

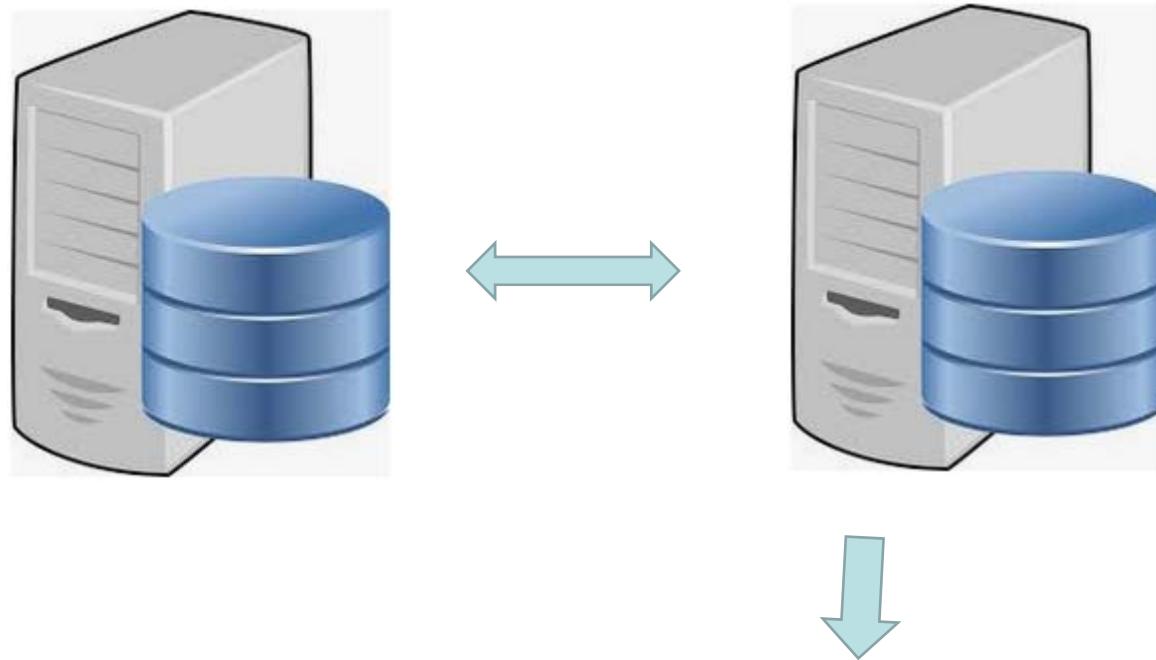
# 5. Punktmechanik

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18 Okt. 2019

# Teach Center

# Physik



exam_id	number	user_id	params
ball	ball	73957	{"name": "ball", "v": 19, "m": 205, "theta": 43, "vx": "13....
G	ball	76103	{"name": "ball", "v": 12, "m": 231, "theta": 45, "vx": "8.4....
v2f	ball	76103	{"name": "ball", "v": 12, "m": 231, "theta": 45, "vx": "8.4....
G	G	74962	{"name": "G", "x1": 2, "y1": -3, "z1": -4, "x2": -5, "y2": 3, ...}

# Python

```
import math;

# define some constants
e = -1.6022E-19; # electron charge in C
eps = 8.85E-12; # permititivity in F/m
pi = math.pi;
|
# position of electron 1 in m
r1x = 1E-9;
r1y = 2E-9;
r1z = 3E-9;

#position of electron 2 in m
r2x = -2E-9;
r2y = 4E-9;
r2z = -1E-9;

# vector pointing from 1 to 2
r12x = r2x -r1x;
r12y = r2y -r1y;
r12z = r2z -r1z;

# length of vector r12
r_len = math.sqrt(r12x*r12x + r12y*r12y + r12z*r12z);
```

