Carrier Transport

Ballistic transport
Drift
Diffusion
Generation and recombination
The continuity equation
High field effects

Review

n-type

$N_D > N_A \qquad n = N_D - N_A$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D - N_A}$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = N_D - N_A$$

$$\Downarrow$$

$$E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D - N_A}\right)$$

p-type

$N_A > N_D \qquad p = N_A - N_D$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A - N_D}$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right) = N_A - N_D$$

$$\Downarrow$$

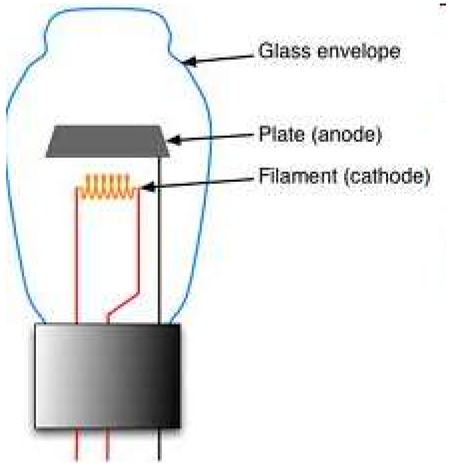
$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A - N_D}\right)$$

Ballistic transport

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0 t + \vec{x}_0$$



Electrons moving in an electric field follow parabolic trajectories like a ball in a gravitational field.

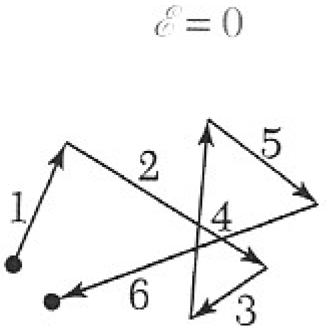
Drift

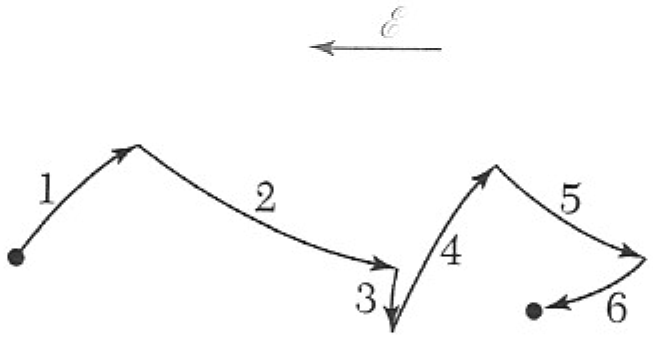
The electrons scatter and change direction after a time τ_{sc}.

Classical equipartition: $\frac{1}{2} m v_{th}^2 = \frac{3}{2} k_B T$

At 300 K, $v_{th} \sim 10^7$ cm/s.

mean free path: $\ell = v_{th} \tau_{sc}$ ~ 10 nm ~ 200 atoms



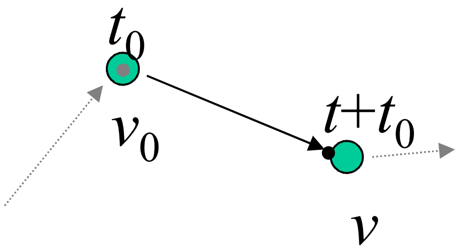


Drift (diffusive transport)

$$\vec{F} = -e\vec{E} = m^*\vec{a} = m^*\frac{d\vec{v}}{dt}$$

$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*}(t - t_0)$$

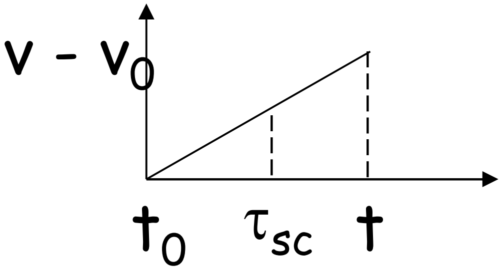
time between two collisions



$\langle v_0 \rangle = 0$

$\langle t - t_0 \rangle = \tau_{sc}$ < average time between scattering events

$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v}$$



drift velocity: $\vec{v}_{d,n} = -\mu_n\vec{E}$ $\vec{v}_{d,p} = \mu_p\vec{E}$

Drift

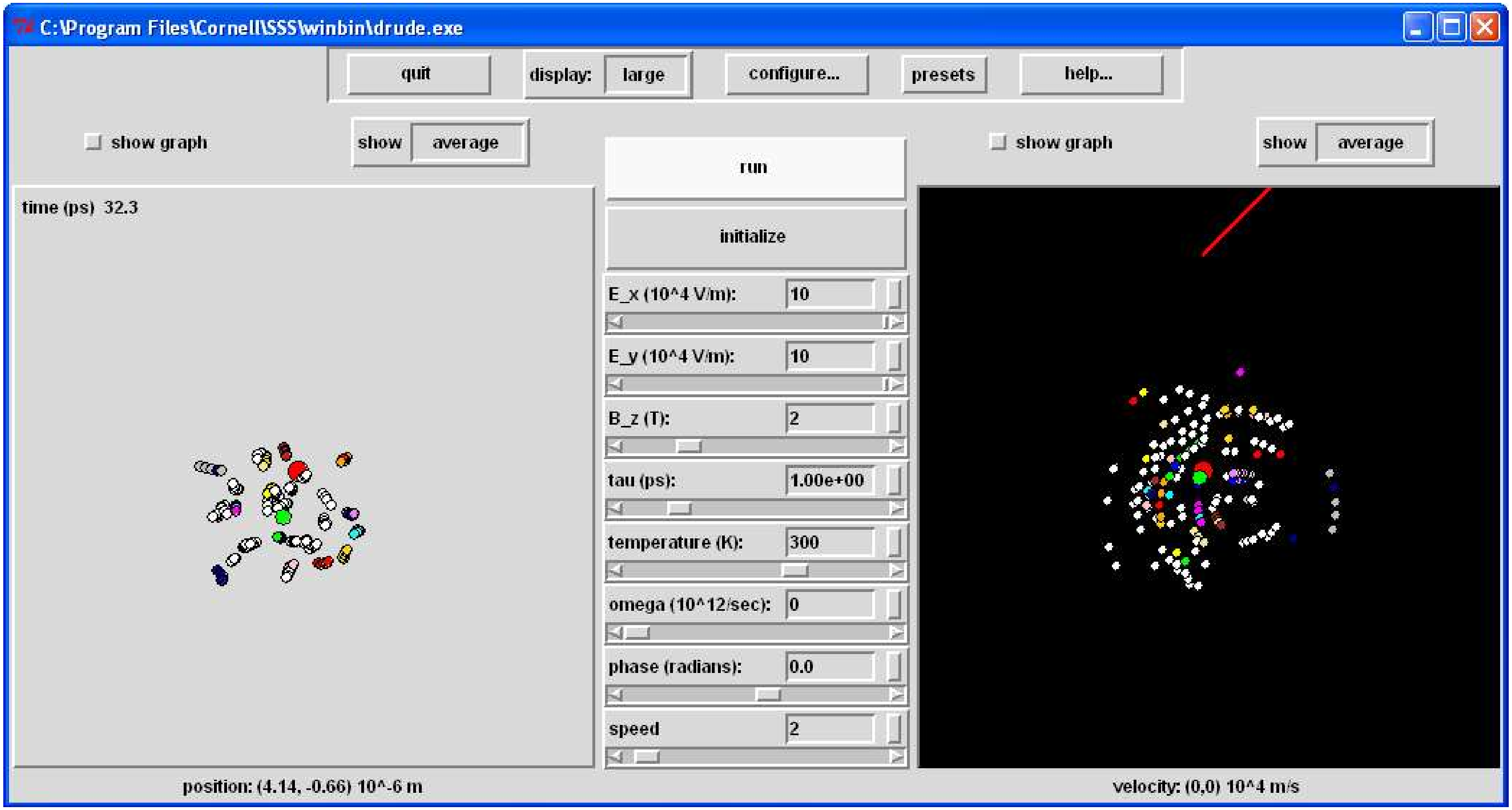
drift velocity: $\overrightarrow{v}_{d,n} = -\mu_n \overrightarrow{E}$ $\overrightarrow{v}_{d,p} = \mu_p \overrightarrow{E}$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = \left(ne\mu_n + pe\mu_p\right)\vec{E} = \sigma\vec{E}$$

$$\mu = \frac{-e\tau_{sc}}{m^*} = \frac{-e\ell}{m^* v}$$

for Si: $\mu_n = 1500 \text{ cm}^2/\text{Vs}$
$\mu_p = 450 \text{ cm}^2/\text{Vs}$

