

# Metal / n-semiconductor Schottky contact in the depletion approximation

$\phi_b = 0.6$  eV

$N_D = 1E15$  1/cm<sup>3</sup>

$V = -0.5$  V

$T = 300$  K

$E_g = 1.12$  eV

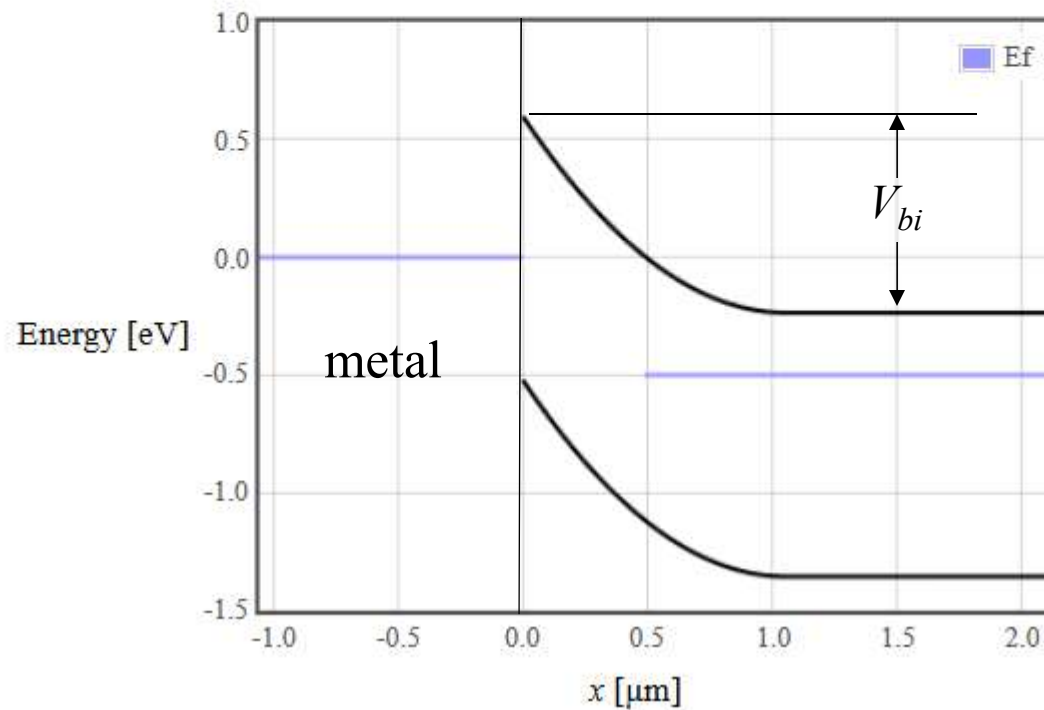
$N_c(300) = 2.78E19$  1/cm<sup>3</sup>

$\epsilon_r = 12$

Submit

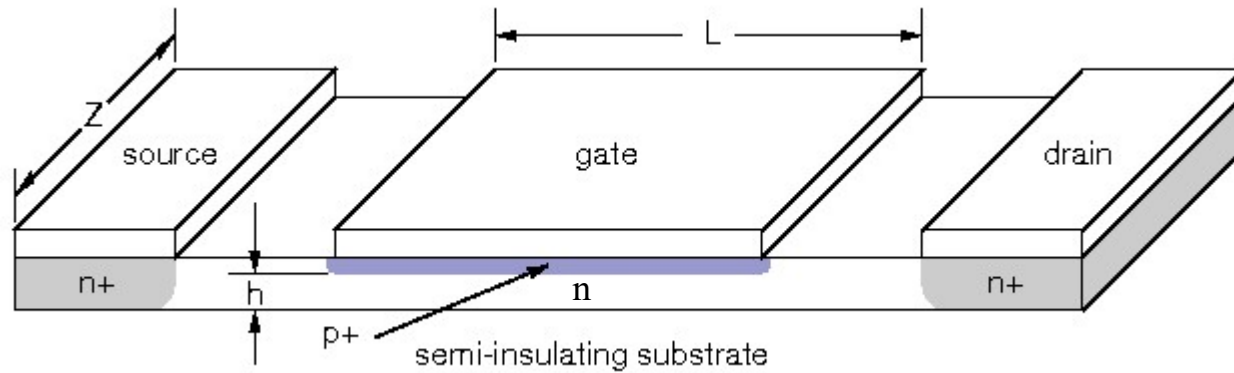
$W = 1.05$   $\mu\text{m}$      $V_{bi} = 0.335$  V     $C_j = 10.1$  nF/cm<sup>2</sup>

Band diagram

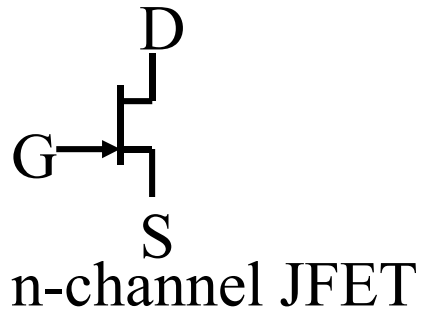


<http://lampx.tugraz.at/~hadley/psd/L6/abruptschottky.html>

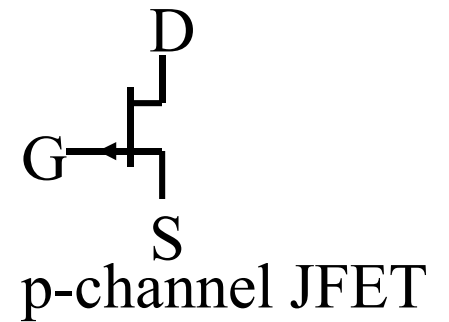
# JFET



n-channel JFET



$$x_n = \sqrt{\frac{2\epsilon(V_{bi} - V)}{eN_D}}$$

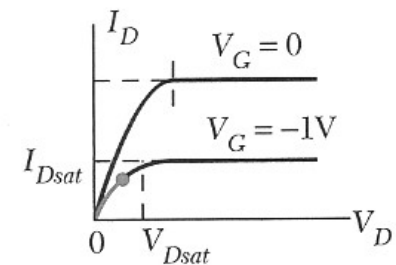
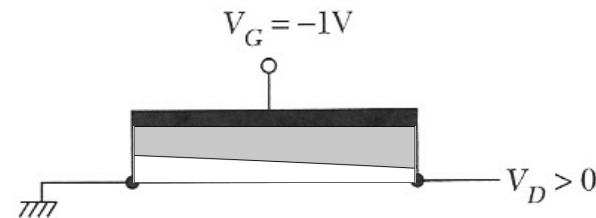
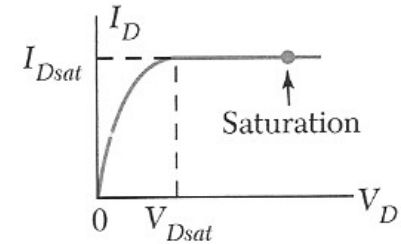
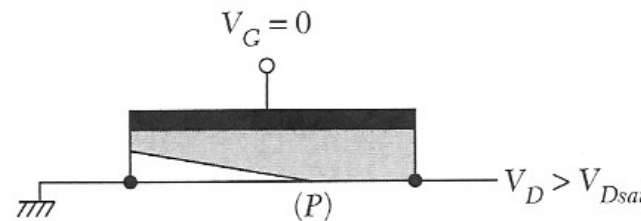
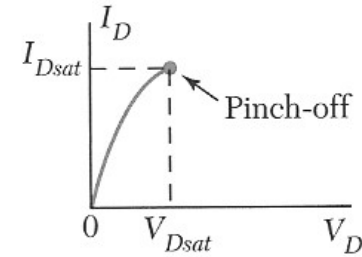
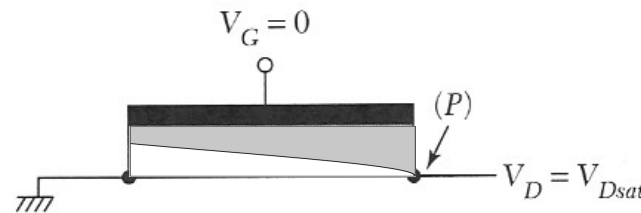
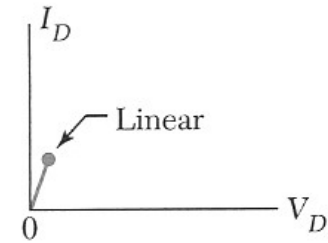
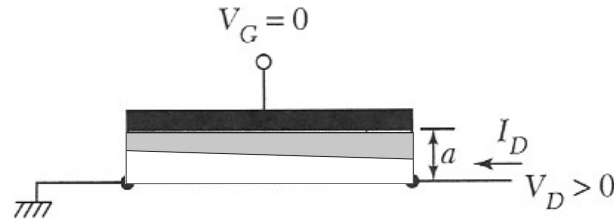


Pinch-off at  $h = x_n$        $V_p = V_{bi} - V$ .

$V_p =$  pinch-off voltage       $V_p = \frac{eN_D h^2}{2\epsilon}$

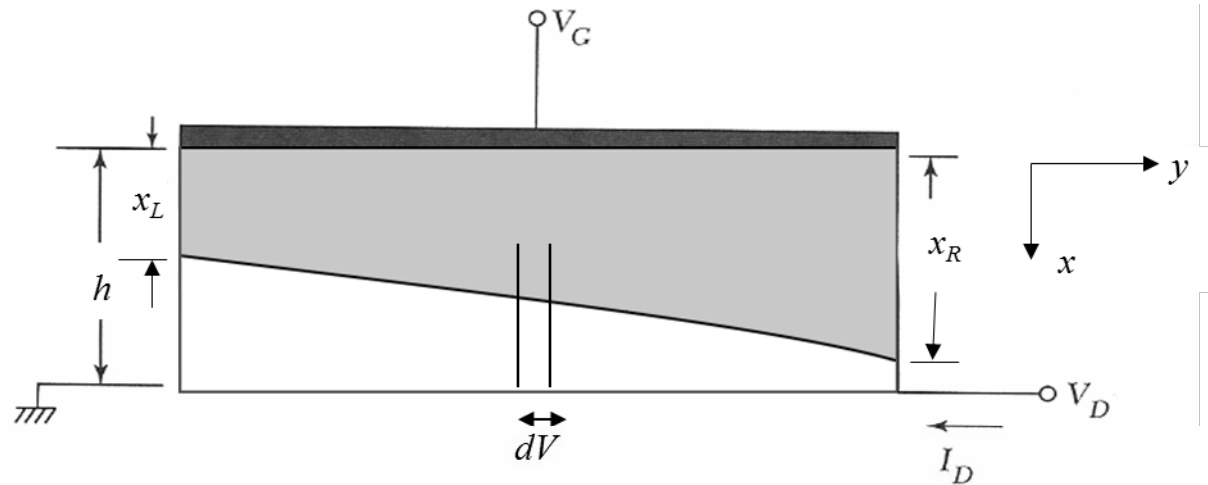
# JFET

The drain is the side of the transistor that gets pinched off.



