# Exams

February 3
March 3
April 28
June 30

# Exam

Four questions, two from the online list.

Calculator is ok. No notes.

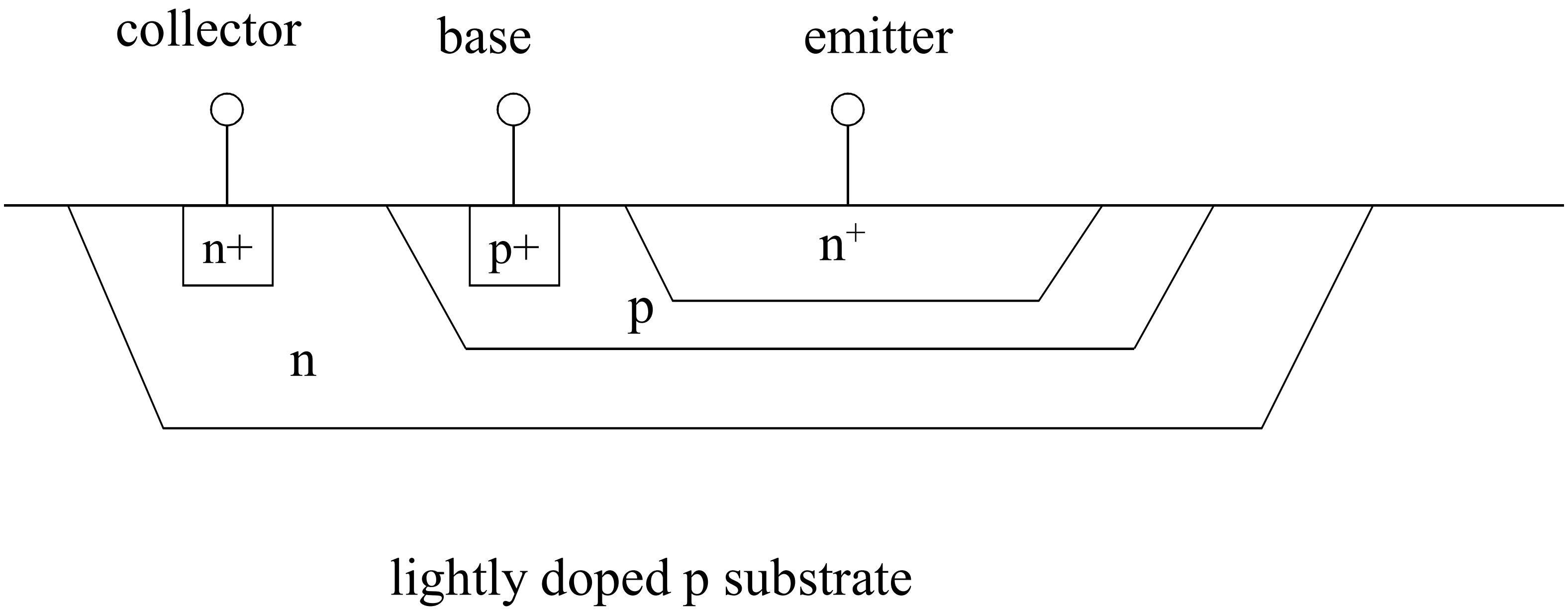
Explain some concept:
(tunnel contact, indirect band gap, thermionic emission, inversion, threshold voltage, ...)

Perform a calculation:
(concentration of minority carriers, integrate charge density to find electric field, ...)

Explain how a device works:
(JFET, MESFET, MOSFET, laser diode, bipolar transistor, LED, Schottky diode, Heterojunction bipolar transistor, ...)

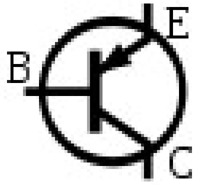
# bipolar transistors

npn transistor

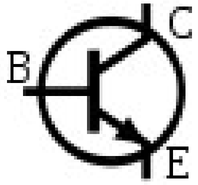


Used in front-end high-frequency receivers (mobile telephones).

# bipolar transistors



PNP



NPN

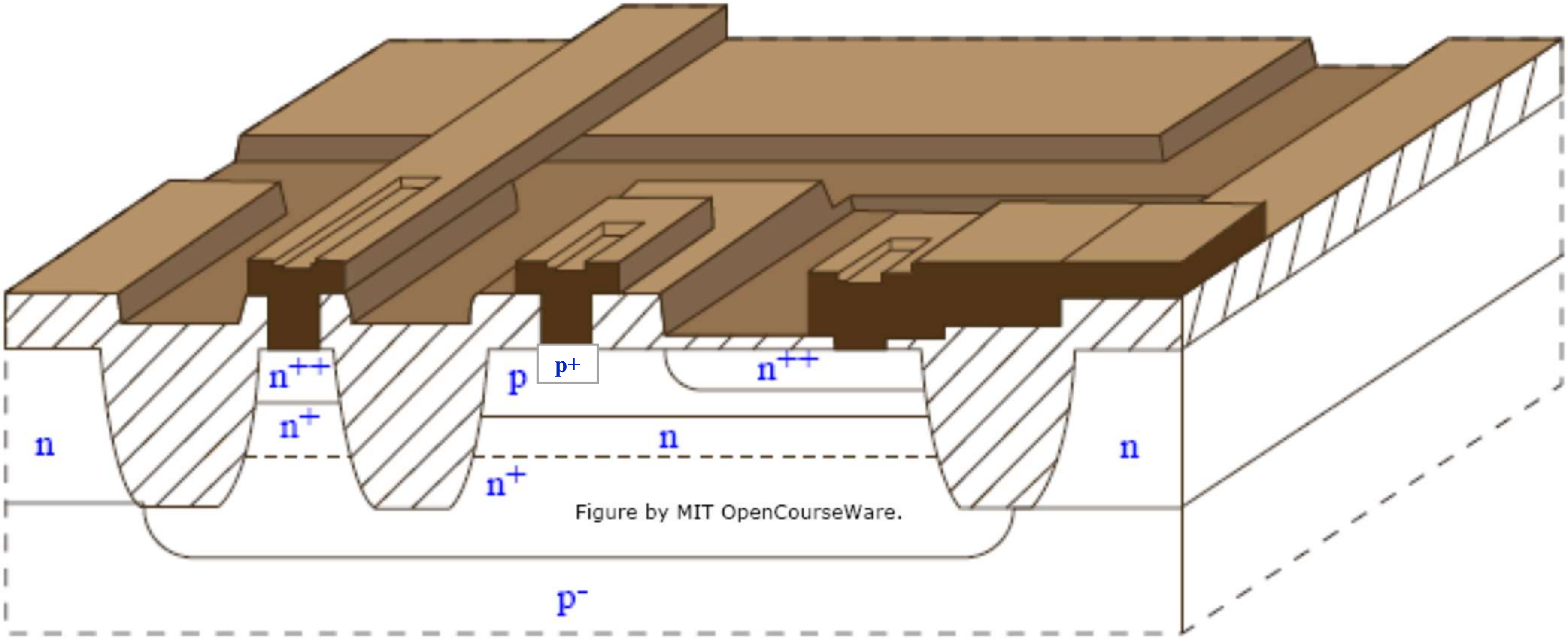
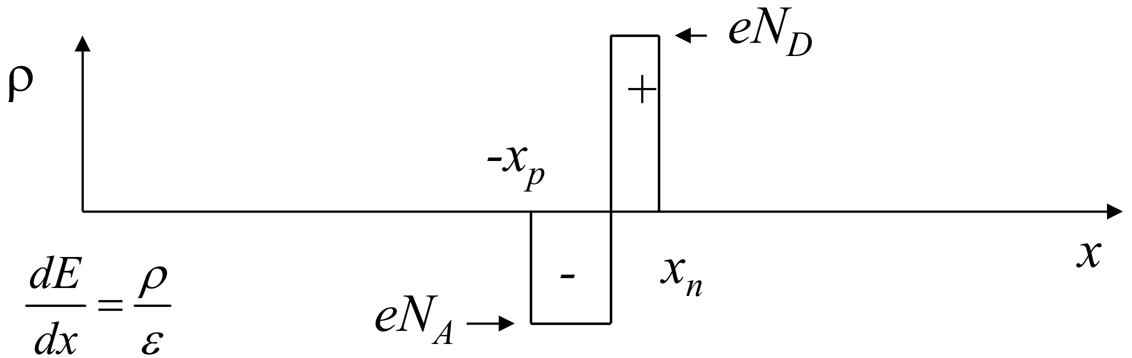


Figure by MIT OpenCourseWare.

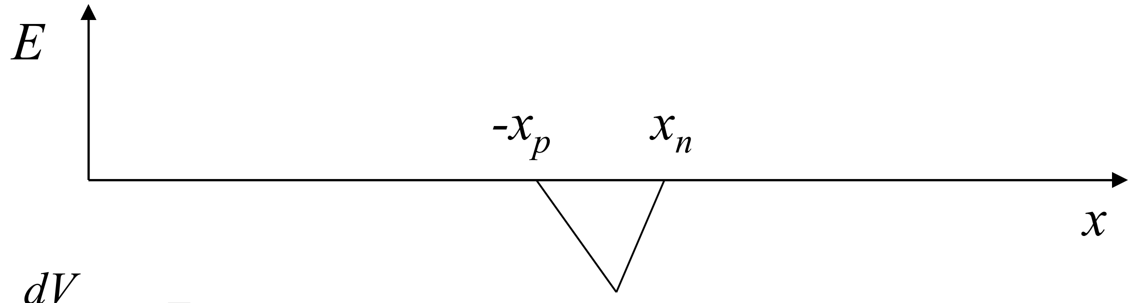
Oxide isolated integrated BJT - a modern process

# abrupt junction

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right)$$



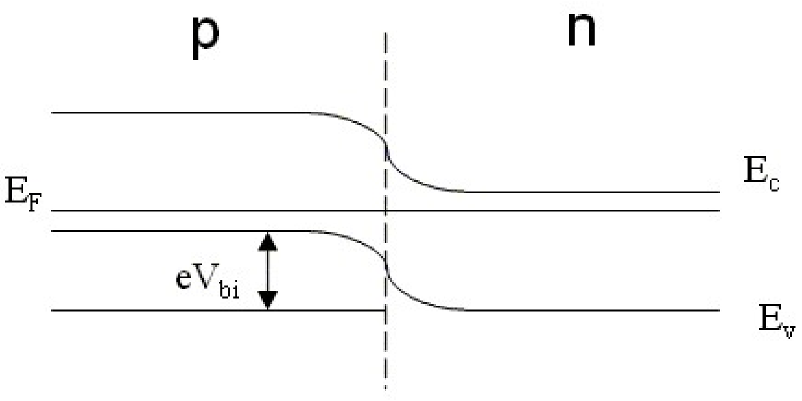
$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$



$$E = -\frac{eN_A}{\varepsilon}\left(x + x_p\right) \qquad -x_p > x > 0$$

$$E = \frac{eN_D}{\varepsilon}\left(x - x_n\right) \qquad 0 > x > x_n$$

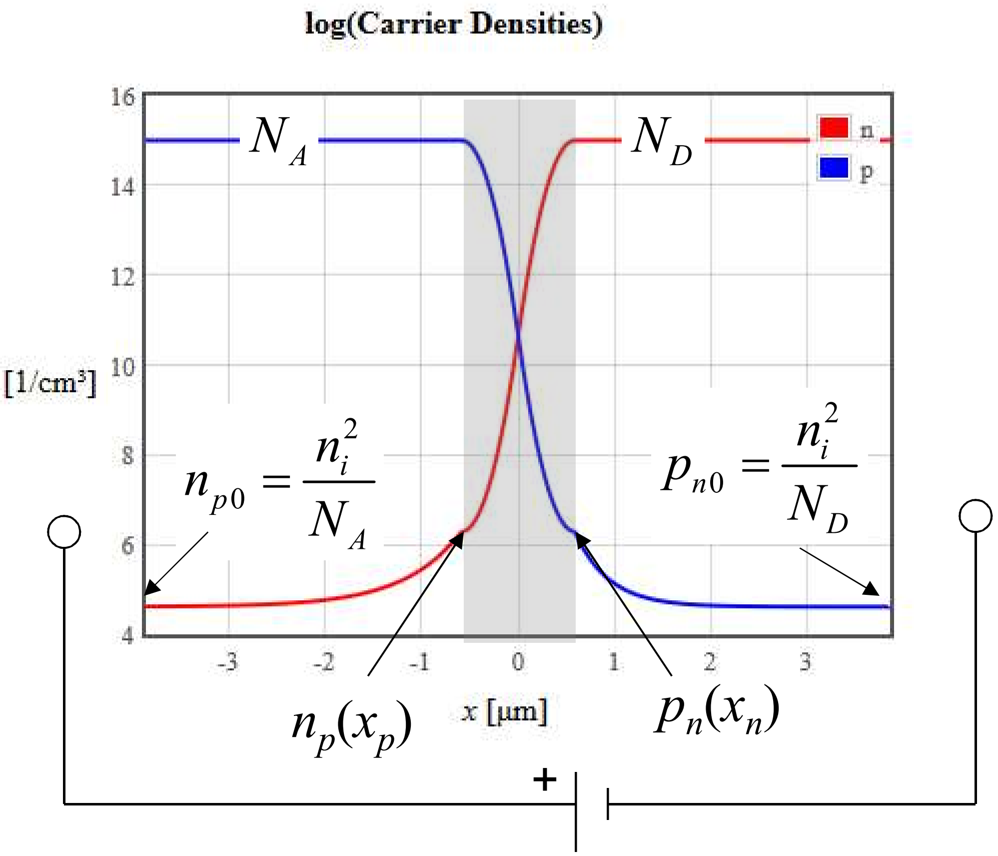
$$\frac{dV}{dx} = -E$$



$$V = \frac{eN_A}{\varepsilon}\left(\frac{x^2}{2} + xx_p\right) \qquad -x_p > x > 0$$

$$V = \frac{-eN_D}{\varepsilon}\left(\frac{x^2}{2} - xx_n\right) \qquad 0 > x > x_n$$

# Forward bias, V > 0



Electrons and holes are driven towards the junction.