# Python

```
# vector pointing from 1 to 2
r12x = r2x -r1x;
r12y = r2y -r1y;
r12z = r2z -r1z;

# length of vector r12
r_len = math.sqrt(r12x*r12x + r12y*r12y + r12z*r12z);

# unit vector pointing from 1 to 2
ex = r12x/r_len;
ey = r12y/r_len;
ez = r12z/r_len;

# Force in N
Fx= e*e*ex/(4*pi*eps*r_len*r_len);
Fy= e*e*ey/(4*pi*eps*r_len*r_len);
Fz= e*e*ez/(4*pi*eps*r_len*r_len);

print('Fx = ' + str(Fx) + 'N');
print('Fy = ' + str(Fy) + 'N');
print('Fz = ' + str(Fz) + 'N');
```

# Javascript

```
<script>

// define some constants
e = -1.6022E-19; // electron charge in C
eps = 8.85E-12; // permitivity in F/m
pi = Math.PI;

// position of electron 1 in m
r1x = 1E-9;
r1y = 2E-9;
r1z = 3E-9;

// position of electron 2 in m
r2x = -2E-9;
r2y = 4E-9;
r2z = -1E-9;

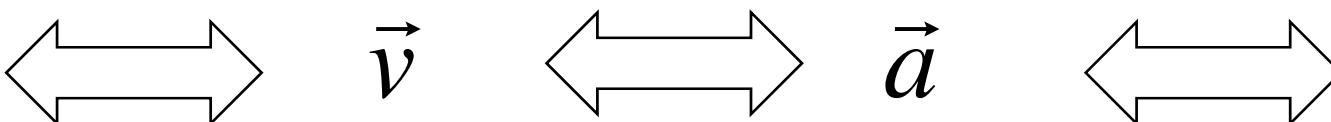
// vector pointing from 1 to 2
r12x = r2x -r1x;
r12y = r2y -r1y;
r12z = r2z -r1z;

// length of vector r12
r_len = Math.sqrt(r12x*r12x + r12y*r12y + r12z*r12z);

// unit vector pointing from 1 to 2
ex = r12x/r_len;
```

# Punktmechanik

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$$\vec{r} \quad \vec{v} \quad \vec{a} \quad \vec{F}$$


The diagram shows four pairs of horizontal arrows pointing in opposite directions, each pair enclosed in a small bracket below it. The first pair is under  $\vec{r}$ , the second under  $\vec{v}$ , the third under  $\vec{a}$ , and the fourth under  $\vec{F}$ . This visual representation indicates that each of these variables is a vector quantity.

# Punktmechanik

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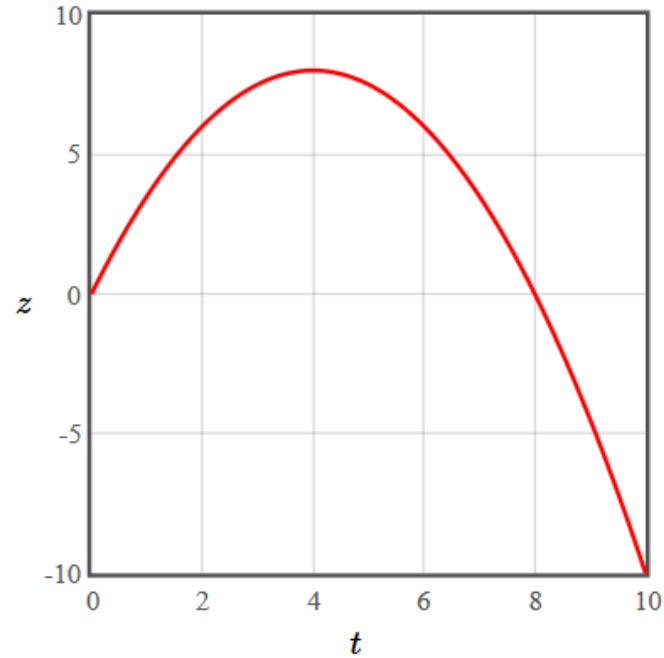
$$\vec{r} \quad \vec{v} = \frac{d\vec{r}}{dt} \quad \vec{v} \quad \vec{a} = \frac{d\vec{v}}{dt} \quad \vec{a} \quad \vec{F} = m\vec{a} \quad \vec{F}$$

# Punktmechanik

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$$\vec{r} \quad \vec{v} \quad \vec{a} \quad \vec{F}$$
$$\vec{r}(t) = \int_{t_0}^t \vec{v}(t') dt' + \vec{r}(t_0) \quad \vec{v}(t) = \int_{t_0}^t \vec{a}(t') dt' + \vec{v}(t_0)$$
$$\vec{a} = \frac{\vec{F}}{m}$$