# JFET

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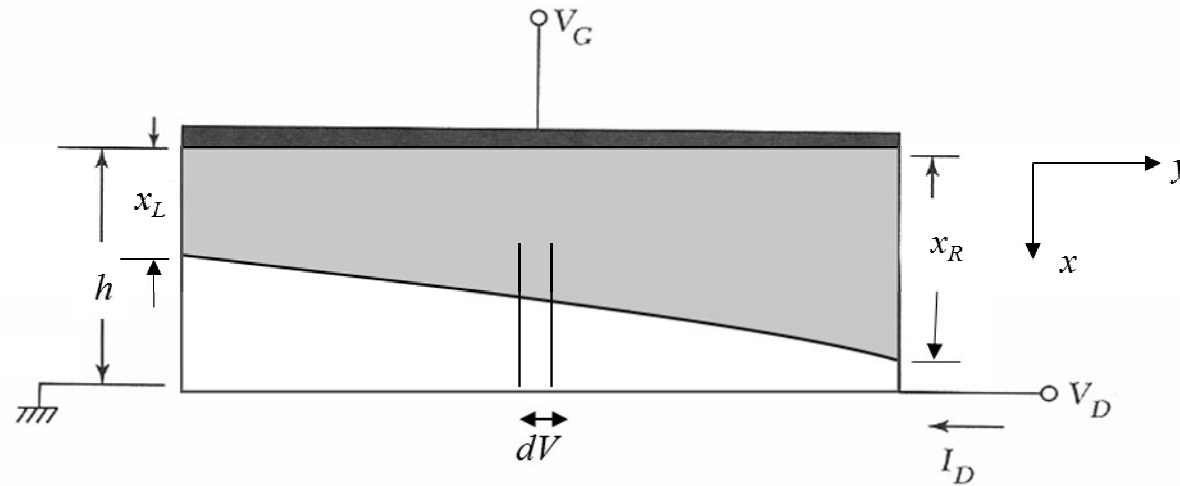


There is a long derivation to determine how the current depends on  $V_G$  and  $V_D$ .

We will find a relatively simple formula (probably familiar to electrical engineers).

Understanding the derivation is important for knowing when this formula is valid.

# JFET



$$dV = I_D dR = I_D \frac{\rho dy}{Z(h - x_n(y))}$$

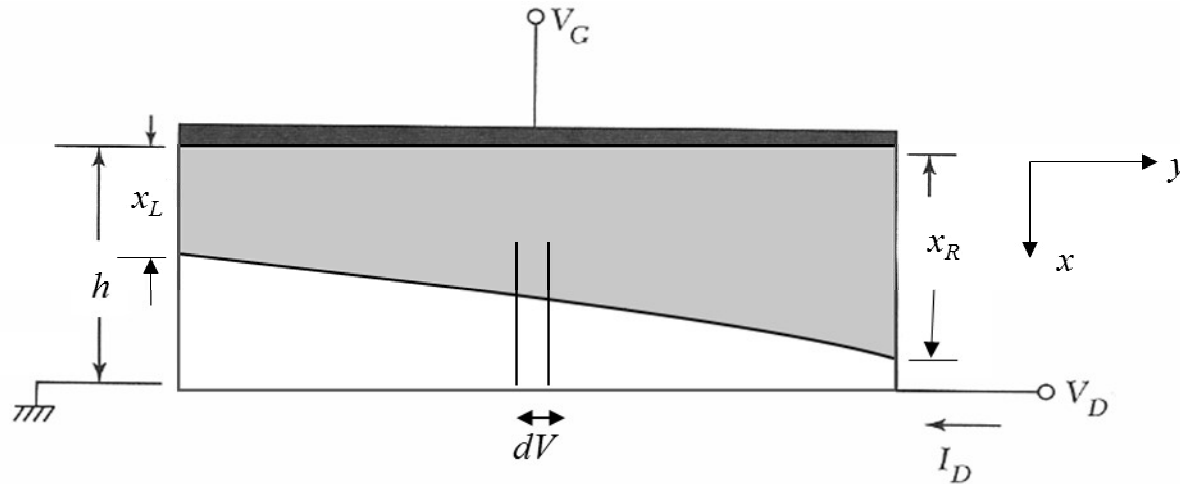
$$\rho = \frac{1}{\sigma} = \frac{1}{ne\mu_n} = \frac{1}{N_D e \mu_n}$$

$$dV = I_D \frac{dy}{e\mu_n N_D Z(h - x_n(y))}$$

$I_D$  is constant throughout the transistor  $\longrightarrow I_D dy = e\mu_n N_D Z(h - x_n(y)) dV$

# JFET

$$I_D dy = e\mu_n N_D Z (h - x_n(y)) dV$$



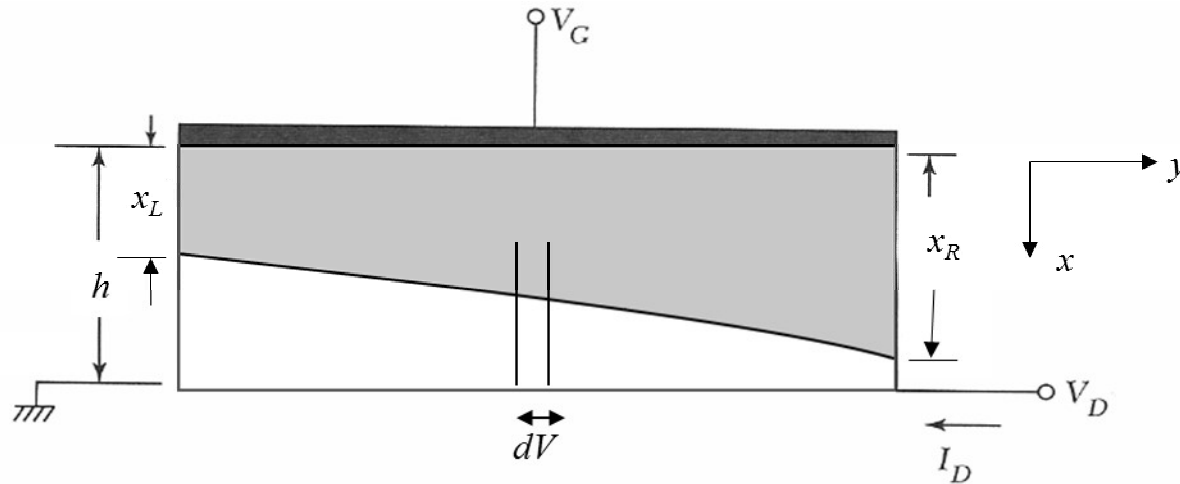
$V_G$  is a forward bias  
 $V(y)$  is a reverse bias.

depletion width is a function of position  $x_n(y) = \sqrt{\frac{2\varepsilon(V_{bi} + V(y) - V_G)}{eN_D}}$

differentiate  $\frac{dx_n(y)}{dV} =$

# JFET

$$I_D dy = e\mu_n N_D Z (h - x_n(y)) dV$$



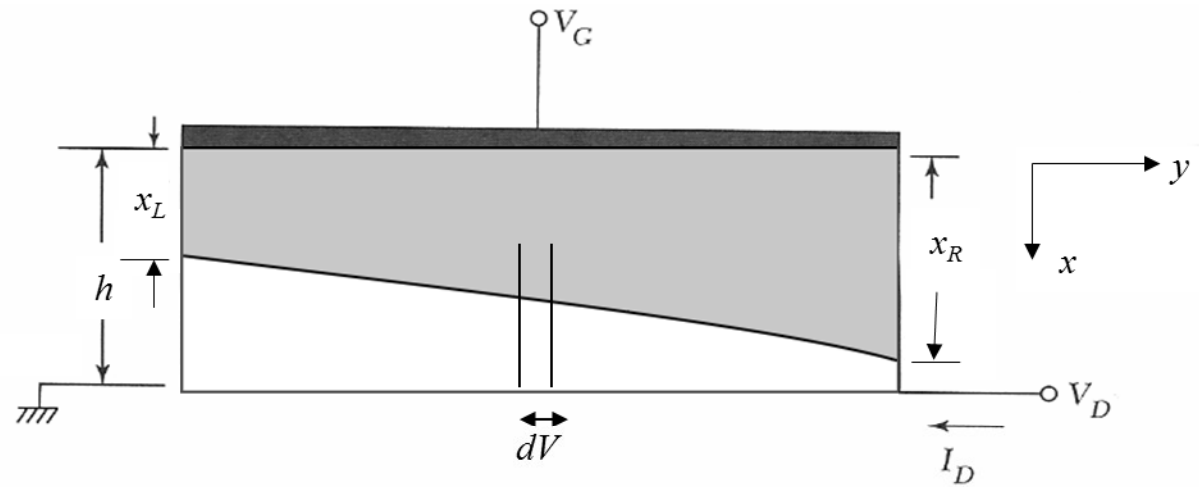
$V_G$  is a forward bias  
 $V(y)$  is a reverse bias.

depletion width is a function of position  $x_n(y) = \sqrt{\frac{2\epsilon(V_{bi} + V(y) - V_G)}{eN_D}}$

differentiate  $\frac{dx_n(y)}{dV} = \frac{1}{2} \left( \frac{2\epsilon(V_{bi} + V(y) - V_G)}{eN_D} \right)^{-1/2} \frac{2\epsilon}{eN_D} = \frac{\epsilon}{eN_D x_n(y)}$

$$dV = \frac{eN_D x_n}{\epsilon} dx_n$$

# JFET



$$I_D dy = e\mu_n N_D Z (h - x_n(y)) dV \quad \leftarrow \text{from last slide}$$

$$dV = \frac{eN_D x_n}{\epsilon} dx_n$$

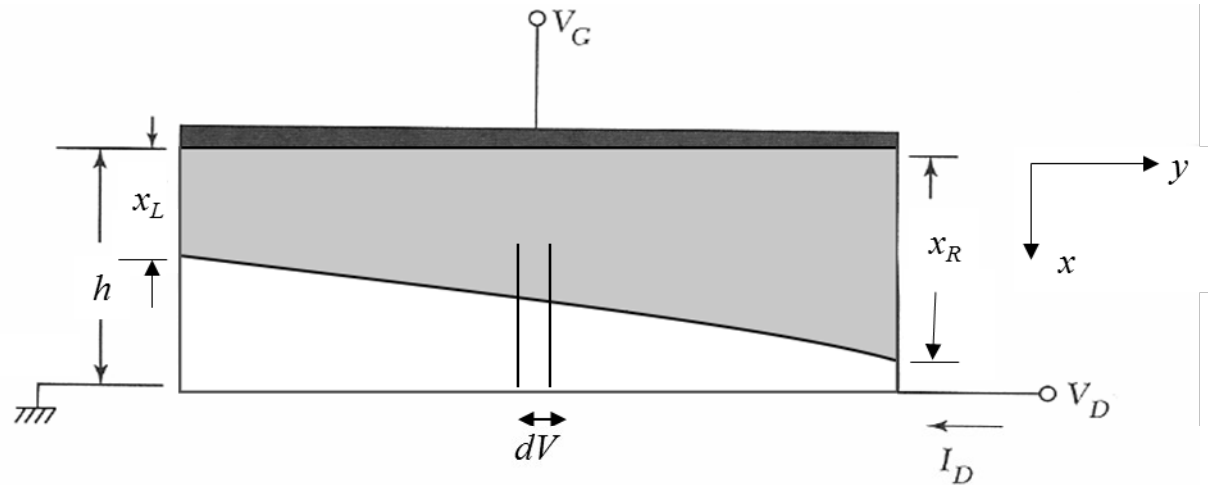
$$I_D dy = e\mu_n N_D Z (h - x_n(y)) \frac{eN_D}{\epsilon} x_n dx_n$$

$$I_D \int_0^L dy = e\mu_n N_D Z \frac{eN_D}{\epsilon} \int_{x_L}^{x_R} (h - x_n(y)) x_n dx_n$$

$$I_D = \frac{\mu_n N_D^2 Z e^2}{2L\epsilon} \left[ h(x_R^2 - x_L^2) - \frac{2}{3}(x_R^3 - x_L^3) \right]$$