The depletion region becomes narrower

$$n_p(x_p) = N_D \exp\left(\frac{-e\left(V_{bi} - V\right)}{k_B T}\right)$$

$$= n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

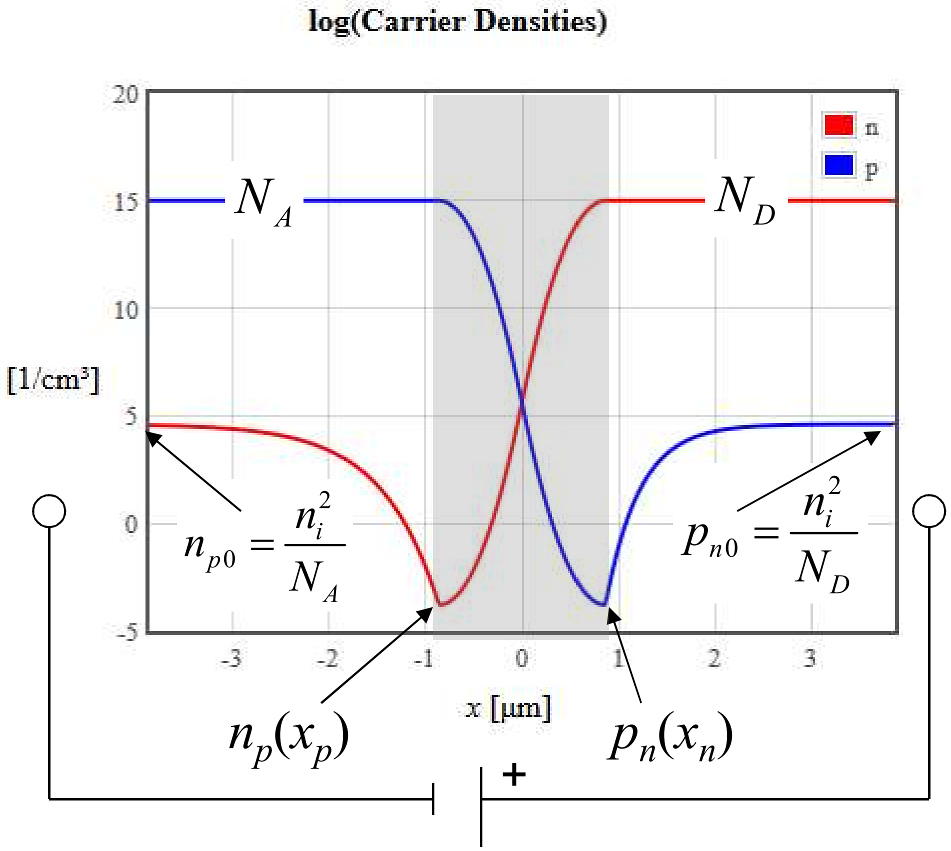
$$p_n(x_n) = N_A \exp\left(\frac{-e\left(V_{bi} - V\right)}{k_B T}\right)$$

$$= p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

Minority electrons are injected into the p-region

Minority holes are injected into the n-region

# Reverse bias, V < 0



Electrons and holes are driven away from the junction.
The depletion region becomes wider

$$n_p(x_p) = N_D \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$

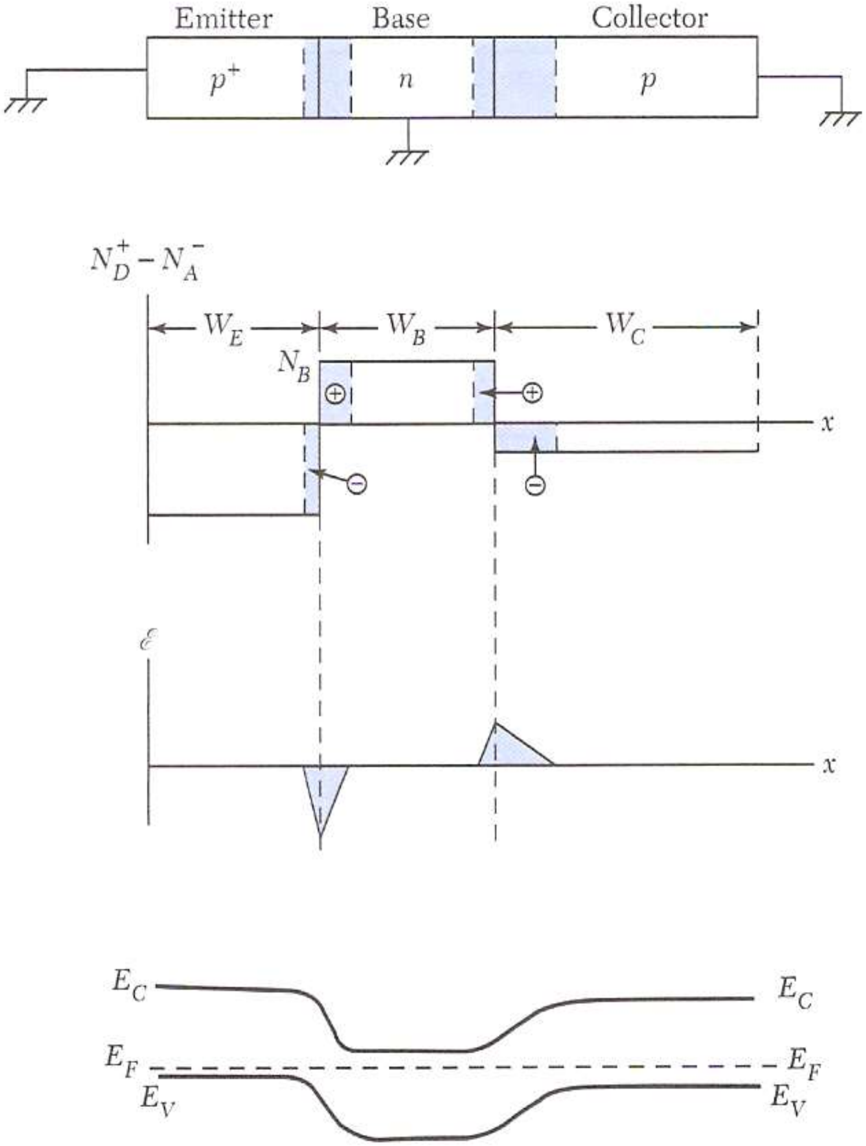
$$= n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_n(x_n) = N_A \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$

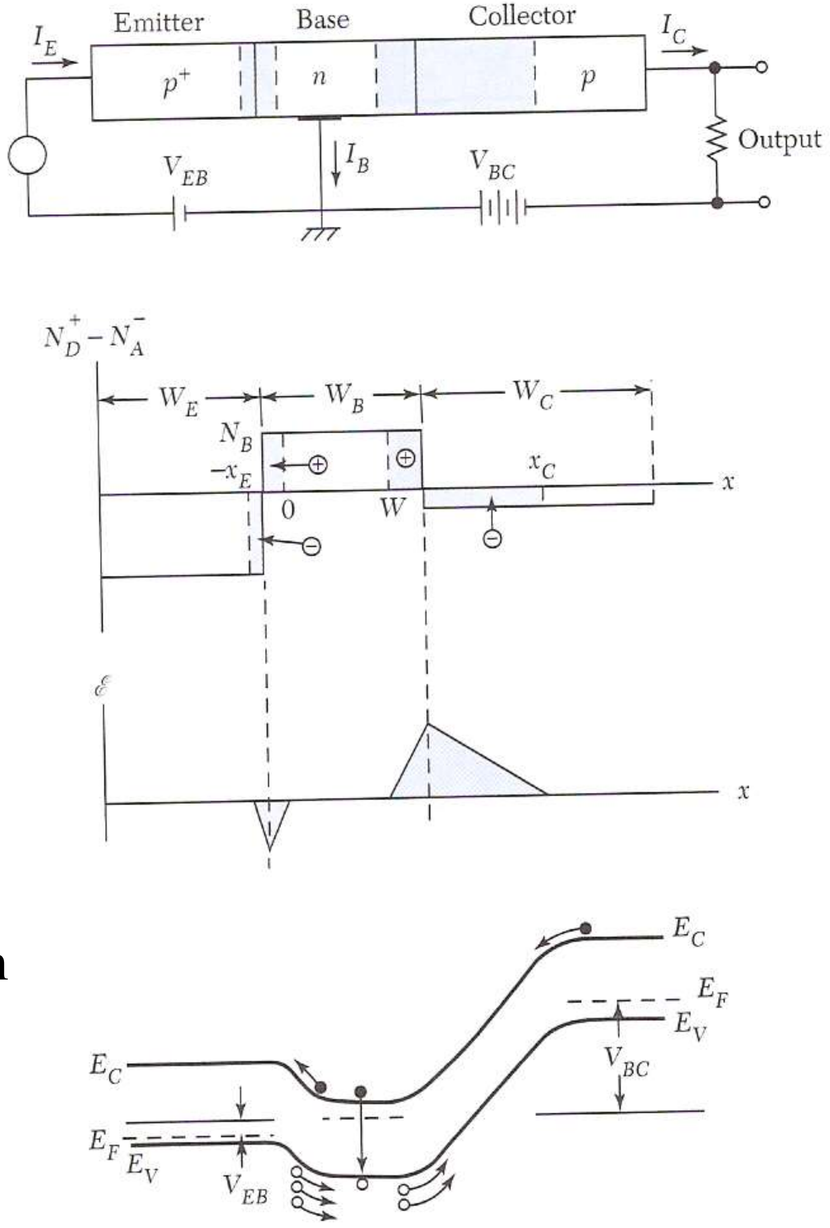
$$= p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

Minority electrons are extracted from the p-region by the electric field
Minority holes are extracted from the n-region by the electric field

# pnp transistor, no bias



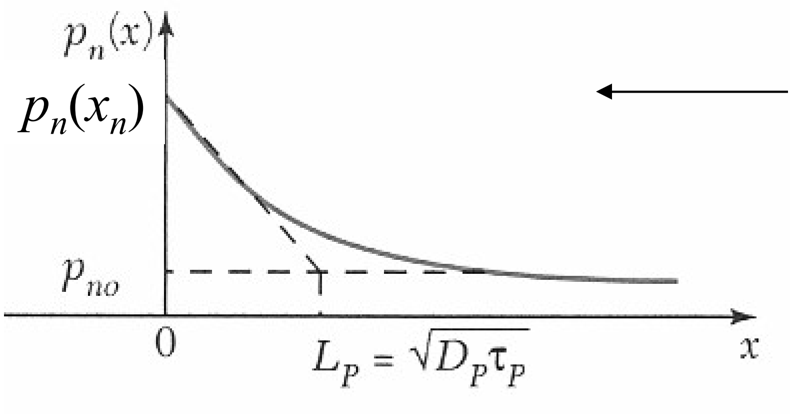
# pnp transistor, forward active bias



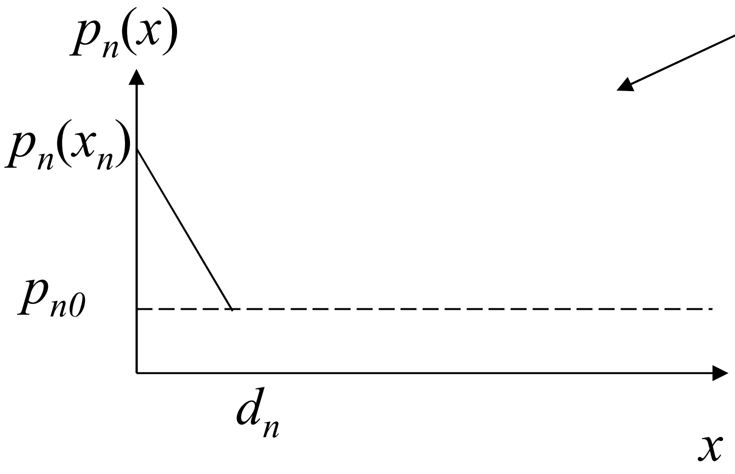
Always dissipate power due to the forward bias

The base-emitter voltage controls the minority carriers injected from the emitter to the base. These diffuse to the base-collector junction and are swept into the collector.

