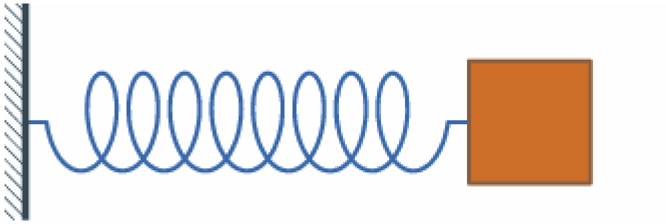
16. Schwingungen

30 Nov. 2018

Freie Schwingung

$$m\frac{d^2x}{dt^2} = -kx$$



Freie Schwingung

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-x}$$

Anfangsbedingungen:

$x(t_0) = 0$

$v_x(t_0) = 1$

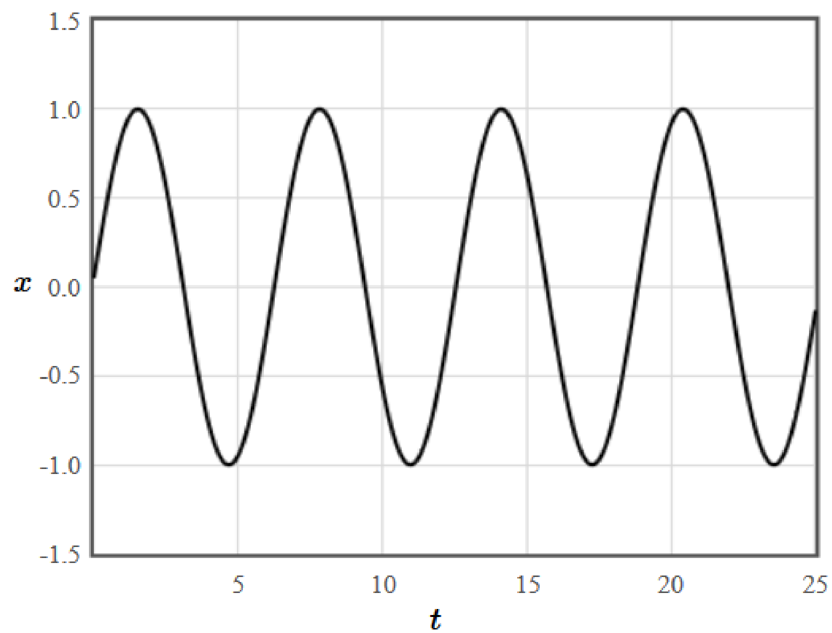
$t_0 = 0$

$\Delta t = 0.05$

N_{steps} 500

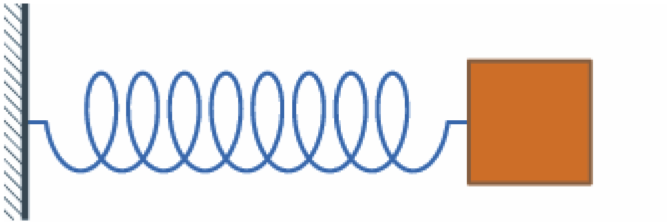
Graphische Darstellung: x vs. t

Absenden

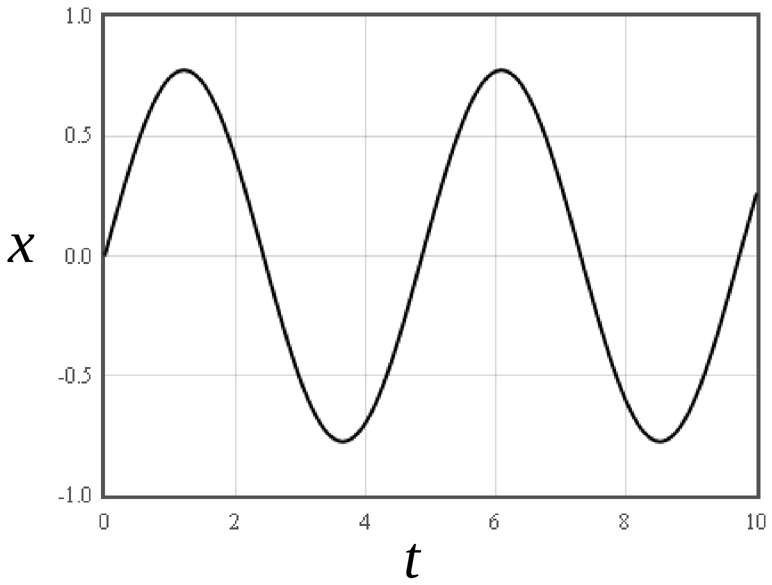


Freie Schwingung

$$m\frac{d^2x}{dt^2} = -kx$$



Lösung: $x(t) = A\cos(\omega t + \theta)$

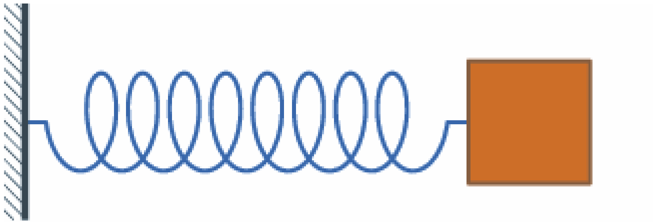


$$\omega = \sqrt{\frac{k}{m}}$$