# JFET



$$I_D = \frac{\mu_n N_D^2 Z e^2}{2L\epsilon} \left[ h(x_R^2 - x_L^2) - \frac{2}{3}(x_R^3 - x_L^3) \right]$$

$$h = \sqrt{\frac{2\epsilon V_p}{eN_D}}$$

$$x_L = \sqrt{\frac{2\epsilon(V_{bi} - V_G)}{eN_D}}$$

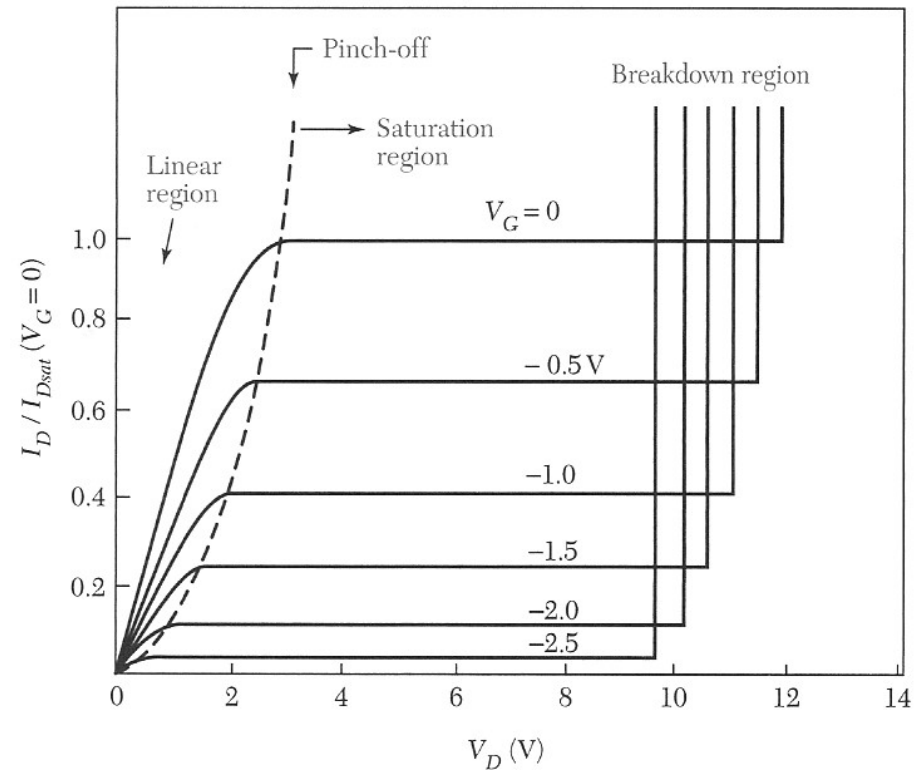
$$x_R = \sqrt{\frac{2\epsilon(V_{bi} - V_G + V_D)}{eN_D}}$$

# JFET - drain current

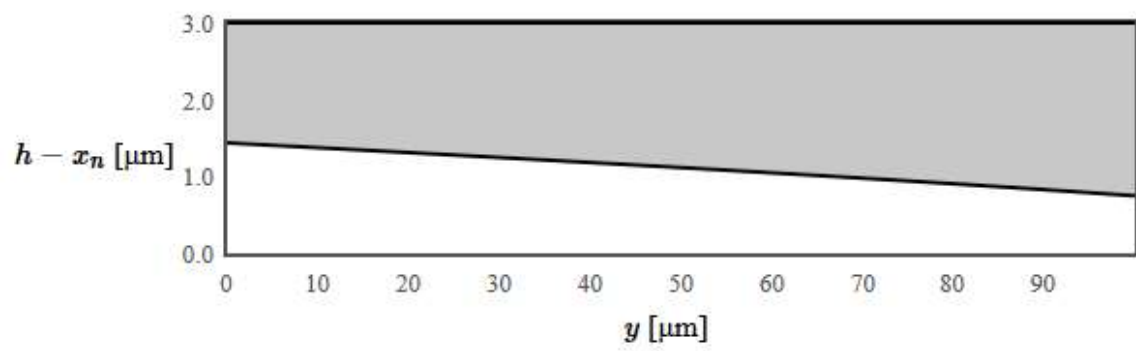
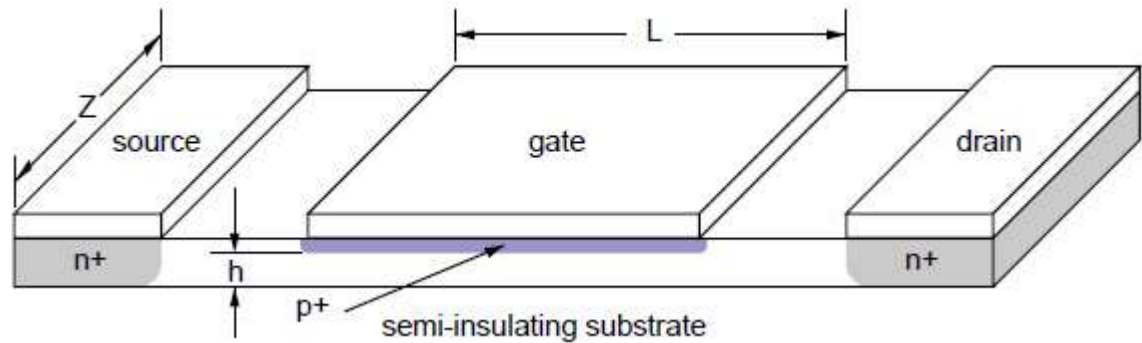
$$I_D = I_p \left[ \frac{V_D}{V_p} - \frac{2}{3} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{3/2} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$

$$I_p = \frac{\mu_n N_D^2 Z e^2 h^3}{2L\epsilon}$$

valid in the linear regime  
(until pinch-off)



# JFET Gradual Channel Approximation



$N_c(300K)$	<input type="text" value="2.78E19"/>	$\text{cm}^{-3}$
$N_v(300K)$	<input type="text" value="9.84E18"/>	$\text{cm}^{-3}$
$E_g$	<input type="text" value="1.166-4.73E-4*T*T/(T+636)"/>	eV
$N_D$	<input type="text" value="1E15"/>	$\text{cm}^{-3}$
$N_A$	<input type="text" value="1E19"/>	$\text{cm}^{-3}$
$\mu_n$	<input type="text" value="1350"/>	$\text{cm}^2/\text{Vs}$
$h$	<input type="text" value="3"/>	$\mu\text{m}$
$L$	<input type="text" value="100"/>	$\mu\text{m}$
$Z$	<input type="text" value="100"/>	$\mu\text{m}$
$\epsilon_r$	<input type="text" value="11.9"/>	
$T$	<input type="text" value="300"/>	K
$V_D$	<input type="text" value="2"/>	V
$V_g$	<input type="text" value="-1"/>	V

# JFET - Linear regime

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$$I_D = I_p \left[ \frac{V_D}{V_p} - \frac{2}{3} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{3/2} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$

In the linear regime  $V_D \ll V_{sat}$ .

$$\frac{dI_D}{dV_D} =$$

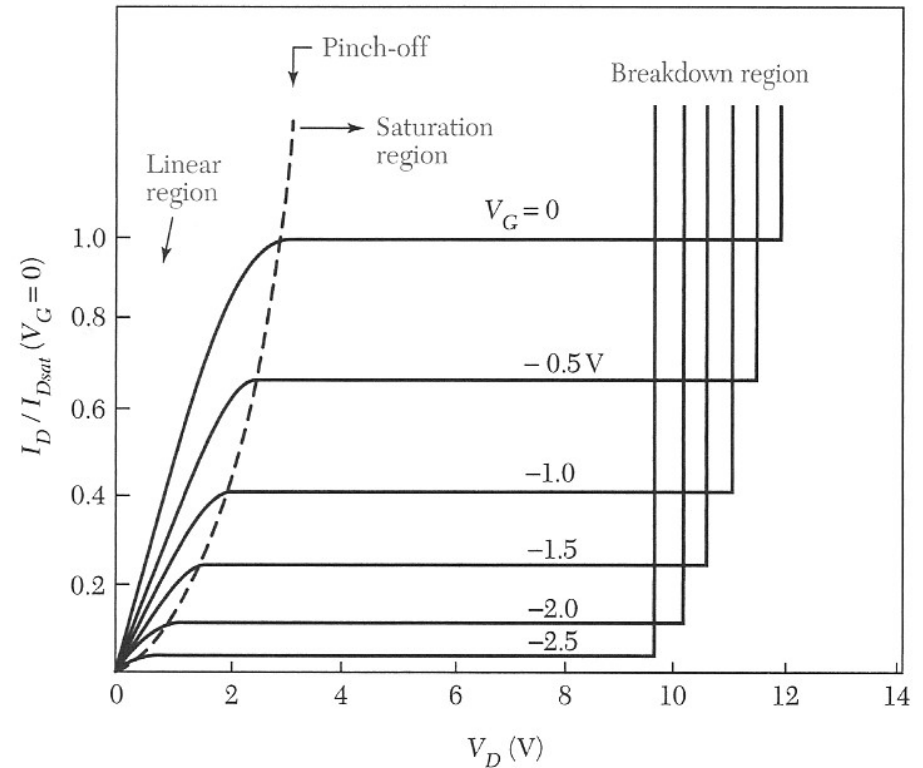
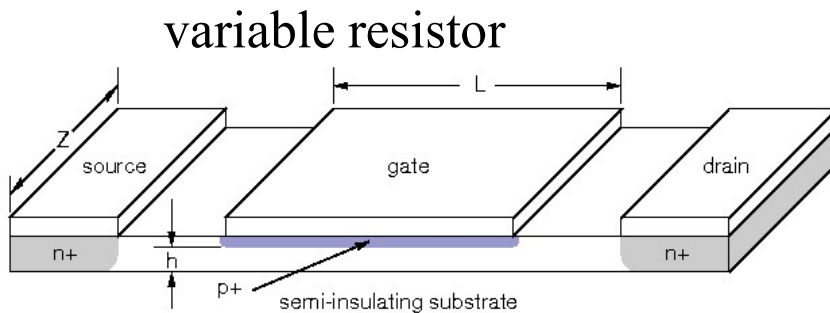
# JFET - Linear regime

$$I_D = I_p \left[ \frac{V_D}{V_p} - \frac{2}{3} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{3/2} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$

In the linear regime  $V_D \ll V_{sat}$ .

$$\frac{dI_D}{dV_D} = I_p \left[ \frac{1}{V_p} - \frac{1}{V_p} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{1/2} \right]$$

$$I_D \approx \left. \frac{dI_D}{dV_D} \right|_{V_D=0} V_D = \frac{I_p}{V_p} \left[ 1 - \sqrt{\frac{V_{bi} - V_G}{V_p}} \right] V_D$$



