_2|-\omega t)} = 0.0248(\cos(2.08 - 8t) + i\sin(2.08 - 8t)).$$

Die Summation dieser beiden Phasoren ergibt einen einzelnen Phasor, der in der komplexen Ebene mit der Kreisfrequenz 8 rad/s rotiert. Die Länge dieses Phasors entspricht der Amplitude der Schwingungen am Punkt $\vec{r}$, welche sich aufgrund der Interferenz der beiden Einzelwellen einstellt.

$$z_{\text{amplitude}} = 0.129 \text{ cm}.$$

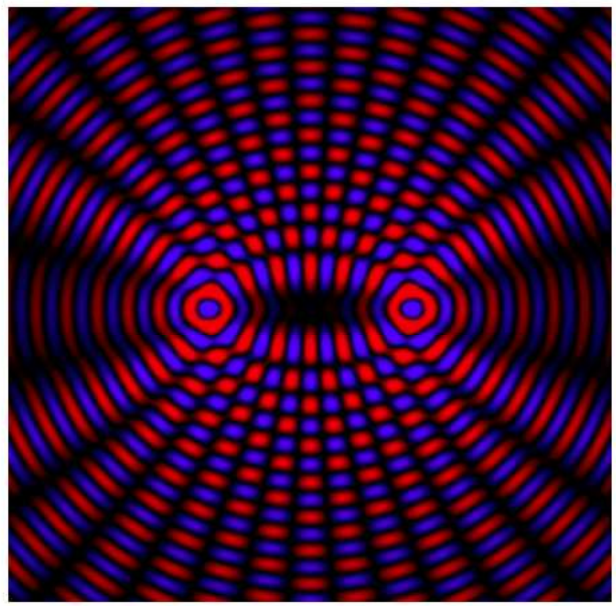
ie auch die APP: Interferenz zweier Oberflächenwellen



Physik M
07.12.2017

Problem 5

Zweidimensionale Wellen werden von zwei punktförmigen Quellen, die sich an $\vec{r}_1 = -2\hat{x}$ [m] und $\vec{r}_2 = 2\hat{x}$ [m] befinden ausgesendet. Die Amplitude der Wellen ist $1/\sqrt{|\vec{r} - \vec{r}_j|}$, wobei $|\vec{r} - \vec{r}_j|$ die Entfernung von der Quelle ist. Die Wellenlänge der Wellen ist $\lambda = 9$ m. Die Winkelfrequenz der Wellen ist $\omega = 8$ rad/s.



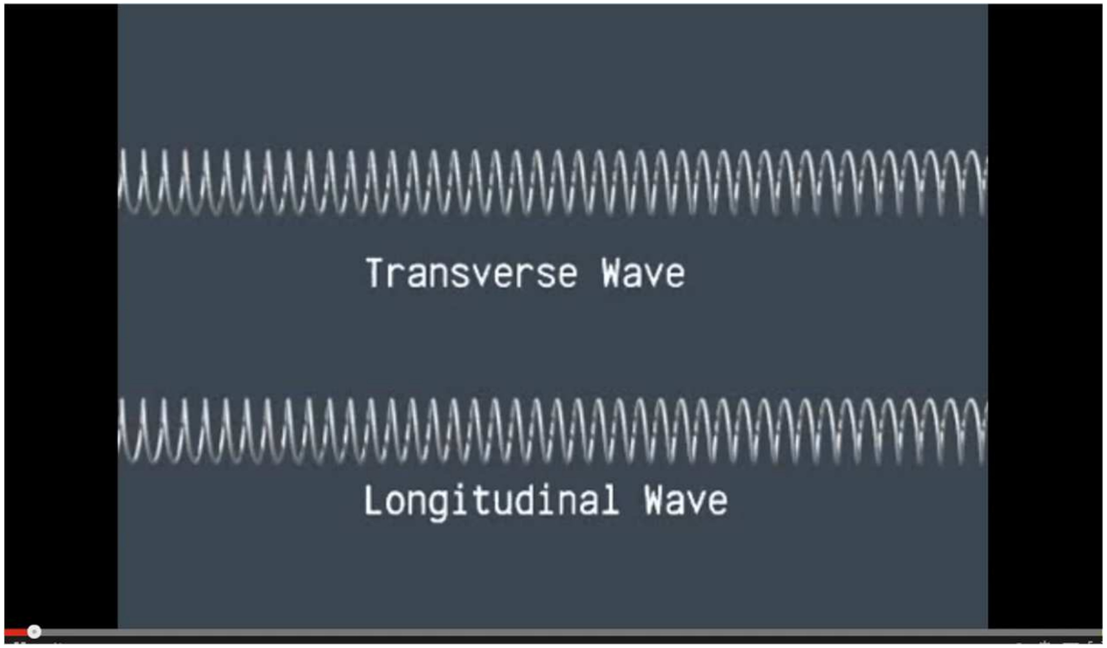
Wie groß ist die Wellengeschwindigkeit?

$v =$ ______ [m/s]

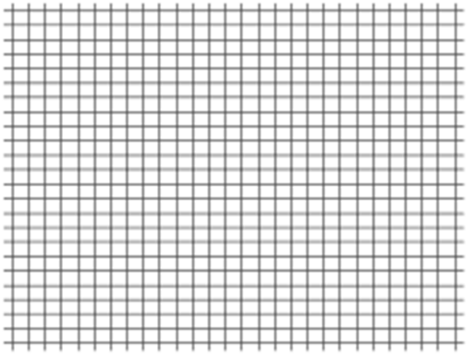
Eine periodische Schwingung wird auf der Stelle $\vec{r} = 3\hat{x} + 3\hat{y}$ [m] beobachtet. Wie groß ist die Amplitude dieser Schwingungen?