# Long/Short diode



Long diode $d_n >> L_p$



Short diode

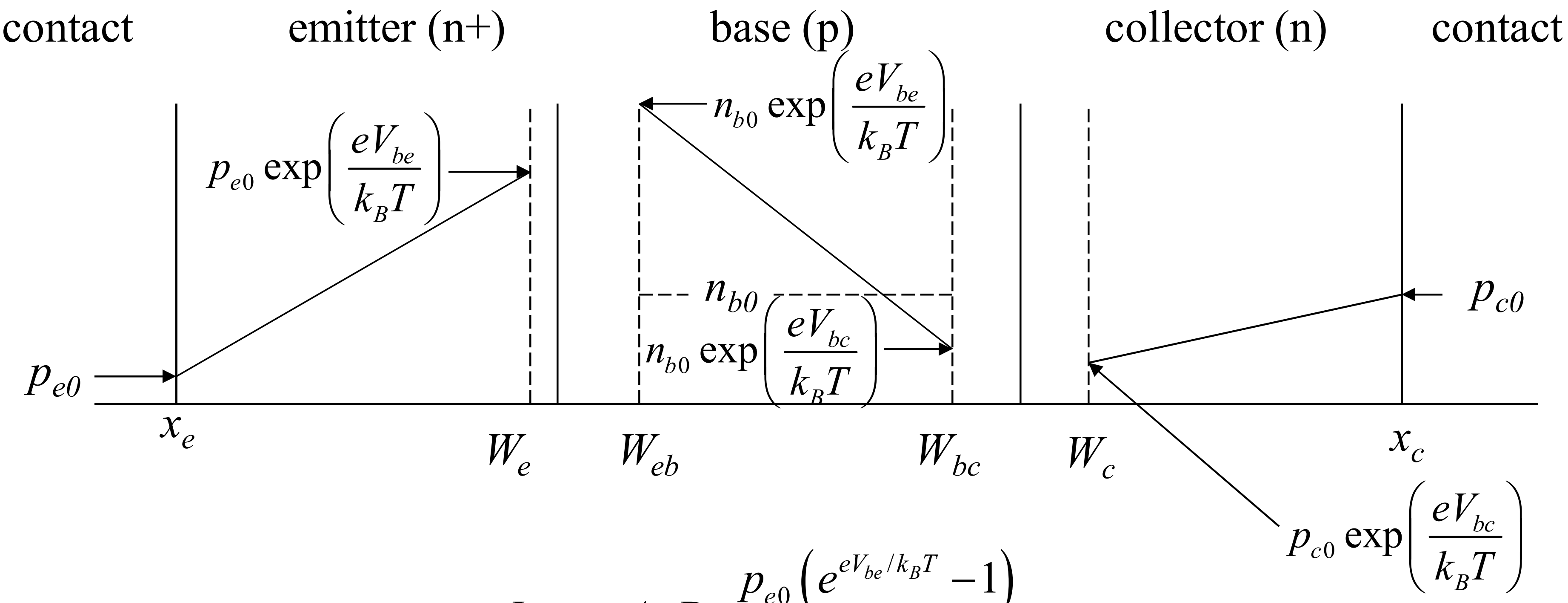
$d_n << L_p$

Metal contact is much closer to the depletion region than the diffusion length

$$J_{diff,p} = eD_p \frac{dp}{dx}$$

$$J_{diff,p} = eD_p \frac{dp}{dx} = eD_p \frac{\left(p_n(x_n) - p_{n0}\right)}{d_n}$$

# Minority carrier concentration



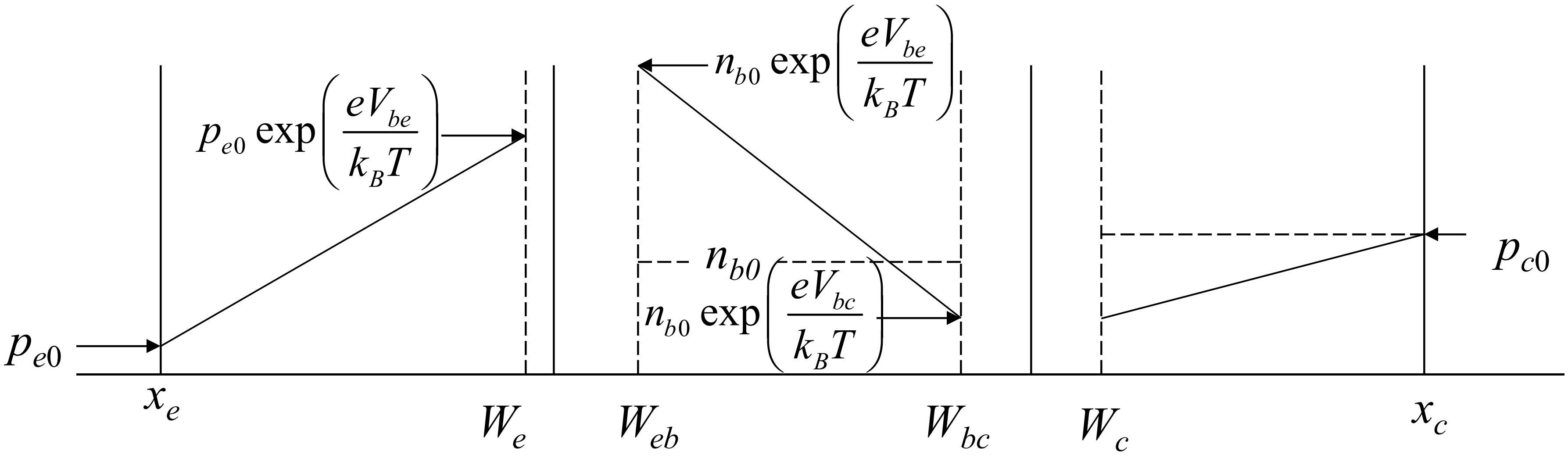
$$I_{Ep} = eA_{be}D_p \frac{p_{e0}\left(e^{eV_{be}/k_BT} - 1\right)}{W_e - x_e}$$

$$I_{En} = -eA_{be}D_n \frac{n_{b0}\left(e^{eV_{be}/k_BT} - e^{eV_{bc}/k_BT}\right)}{W_{bc} - W_{be}}$$

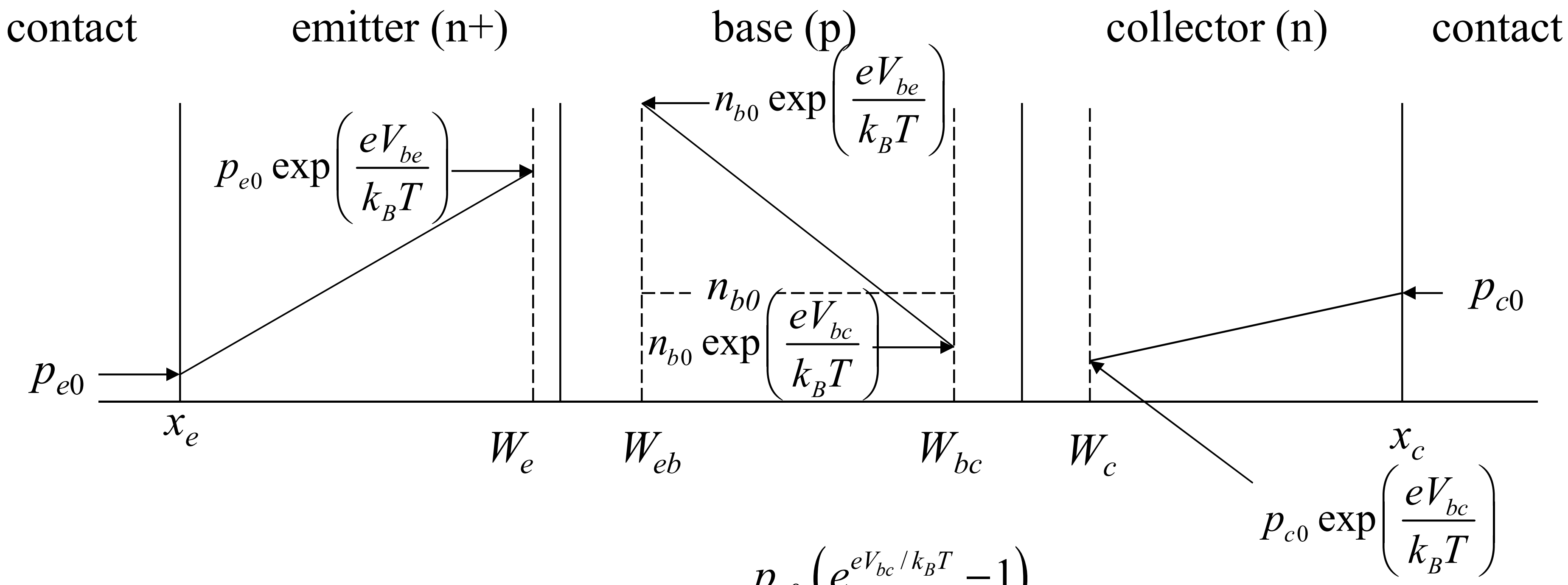
# Emitter current

$$I_E = I_{En} + I_{Ep} = \left[\frac{eA_{be}D_p p_{e0}}{W_{eb}-x_e} + \frac{eA_{be}D_n n_{b0}}{W_{bc}-W_{be}}\right]\left(e^{eV_{be}/k_BT}-1\right) - \frac{eA_{be}D_n n_{b0}}{W_{bc}-W_{be}}\left(e^{eV_{bc}/k_BT}-1\right)$$

$$I_E = I_{ES}\left(e^{eV_{be}/k_BT}-1\right) - \alpha_R I_{CS}\left(e^{eV_{bc}/k_BT}-1\right)$$



# Collector current



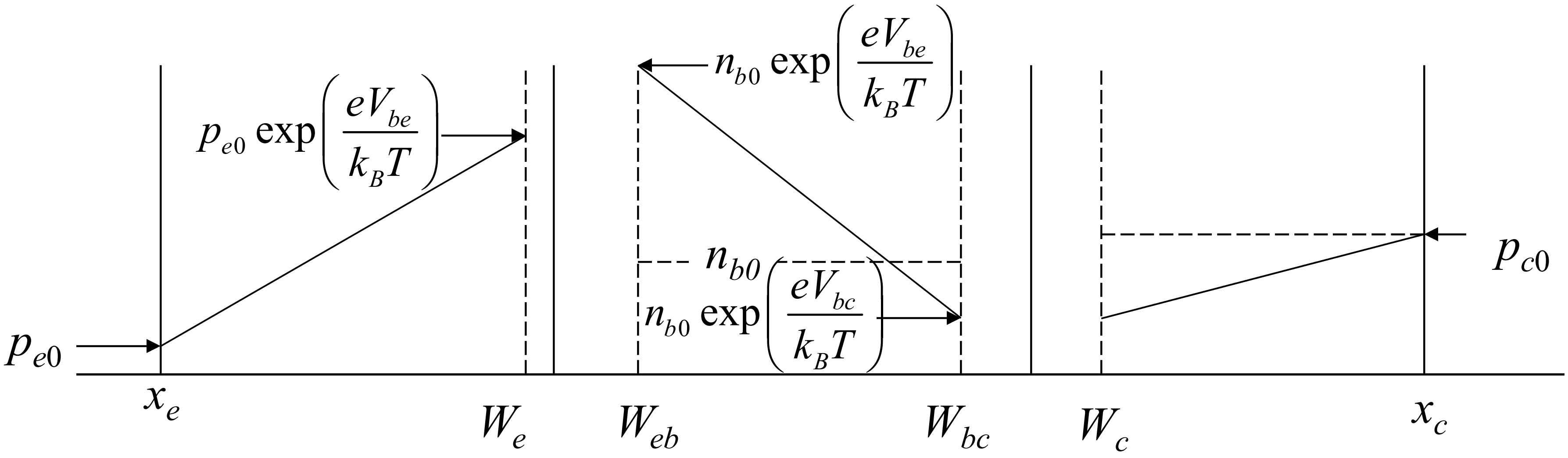
$$I_{cp} = -eA_{bc}D_p \frac{p_{c0}\left(e^{eV_{bc}/k_BT}-1\right)}{x_c - W_c}$$

$$I_{cn} = -eA_{bc}D_n \frac{n_{b0}\left(e^{eV_{be}/k_BT}-e^{eV_{bc}/k_BT}\right)}{W_{bc}-W_{eb}}$$