## Konstante Kraft = Parabelbewegung



$$\vec{r} = \left( x_0 + v_{x0}t + \frac{F_x}{2m}t^2 \right) \hat{x} + \left( y_0 + v_{y0}t + \frac{F_y}{2m}t^2 \right) \hat{y} + \left( z_0 + v_{z0}t + \frac{F_z}{2m}t^2 \right) \hat{z}$$

$$\vec{v} = \left( v_{x0} + \frac{F_x}{m}t \right) \hat{x} + \left( v_{y0} + \frac{F_y}{m}t \right) \hat{y} + \left( v_{z0} + \frac{F_z}{m}t \right) \hat{z}$$

$$\vec{a} = \frac{F_x}{m} \hat{x} + \frac{F_y}{m} \hat{y} + \frac{F_z}{m} \hat{z}$$

$$\vec{F} = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}$$

$$z_0 = 0 \text{ m} \qquad m = 1 \text{ kg}$$

$F_z = -1.00 \text{ [N]}$

$v_{z0} = 4.00 \text{ [N]}$

Erläuterung:

# Fähigkeiten

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## Mechanik punktartiger Teilchen

Bei gegebener Position  $\vec{r}$  [m], Geschwindigkeit  $\vec{v}$  [m/s], Beschleunigung  $\vec{a}$  [ $\text{m/s}^2$ ], oder Kraft  $\vec{F}$  [N] als Funktion der Zeit eines Teilchens, müssen Sie in der Lage dazu sein, jede der vier Größen durch Integrieren oder Ableiten der anderen Größen zu erhalten.

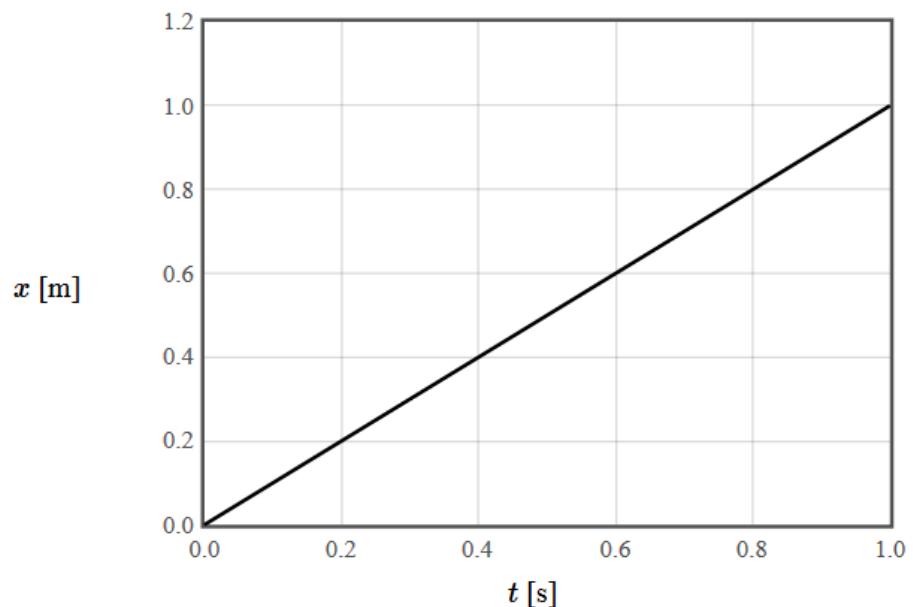
App: Numerische Integration und Differentiation von Funktionen in Abhängigkeit von  $t$ .

# Position → Velocity → Acceleration

$x(t) =$   [m]

Calculate from formula in the range from  $t_1 =$   [s] to  $t_2 =$   [s].

$t$ [s]	$x$ [m]
0	0
0.001	0.001
0.002	0.002
0.003	0.003
0.004	0.004
0.005	0.005
0.006	0.006
0.007	0.007
0.008	0.008
0.009	0.009
0.01	0.01



## Position → Kraft (numerisch)

Ein auf einer geraden Straße fahrendes Auto hat ein GPS-Gerät installiert, welches die Position des Autos speichert. Die Masse des Autos ist 1175 kg. Welche Kraft wirkt auf das Auto zur Zeit  $t = 20$  s?