$A =$ ______ [m]

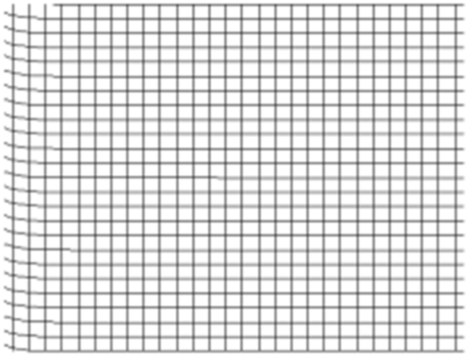
Longitudinalwellen und Transversalwellen



Longitudinalwellen: Schallwellen

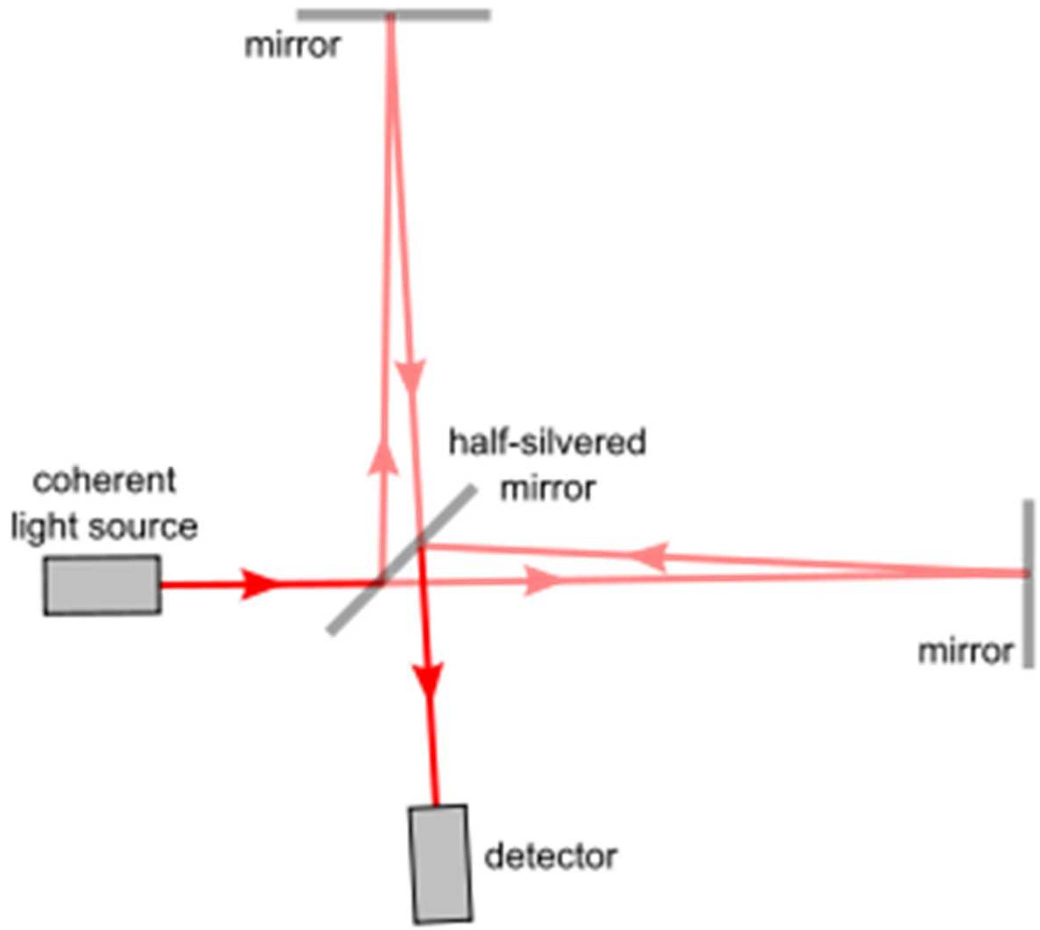


Transversalwellen: Seilwelle, Licht



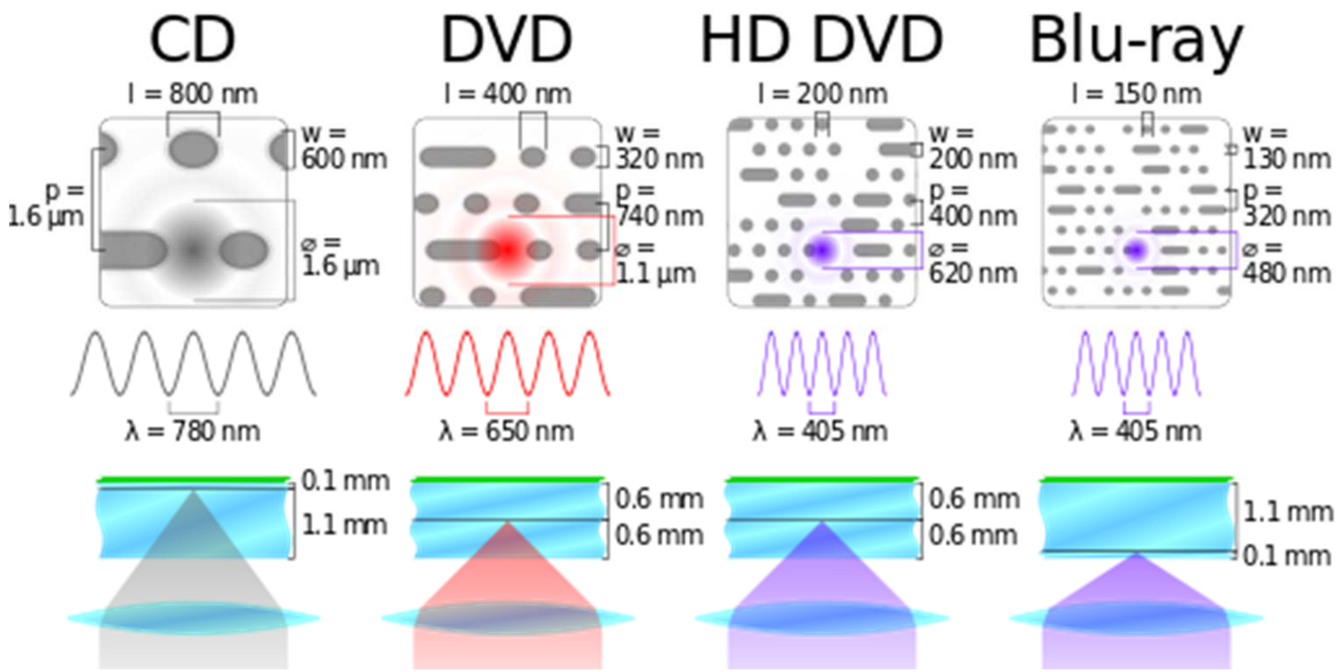
https://www.youtube.com/watch?v=Rbuhdo0AZDU