For $E = 1000$ V/cm $v_d = 10^6$ cm/s



C:\Program Files\Cornell\SSS\winbin\drude.exe

time (ps) 32.3

E_x (10^4 V/m): 10

E_y (10^4 V/m): 10

B_z (T): 2

tau (ps): 1.00e+00

temperature (K): 300

omega (10^12/sec): 0

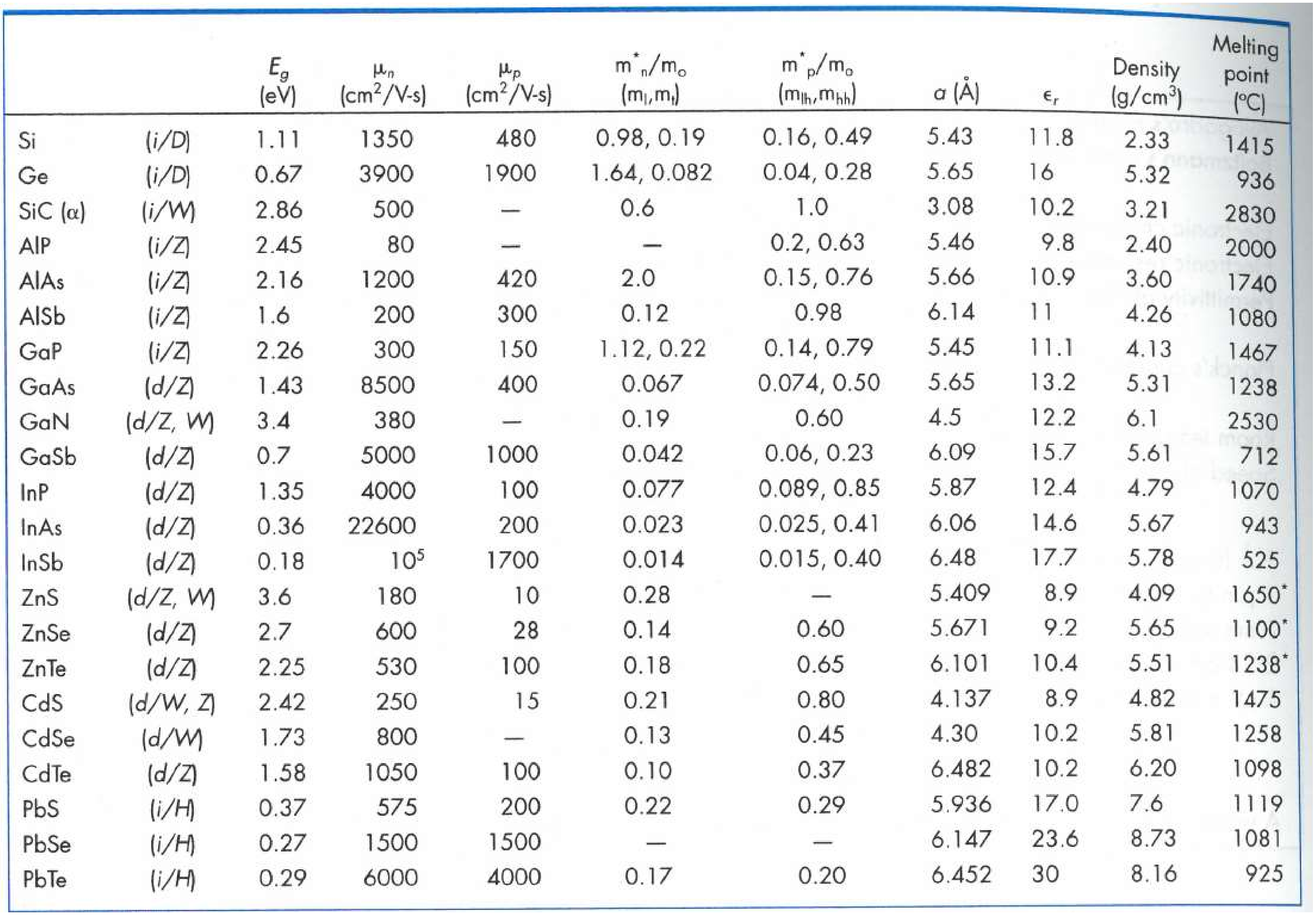
phase (radians): 0.0

speed 2

position: (4.14, -0.66) 10^-6 m

velocity: (0,0) 10^4 m/s

Drift



		E_g (eV)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)	m^*_n/m_o (m_l, m_t)	m^*_p/m_o (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm^3)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10^5	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

Solid state electronic devices, Streetman and Banerjee

$$\vec{v}_{d,n} = -\mu_n \vec{E} \quad \vec{v}_{d,p} = \mu_p \vec{E}$$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = \left(ne\mu_n + pe\mu_p\right)\vec{E} = \sigma\vec{E}$$

Matthiessen's rule

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

mostly temperature independent

phonons, temperature dependent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

doping increases the conductivity
by increasing the carrier density
but decreases the mobility

Mobility calculator

$$\mu = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1+(N/N_{ref})^{\gamma}}$$

For Electrons:

$$\mu_{min} = 47\left(\frac{T}{300}\right)^{-1{,}23} \frac{\mathrm{cm}^2}{\mathrm{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 1373\left(\frac{T}{300}\right)^{-2{,}38} \frac{cm^2}{Vs}$$