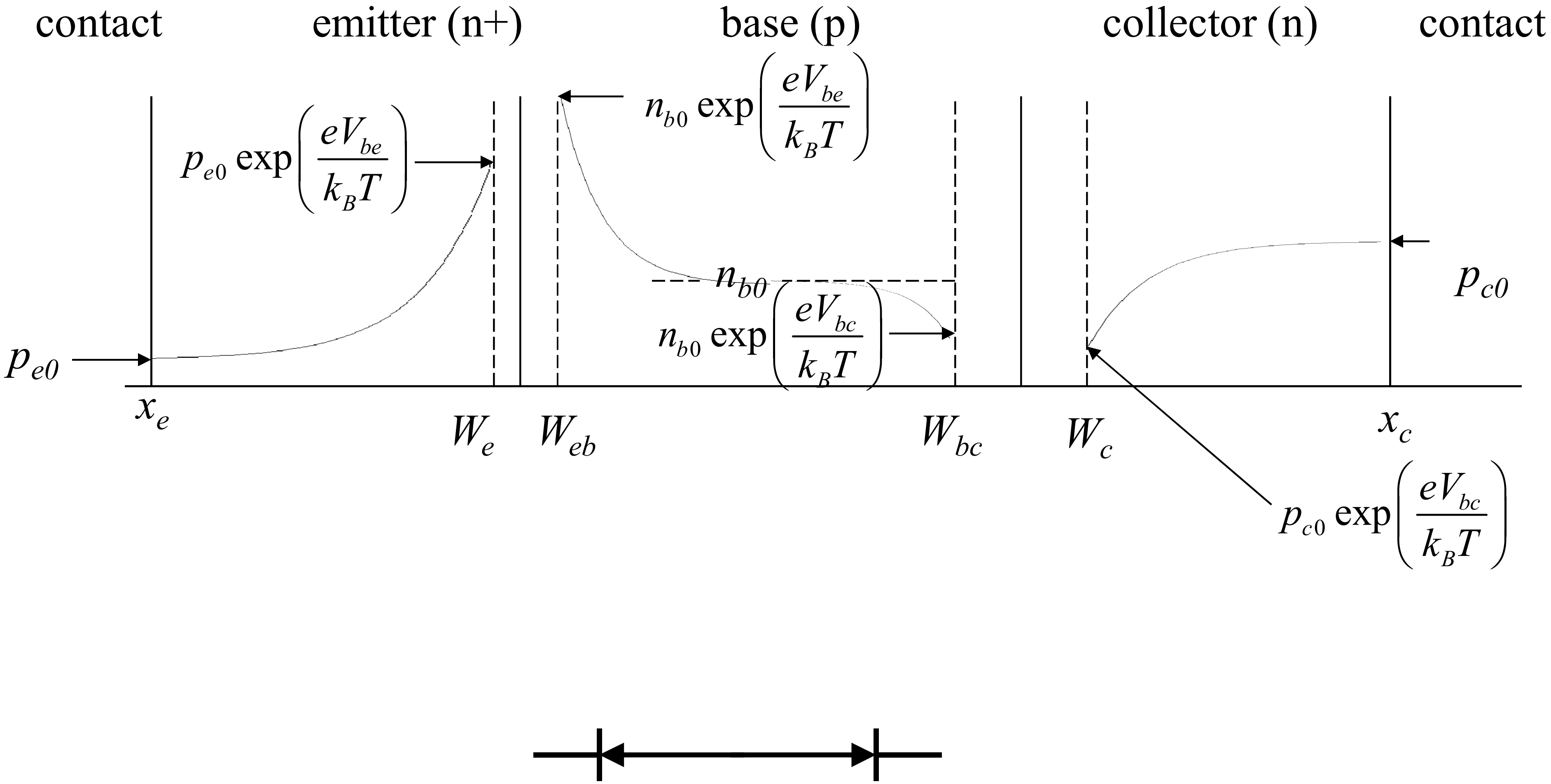
# Collector current

$$I_c = I_{cp} + I_{cn} = \frac{eA_{bc}D_n n_{b0}}{W_{bc} - W_{be}}\left(e^{eV_{be}/k_BT} - 1\right) - \left[\frac{eA_{bc}D_p p_{c0}}{x_c - W_c} + \frac{eA_{bc}D_n n_{b0}}{W_{bc} - W_{be}}\right]\left(e^{eV_{bc}/k_BT} - 1\right)$$

$$I_c = I_{cp} + I_{cn} = \alpha_F I_{ES}\left(e^{eV_{be}/k_BT} - 1\right) - I_{CS}\left(e^{eV_{bc}/k_BT} - 1\right)$$



# Not an npn transistor

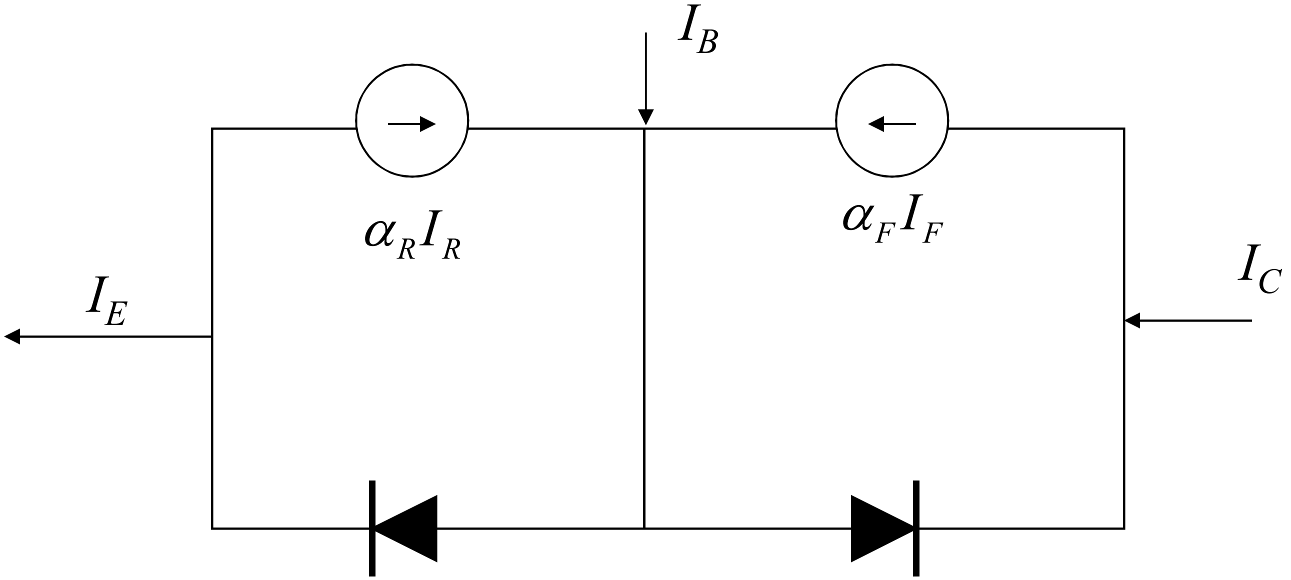


# Ebers-Moll model

$$I_E = I_{ES}\left(e^{eV_{be}/k_BT} - 1\right) - \alpha_R I_{CS}\left(e^{eV_{bc}/k_BT} - 1\right)$$

$$I_C = \alpha_F I_{ES}\left(e^{eV_{be}/k_BT} - 1\right) - I_{CS}\left(e^{eV_{bc}/k_BT} - 1\right)$$

$$I_B = I_E - I_C$$



$$I_F = I_{ES}\left(e^{eV_{be}/k_BT} - 1\right) \qquad I_R = I_{CS}\left(e^{eV_{bc}/k_BT} - 1\right)$$

# Emitter efficiency

$$\gamma_e = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{1}{1 + I_{Ep} / I_{En}}$$

for npn

$$I_{Ep} = eA_{be} D_p \frac{p_{e0}\left(e^{eV_{be}/k_B T} - 1\right)}{W_{eb} - x_e}$$

$$I_{En} = -eA_{be} D_n \frac{n_{b0}\left(e^{eV_{be}/k_B T} - e^{eV_{bc}/k_B T}\right)}{W_{bc} - W_{be}}$$

For $\gamma_e \sim 1$, $W_{bc} - W_{be} << L_b$, $W_{eb} - x_e$ and $n_{b0} >> p_{e0}$

neutral base width

$$\frac{n_i^2}{N_{Ab}}$$

$$\frac{n_i^2}{N_{De}}$$

Small base width and heavy emitter doping

# Base transport factor

$$B = \frac{I_c}{I_{En}}$$

ratio of the injected current to the collected current

recombination in the base would reduce the base transport factor

A thin base with low doping results in a base transport factor ~ 1

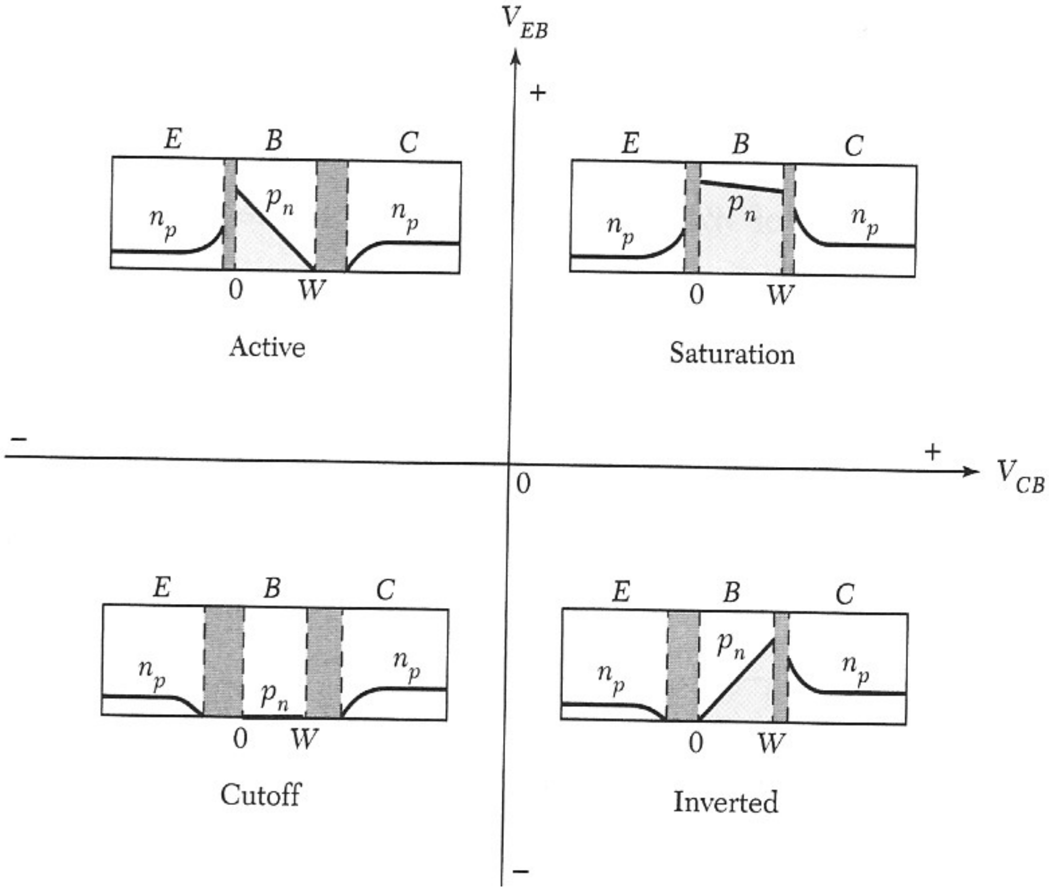
# Current transfer ratio

$$\alpha = \frac{I_C}{I_E} = B\gamma_e$$

$\alpha \sim 1$ for a good BJT

# Transistor modes

1. Forward active: emitter-base **forward**, base-collector **reverse**
2. Saturation: emitter-base **forward**, base-collector **forward**
3. Reverse active: emitter-base **reverse**, base-collector **forward**
4. Cut-off: emitter-base **reverse**, base-collector **reverse**