Interferometrie



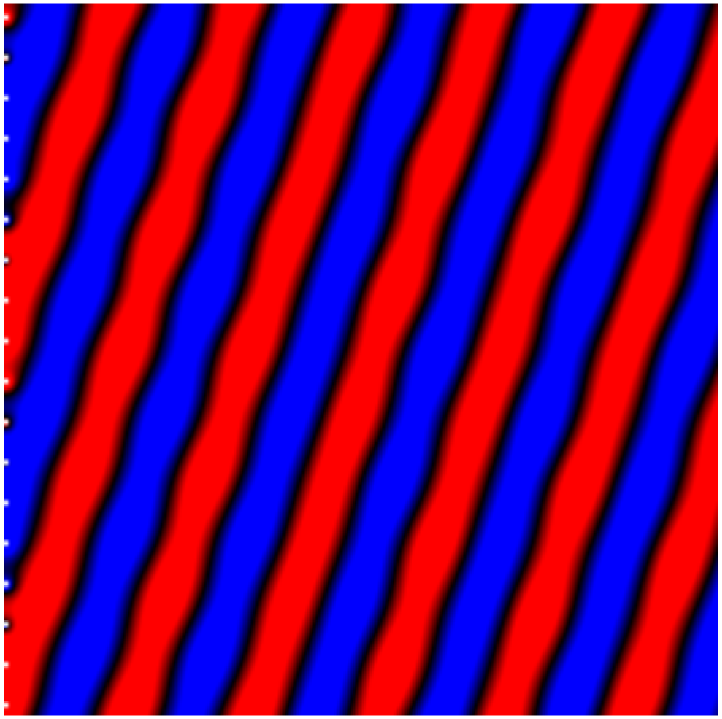
http://en.wikipedia.org/wiki/Interferometry

Interferenz



http://en.wikipedia.org/wiki/Compact_disc#mediaviewer/File:Comparison_CD_DVD_HDDVD_BD.svg

ebene Wellen



$n =$ 3 [cm^{-1}]

$\lambda =$ 1 [cm]

$T =$ 0.5 [s]

$\Delta\phi =$ 2 [$\mathrm{rad/cm}^{-1}$]

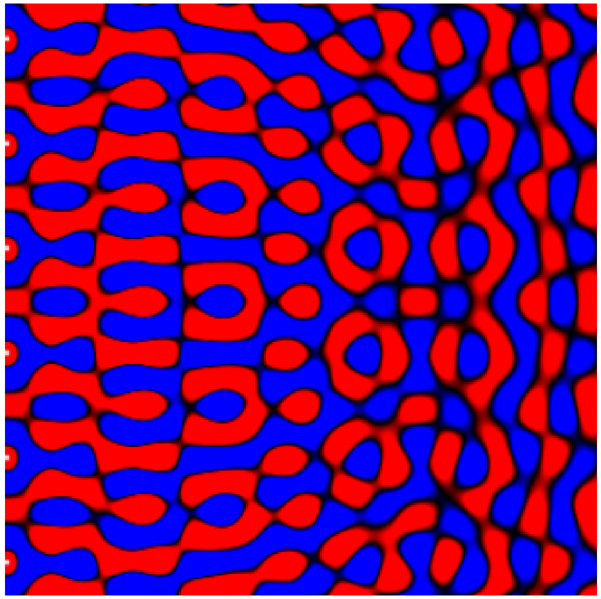
plot at $t =$ 0 [s].

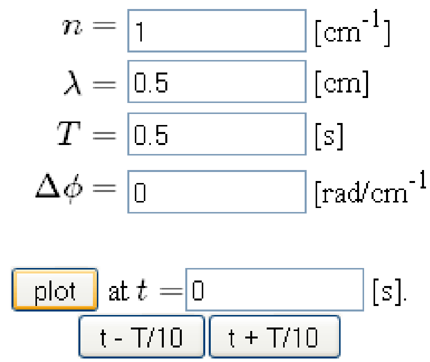
t - T/10 t + T/10

APP “Huygenssches Prinzip: Ebene Wellen”