# JFET - Saturation regime

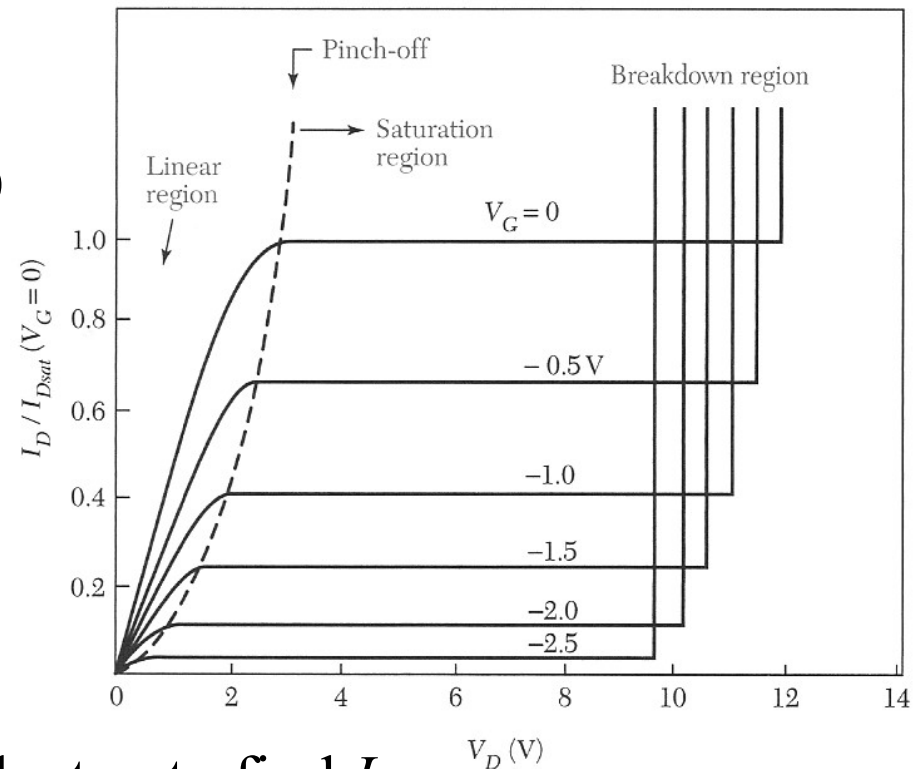
$$I_D = I_p \left[ \frac{V_D}{V_p} - \frac{2}{3} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{3/2} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$

set  $dI_D/dV_D = 0$  to find  $V_{sat}$

$$\frac{dI_D}{dV_D} = I_p \left[ \frac{1}{V_p} - \frac{1}{V_p} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{1/2} \right] = 0$$

$$dI_D/dV_D = 0 \text{ when } \frac{V_{bi} + V_D - V_G}{V_p} = 1$$

$$V_{sat} = V_p - V_{bi} + V_G$$



Substitute  $V_{sat}$  into the equation at the top to find  $I_{sat}$

# JFET - Saturation regime

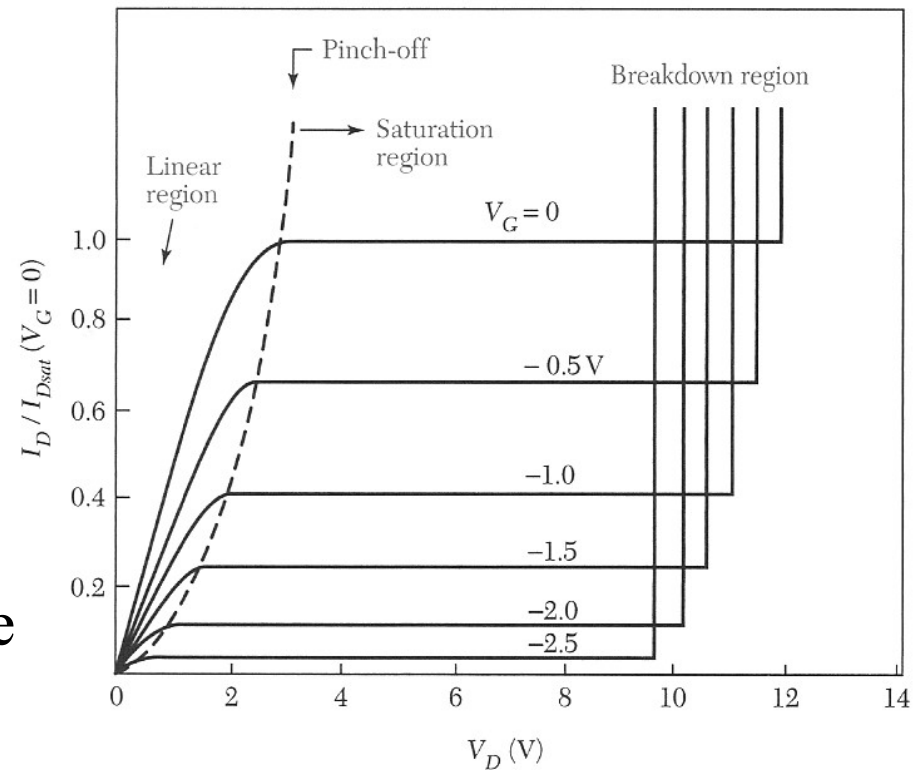
$$V_{sat} = V_p - V_{bi} + V_G$$

$$I_D = I_p \left[ \frac{V_D}{V_p} - \frac{2}{3} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{3/2} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$

$$I_{sat} = I_p \left[ \frac{1}{3} - \frac{V_{bi} - V_G}{V_p} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$

No  $V_D$  dependence

Voltage controlled current source



# JFET - transconductance

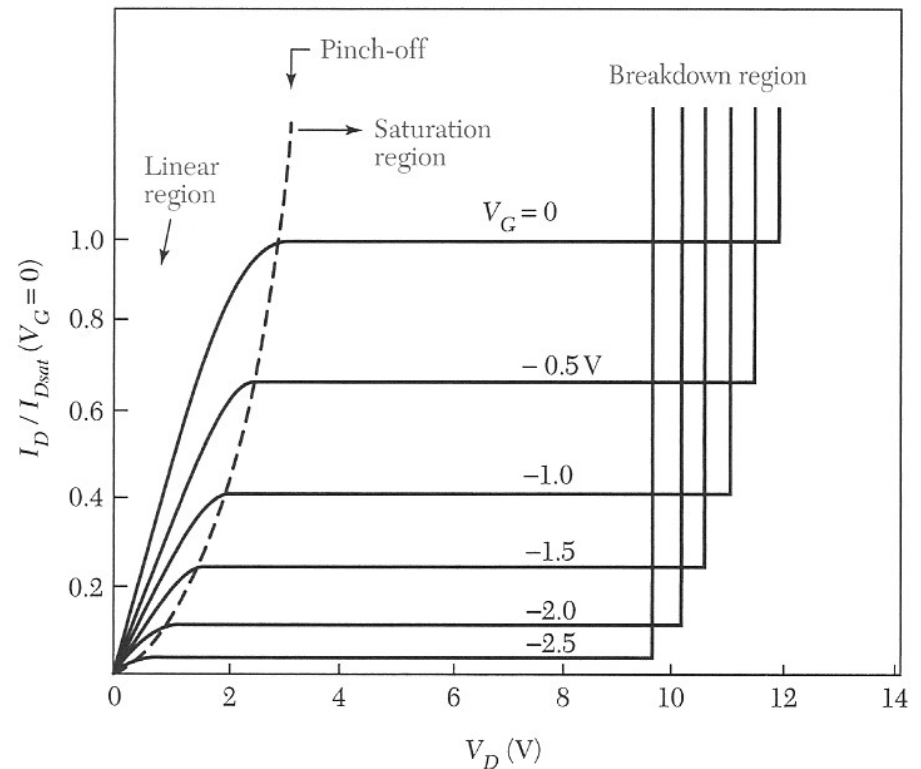
In the saturation regime,

$$I_{sat} = I_p \left[ \frac{1}{3} - \frac{V_{bi} - V_G}{V_p} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$

transconductance (describes how good the voltage controlled current source is)

$$g_m = \frac{dI_{sat}}{dV_G} = \frac{I_p}{V_p} \left( 1 - \sqrt{\frac{V_{bi} - V_G}{V_p}} \right)$$

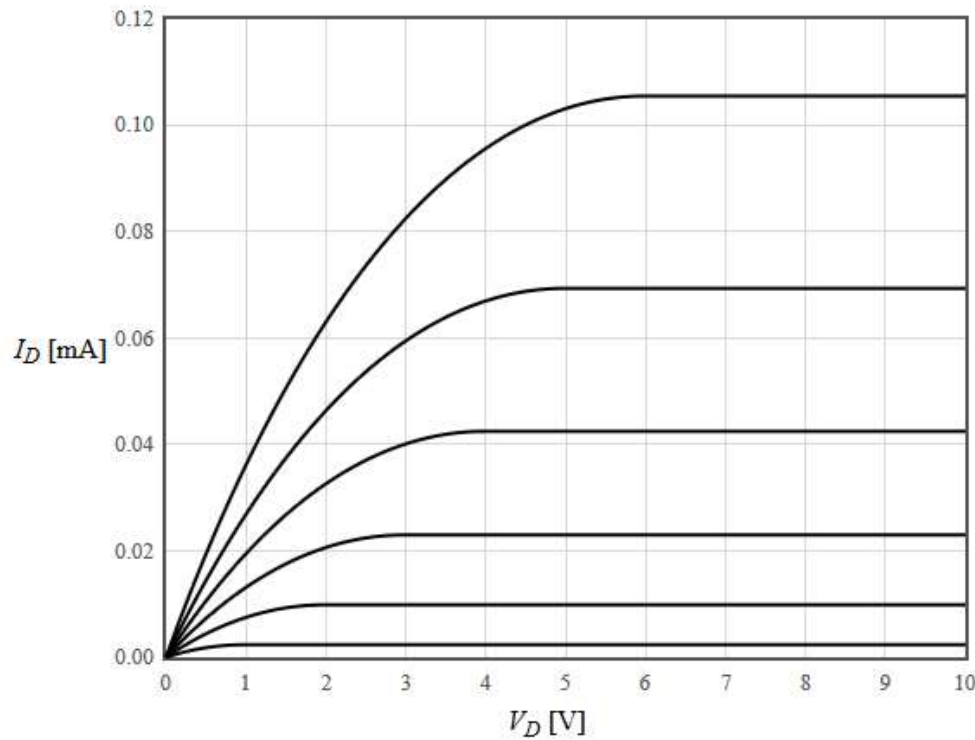
$$g_m = \frac{dI_{sat}}{dV_G} = \frac{2Z\mu_n e N_D h}{L} \left( 1 - \sqrt{\frac{V_{bi} - V_G}{V_p}} \right)$$





# JFET

$$I_D = I_p \left[ \frac{V_D}{V_p} - \frac{2}{3} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{3/2} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$



$N_c(300K)$	2.78E19	cm <sup>-3</sup>
$N_v(300K)$	9.84E18	cm <sup>-3</sup>
$E_g$	1.166-4.73E-4*T*T/(T+636)	eV
$N_D$	1E15	cm <sup>-3</sup>
$N_A$	1E19	cm <sup>-3</sup>
$\mu_n$	1350	cm <sup>2</sup> /Vs
$h$	3	$\mu$ m
$L$	100	$\mu$ m
$Z$	100	$\mu$ m
$\epsilon_r$	11.9	
$T$	300	K
$V_D(\text{max})$	10	V
$V_g$ [1]	0	V
$V_g$ [2]	-1	V
$V_g$ [3]	-2	V
$V_g$ [4]	-3	V
$V_g$ [5]	-4	V
$V_g$ [6]	-5	V

Replot

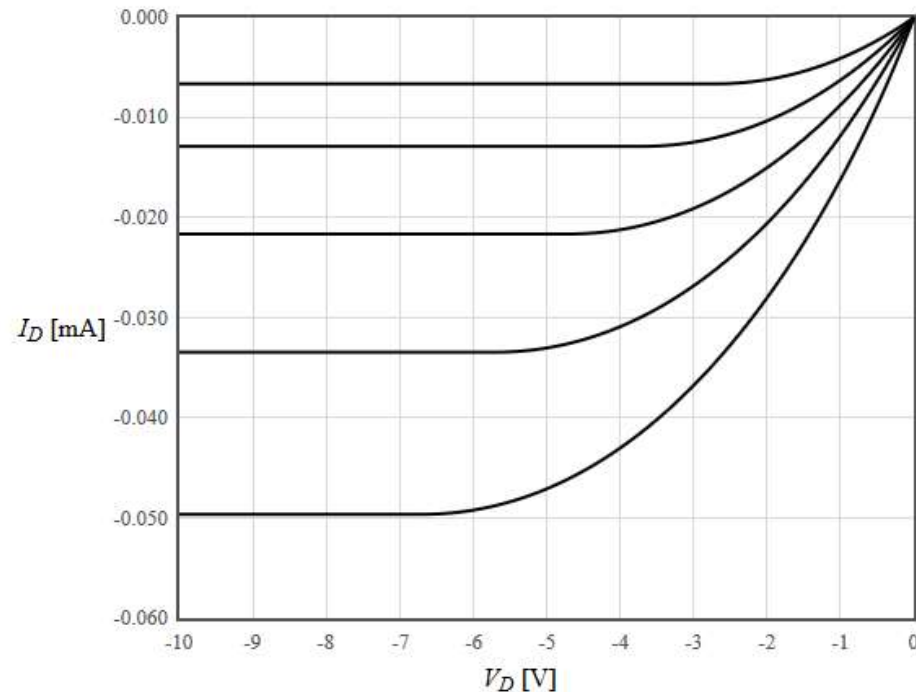
Si Ge GaAs

$E_g = 1.12$  eV;  $n_i = 6.41e+9$  cm<sup>-3</sup>;  $V_{bi} = 0.856$  V;  $I_p = 0.000444$  A;  $V_p = 6.84$  V.