Differenzieren Sie mittels der APP Numerische Integration.

$t$ [s]	$x$ [m]
0.00	7.0000000
0.500	14.191468
1.00	21.556045
1.50	29.073493
2.00	36.721801
2.50	44.477305
3.00	52.314830
3.50	60.207848
4.00	68.128647
4.50	76.048522
5.00	83.937969

solution

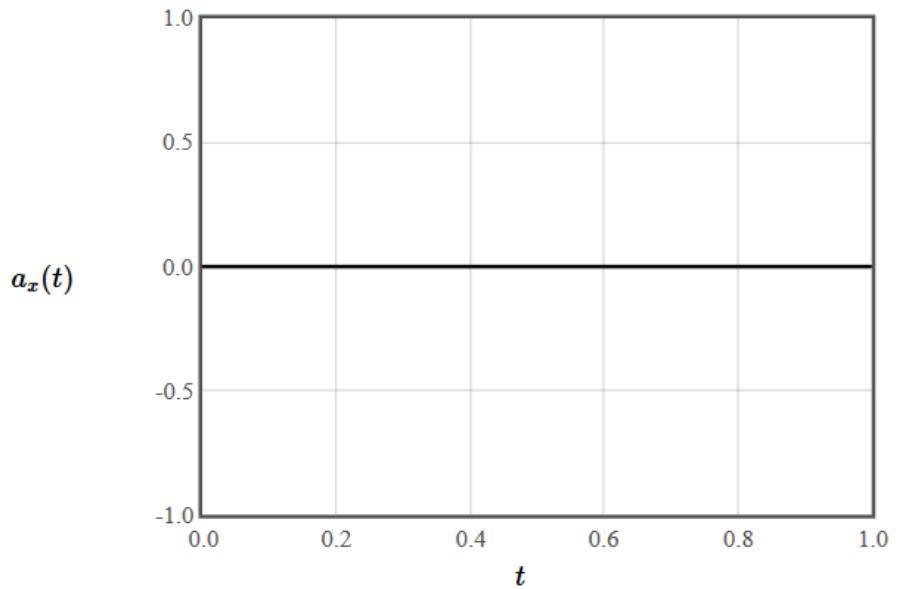
# Acceleration → Velocity → Position

$a(t) = 0$  [m/s<sup>2</sup>] ?

Calculate from formula in the range from  $t_1 = 0$  [s] to  $t_2 = 1$  [s].

Zero force Constant force Terminal velocity Harmonic motion

$t$	$a_x(t)$
0	0
0.001	0
0.002	0
0.003	0
0.004	0
0.005	0
0.006	0
0.007	0
0.008	0
0.009	0
0.01	0