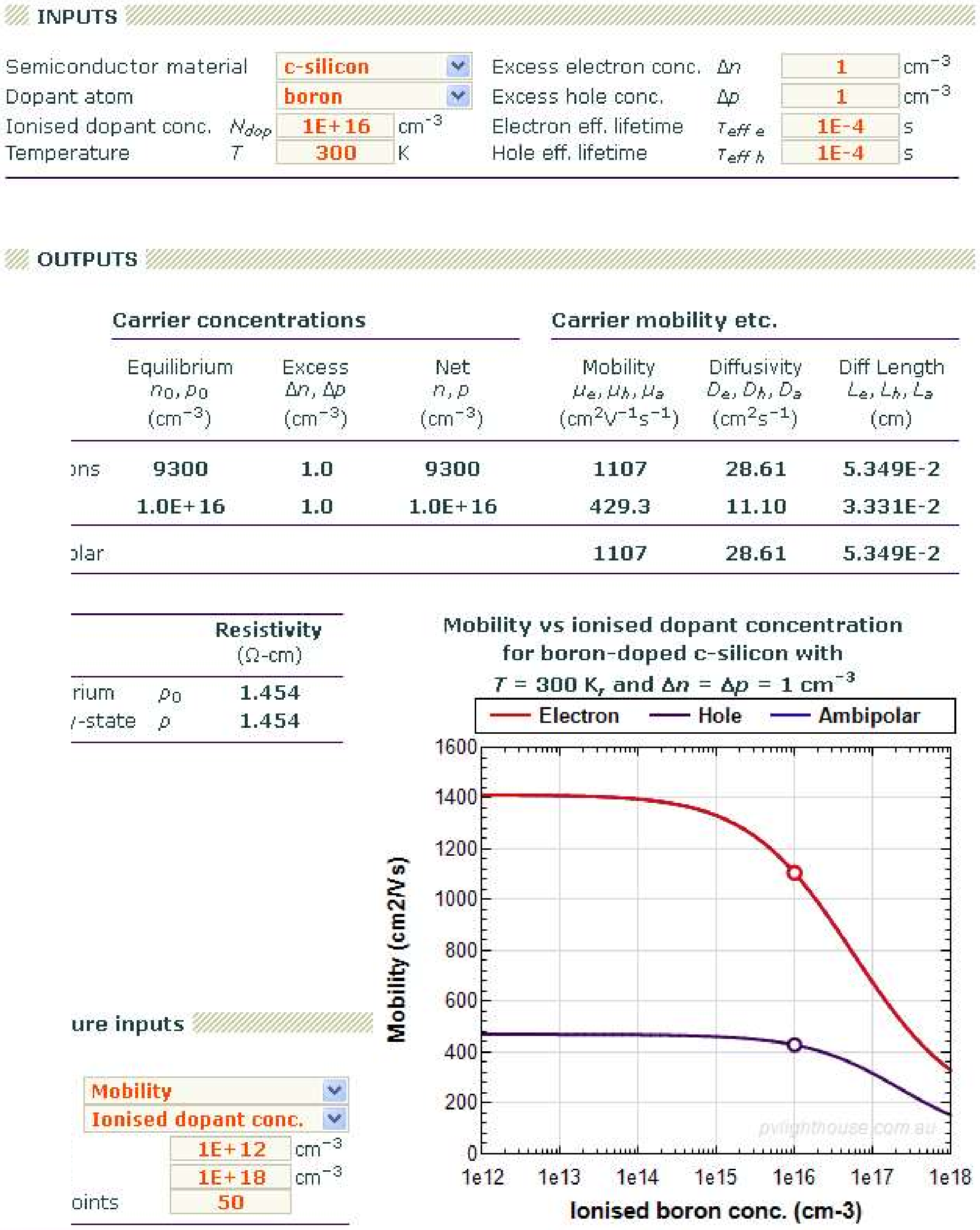
$$N_{ref} = 1{,}05 \cdot 10^{17}\left(\frac{T}{300}\right)^{3{,}65} cm^{-3}\ ;\ \gamma = 0{,}68\left(\frac{T}{300}\right)^{-0{,}32}$$

For Holes:

$$\mu_{min} = 36\left(\frac{T}{300}\right)^{-0{,}87} \frac{\mathrm{cm}^2}{\mathrm{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 438\left(\frac{T}{300}\right)^{-2{,}01} \frac{cm^2}{Vs}$$

$$N_{ref} = 2{,}85 \cdot 10^{17}\left(\frac{T}{300}\right)^{2{,}93} cm^{-3}\ ;\ \gamma = 0{,}65\left(\frac{T}{300}\right)^{0{,}26}$$



INPUTS

Semiconductor material		c-silicon		Excess electron conc.	Δn	1	cm^{-3}
Dopant atom		boron		Excess hole conc.	Δp	1	cm^{-3}
Ionised dopant conc.	N_{dop}	1E+16	cm^{-3}	Electron eff. lifetime	$\tau_{eff\ e}$	1E-4	s
Temperature	T	300	K	Hole eff. lifetime	$\tau_{eff\ h}$	1E-4	s

OUTPUTS

	Carrier concentrations			Carrier mobility etc.		
	Equilibrium n_0, p_0 (cm^{-3})	Excess $\Delta n, \Delta p$ (cm^{-3})	Net n, p (cm^{-3})	Mobility μ_e, μ_h, μ_a ($\mathrm{cm}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$)	Diffusivity D_e, D_h, D_a ($\mathrm{cm}^2\mathrm{s}^{-1}$)	Diff Length L_e, L_h, L_a (cm)
ons	9300	1.0	9300	1107	28.61	5.349E-2
	1.0E+16	1.0	1.0E+16	429.3	11.10	3.331E-2
olar				1107	28.61	5.349E-2

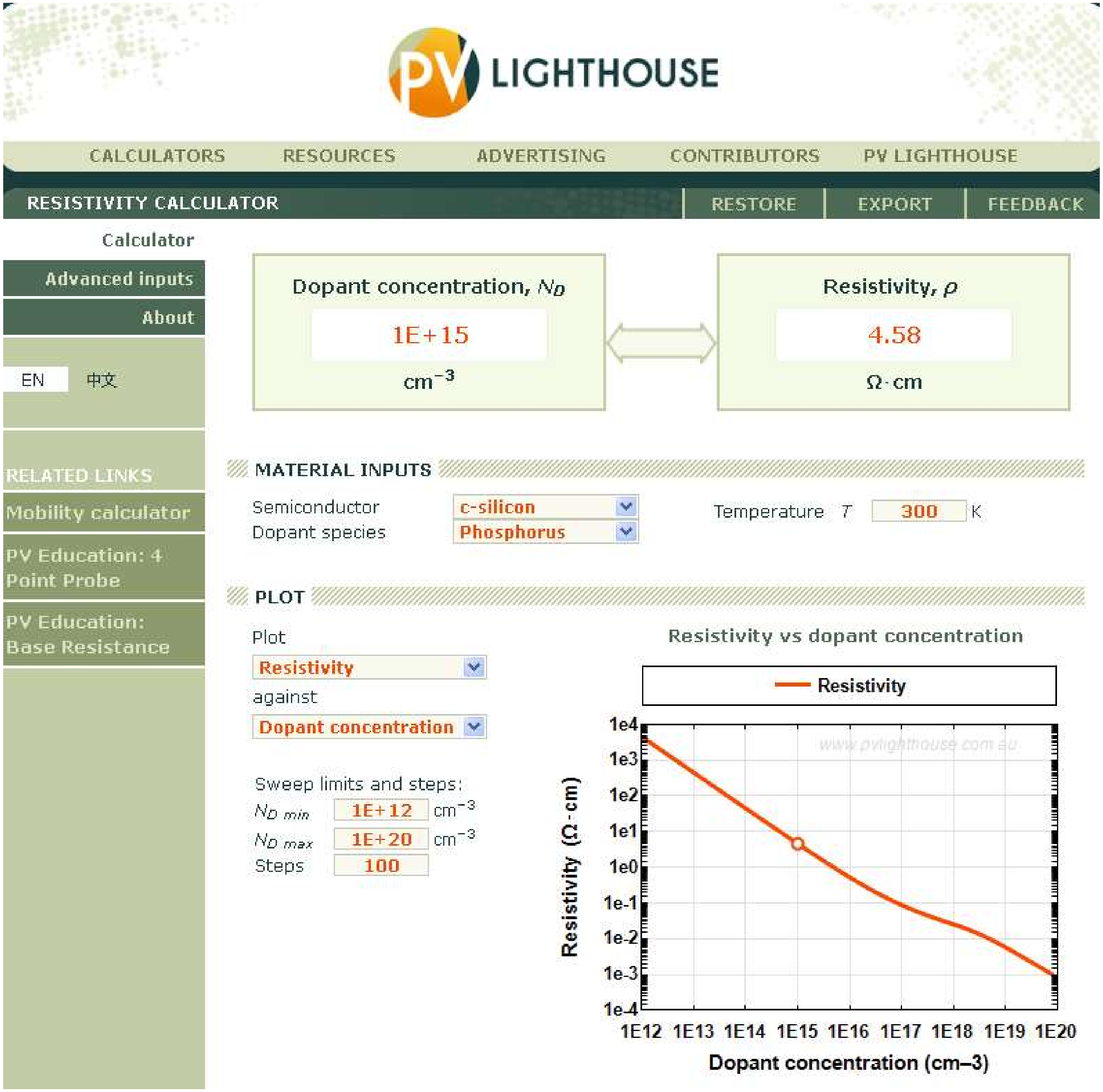
		Resistivity (Ω-cm)
rium	p_0	1.454
/-state	p	1.454