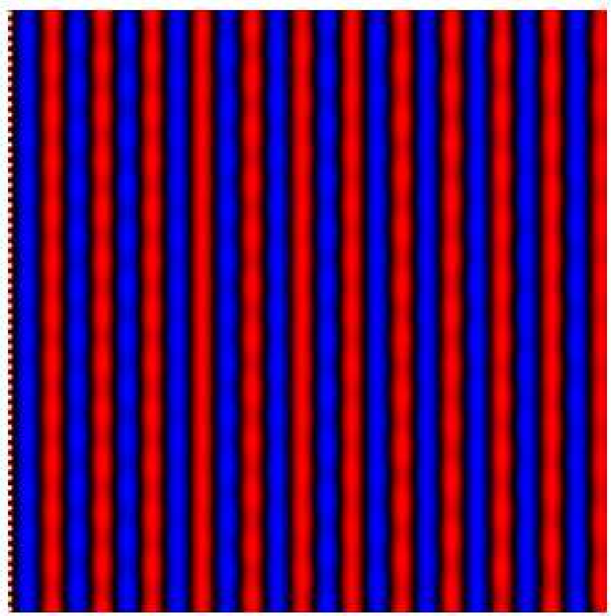
ebene Wellen

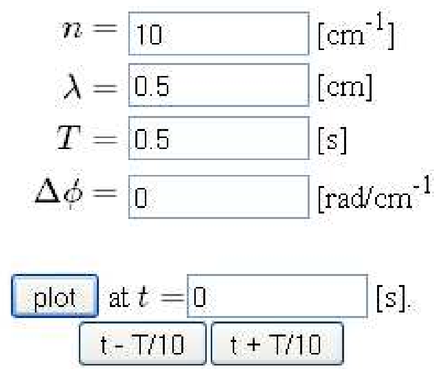
1 Quelle/cm



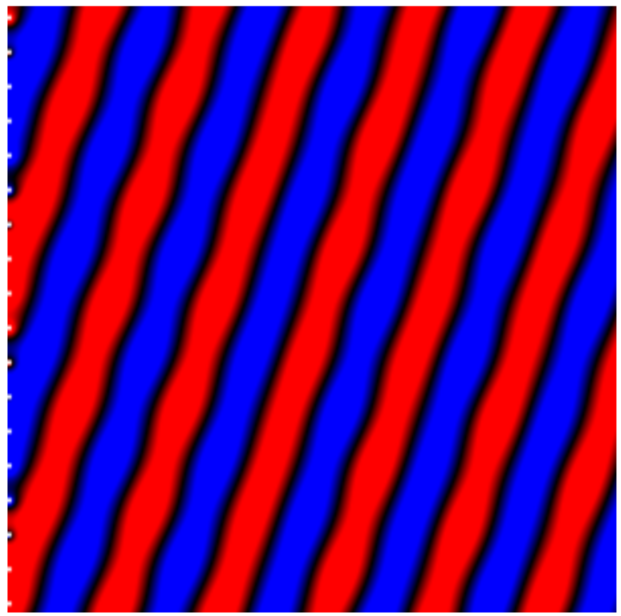


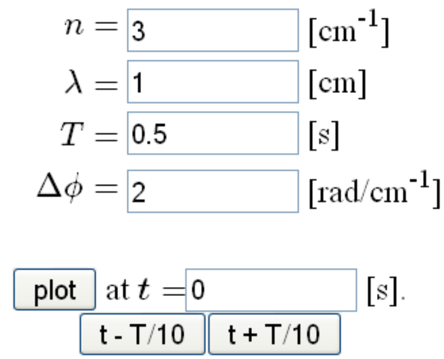
10 Quellen/cm

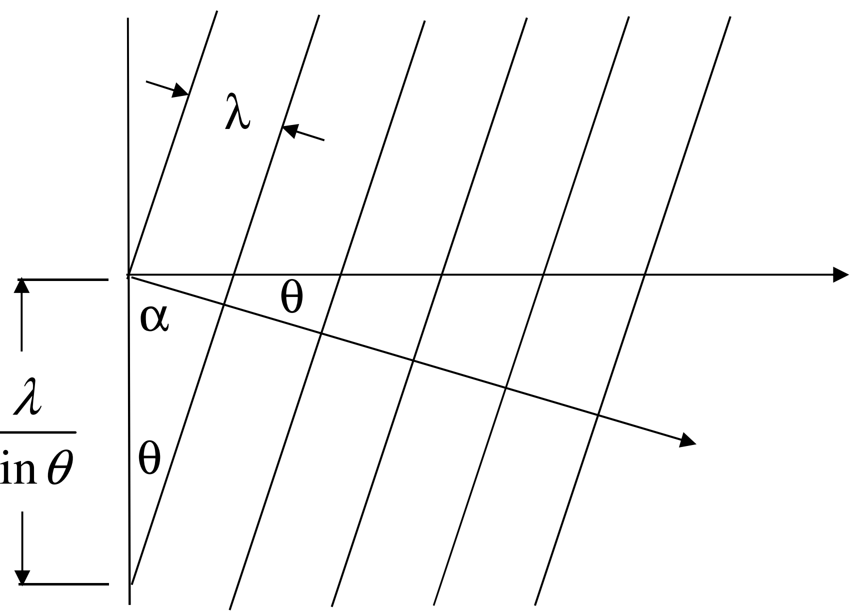




ebene Wellen



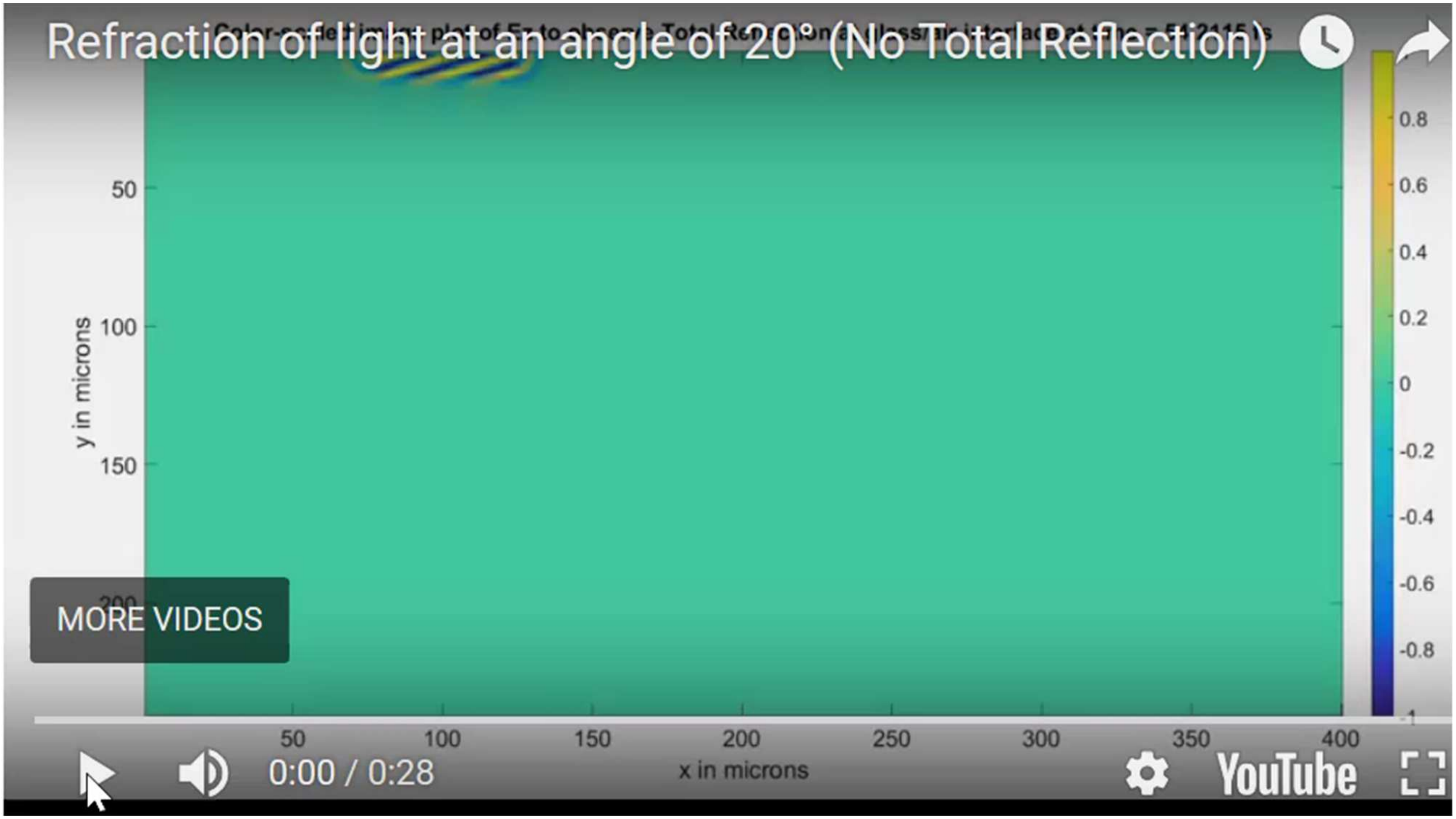




$$d = \frac{\lambda}{\sin\theta}$$

