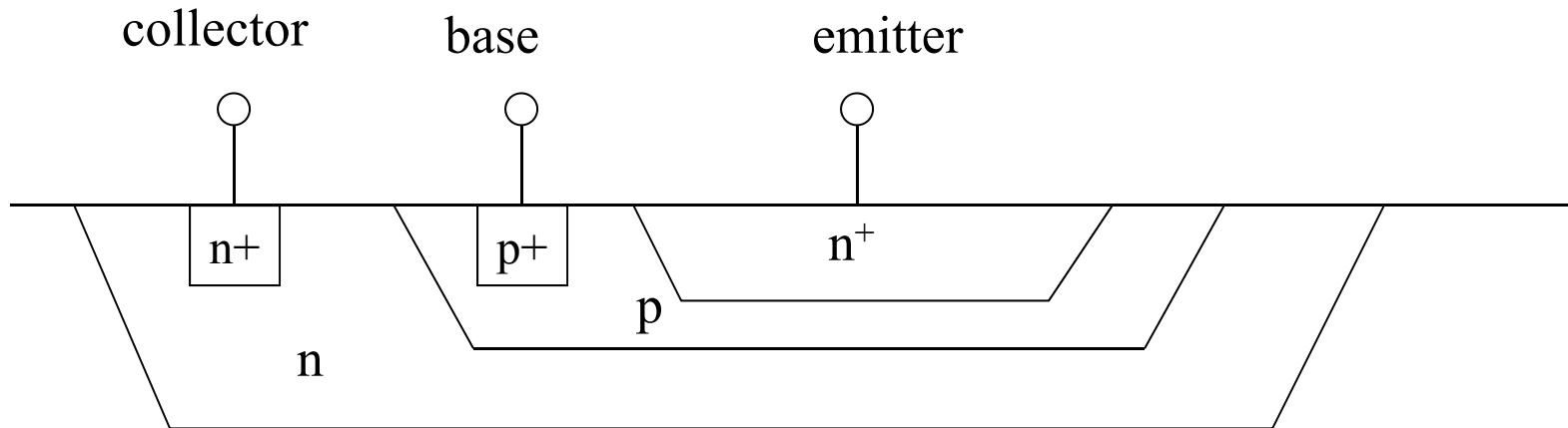


Bipolar transistors

bipolar transistors

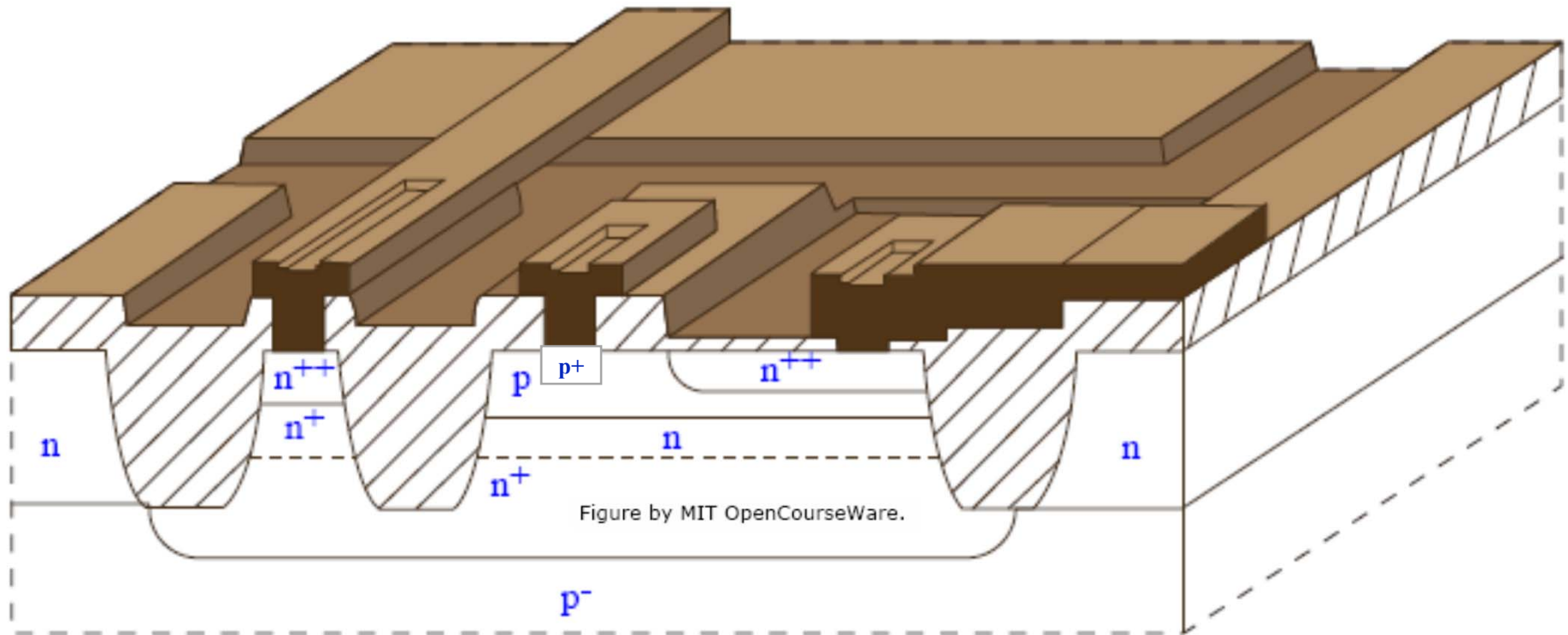
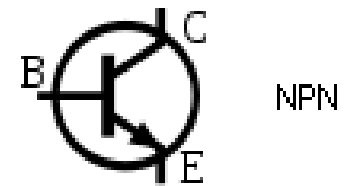
npn transistor