Freie Schwingung

Lösung: $x(t) = A\cos\left(\omega t + \theta\right)$

Anfangsbedingungen: x_0, v_{xo} bei $t = 0$

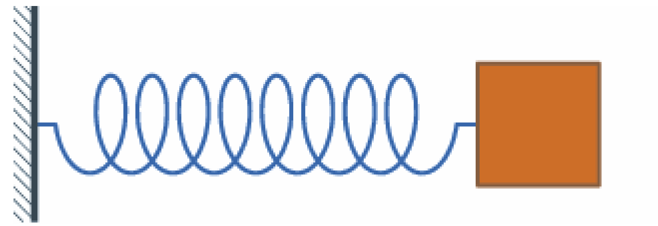


$$\theta = \tan^{-1}\left(\frac{-v_{x0}}{\omega x_0}\right)$$

$$A = \sqrt{x_0^2 + v_{x0}^2 / \omega^2}$$

$b^2 < 4km$ Schwingfall

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$



Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} =$$ -x-0.1*vx

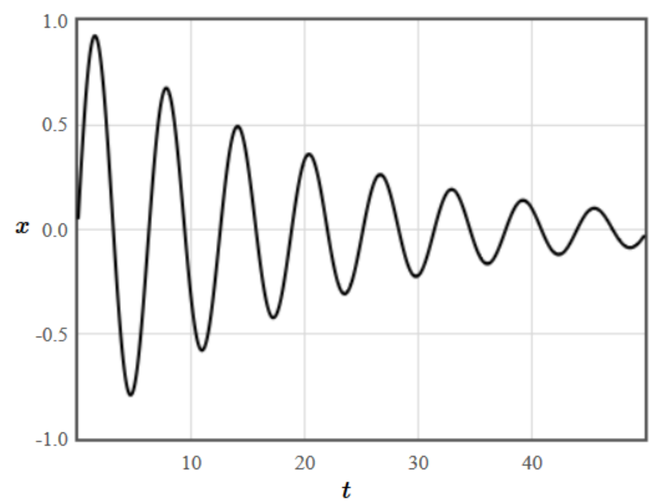
Anfangsbedingungen:

$x(t_0) =$ 0 $\Delta t =$ 0.05

$v_x(t_0) =$ 1 N_{steps} 1000

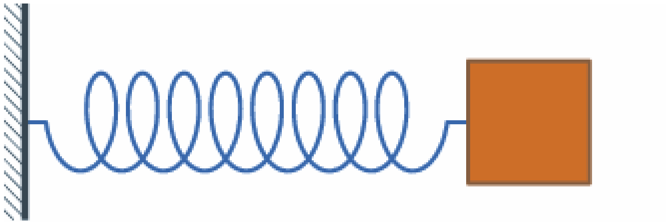
$t_0 =$ 0 Graphische Darstellung: x vs. t