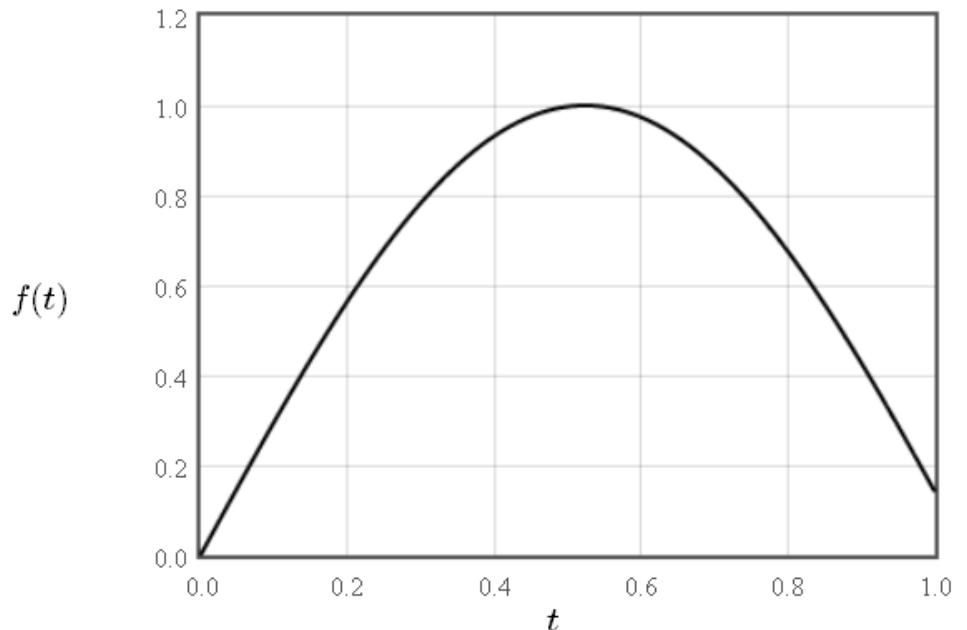


## Numerische Integration and Differentiation

$$f(t) = \sin(3*t)$$

[?](#)
 from  $t_1 = 0$  to  $t_2 = 1$ .

$t$	$f(t)$
0.02000	0.05996
0.02333	0.06994
0.02667	0.07991
0.03000	0.08988
0.03333	0.09983
0.03667	0.1098
0.04000	0.1197
0.04333	0.1296
0.04667	0.1395
0.05000	0.1494
0.05333	0.1593



### Die 1. Ableitung

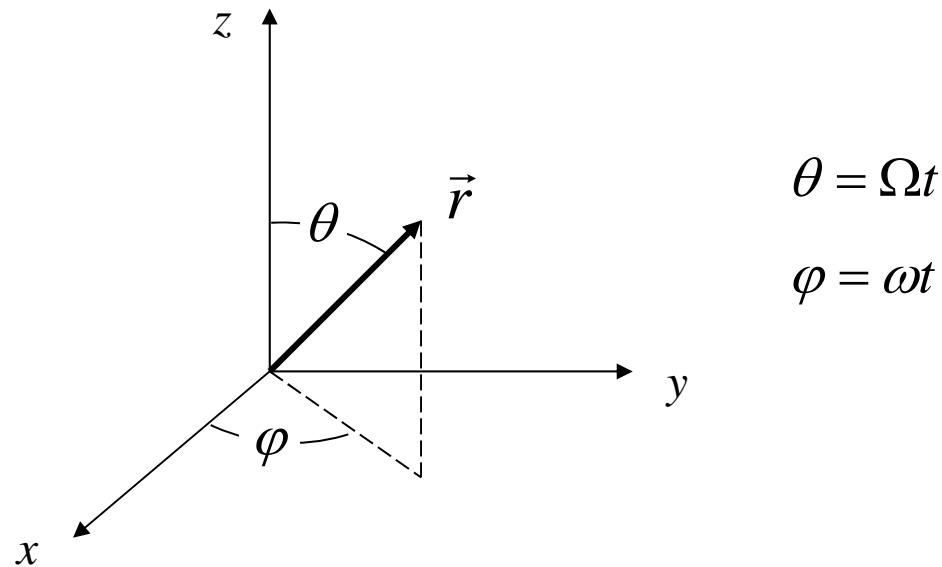
Die Ableitung von  $f(t)$  wird berechnet aus

$$\frac{df}{dt} \approx \frac{f(t+\Delta t) - f(t)}{\Delta t}.$$

# Corioliskraft

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$$\vec{r}(t) = R \sin(\Omega t) \cos(\omega t) \hat{x} + R \sin(\Omega t) \sin(\omega t) \hat{y} + R \cos(\Omega t) \hat{z}$$



$$\theta = \Omega t$$

$$\varphi = \omega t$$

$\vec{F}$  ?