lightly doped p substrate

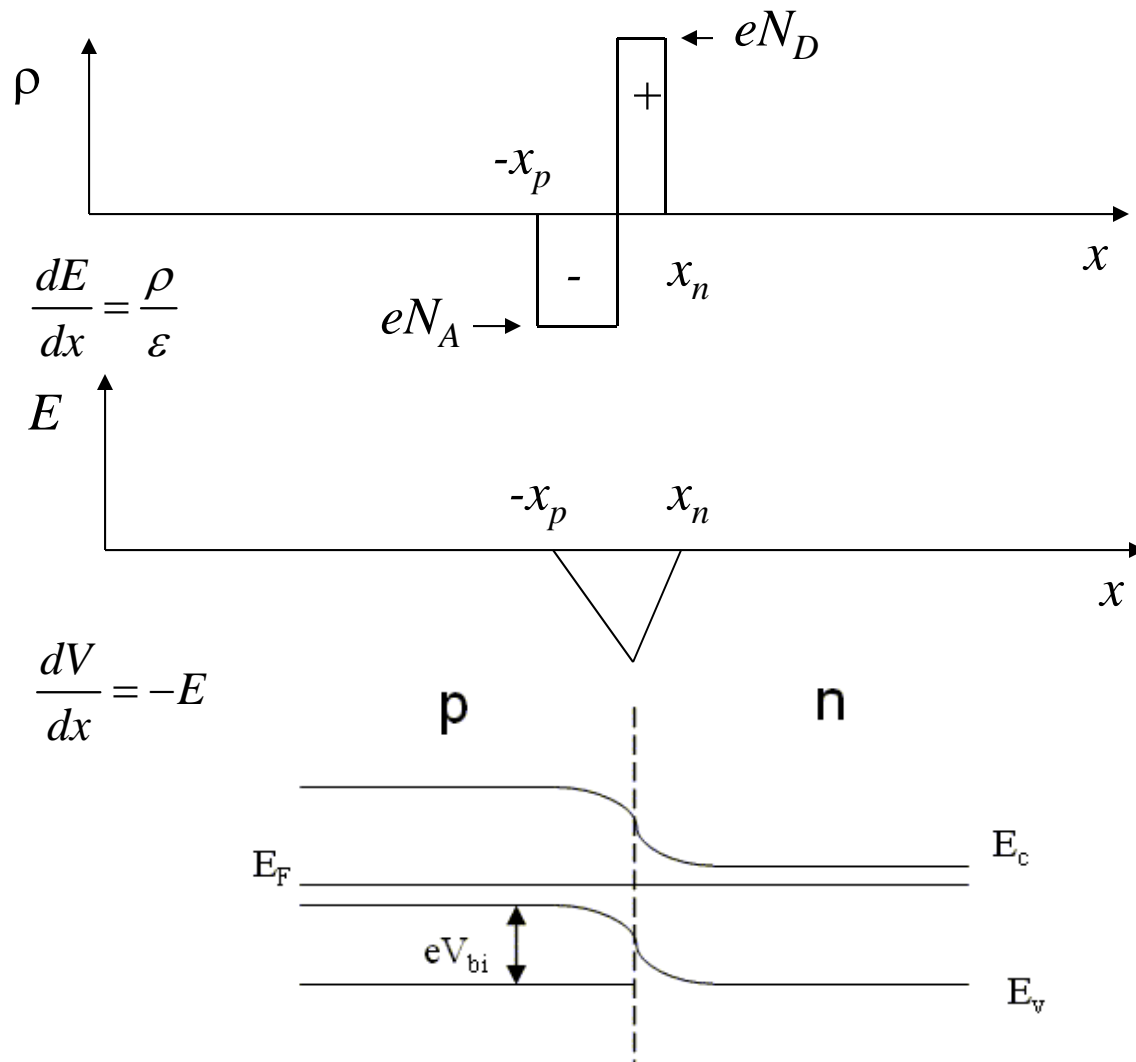
Used in front-end high-frequency receivers (mobile telephones).