Absenden



Schwingfall $b^2 < 4km$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = -mg$$



Lösung: $$x(t) = Ae^{-t/\tau}\cos(\omega t + \theta)$$

$$\tau = \frac{2m}{b}$$

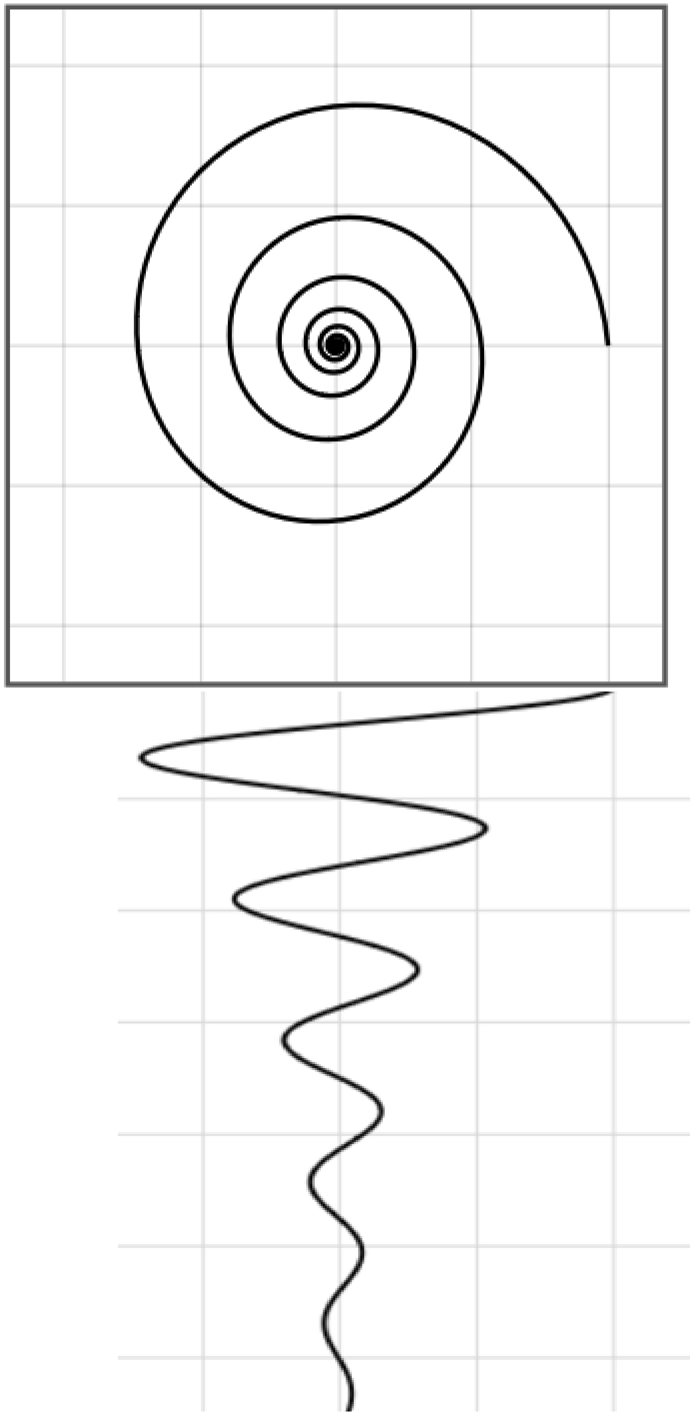
$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

komplexe Lösung

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=0$$

$$z=x+iy$$

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$



komplexe Lösung

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=0$$

Ansatz: $$z=e^{\lambda t}$$

$$\lambda_\pm=\frac{-b\pm\sqrt{b^2-4km}}{2m}$$

Schwingfall $b^2 < 4km$

$$z_\pm = \exp\left(\frac{-b \pm \sqrt{b^2 - 4km}}{2m} t\right) = \exp\left(\frac{-b}{2m} t\right) \exp\left(\frac{\pm i \sqrt{4km - b^2}}{2m} t\right)$$

$$= \exp\left(\frac{-b}{2m} t\right)\left(\cos\left(\frac{\sqrt{4km - b^2}}{2m} t\right) + i \sin\left(\frac{\sqrt{4km - b^2}}{2m} t\right)\right)$$