ure inputs

Mobility		
Ionised dopant conc.		
	1E+12	cm^{-3}
	1E+18	cm^{-3}
oints	50	

http://www.pvlighthouse.com.au/calculators/mobility%20calculator/mobility%20calculator.aspx

Resistivity calculator



Dopant concentration, N_D: 1E+15 cm^{-3}

Resistivity, ρ: 4.58 Ω·cm

Semiconductor: c-silicon

Dopant species: Phosphorus

Temperature T: 300 K

Plot: Resistivity

against: Dopant concentration

Sweep limits and steps:

$N_{D\ min}$: 1E+12 cm^{-3}

$N_{D\ max}$: 1E+20 cm^{-3}

Steps: 100

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

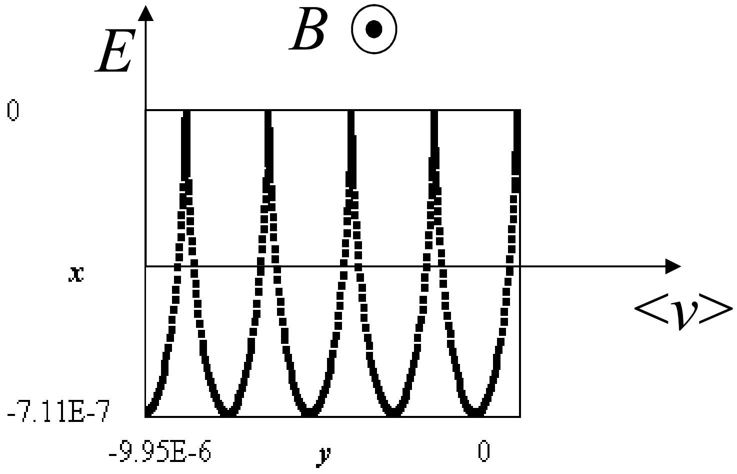
$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

http://www.pvlighthouse.com.au/calculators/Resistivity%20calculator/Resistivity%20calculator.aspx

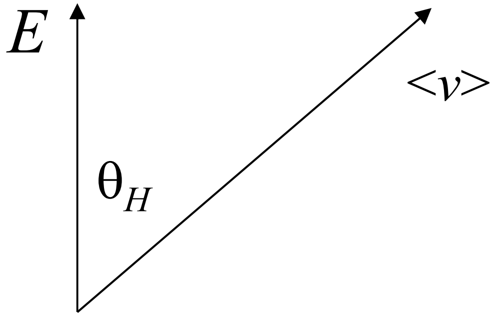
Crossed *E* and *B* fields

Ballistic transport

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



Diffusive transport



Hall angle: $$\theta_H = \tan^{-1}\left(-\frac{eB_z\tau_{sc}}{m^*}\right)$$

Magnetic field (diffusive transport)

$$\vec{F} = m\vec{a} = -e\vec{E} = e\frac{\vec{v}_d}{\mu}$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right) = e\frac{\vec{v}_d}{\mu}$$

If B is in the z-direction, the three components of the force are

$$-\mu\left(E_x + v_{dy}B_z\right) = v_{dx}$$

$$-\mu\left(E_y - v_{dx}B_z\right) = v_{dy}$$

$$-\mu E_z = v_{dz}$$

Magnetic field

$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

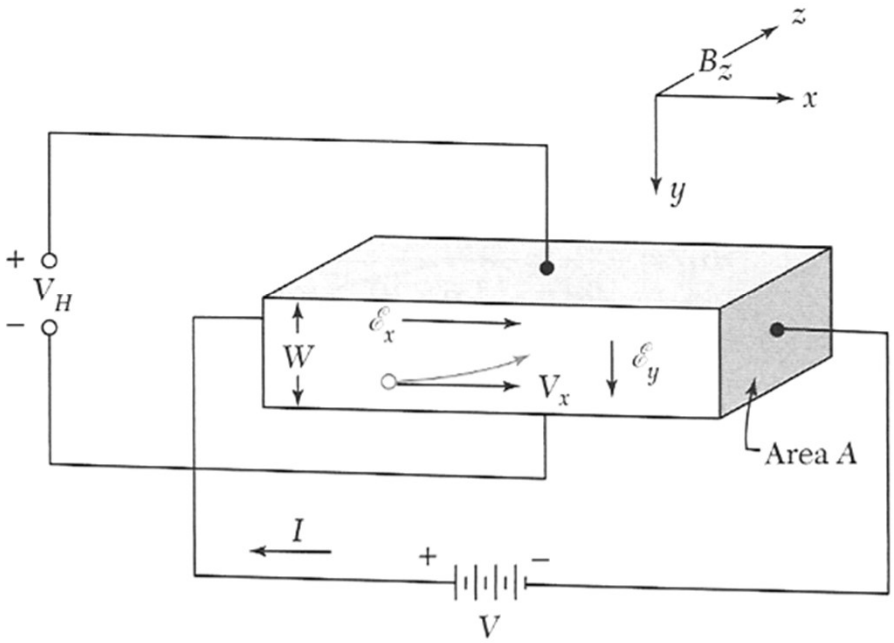
$$v_{d,z} = -\mu E_z$$

If $E_y = 0$,

$$v_{d,y} = -\mu B_z v_{d,x}$$

$$\tan \theta_H = -\mu B_z$$

The Hall Effect (diffusive regime)



$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

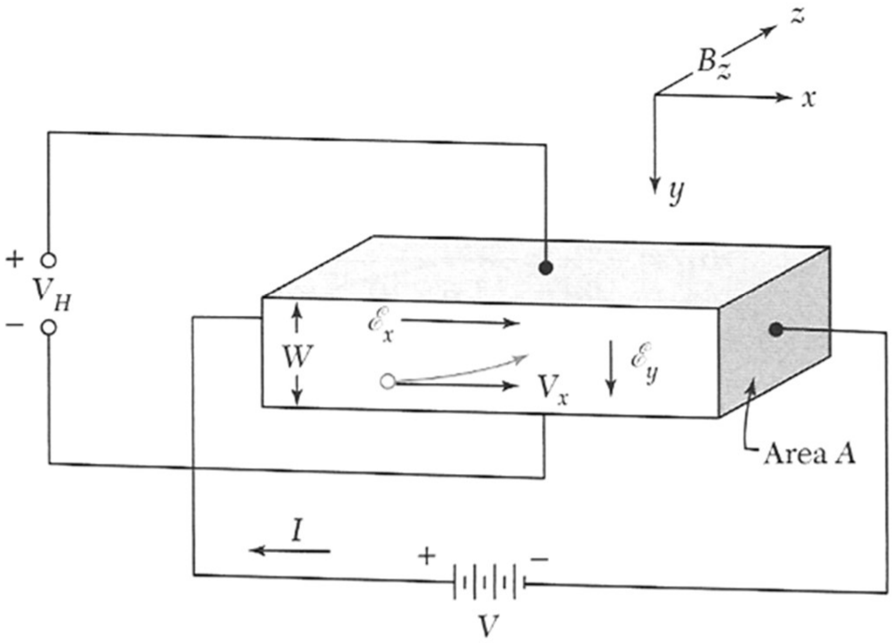
$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

$$v_{d,z} = -\mu E_z$$

If $v_{d,y} = 0$,

$E_y = v_x B_z = V_H/W = R_H j_x B_z$ V_H = Hall voltage, R_H = Hall Constant

The Hall Effect (diffusive regime)