# Müssen wir die Coriolis-Kraft für die Prüfung wissen?

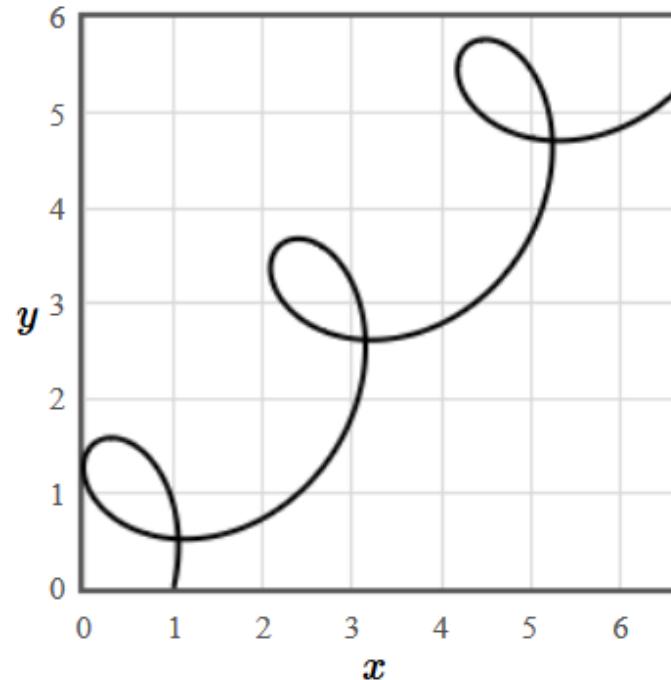
## Problem 2

Die Bahnkurve eines Teilchens der Masse  $m = 33 \text{ g}$  ist,

$$\vec{r}(t) = (t + \cos(3t)) \hat{x} + (t + \sin(3t)) \hat{y} \quad [\text{m}].$$

Dabei ist  $t$  die Zeit in Sekunden.

immer Radian

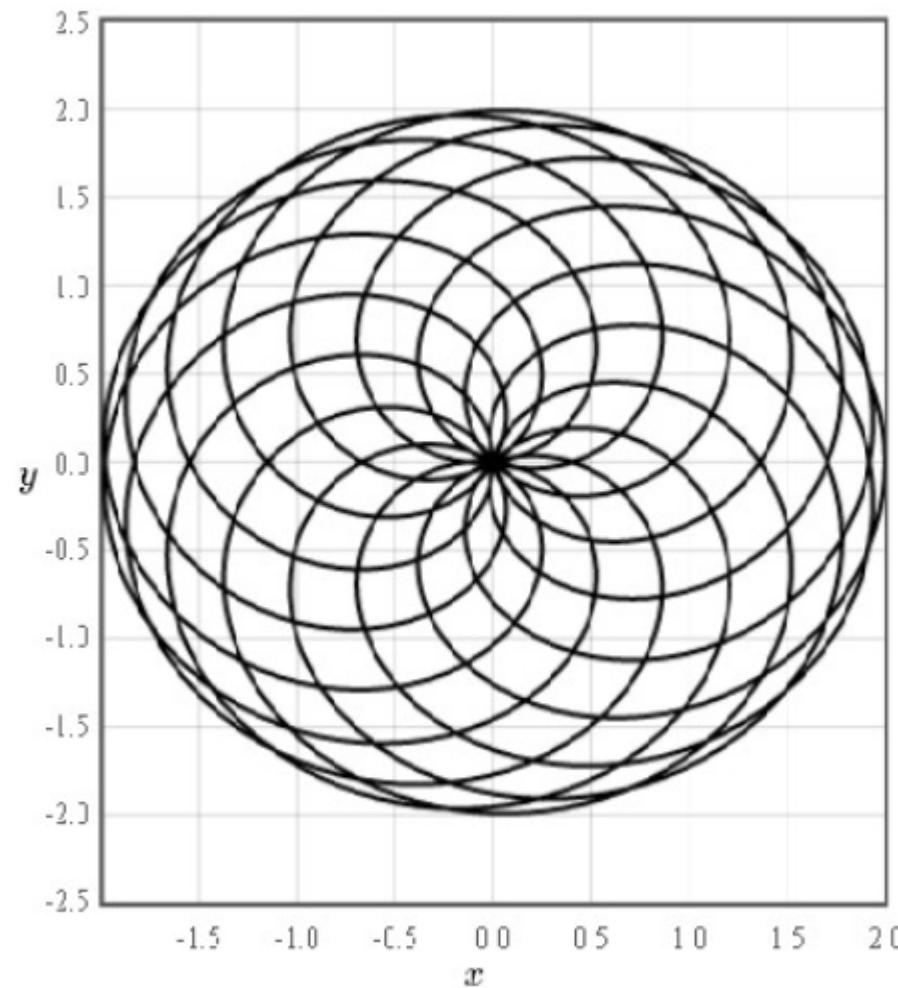


Welche Kraft wirkt auf das Teilchen zur Zeit  $t = 1 \text{ s}$ ?