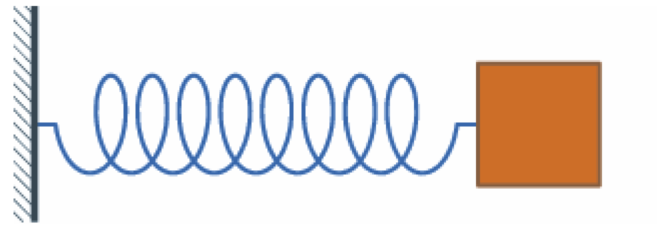
Lösung: $$x(t) = Ae^{-t/\tau} \cos\left(\omega t + \theta\right)$$

$$\tau = \frac{2m}{b}$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$b^2 > 4km$ Kriechfall

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$



Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-x-3*vx}$$

Anfangsbedingungen:

$x(t_0) =$ 0

$v_x(t_0) =$ 1

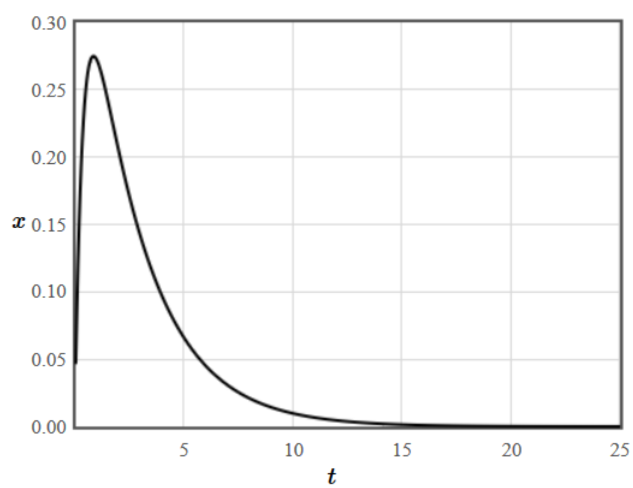
$t_0 =$ 0

$\Delta t =$ 0.05

N_{steps} 500

Graphische Darstellung: x vs. t

Absenden



$b^2 > 4km$ Kriechfall

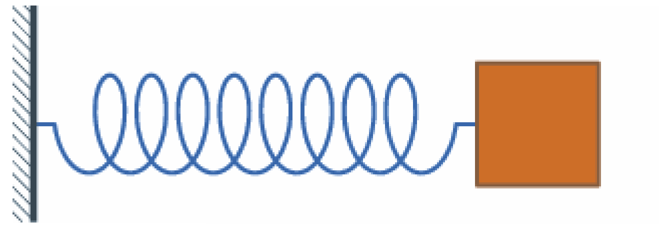
$$z_\pm = \exp\left(\frac{-b \pm \sqrt{b^2 - 4km}}{2m} t\right)$$

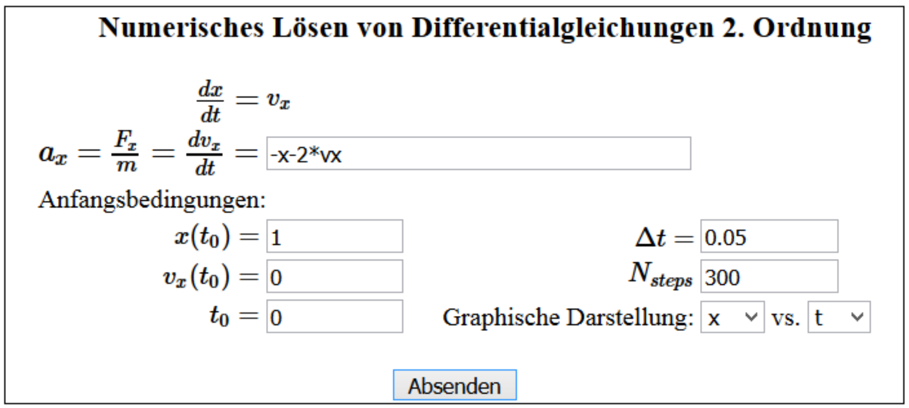
Lösung: $$x(t) = C_+ \exp\left(\frac{-t}{\tau_+}\right) + C_- \exp\left(\frac{-t}{\tau_-}\right)$$