# p-channel JFET

The expression for the drain current of a p-channel JFET in the linear regime is,

$$I_D = I_p \left[ \frac{V_D}{V_p} - \frac{2}{3} \left( \frac{V_{bi} + V_D - V_G}{V_p} \right)^{3/2} + \frac{2}{3} \left( \frac{V_{bi} - V_G}{V_p} \right)^{3/2} \right]$$



$N_c(300K)$	<input type="text" value="2.78E19"/>	cm <sup>-3</sup>
$N_v(300K)$	<input type="text" value="9.84E18"/>	cm <sup>-3</sup>
$E_g$	<input type="text" value="1.166-4.73E-4*T*T/(T+636)"/>	eV
$N_D$	<input type="text" value="1E19"/>	cm <sup>-3</sup>
$N_A$	<input type="text" value="1E15"/>	cm <sup>-3</sup>
$\mu_p$	<input type="text" value="480"/>	cm <sup>2</sup> /Vs
$h$	<input type="text" value="3"/>	μm
$L$	<input type="text" value="100"/>	μm
$Z$	<input type="text" value="100"/>	μm
$\epsilon_r$	<input type="text" value="11.9"/>	
$T$	<input type="text" value="300"/>	K
$V_D(\text{min})$	<input type="text" value="-10"/>	V
$V_g [1]$	<input type="text" value="0"/>	V
$V_g [2]$	<input type="text" value="1"/>	V
$V_g [3]$	<input type="text" value="2"/>	V
$V_g [4]$	<input type="text" value="3"/>	V
$V_g [5]$	<input type="text" value="4"/>	V
$V_g [6]$	<input type="text" value="5"/>	V

Replot

Si   Ge   GaAs

$E_g = 1.12 \text{ eV}; \quad n_i = 6.41 \times 10^9 \text{ cm}^{-3}; \quad V_{bi} = 0.856 \text{ V}; \quad I_p = -0.000158 \text{ A}; \quad V_p = -6.84 \text{ V}.$

# High frequencies

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$$\tilde{i}_{in} = 2\pi f C_G \tilde{v}_G$$

$$\tilde{i}_{out} = g_m \tilde{v}_G$$

for gain:  $\tilde{i}_{in} < \tilde{i}_{out}$

$$f < \frac{g_m}{2\pi C_G} = f_T$$

$f_T$  is the frequency  
where the gain drops  
below 1

average capacitance:

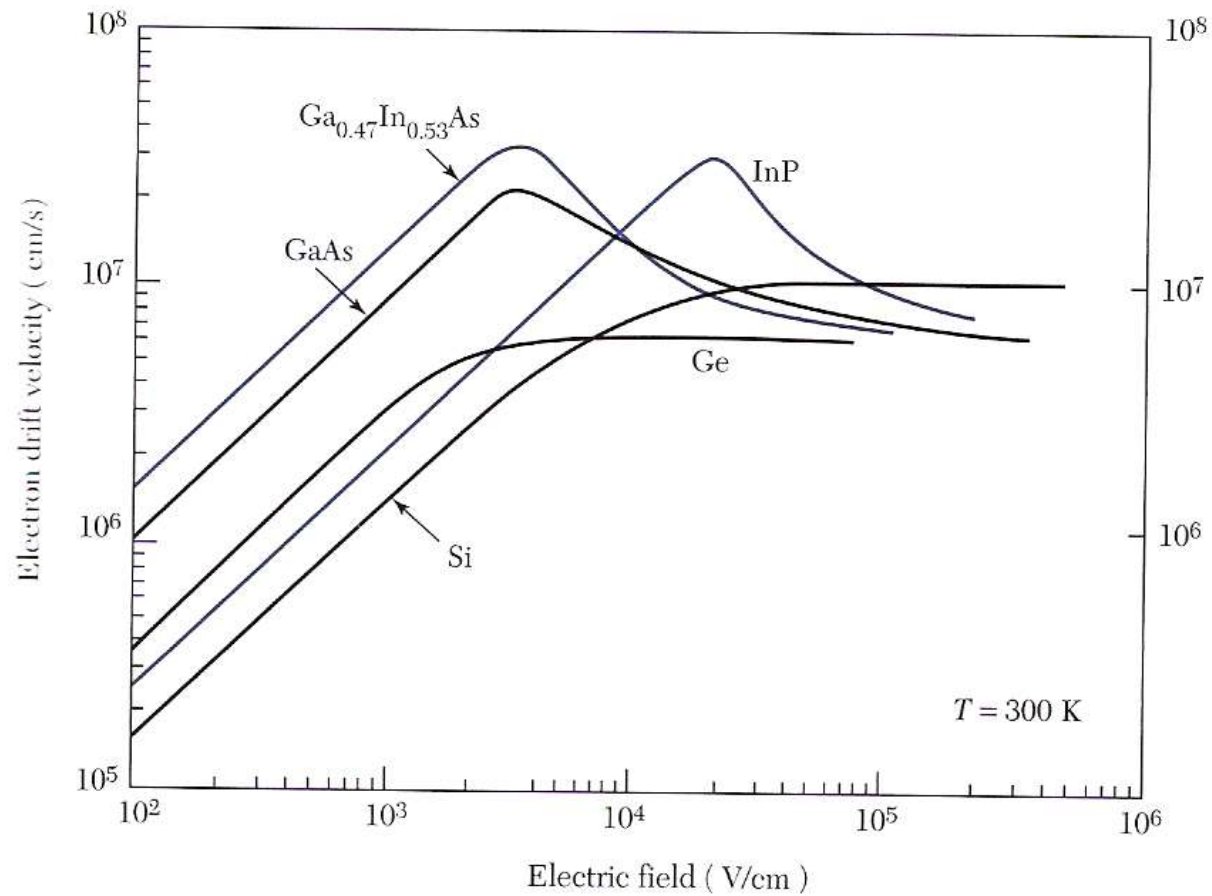
$$C_G = ZL \frac{\epsilon}{\bar{x}_n}$$

$$f_T = \frac{\mu_n e N_D h^2}{2\pi \epsilon L^2}$$

For velocity saturation, the approximation  $dV = I_D \frac{\rho dy}{Z(h - x_n(y))}$  is not valid

Ohm's law assumes  $v_d = \mu E$

$$f_T \approx \frac{v_s}{L}$$



# JFET/MESFET

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JFET: small gate current (reverse leakage of the gate-to-channel junction)

More gate leakage than MOSFET, less than bipolar.

JFET has higher transconductance than the MOSFET.

Used in low-noise, high input-impedance op-amps and sometimes used in switching applications.

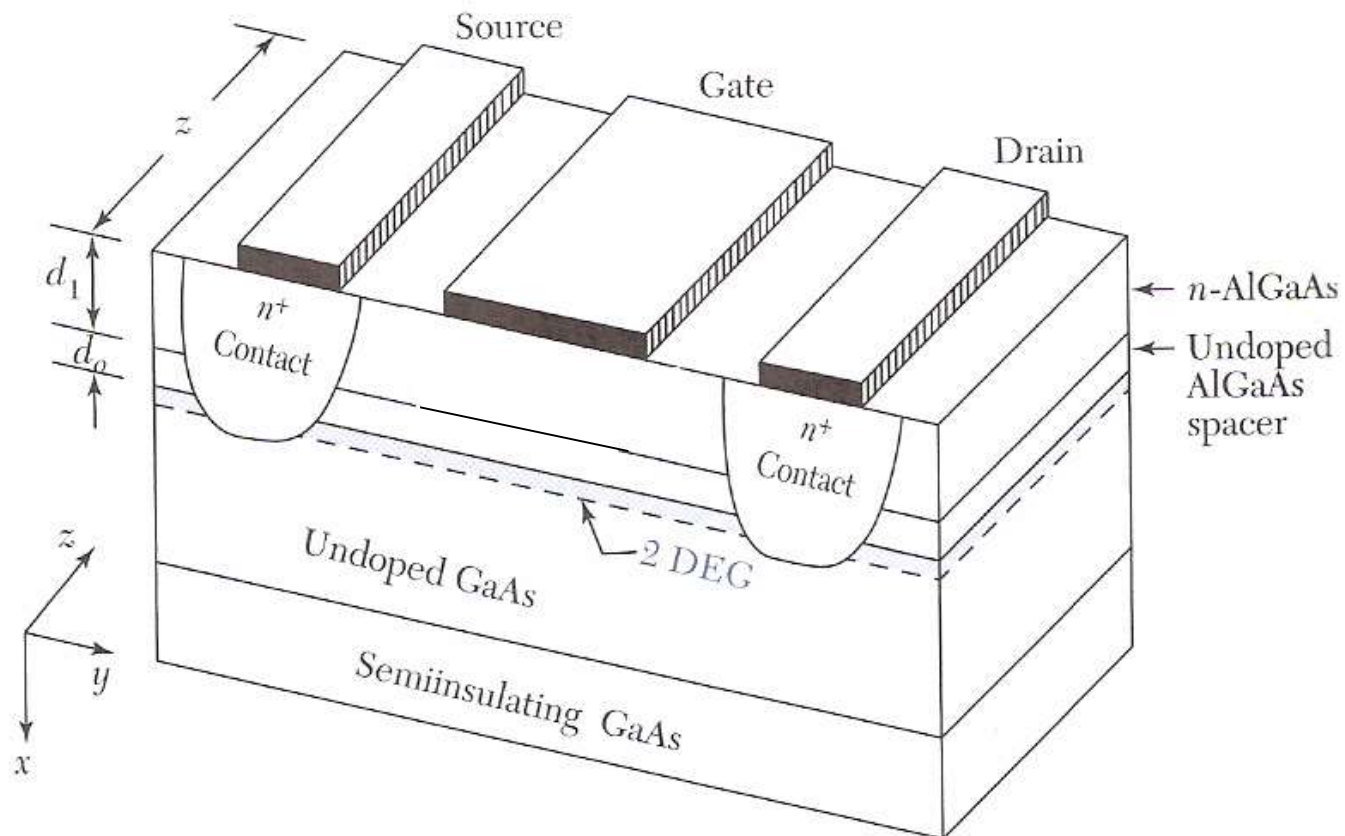
MESFET: usually constructed in compound semiconductor technologies lacking high quality surface passivation such as GaAs, InP, or SiC, and are faster but more expensive than silicon-based JFETs or MOSFETs.

Production MESFETs are operated up to approximately 30 GHz, and are commonly used for microwave frequency communications and radar.

Majority carrier device (like Schottky diode).