$$\vec{F} = \boxed{\phantom{00}} \hat{x} + \boxed{\phantom{00}} \hat{y} + \boxed{\phantom{00}} \hat{z} \quad [\text{N}]$$

# Bahnkurve

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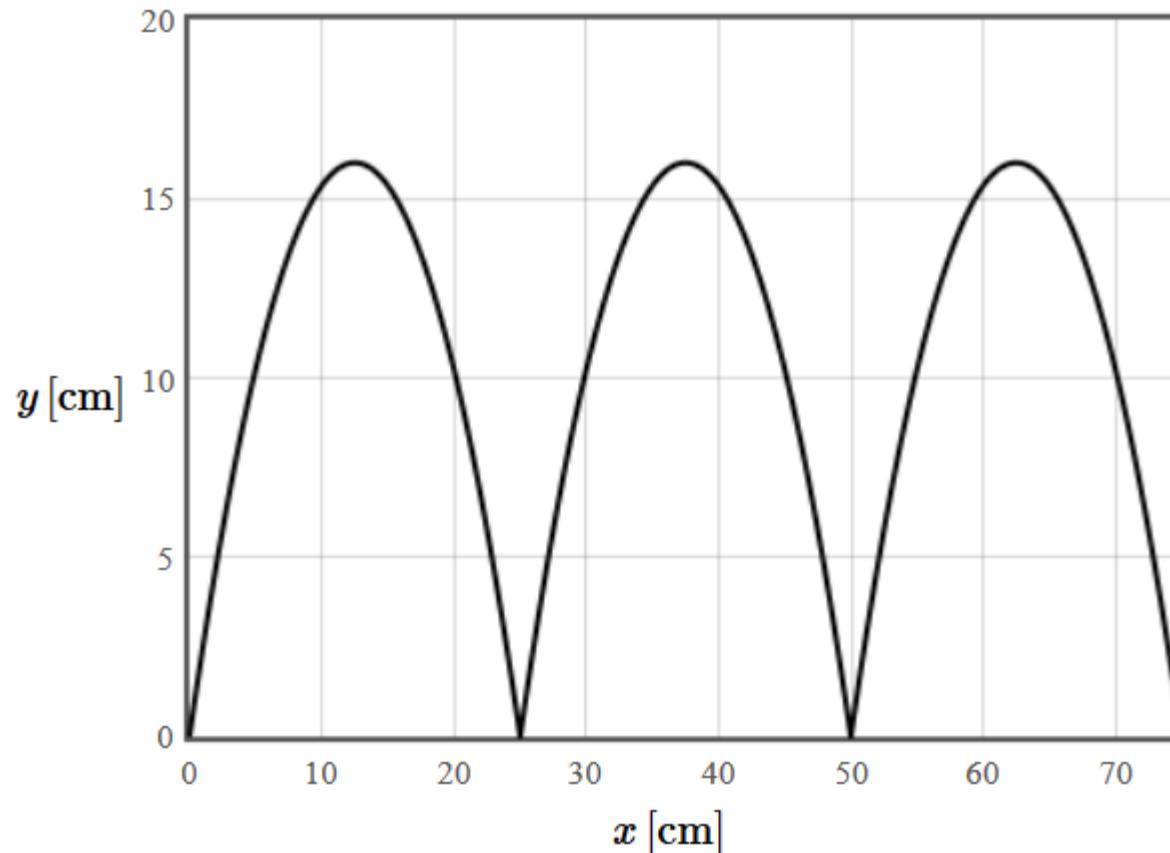


$$\vec{r}(t) = (\cos(2\pi t) + \cos(7.2\pi t)) \hat{x} + (\sin(2\pi t) + \sin(7.2\pi t)) \hat{y}$$

←                      ↑  
                        immer Radian

# Pruefung 29.06.2017

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$$\vec{r} = t\hat{x} + \frac{64}{625}(25t - t^2)\hat{y} + 0\hat{z} \quad [\text{cm}]$$