$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

$E_y = v_x B_z = V_H/W = R_H j_x B_z$ $\quad$ V_H = Hall voltage, R_H = Hall Constant

$j_x = I/A$

$v_x = -j_x/ne$ for n-type $\quad$ $v_x = j_x/pe$ for p-type

$R_H = -1/ne$ for n-type $\quad$ $R_H = 1/pe$ for p-type

Ballistic transport in transistors

The mean free path ~100 nm > gate length ~ 20 nm

v not proportional to E $$\cancel{\vec{v} = \mu\vec{E}}$$

j not proportional to E $$\cancel{\vec{j} = \sigma\vec{E}}$$

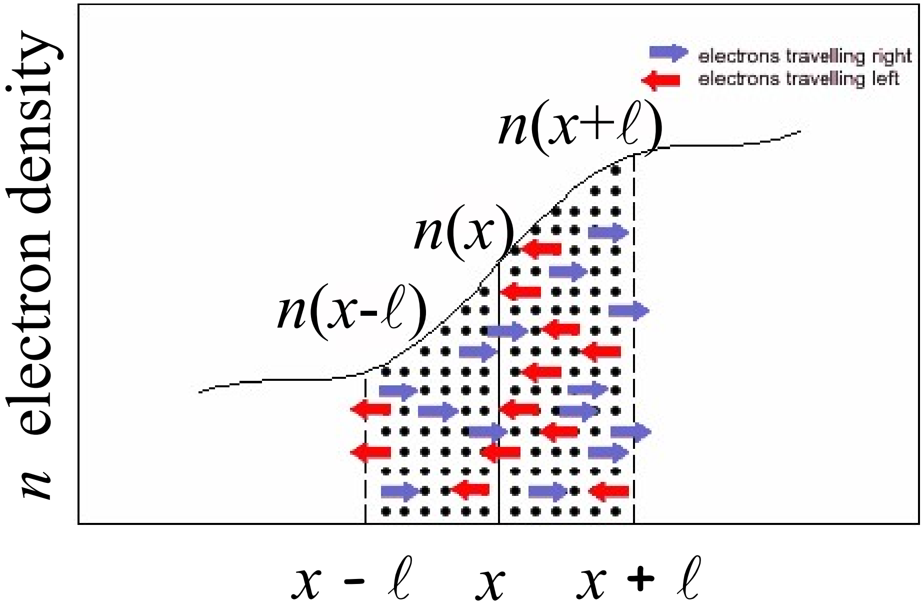
nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

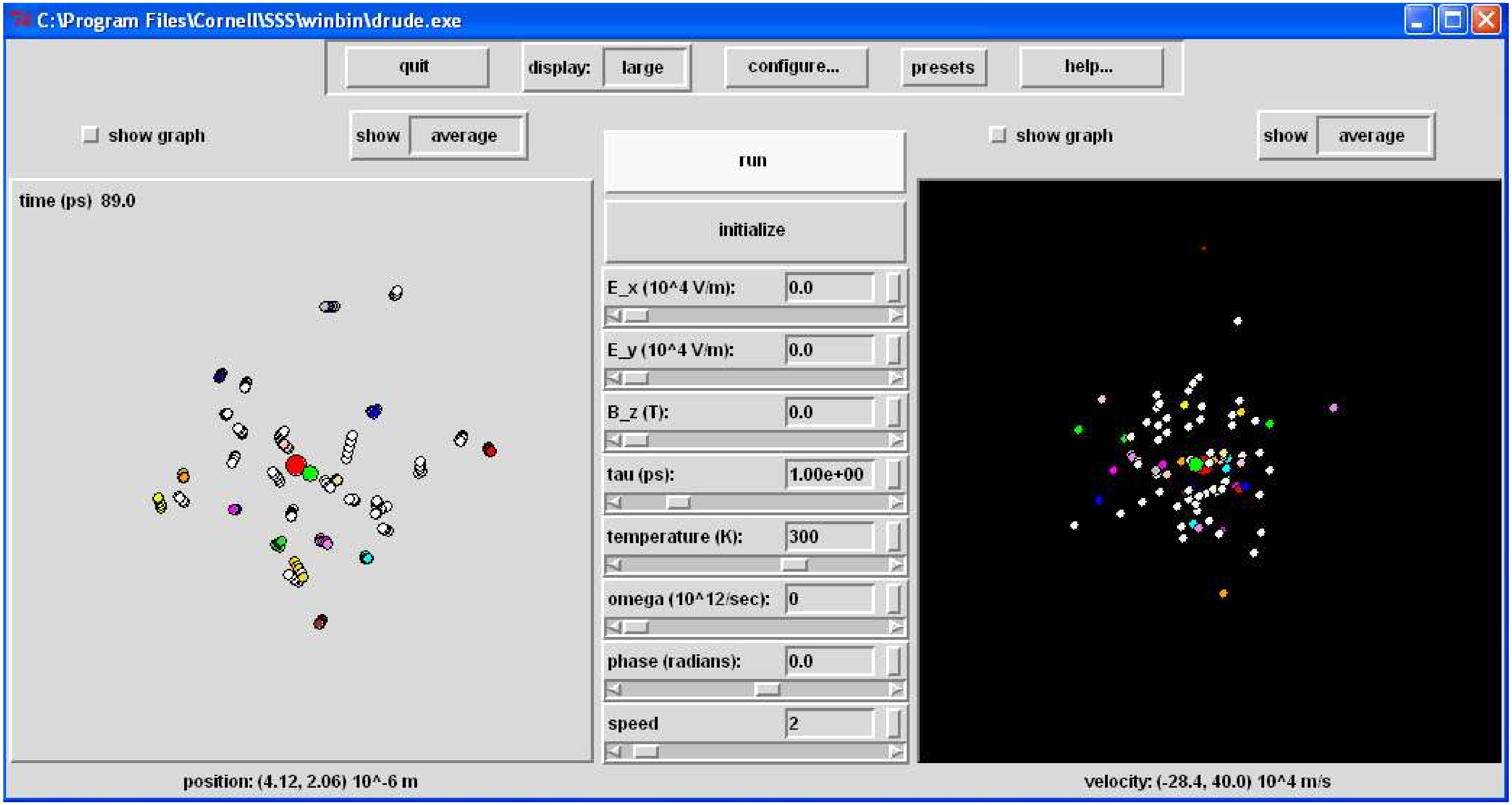
Diffusion

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -|e| D_p \frac{dp}{dx}$$



Diffusion is from high concentration to low concentration.