$$\alpha = \pi/2 - \theta$$

Snelliussches Brechungsgesetz

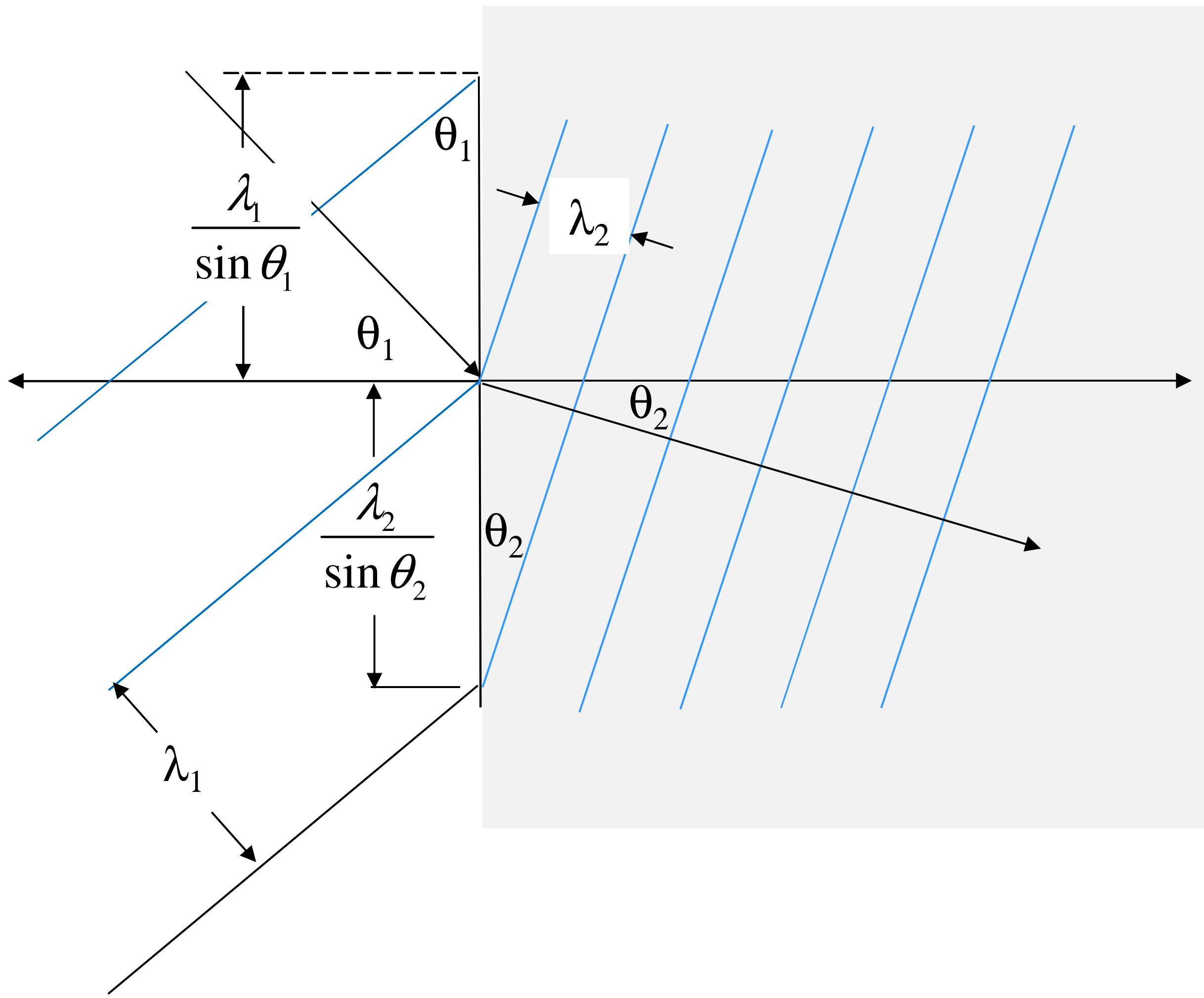


Snelliussches Brechungsgesetz

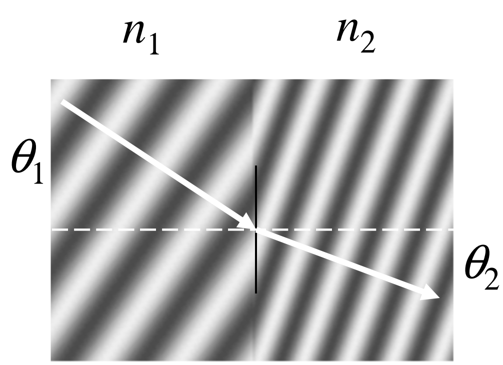
$$\frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$

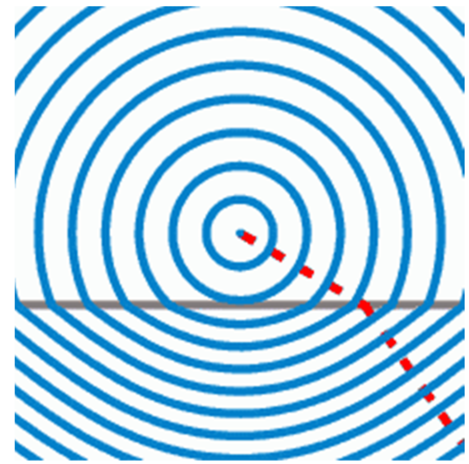
$$\lambda_1 = \frac{c_1}{f} \qquad \lambda_2 = \frac{c_2}{f}$$