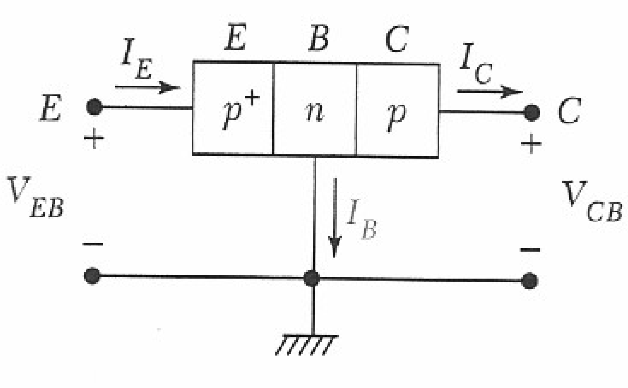


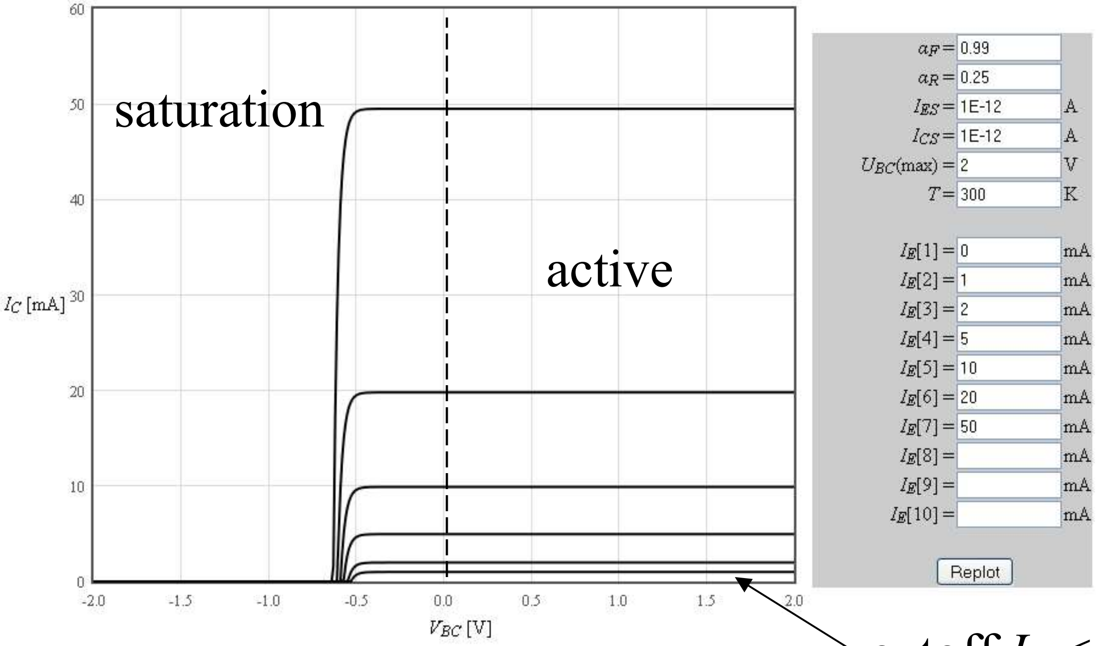
# Common base configuration

$$I_E = I_{ES}\left(e^{eV_{be}/k_BT}-1\right)-\alpha_R I_{CS}\left(e^{eV_{bc}/k_BT}-1\right)$$

solve for $V_{be}$

$$I_c = \alpha_F I_{ES}\left(e^{eV_{be}/k_BT}-1\right)-I_{CS}\left(e^{eV_{bc}/k_BT}-1\right)$$





saturation

active

cutoff $I_E < 0$

$\alpha_F =$ 0.99

$\alpha_R =$ 0.25

$I_{ES} =$ 1E-12 A

$I_{CS} =$ 1E-12 A

$U_{BC}$(max) = 2 V

$T =$ 300 K

$I_E[1] =$ 0 mA

$I_E[2] =$ 1 mA

$I_E[3] =$ 2 mA

$I_E[4] =$ 5 mA

$I_E[5] =$ 10 mA

$I_E[6] =$ 20 mA

$I_E[7] =$ 50 mA

$I_E[8] =$ mA

$I_E[9] =$ mA

$I_E[10] =$ mA

http://lamp.tu-graz.ac.at/~hadley/psd/L13/commonbase/pnp_current.html

# Ebers - Moll Model

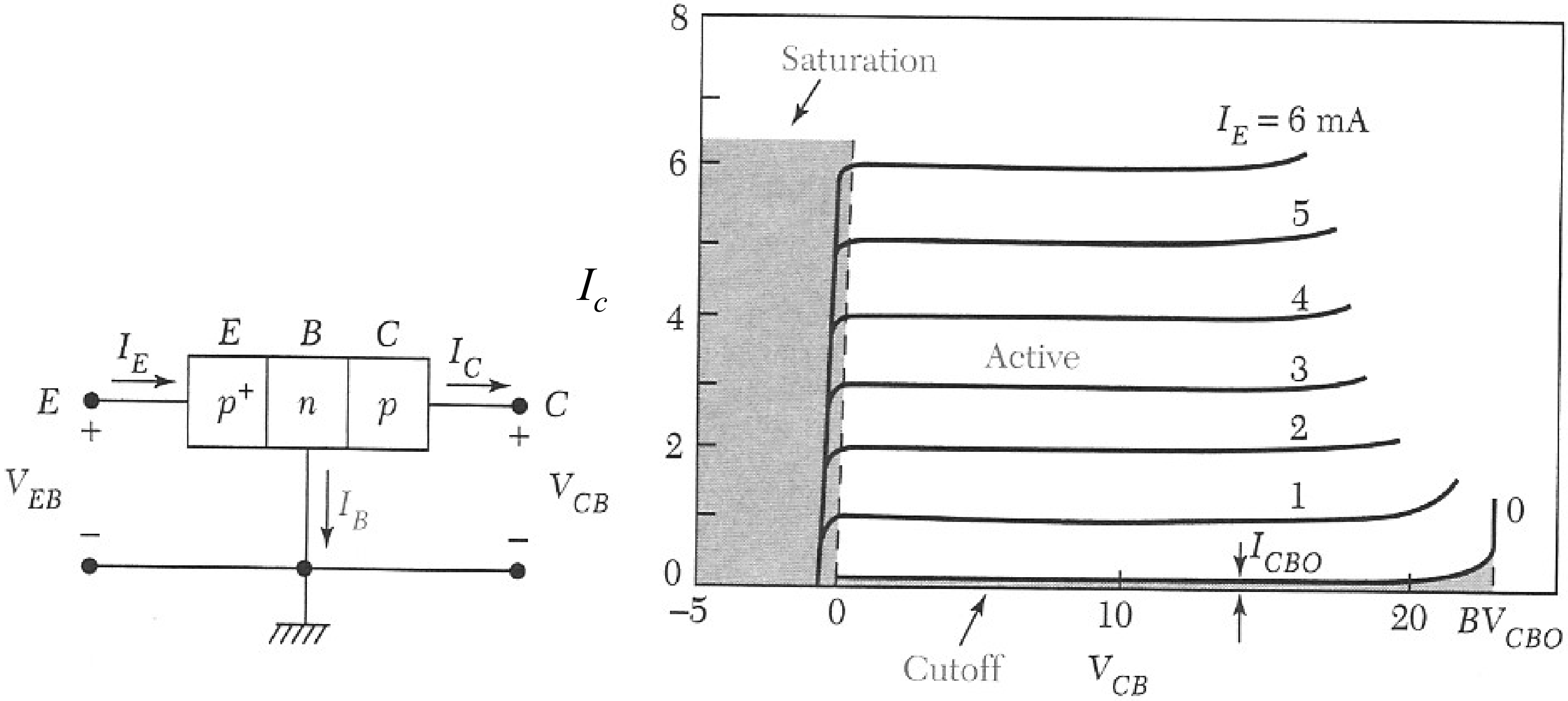
$$I_{ES} = \left[ \frac{eA_{be} D_p p_{e0}}{W_{eb} - x_e} + \frac{eA_{be} D_n n_{b0}}{W_{bc} - W_{be}} \right]$$

$$\alpha_R I_{CS} = \frac{eA_{be} D_n n_{b0}}{W_{bc} - W_{be}}$$

$$\alpha_F I_{ES} = \frac{eA_{bc} D_n n_{b0}}{W_{bc} - W_{be}}$$

$$I_{CS} = \left[ \frac{eA_{bc} D_p p_{c0}}{x_c - W_c} + \frac{eA_{bc} D_n n_{b0}}{W_{bc} - W_{be}} \right]$$

# Common base configuration

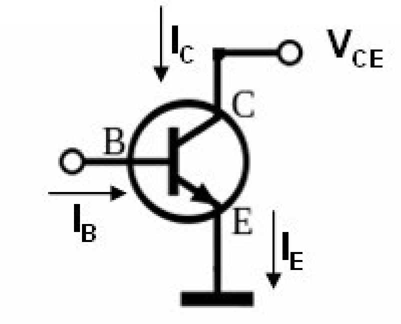


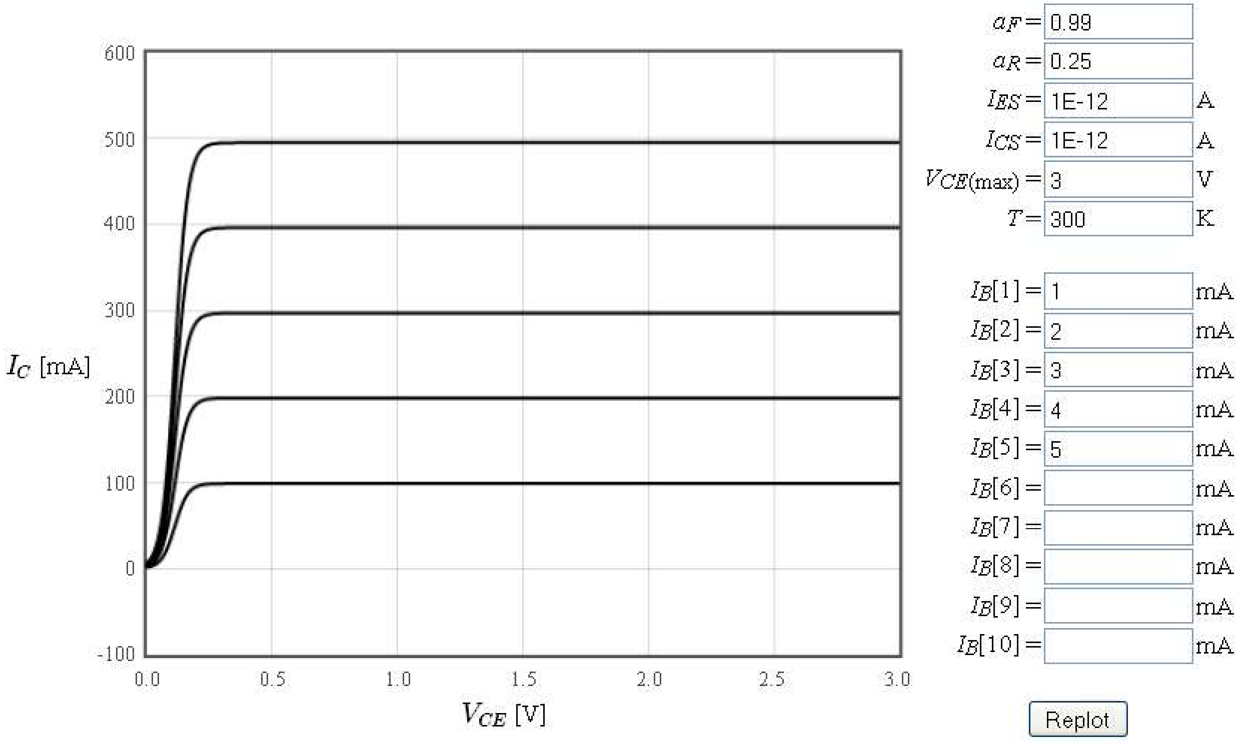
$I_C \sim I_E$ buffer circuit: the output current is constant over a wide range of output voltages