time (ps) 89.0

E_x (10^4 V/m): 0.0

E_y (10^4 V/m): 0.0

B_z (T): 0.0

tau (ps): 1.00e+00

temperature (K): 300

omega (10^12/sec): 0

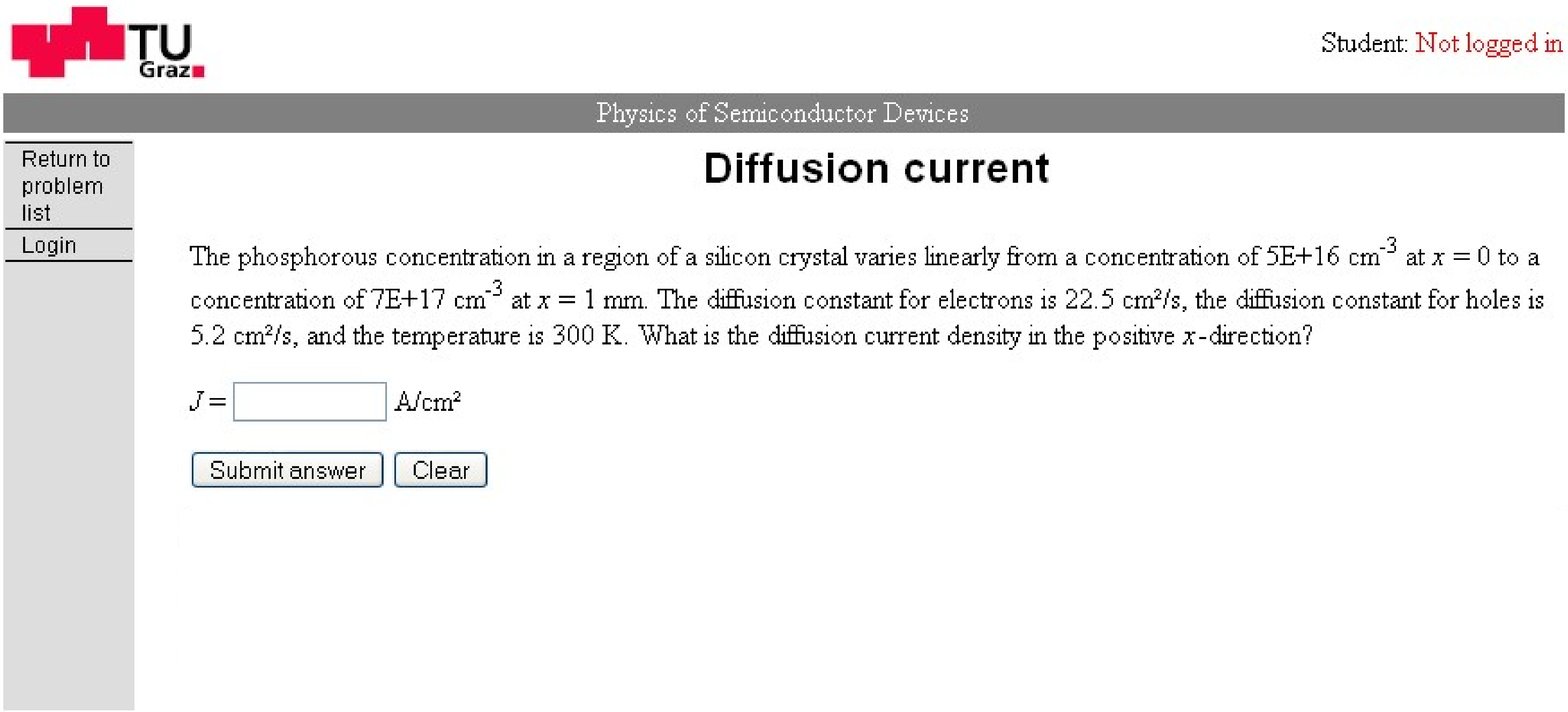
phase (radians): 0.0

speed 2

position: (4.12, 2.06) 10^-6 m

velocity: (-28.4, 40.0) 10^4 m/s

If no forces are applied, the electrons diffuse.
The average velocity moves against an electric field.
In just a magnetic field, the average velocity is zero.
In an electric and magnetic field, the electrons move in a straight line at the Hall angle.



Student: Not logged in

Physics of Semiconductor Devices

Diffusion current

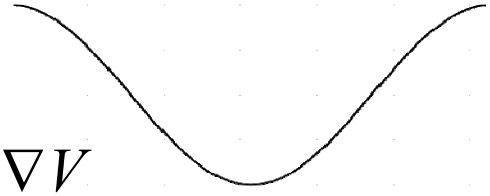
The phosphorous concentration in a region of a silicon crystal varies linearly from a concentration of 5E+16 cm^{-3} at $x = 0$ to a concentration of 7E+17 cm^{-3} at $x = 1$ mm. The diffusion constant for electrons is 22.5 cm²/s, the diffusion constant for holes is 5.2 cm²/s, and the temperature is 300 K. What is the diffusion current density in the positive x-direction?

$J =$ ☐ A/cm²

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

Einstein relation

$$\vec{E} = -\nabla V$$



$$n(x) = A\exp\left(\frac{-eV(x)}{k_B T}\right)$$ Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + D\nabla n = 0$$

$$\nabla n = -\frac{1}{k_B T}A\exp\left(\frac{-eV_{pot}}{k_B T}\right)\nabla V = -\frac{n}{k_B T}\nabla V = \frac{n\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + D\frac{n\vec{E}}{k_B T} = 0$$

$$\boxed{D = \frac{\mu k_B T}{e}}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen

Current Density Equations

Drift

Diffusion

$$\vec{j}_n = -ne\mu_n\vec{E} + eD_n\nabla n$$

$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

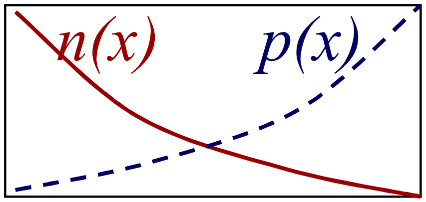
Current Density Equations

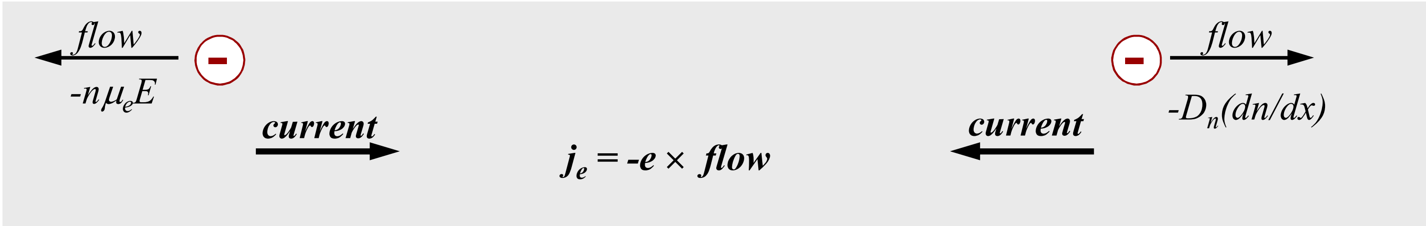
note: electron and hole currents have same direction