$$\boxed{\frac{c_1}{\sin\theta_1} = \frac{c_2}{\sin\theta_2}}$$



Snelliussches Brechungsgesetz





$$\frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$

$$\lambda_1 = \frac{c}{n_1 f} \qquad \lambda_2 = \frac{c}{n_2 f}$$

Brechungsindex

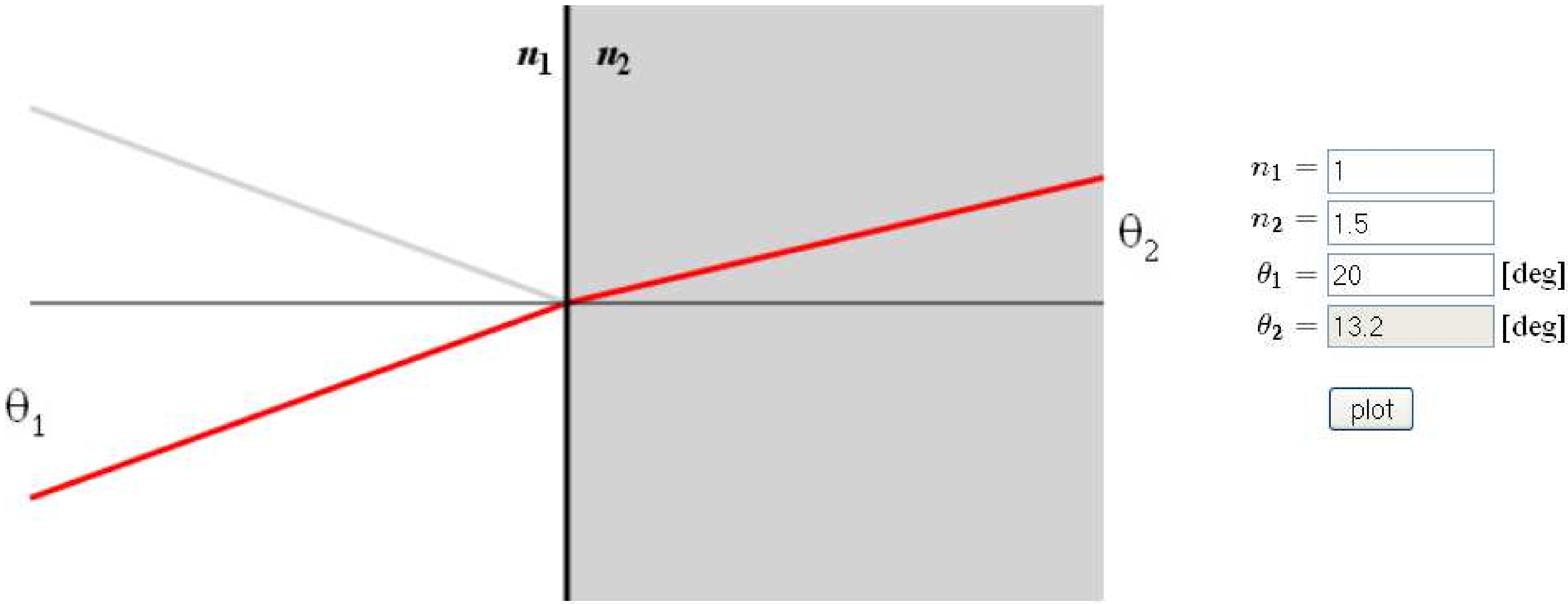
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