# MODFET (HEMT)

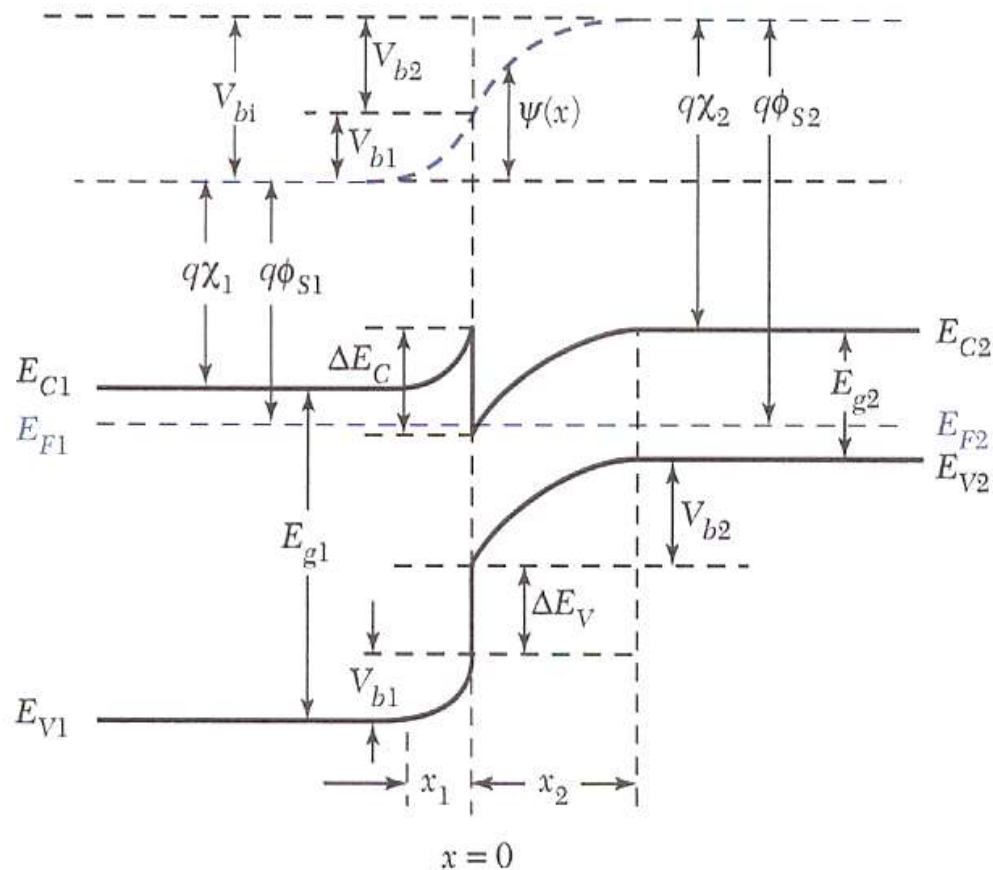
Modulation doped field effect transistor (MODFET)  
High electron mobility transistor (HEMT)



$V_T$  = Threshold voltage = voltage where charge is depleted

# Heterostructure

pn junction formed from two semiconductors with different band gaps



# MODFET/HEMT

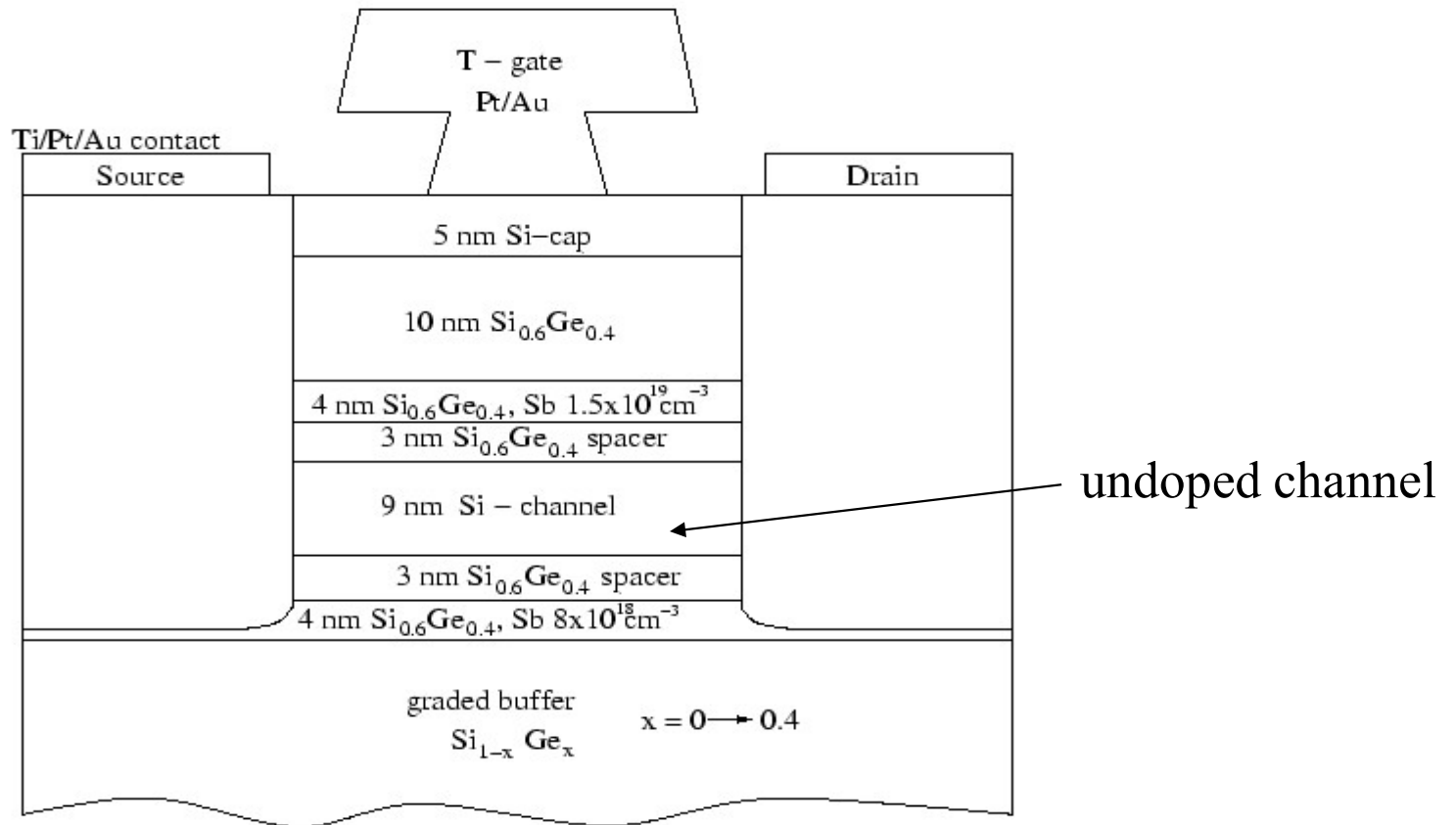


Figure 3.3: S-MODFET structure.

**HEMT:** HEMT devices are found in cell phones, electronic warfare systems, microwave and millimeter wave communications, radar, and radio astronomy.

PhD Thesis Sergey Smirnov

<http://www.iue.tuwien.ac.at/phd/smirnov/node71.html>



# MODFET (HEMT)

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$$j = nev_d = ne\mu_n E_y$$

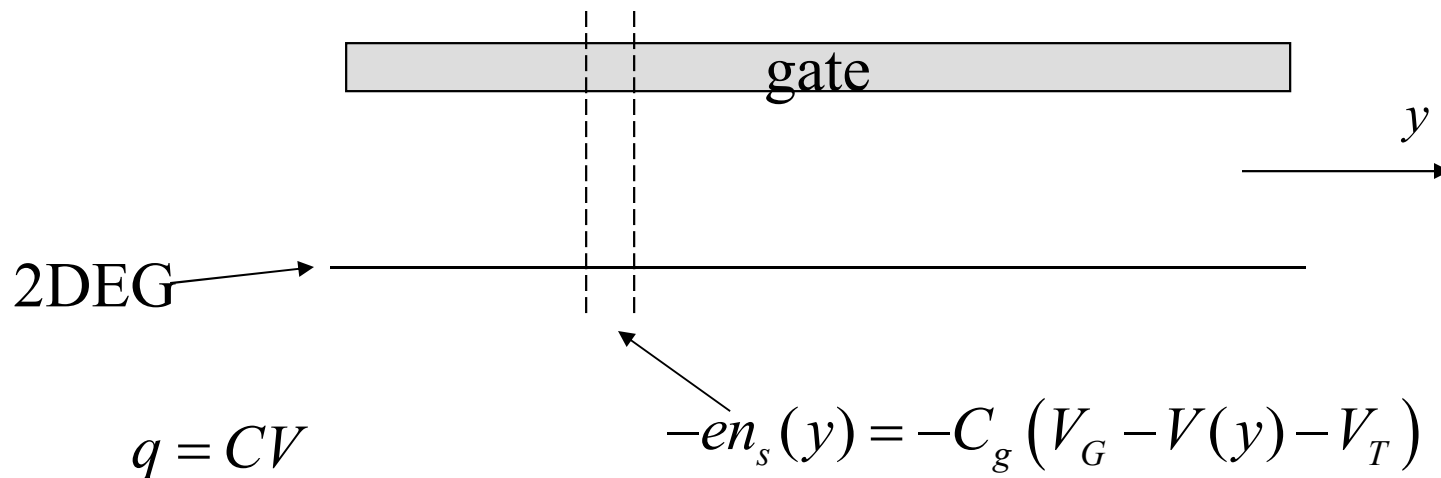
$$I = jZt = Ze\mu_n n_s E_y \quad n = \frac{n_s}{t}$$

$t$  is the thickness of the 2DEG

$n_s$  is the sheet charge at the interface in C/cm<sup>2</sup>.

$V_G - V(y)$  is the voltage between the gate and the 2DEG

$$n_s = 0 \text{ when } V_G - V(y) = V_T$$



# MODFET (HEMT)

---

$-en_s(y) = C_g (V_G - V_B(y) - V_T)$  is the charge on the 2DEG at point  $y$

The charge is zero when  $V_G - V_B(y) = V_T$

solve for  $n_s$

$$n_s(y) = \frac{-C_g (V_G - V_B(y) - V_T)}{e}$$

Substitute this in Ohm's law:

$$I = jZt = Ze\mu_n n_s E_y$$

# MODFET (HEMT)

---

$$I = jZt = Ze\mu_n n_s E_y$$

$$n_s(y) = \frac{-C_g (V_G - V(y) - V_T)}{e}$$

substitute  $n_s$  in the top equation and substitute

$$E_y = \frac{-dV(y)}{dy}$$

$$I = Z\mu_n C (V_G - V_T - V(y)) \frac{dV(y)}{dy}$$

integrate along the length of the channel

$$\int_0^L I dy = \int_0^{V_D} Z\mu_n C (V_G - V_T - V(y)) dV$$

$$I_D = \frac{Z}{L} \mu_n C \left[ (V_G - V_T) V_D - \frac{V_D^2}{2} \right]$$

# MODFET (HEMT)

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$$I = \frac{Z}{L} \mu_n C \left[ (V_G - V_T) V_D - \frac{V_D^2}{2} \right]$$

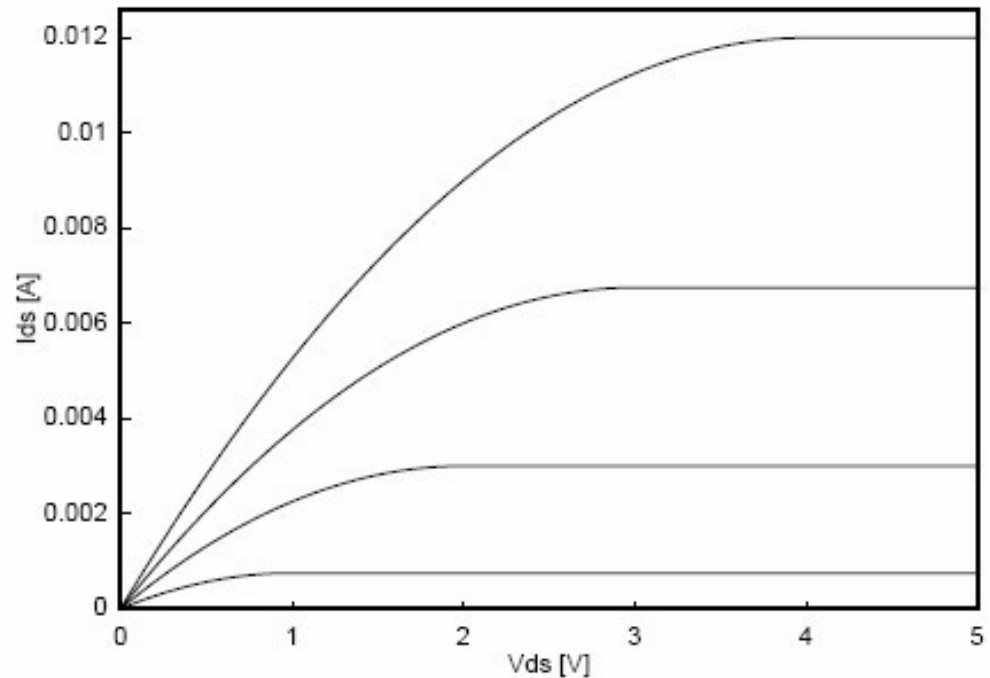
$$\frac{dI}{dV_D} = \frac{Z}{L} \mu_n C \left[ (V_G - V_T) - V_D \right] = 0$$