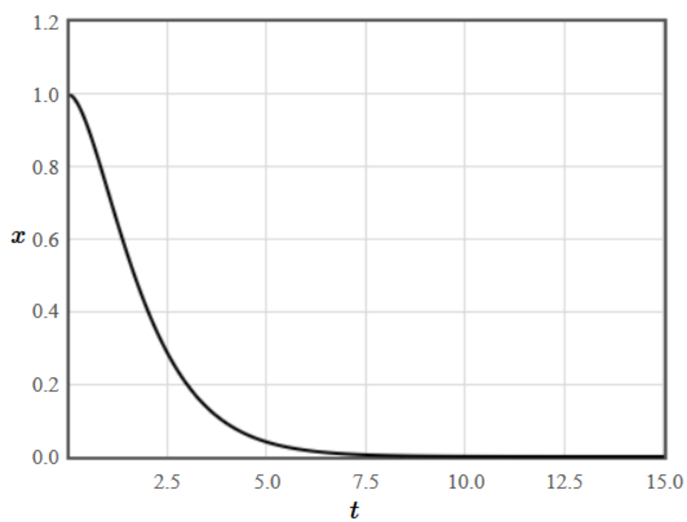
$$\tau_+ = \frac{-1}{\lambda_+} = \frac{2m}{b + \sqrt{b^2 - 4km}} \qquad \tau_- = \frac{-1}{\lambda_-} = \frac{2m}{b - \sqrt{b^2 - 4km}}$$

$b^2 = 4km$ aperiodischer Grenzfall

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

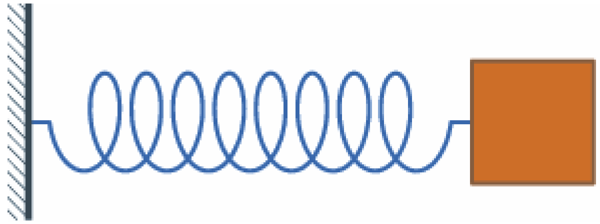






$b^2 = 4km$ aperiodischer Grenzfall

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

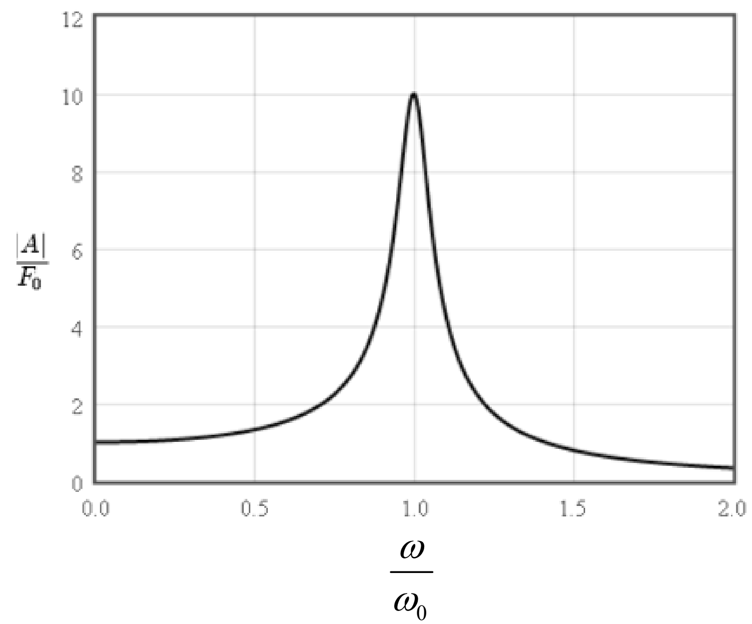


Lösung: $$x(t) = C_1 \exp\left(\frac{-t}{\tau}\right) + C_2 t \exp\left(\frac{-t}{\tau}\right)$$