# Common emitter configuration

$$I_E = I_{ES}\left(e^{eV_{be}/k_BT} - 1\right) - \alpha_R I_{CS}\left(e^{eV_{bc}/k_BT} - 1\right) \qquad I_B = I_E - I_C$$

$$I_c = \alpha_F I_{ES}\left(e^{eV_{be}/k_BT} - 1\right) - I_{CS}\left(e^{eV_{bc}/k_BT} - 1\right)$$





$\alpha_F =$ 0.99

$\alpha_R =$ 0.25

$I_{ES} =$ 1E-12 A

$I_{CS} =$ 1E-12 A

$V_{CE}(\max) =$ 3 V

$T =$ 300 K

$I_B[1] =$ 1 mA

$I_B[2] =$ 2 mA

$I_B[3] =$ 3 mA

$I_B[4] =$ 4 mA

$I_B[5] =$ 5 mA

$I_B[6] =$ mA

$I_B[7] =$ mA

$I_B[8] =$ mA

$I_B[9] =$ mA

$I_B[10] =$ mA

current amplification ~100

http://lamp.tu-graz.ac.at/~hadley/psd/L13/commonemitter.php

# Current amplification factor

$$\beta = h_{fe} = \frac{I_C}{I_B}$$

$$I_B = I_E - I_C$$

$$I_C = \alpha I_E$$

$$I_B = \left(\frac{1}{\alpha} - 1\right) I_C$$

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha} = \frac{B\gamma_e}{1-B\gamma_e}$$

$\beta \sim 50 - 500$

# Transconductance

$$g_m = \frac{\partial I_C}{\partial V_{be}}$$

$$I_c = \alpha_F I_{ES}\left(e^{eV_{be}/k_BT} - 1\right) - I_{CS}\left(e^{eV_{bc}/k_BT} - 1\right)$$

At a typical operating point, the first term usually dominates

$$I_c \approx \alpha_F I_{ES} e^{eV_{be}/k_BT}$$

$$g_m = \frac{e\alpha_F I_{ES}}{k_BT} e^{eV_{be}/k_BT} \approx \frac{eI_C}{k_BT} = \frac{e\beta I_B}{k_BT}$$

The transconductance can be very high.

# Early effect

Ebers - Moll:

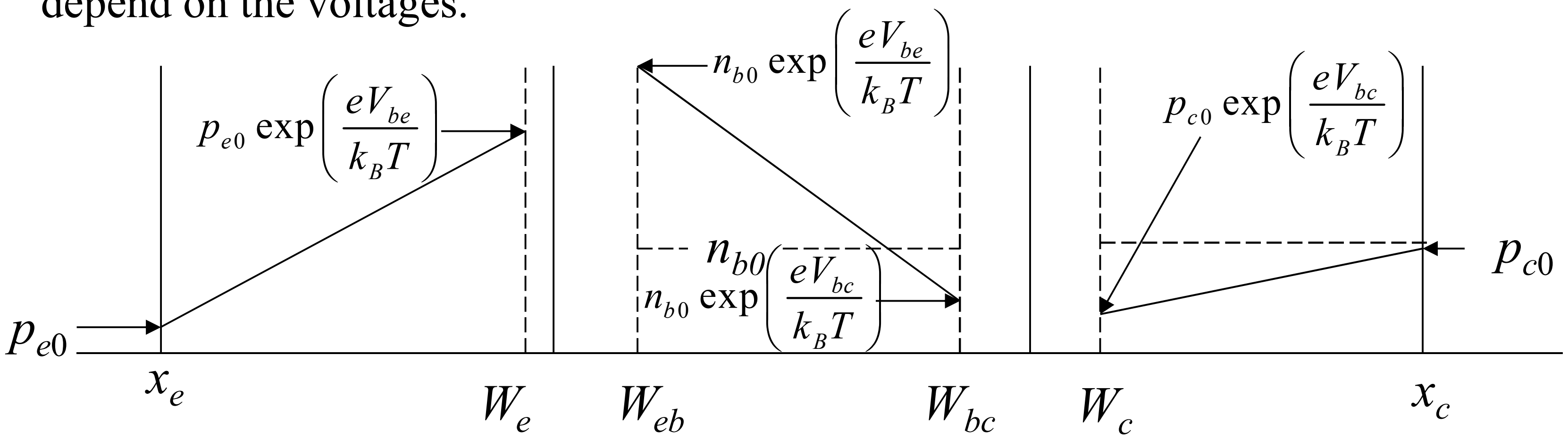
$$I_E = I_{ES}\left(e^{eV_{be}/k_BT}-1\right)-\alpha_R I_{CS}\left(e^{eV_{bc}/k_BT}-1\right)$$

$$I_c = \alpha_F I_{ES}\left(e^{eV_{be}/k_BT}-1\right)-I_{CS}\left(e^{eV_{bc}/k_BT}-1\right)$$

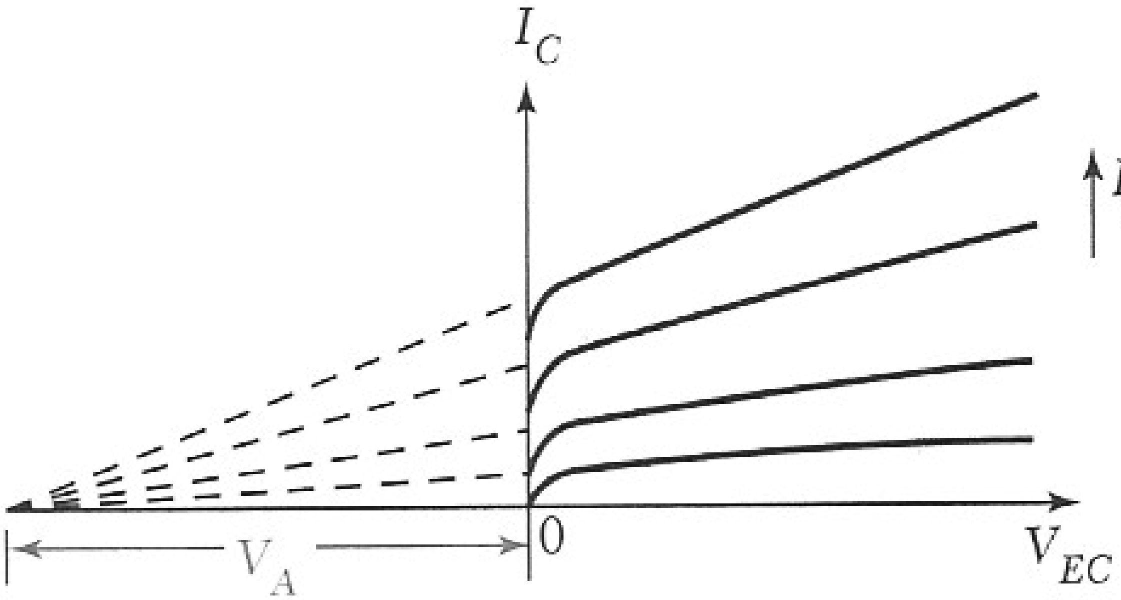
$$I_B = I_E - I_C$$

$$I_{ES} = \left[\frac{eA_{be}D_p p_{e0}}{W_{eb}-x_e}+\frac{eA_{be}D_n n_{b0}}{W_{bc}-W_{be}}\right] \qquad I_{CS} = \left[\frac{eA_{bc}D_p p_{c0}}{x_c-W_c}+\frac{eA_{bc}D_n n_{b0}}{W_{bc}-W_{be}}\right]$$

$I_{ES}$ and $I_{CS}$ are treated as constants but the depletion widths $W_{bc}$, $W_{be}$, $W_c$, and $W_e$ depend on the voltages.



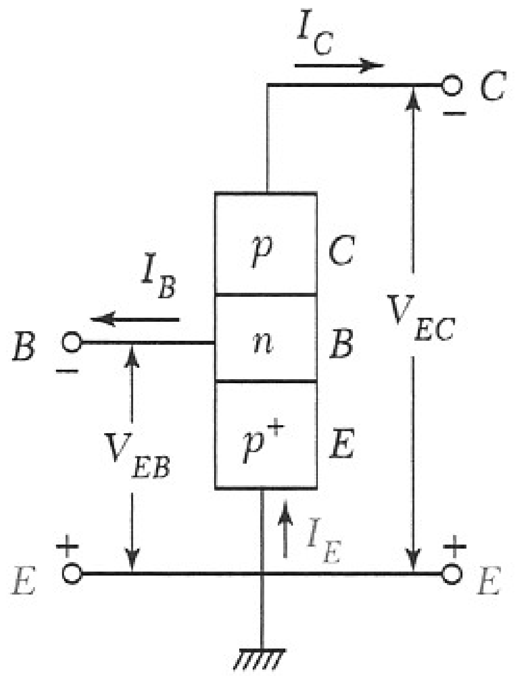
# Early effect

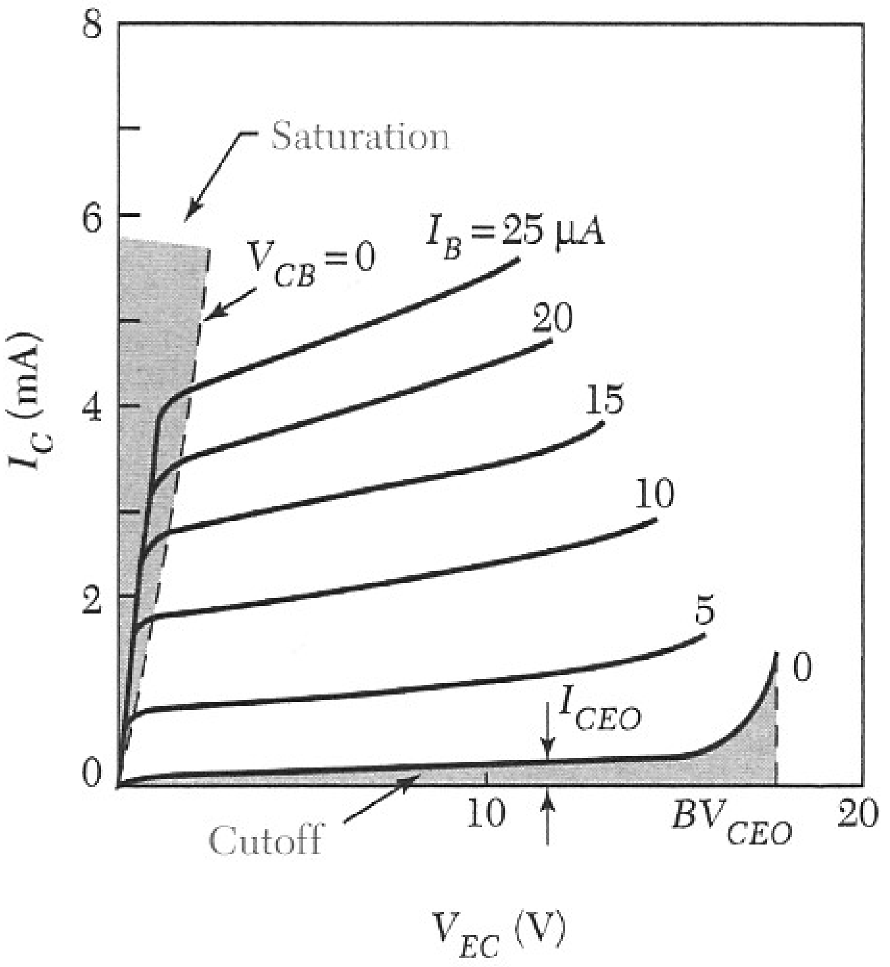


Common emitter configuration
Base width modulation: smaller width increases the diffusion current and increases the gain.

Punchthrough: The neutral base width goes to zero and all gain is lost.
Lightly dope the collector -> voltage drops in collector. Makes circuit slower.