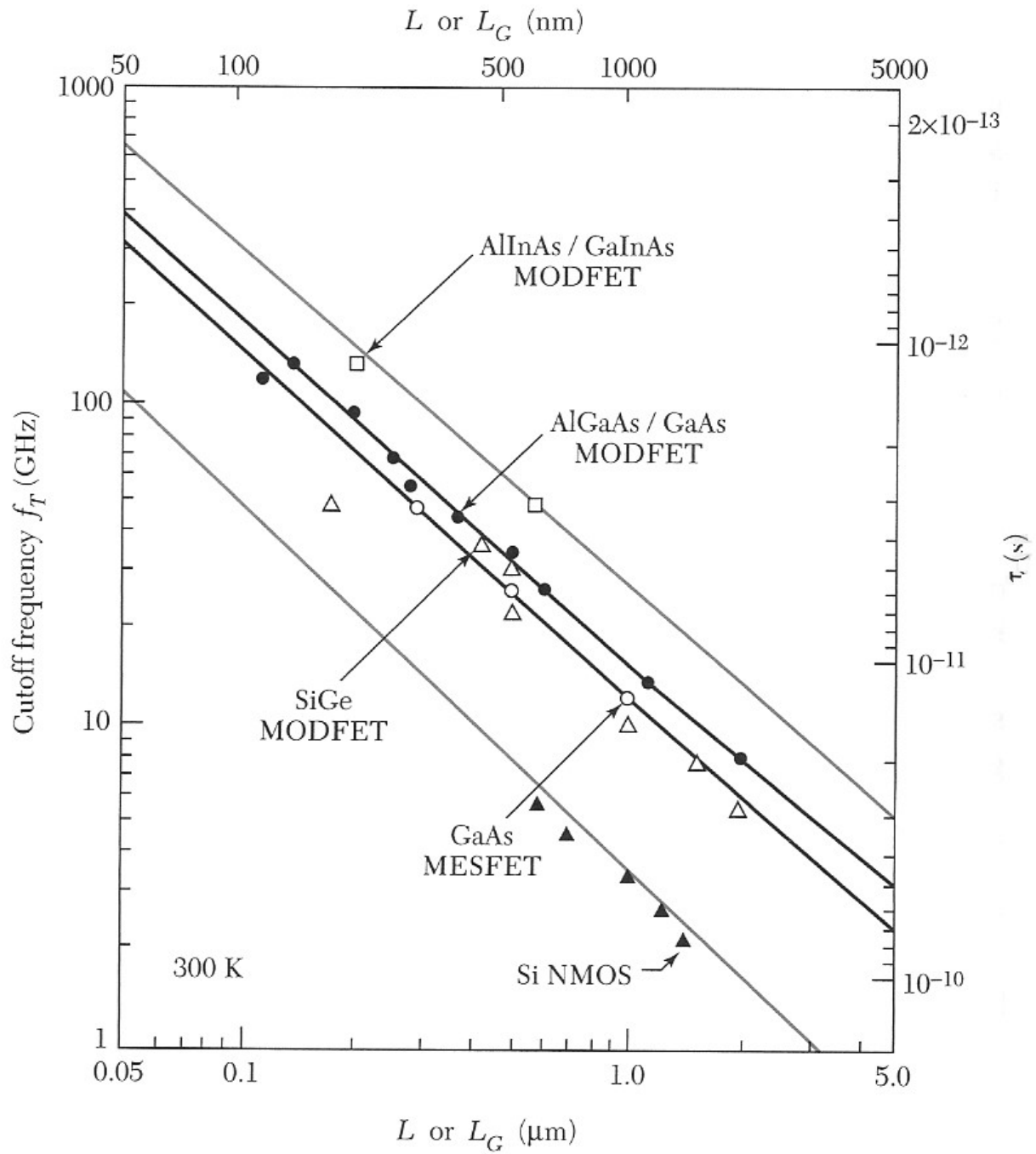
Set the derivative = 0 to find saturation voltage

$$V_{sat} = (V_G - V_T)$$

Substitute the saturation voltage into the formula at the top to find saturation current

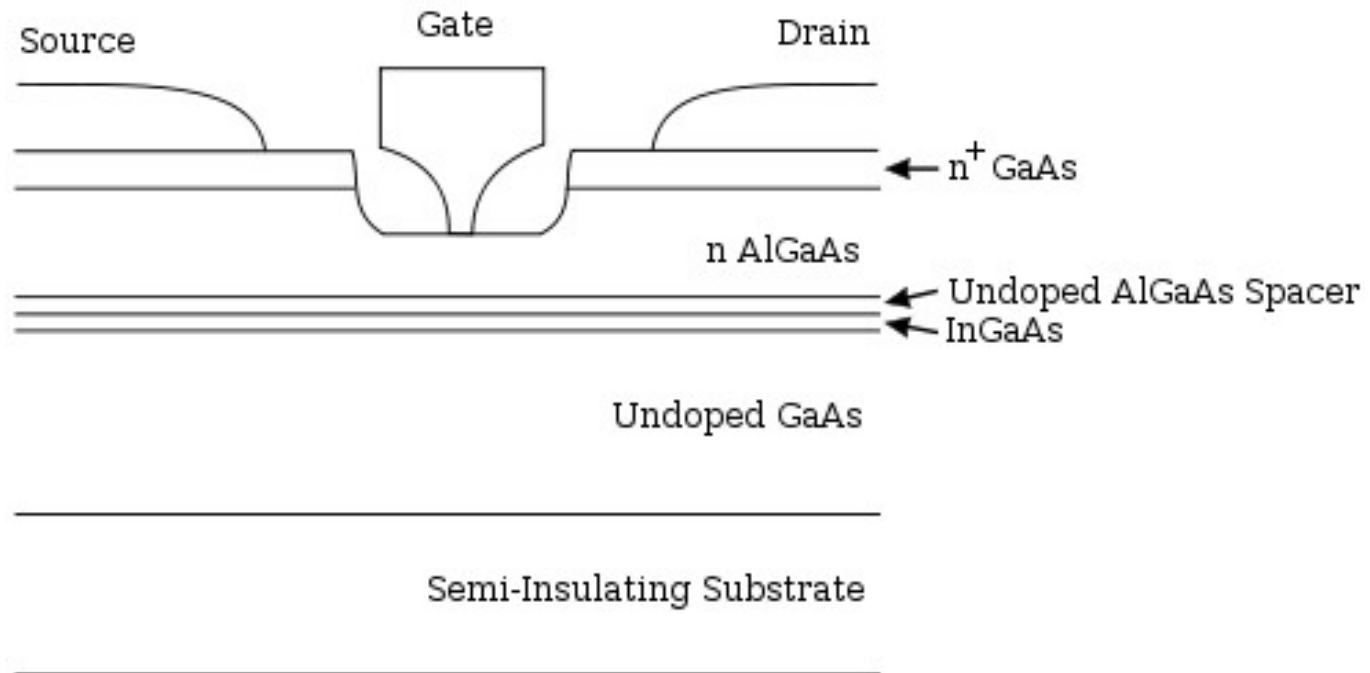
$$I_{sat} = \frac{Z}{2L} \mu_n C (V_G - V_T)^2$$





# MODFET/HEMT

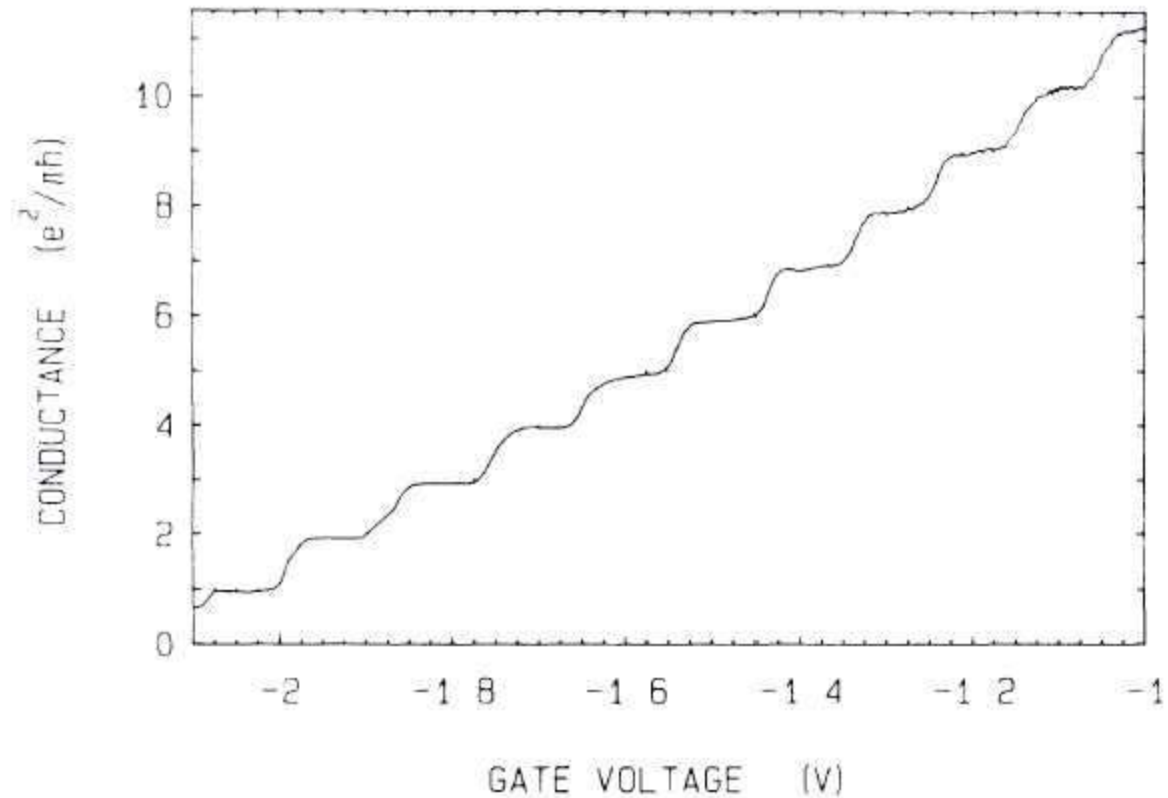
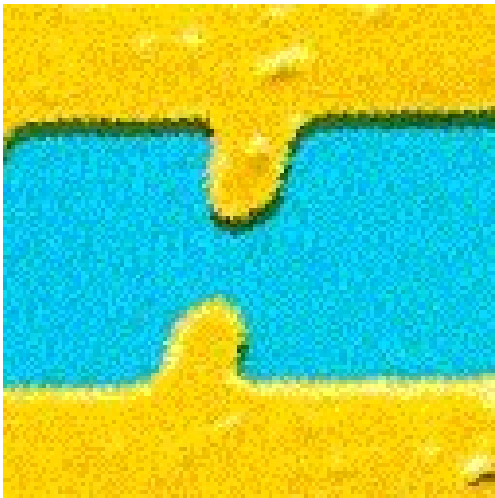
**HEMT:** HEMT devices are found in many types of equipment ranging from cell phones and DBS receivers to electronic warfare systems, microwave and millimeter wave communications, radar, and radio astronomy. 600 GHz