electric current = charge × particle flow

drift

diffusion





← *flow* ⊖

$-n\mu_e E$

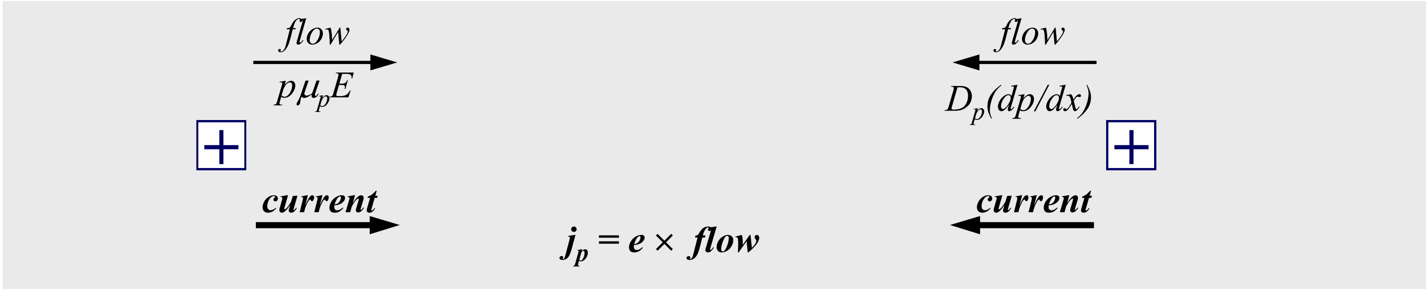
current →

$j_e = -e \times$ ***flow***

current ←

⊖ *flow* →

$-D_n(dn/dx)$



flow →

$p\mu_p E$

⊞

current →

$j_p = e \times$ ***flow***

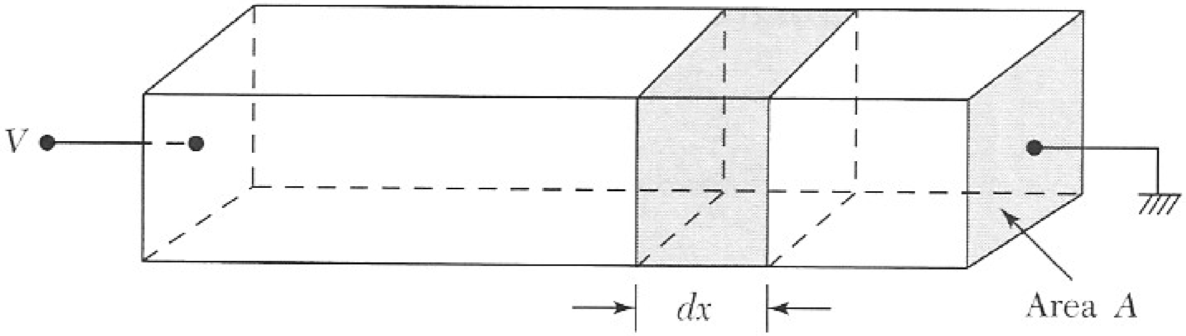
← *flow*

$D_p(dp/dx)$

⊞

current ←

Continuity equations

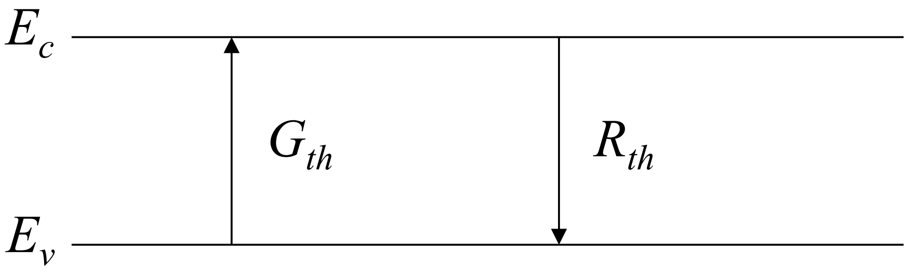


$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{e}\nabla \cdot \vec{j}_p + G_p - R_p$$

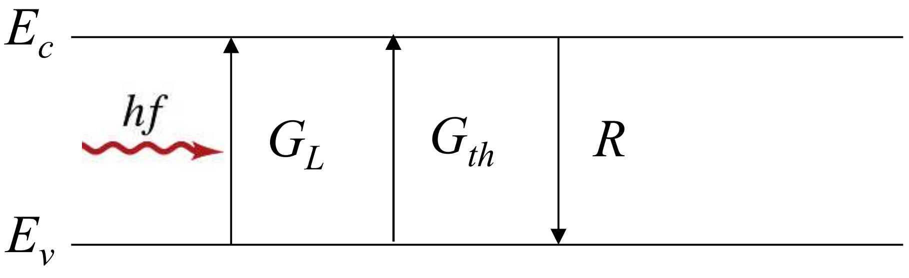
j_n and j_p consist of drift and diffusion terms

Generation and Recombination



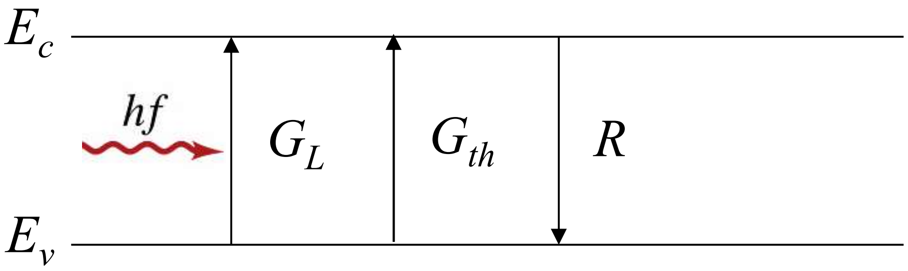
thermal equilibrium

Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution $np \neq n_i^2$



non equilibrium

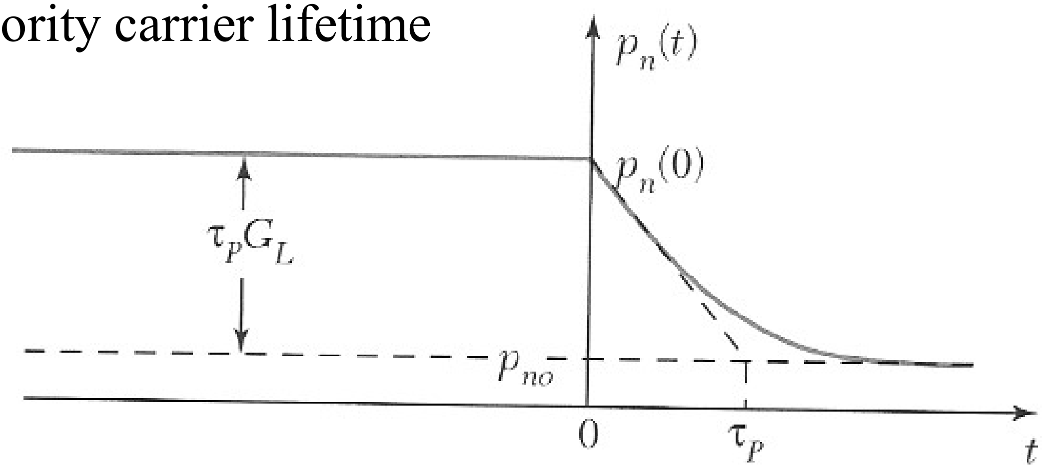
Recombination



$$R - R_{th} = \frac{p_n - p_{n0}}{\tau_p}$$

Recombination rate is limit by the density of minority carriers.
The majority carriers have to find a minority carrier to recombine.