bipolar transistors



Oxide isolated integrated BJT - a modern process

abrupt junction



$$eV_{bi} = k_B T \ln \left(\frac{N_D N_A}{n_i^2} \right)$$

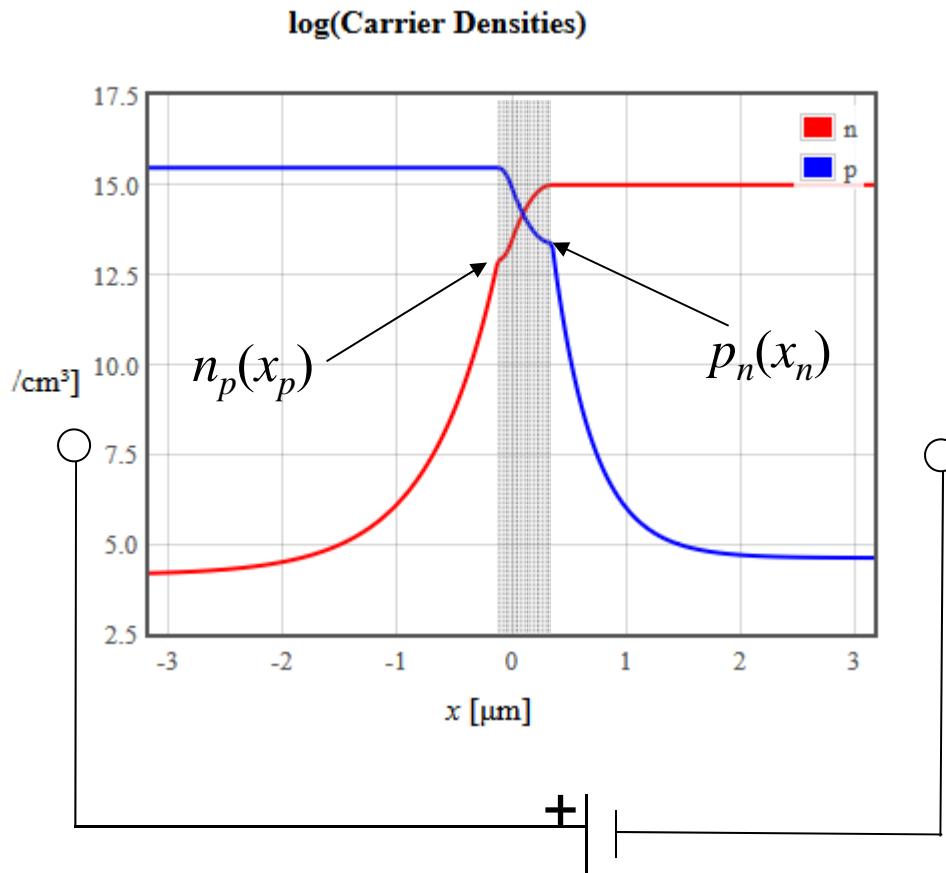
$$E = -\frac{eN_A}{\epsilon}(x + x_p) \quad -x_p > x > 0$$

$$E = \frac{eN_D}{\epsilon}(x - x_n) \quad 0 > x > x_n$$

$$V = \frac{eN_A}{\epsilon} \left(\frac{x^2}{2} + xx_p \right) \quad -x_p > x > 0$$

$$V = \frac{-eN_D}{\epsilon} \left(\frac{x^2}{2} - xx_n \right) \quad 0 > x > x_n$$