# Quantized conduction

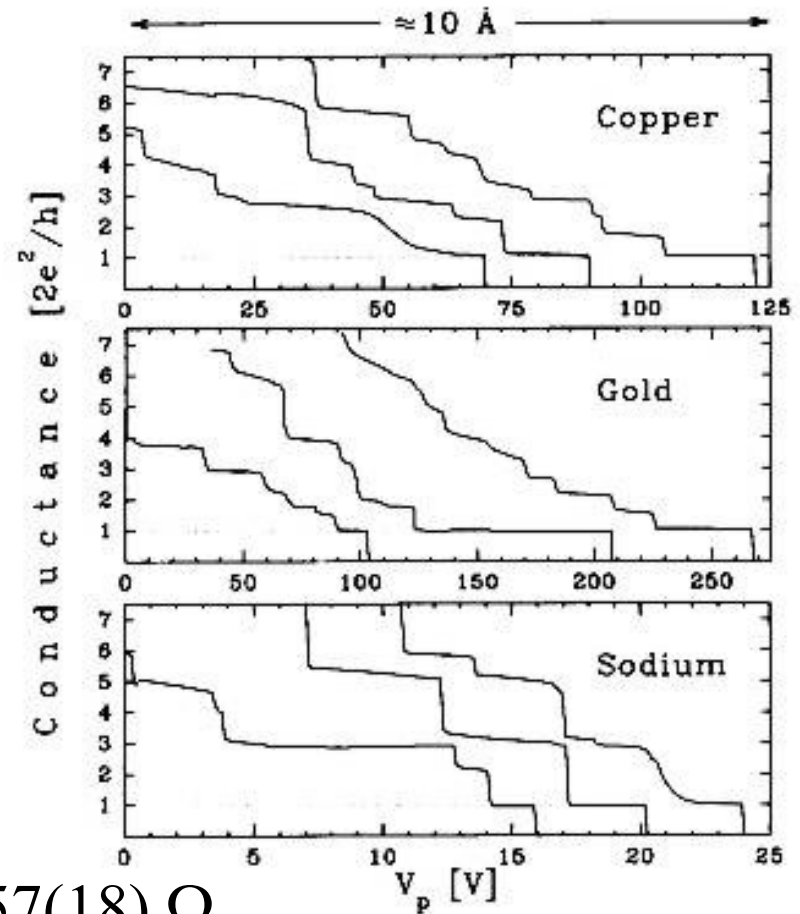
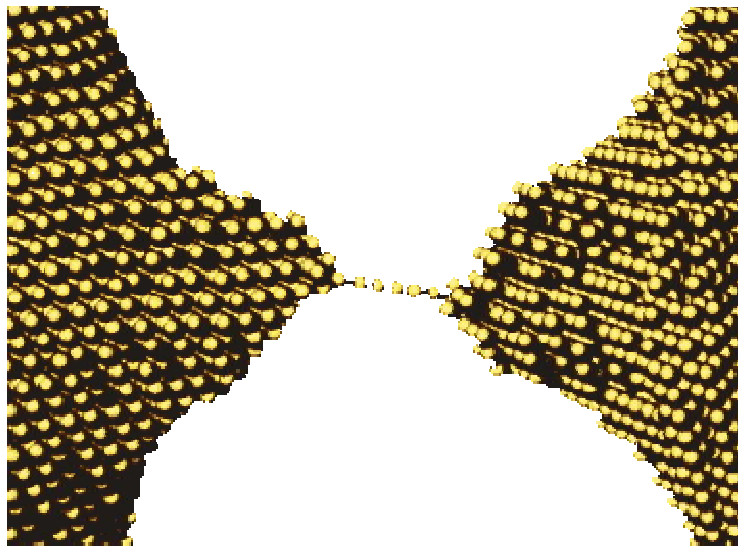
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$$R_K = h/e^2 = 25812.807557(18) \Omega$$



Quantized conductance of point contacts in a two-dimensional electron gas, B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. 60, 848-850 (1988).

# Quantized conduction

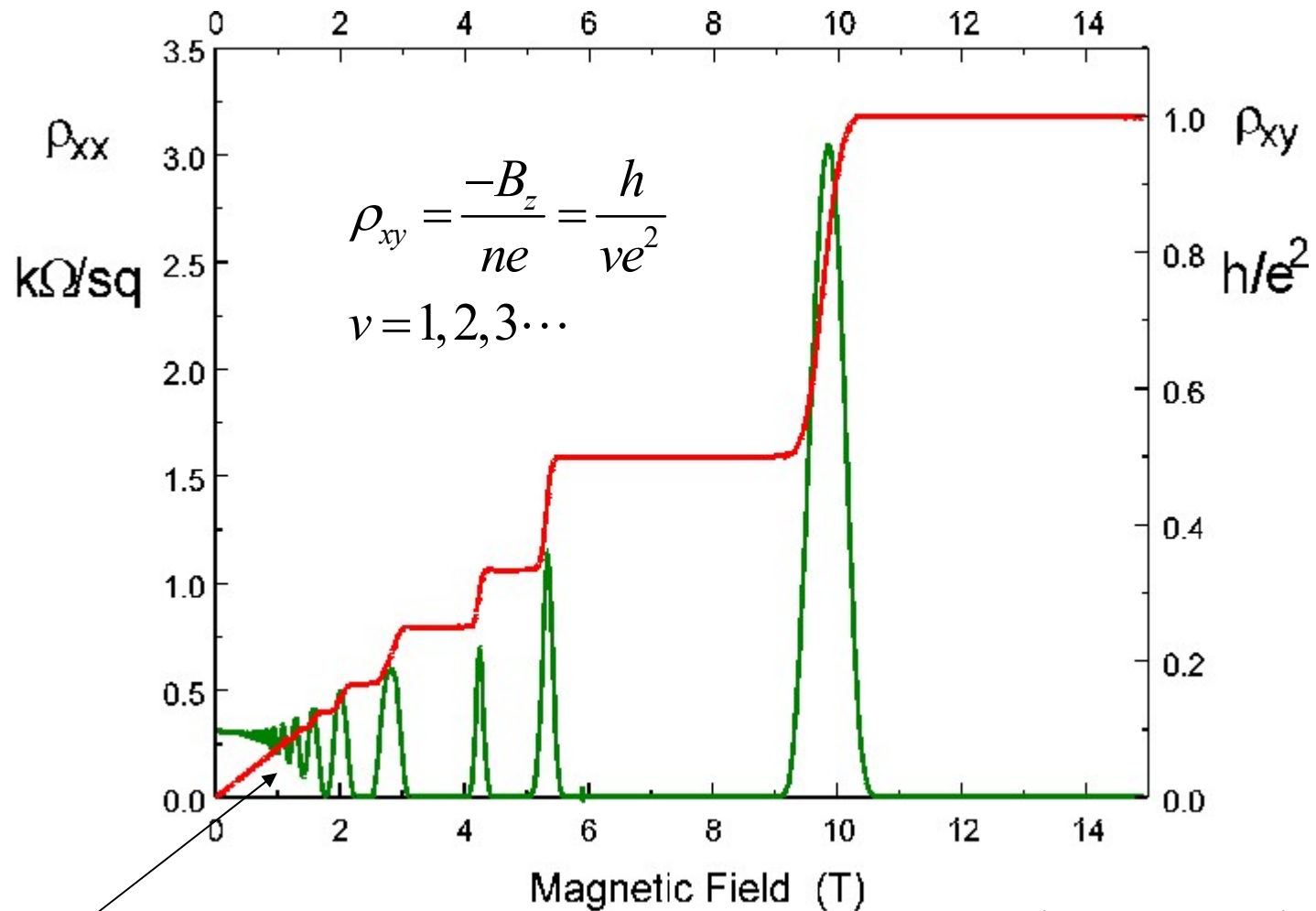


$$R_K = h/e^2 = 25812.807557(18) \Omega$$

Formation and Manipulation of a Metallic Wire of Single Gold Atoms, A. I. Yanson, G. Rubio Bollinger, H.E. van den Brom, N. Agrait, J.M. van Ruitenbeek, Nature Oct. 1998.



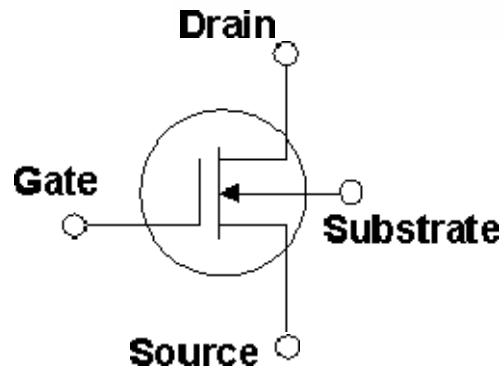
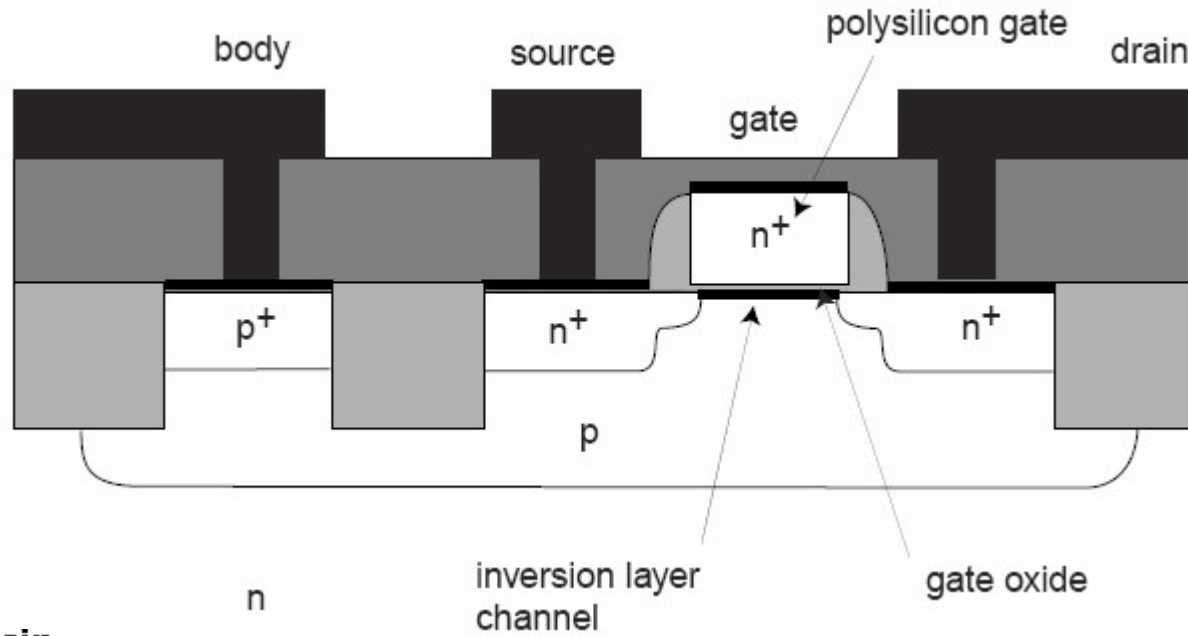
# Quantum Hall Effect



Shubnikov-De Haas oscillations

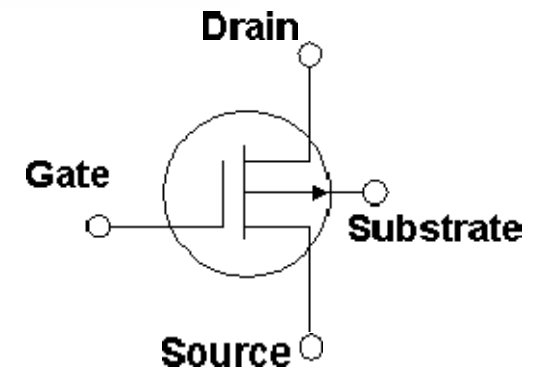
Resistance standard  
25812.807557(18)  $\Omega$

# MOSFETs



n - channel

functions as a switch  
 ~ 1 billion /chip



p - channel