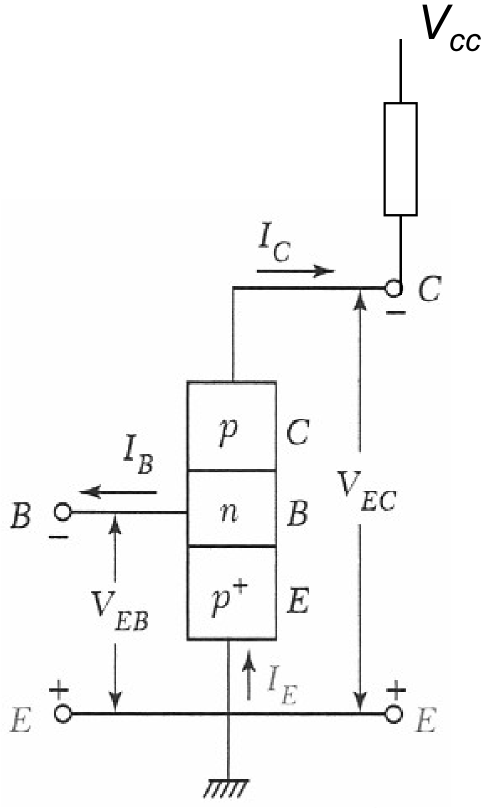
# Common emitter configuration

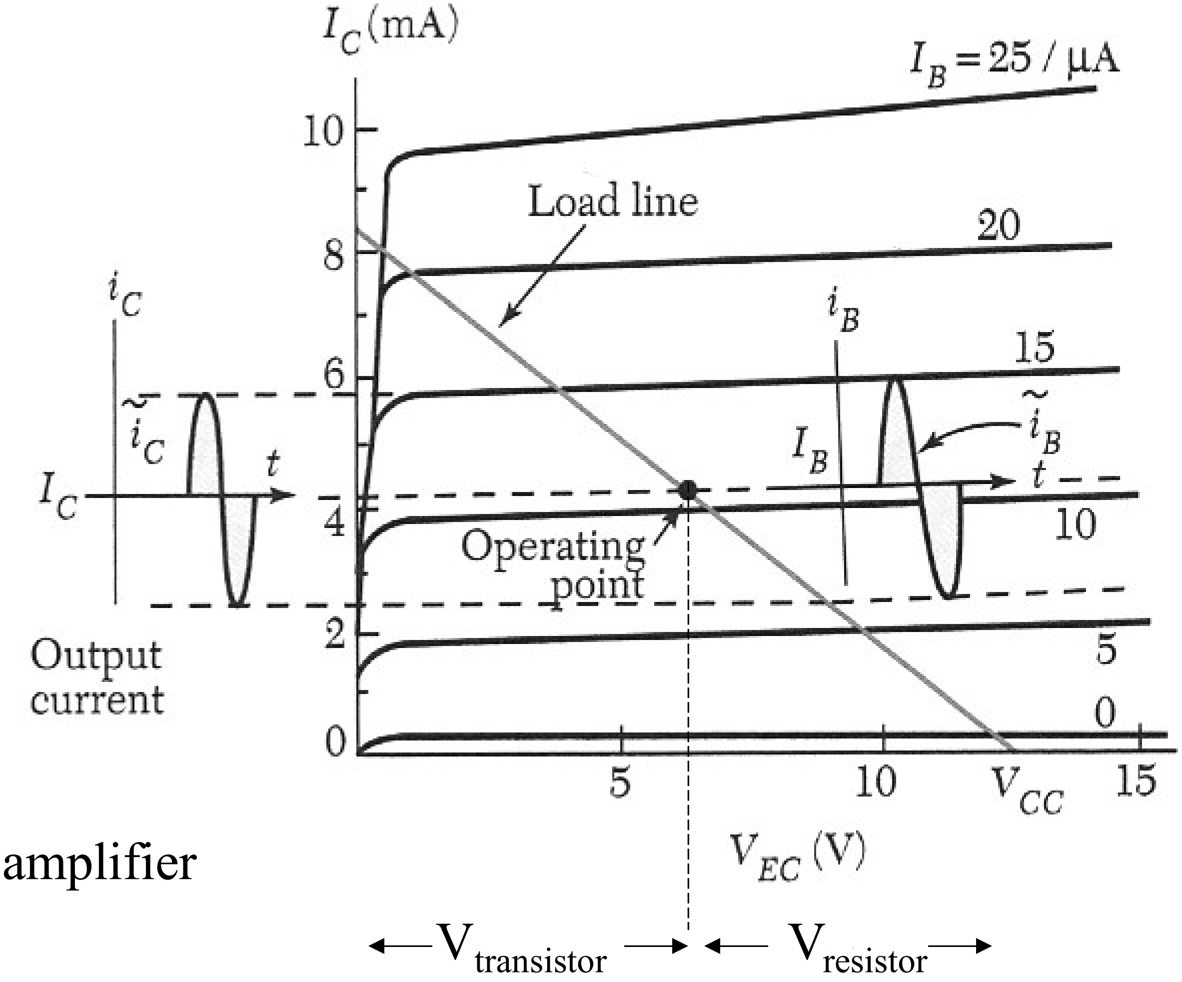




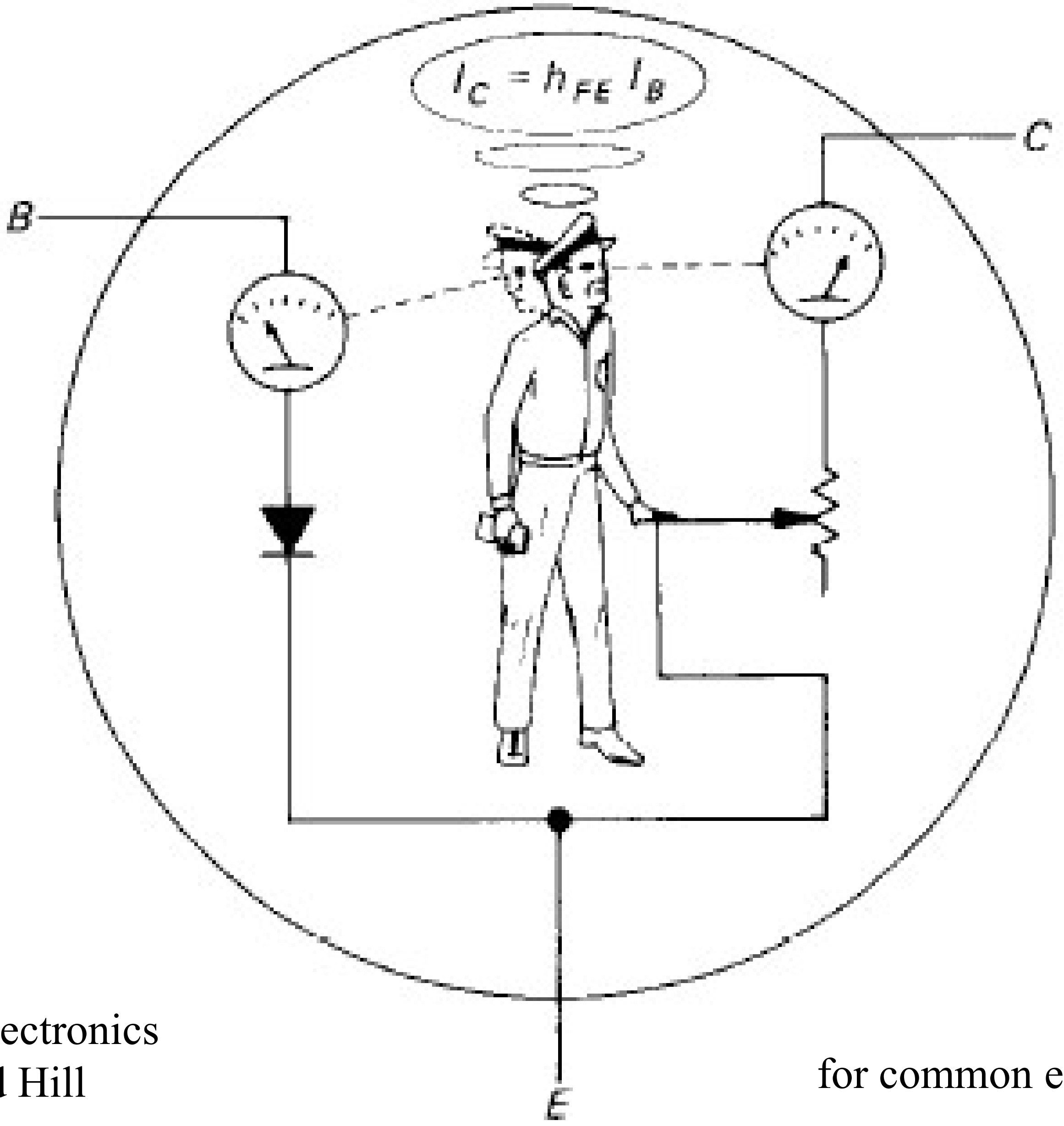
$I_C \sim \beta I_B$ amplifier

# Small signal response





Low input impedance amplifier



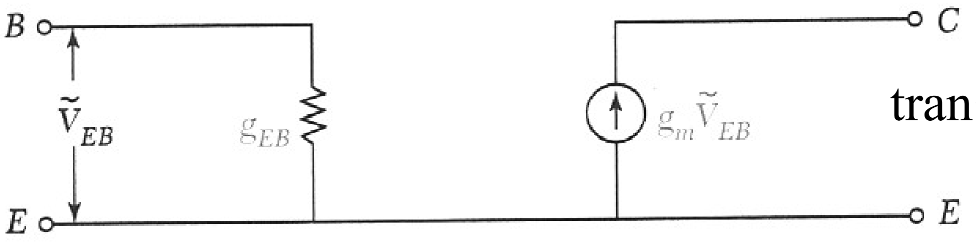
The Art of Electronics
Horowitz and Hill

for common emitter configuration

“Transistor man”

# Small signal response

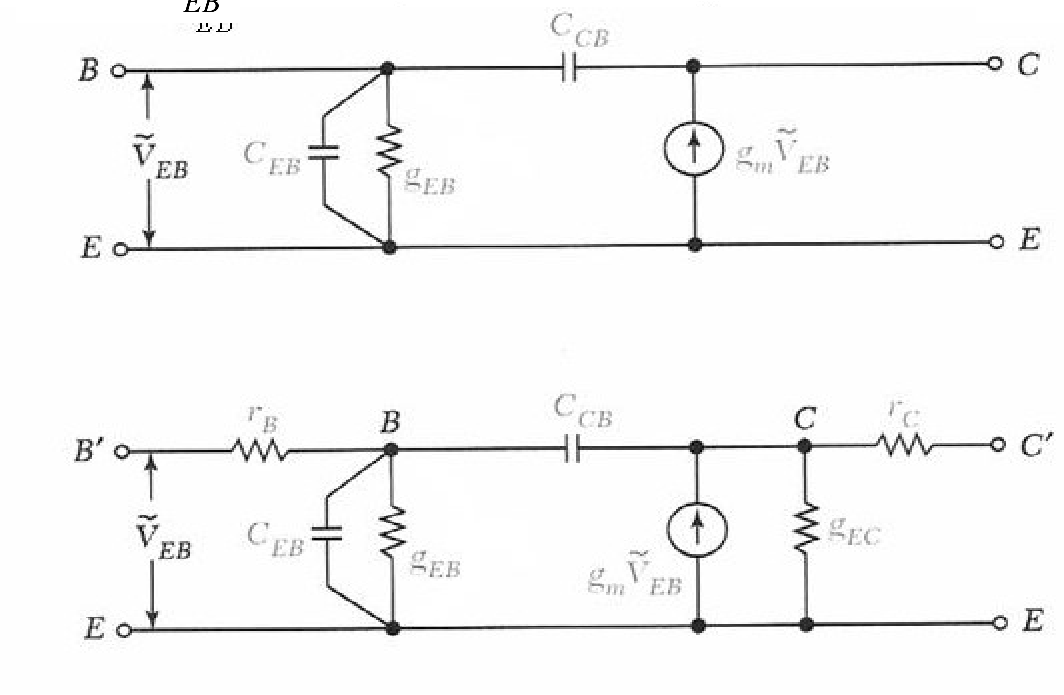
$$\tilde{i}_c = \beta \tilde{i}_B = \beta g_{EB} \tilde{v}_{EB}$$