Forward bias, $V > 0$



Electrons and holes are driven towards the junction.
The depletion region becomes narrower

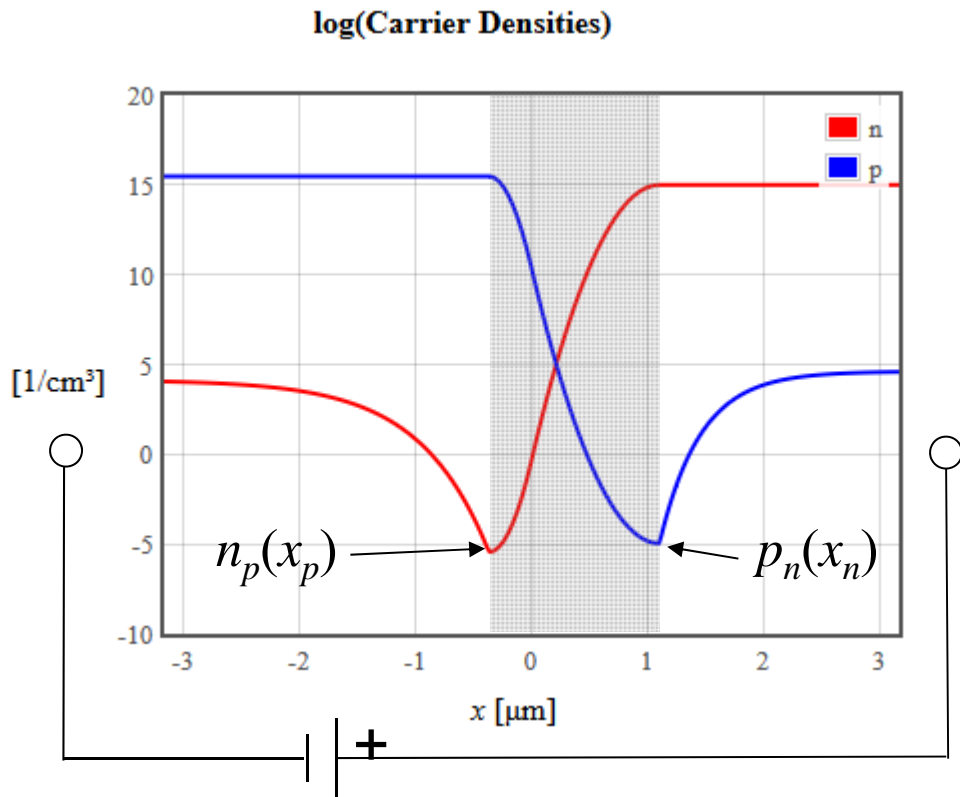
$$n_p(x_p) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_{e0} \exp\left(\frac{eV_{be}}{k_B T}\right)$$

Minority electrons are injected into the p-region
Minority holes are injected into the n-region

Reverse bias, $V < 0$



Electrons and holes are driven away from the junction.