http://commons.wikimedia.org/wiki/File:Wave_refraction.gif

http://commons.wikimedia.org/wiki/File:Snells_law_wavefronts.gif

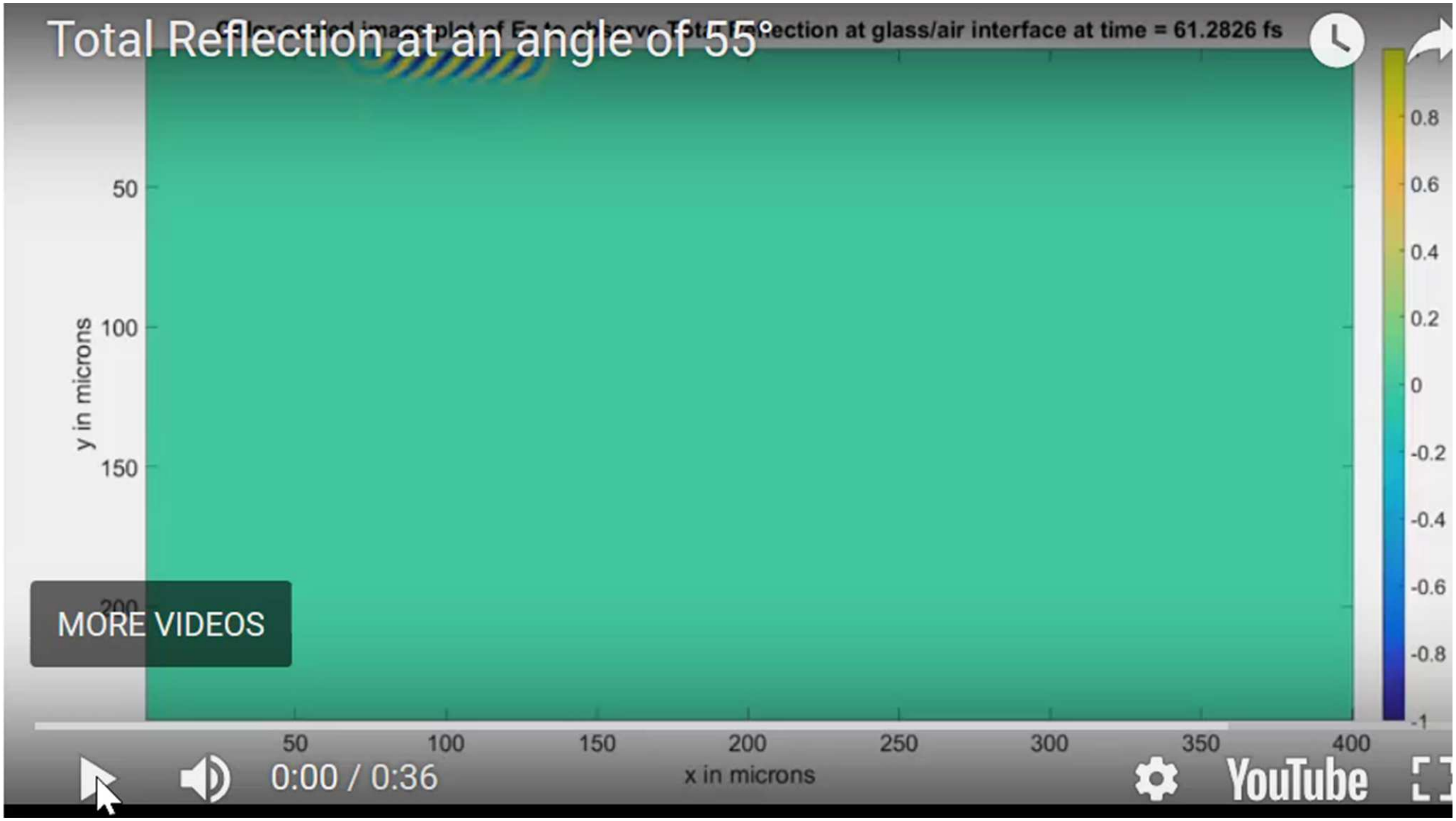
Brechung

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

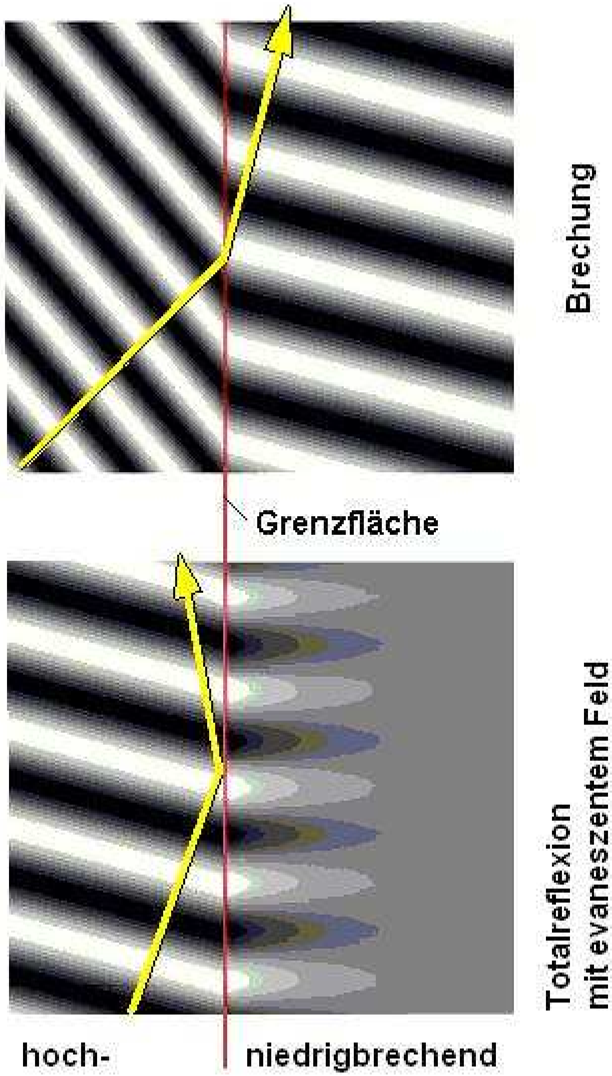


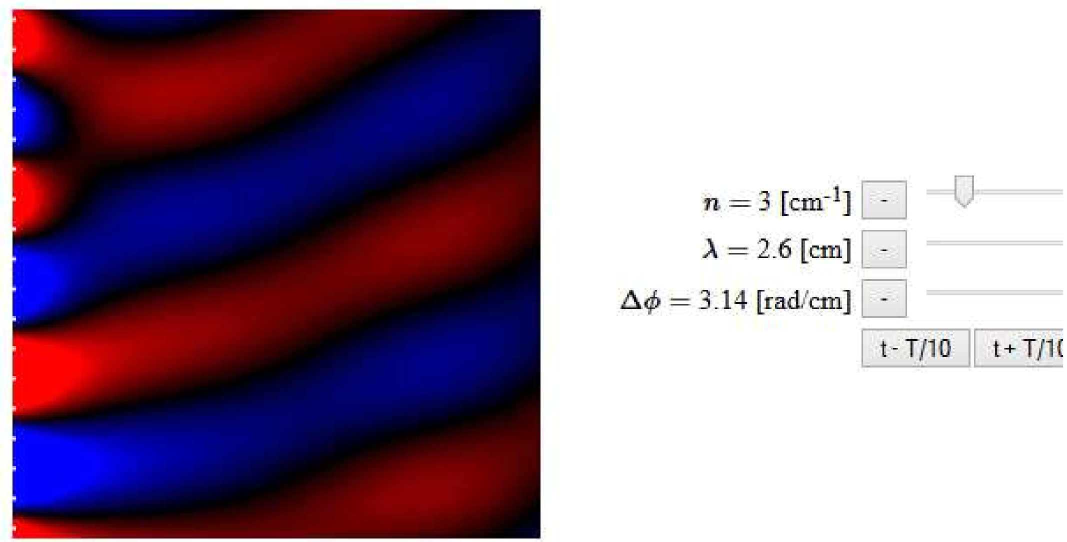
APP "optische Brechung"

Totalreflexion