p_n (or n_p) = minority carrier concentration
p_{n0} (or n_{p0}) = equilibrium minority carrier concentration
τ_p = minority carrier lifetime



minority carrier lifetimes

p-type

$$n_p(t) = n_{excess} \exp(-t/\tau_n) + n_{p0}$$

minority carrier lifetimes

n-type

$$p_n(t) = p_{excess} \exp(-t/\tau_p) + p_{n0}$$

$$np = n_i^2$$

Continuity equations

$$\frac{\partial n}{\partial t}=\frac{1}{e}\nabla\cdot\vec{j}_n+G_n-R_n$$

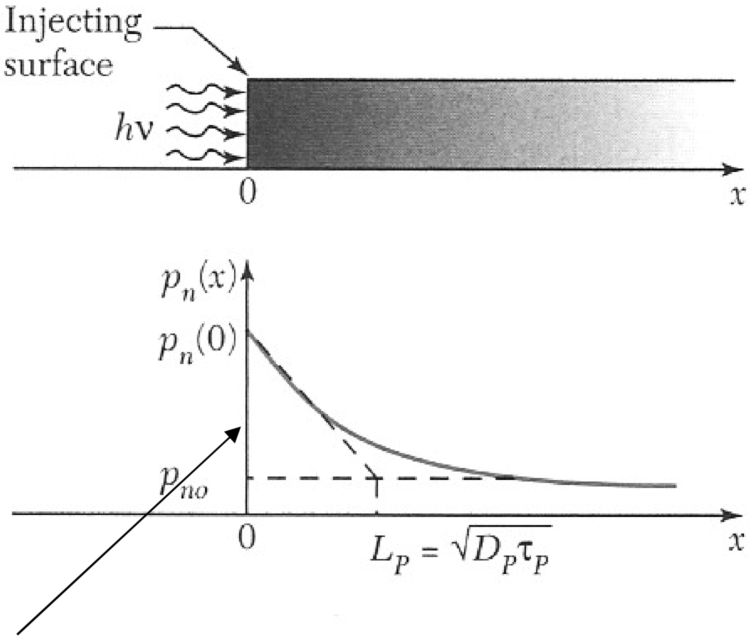
drift: $\vec{j}_n=-ne\mu_n\vec{E}$ $\qquad$ $\nabla\cdot\vec{j}_n=-en\mu_n\nabla\cdot\vec{E}-e\nabla n\mu_n\vec{E}$

diffusion: $\vec{j}_{n,diff}=|e|D_n\nabla n$ $\qquad$ $\nabla\cdot\vec{j}_{n,diff}=|e|D_n\nabla^2 n$

$$\boxed{\begin{aligned}\frac{\partial n}{\partial t}&=n\mu_n\nabla\cdot\vec{E}+\nabla n\mu_n\vec{E}+D_n\nabla^2 n+G_n-\frac{n-n_0}{\tau_n}\\ \frac{\partial p}{\partial t}&=-n\mu_p\nabla\cdot\vec{E}-\nabla n\mu_p\vec{E}+D_p\nabla^2 p+G_p-\frac{p-p_0}{\tau_p}\end{aligned}}$$

Diffusion Length

n-type



Generation only occurs at the surface. There the minority carrier density is $p_n(0)$.

$$\frac{\partial p_n}{\partial t} = \overset{\text{Steady state}}{\overset{\downarrow}{0}} = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

Diffusion Length

$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \qquad \Leftarrow \qquad p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

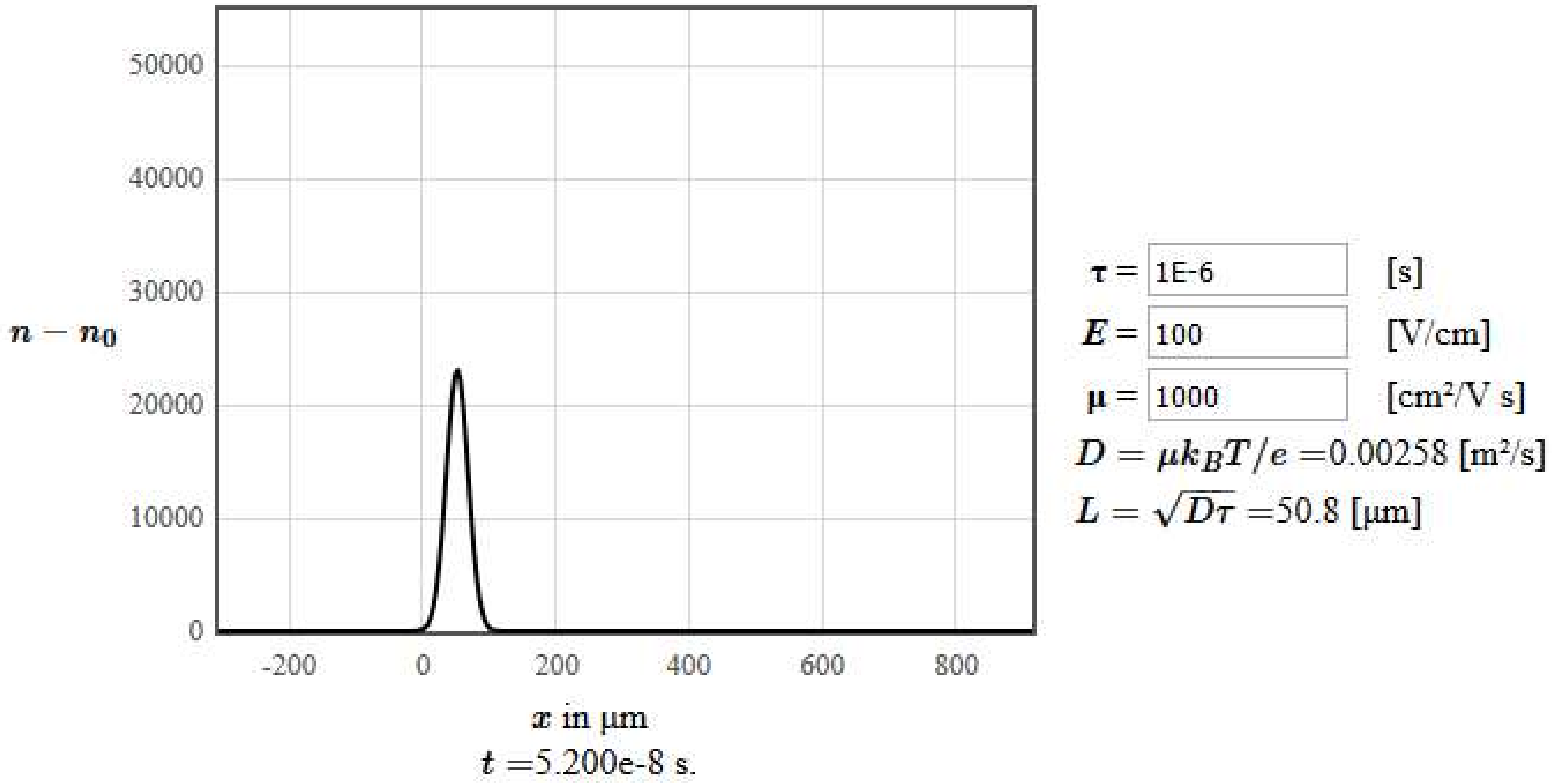
$$0 = \frac{D_p\left(p_n(0) - p_{n0}\right)}{L_p^2}\exp\left(\frac{-x}{L_p}\right) - \frac{\left(p_n(0) - p_{n0}\right)}{\tau_p}\exp\left(\frac{-x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

diffusion length,
typically microns

Haynes Shockley experiment

$$n_p(x,t)=\frac{n_{generated}}{\sqrt{4\pi D_n t}}\exp\left(-\frac{\left(x-\mu_n E t\right)^2}{4D_n t}\right)\exp\left(-\frac{t}{\tau_n}\right)+n_{p0}$$



t =5.200e-8 s.

τ = 1E-6 [s]

E = 100 [V/cm]

μ = 1000 [cm²/V s]

$D=\mu k_B T/e$ =0.00258 [m²/s]

$L=\sqrt{D\tau}$ =50.8 [µm]