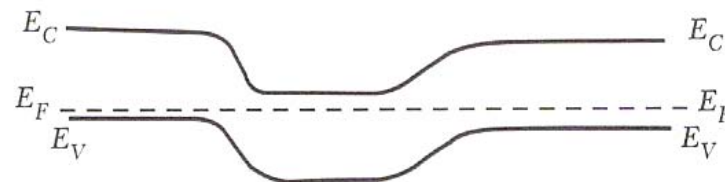
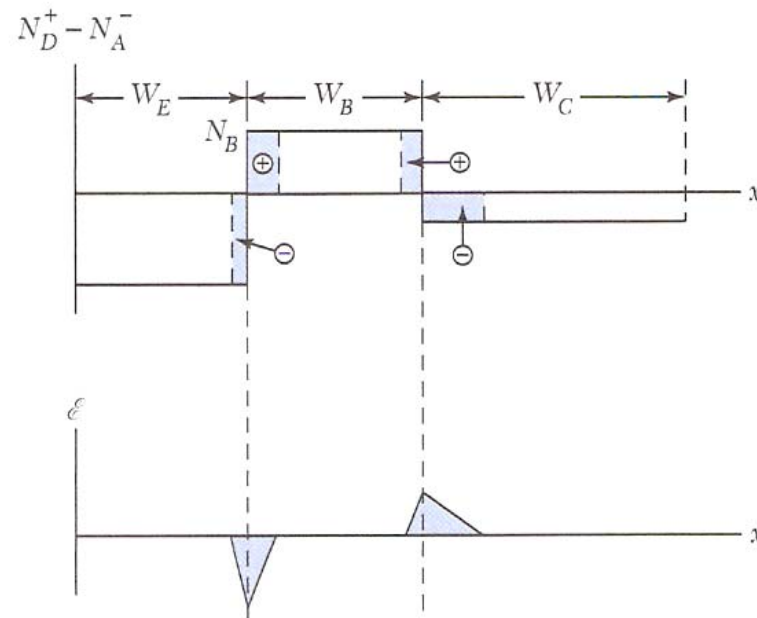
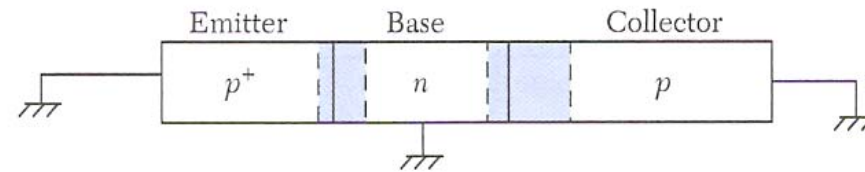
The depletion region becomes wider

$$n_p(x_p) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

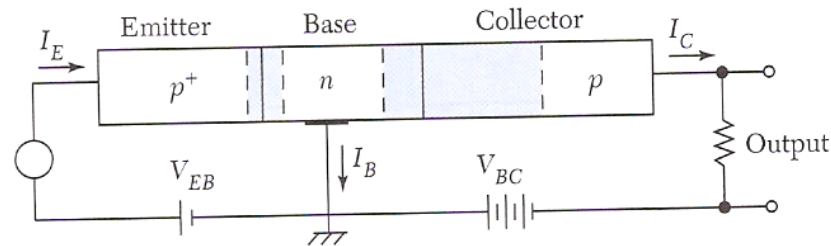
$$p_n(x_n) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

Minority electrons are extracted from the p-region by the electric field
Minority holes are extracted from the n-region by the electric field

pnp transistor, no bias

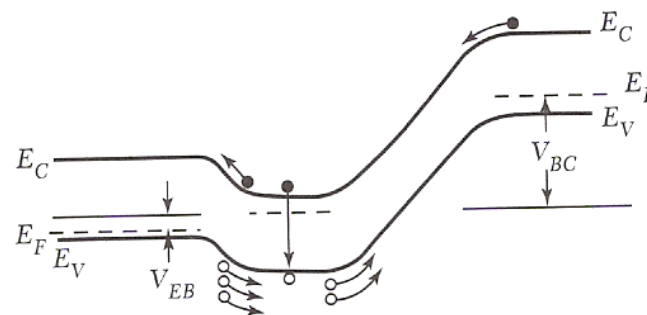
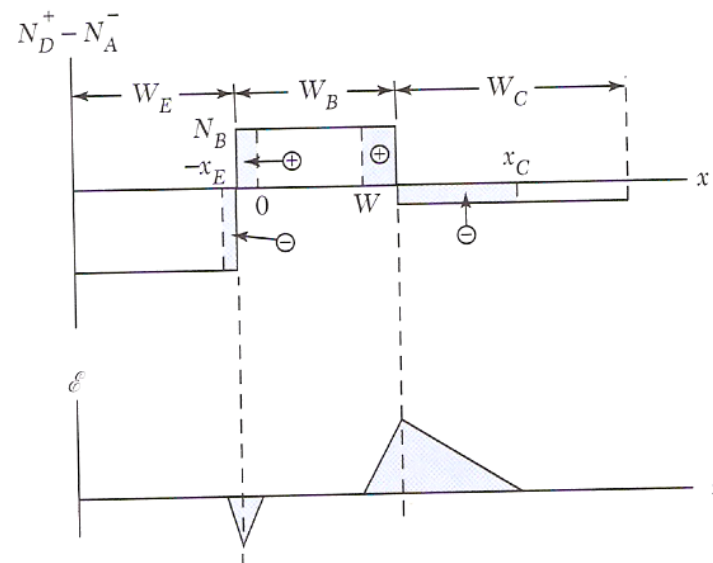