Evaneszenz

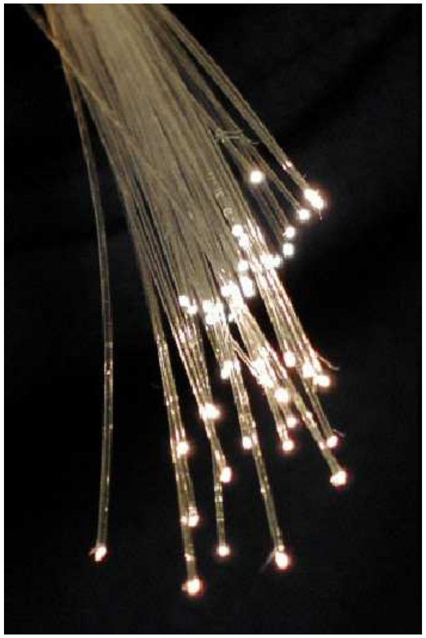




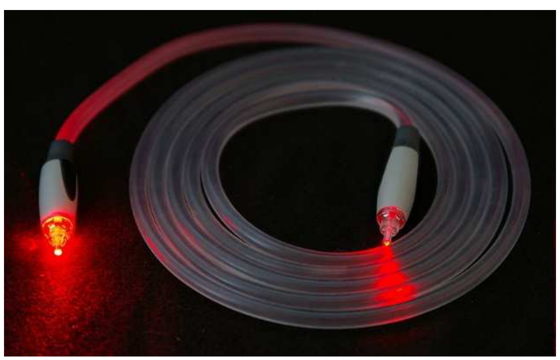
http://de.wikipedia.org/wiki/Evaneszenz#mediaviewer/File:Evaneszenzfeld.jpg

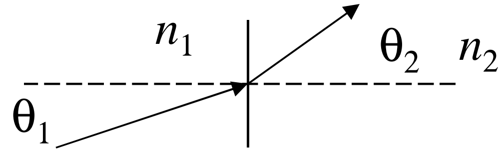
Totalreflexion

http://commons.wikimedia.org/wiki/File:Fibreoptic.jpg



Lichtwellenleiter





$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$