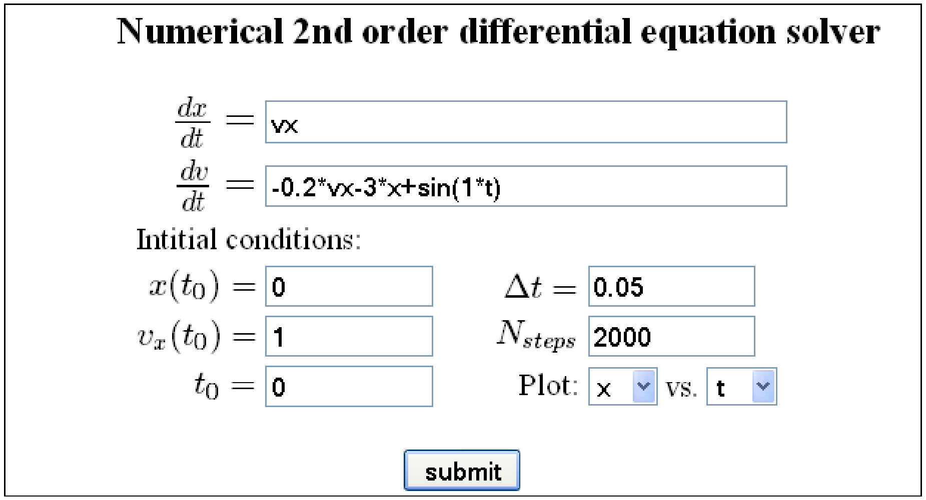
$$\tau = \frac{2m}{b}$$

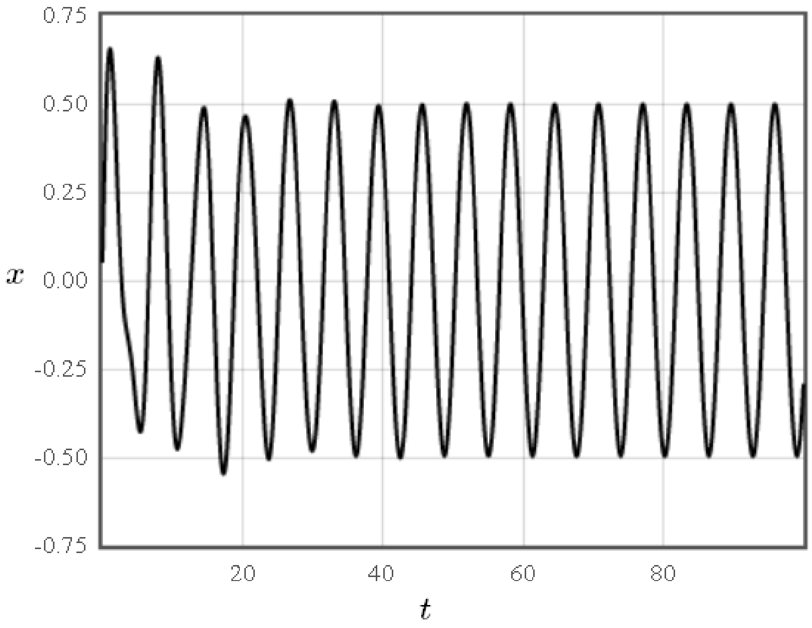
Resonanz

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx + F_0\cos(\omega t)$$



Resonanz


Numerical 2nd order differential equation solver
dx/dt =
vx
dv/dt =
-0.2*vx-3*x+sin(1*t)
Intitial conditions:
x(t0) =
0
vx(t0) =
1
t0 =
0
Δt =
0.05
Nsteps
2000
Plot:
x
vs.
t
submit


0.75
0.50
0.25
x
0.00
-0.25
-0.50
-0.75
20
40
60
80
t