pnp transistor, forward active bias

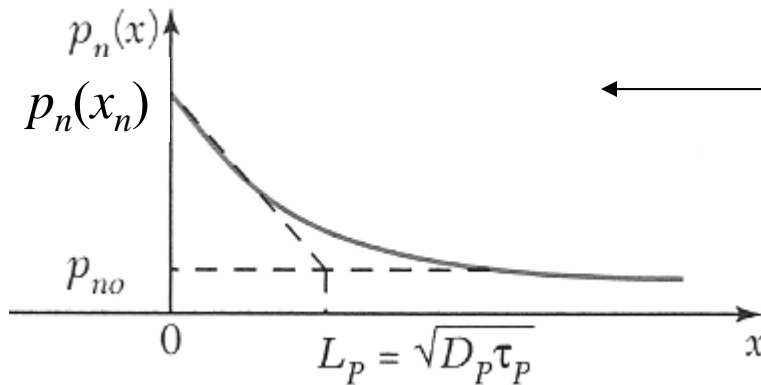


Always dissipate power due to the forward bias

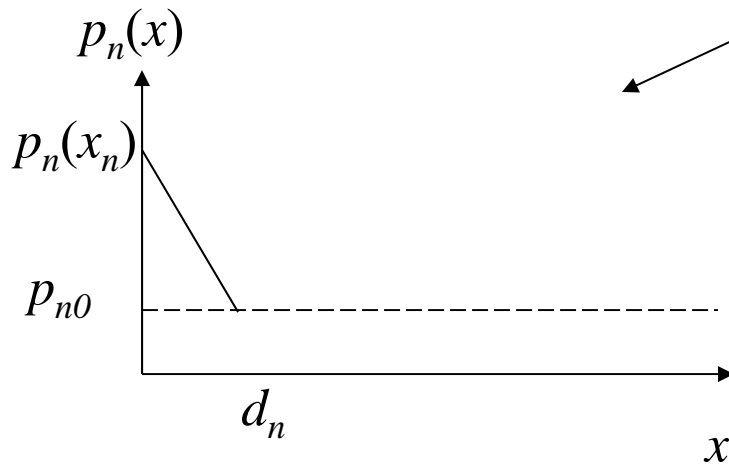
The base-emitter voltage controls the minority carriers injected from the emitter to the base. These diffuse to the base-collector junction and are swept into the collector.