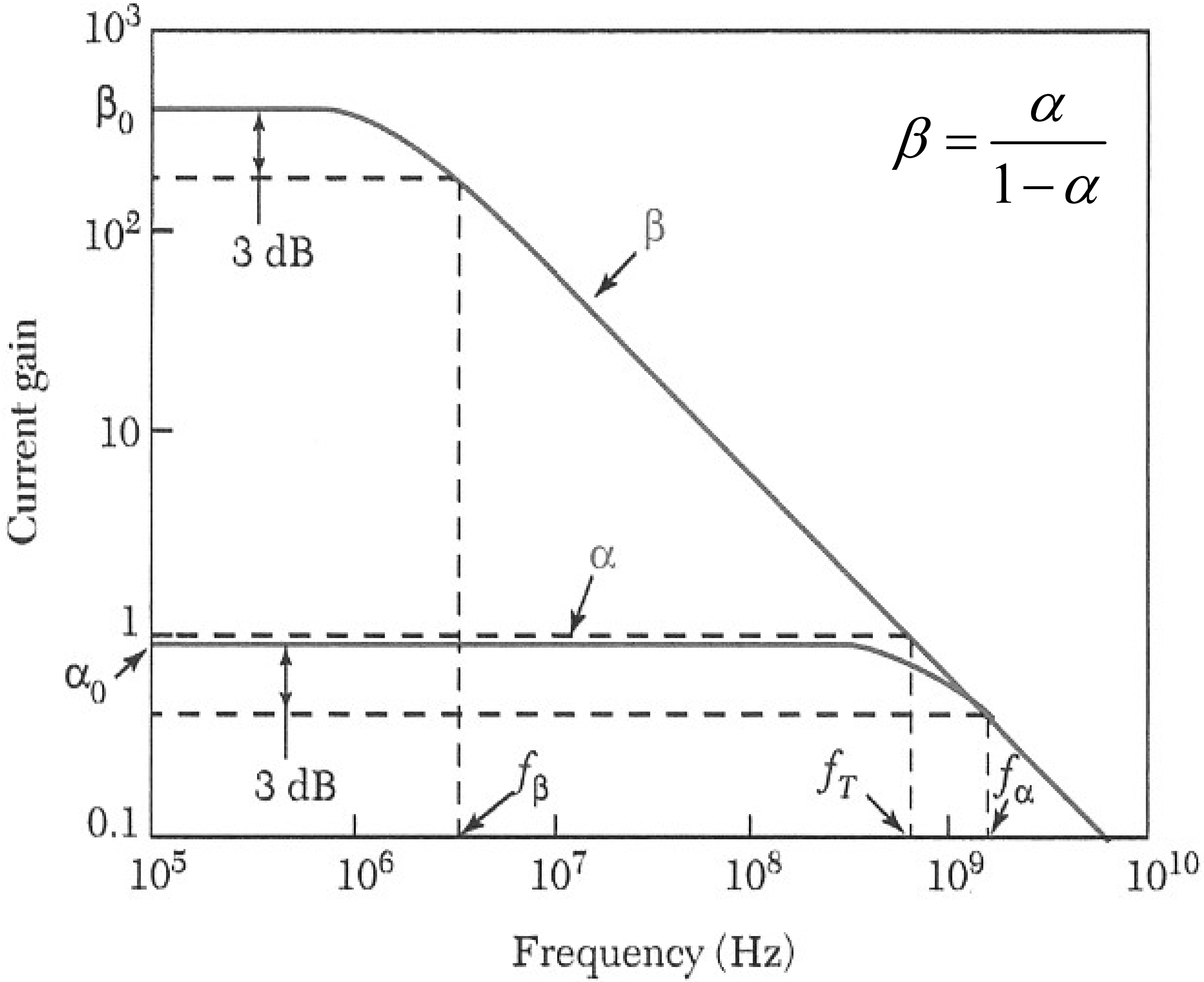
transistor man

input conductance: $$g_{EB} = \frac{\tilde{i}_B}{\tilde{v}_{EB}}$$

transconductance: $$g_m = \frac{\tilde{i}_c}{\tilde{v}_{EB}}$$



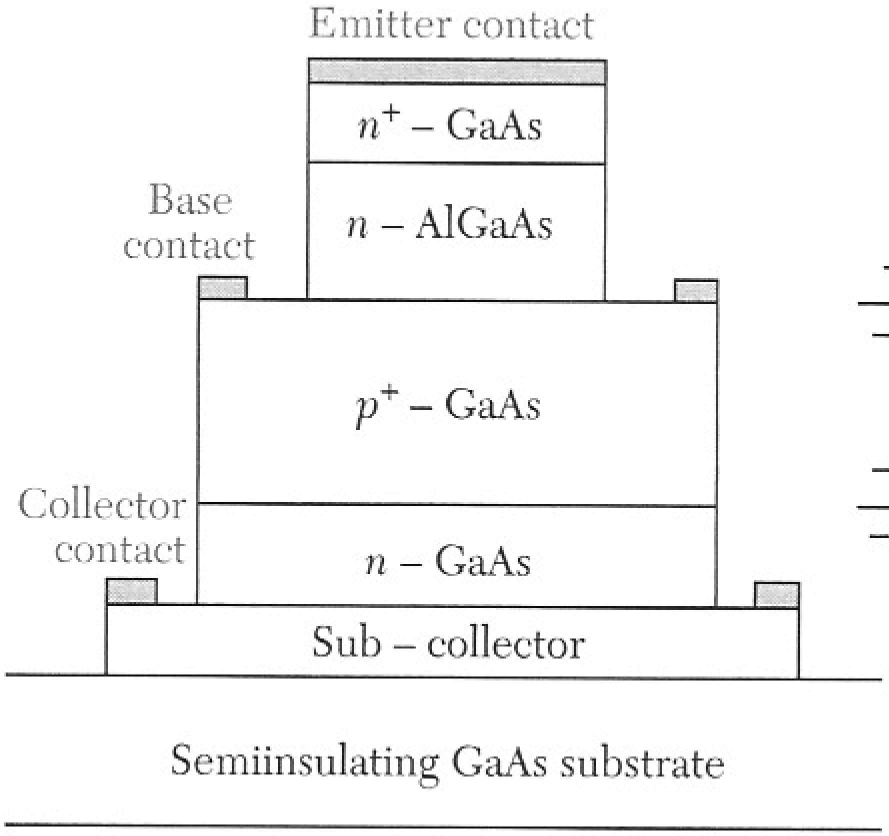
# Small signal response

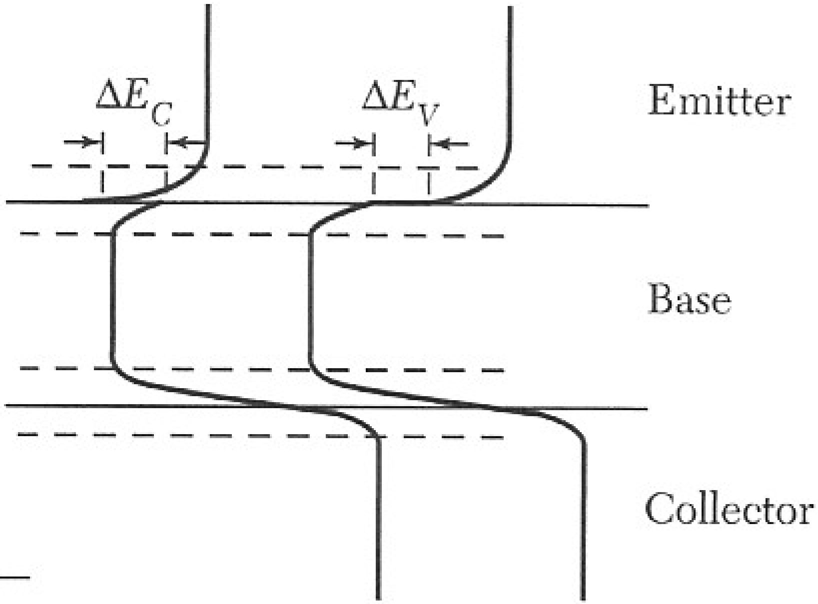


$$\beta = \frac{\alpha}{1-\alpha}$$

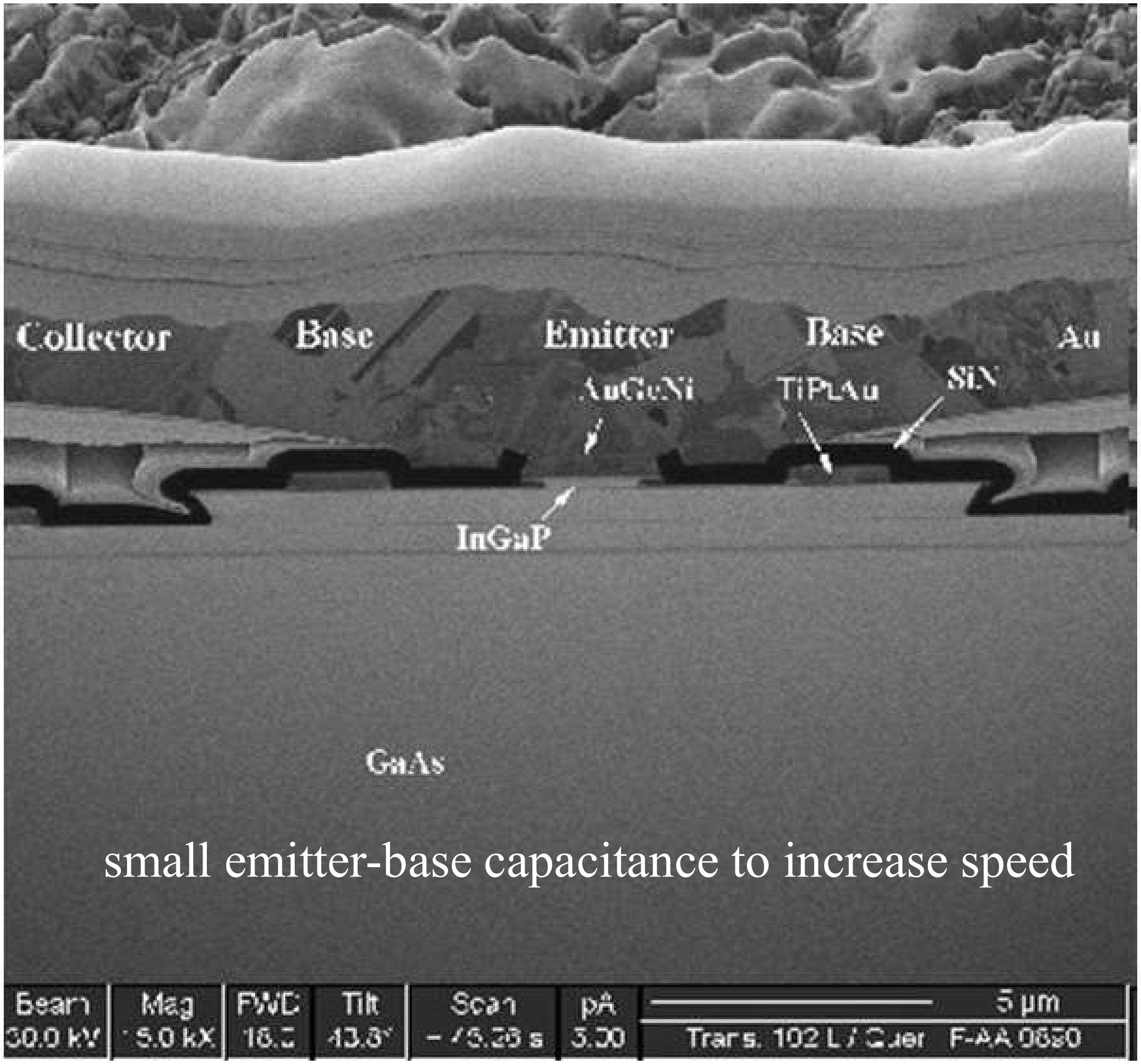
$$f_\beta = (1-\alpha_0)\, f_\alpha \qquad f_T = \alpha_0 f_\alpha$$

# Heterojunction bipolar transistors





# Heterojunction bipolar transistor



small emitter-base capacitance to increase speed

# HBT current gain

$$I_C = \beta I_B$$

$$\beta = \frac{\alpha}{1-\alpha} \approx \frac{n_{B0}}{p_{E0}} \qquad \text{(npn)}$$

Higher doping in the emitter makes the minority carrier concentration lower in the emitter.

$$n_{B0} = \frac{n_i^2}{N_A} = \frac{N_C N_V \exp(-E_{gB}/k_B T)}{N_A}$$

$$p_{E0} = \frac{n_i^2}{N_D} = \frac{N'_C N'_V \exp(-E_{gE}/k_B T)}{N_D}$$

If the emitter and the base have different band gaps

$$\beta = \frac{N_E}{N_B}\frac{N_c N_v}{N'_c N'_v}\exp\left(\frac{\Delta E_g}{k_B T}\right) \sim 100000$$