Long/Short diode



← Long diode $d_n \gg L_p$



Short diode

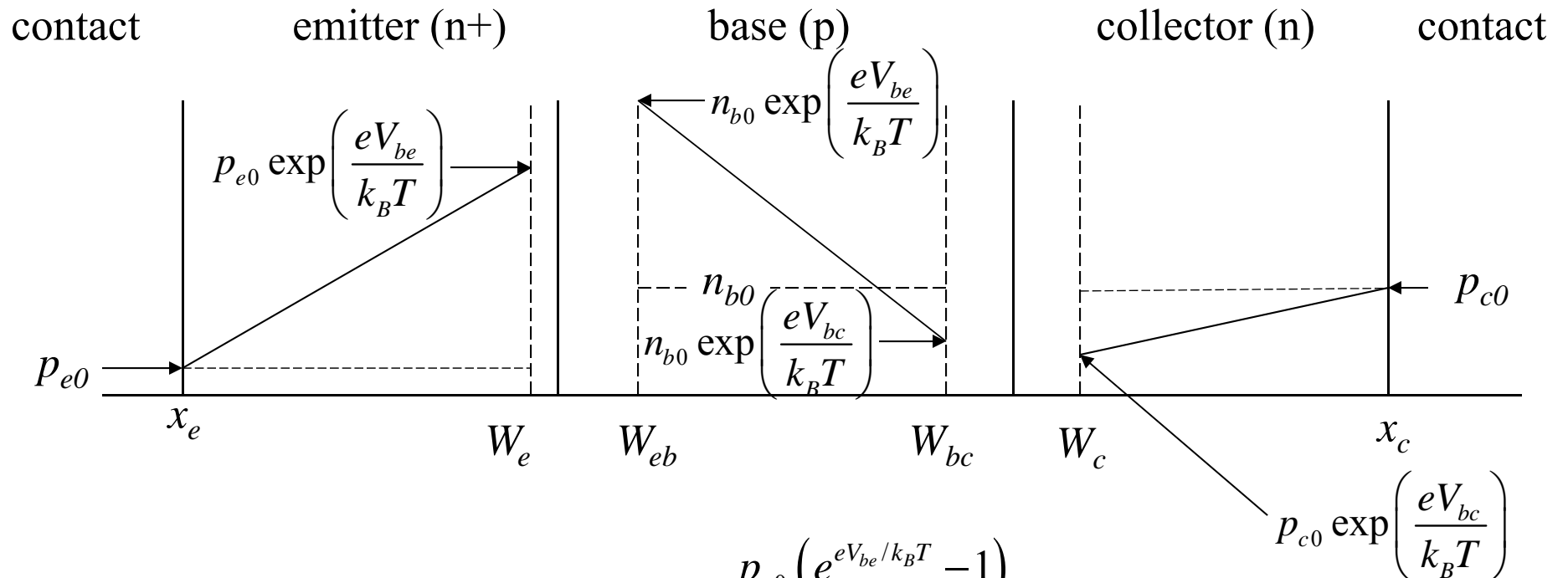
$d_n \ll L_p$

Metal contact is much closer to the depletion region than the diffusion length

$$J_{diff,p} = eD_p \frac{dp}{dx}$$

$$J_{diff,p} = eD_p \frac{dp}{dx} = eD_p \frac{(p_n(x_n) - p_{n0})}{d_n}$$

Minority carrier concentration



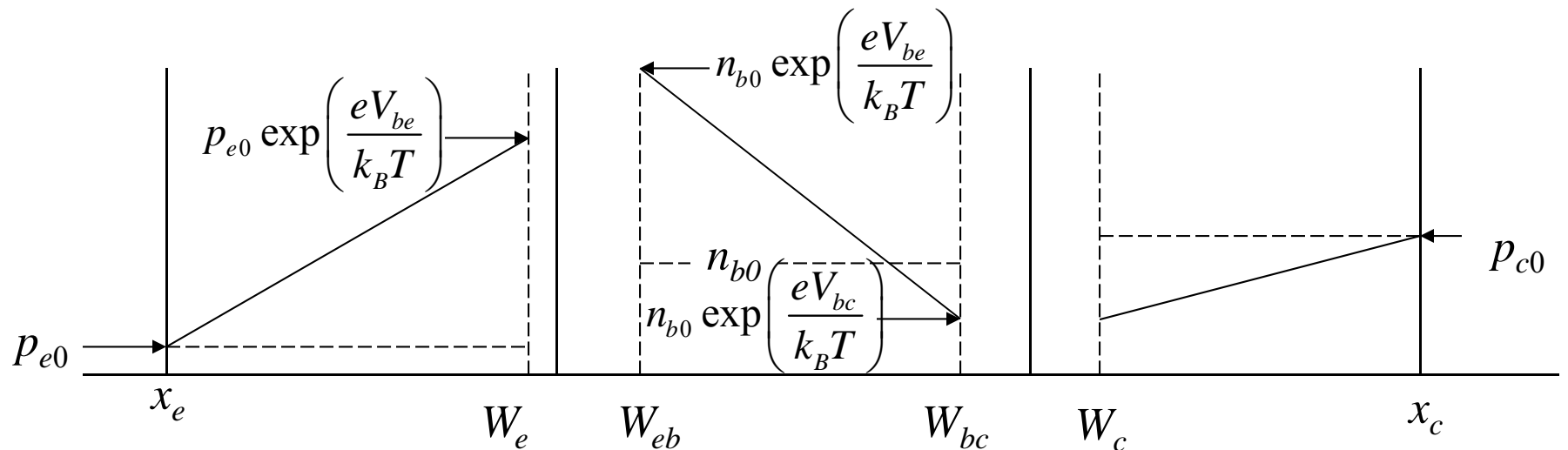
$$I_{Ep} = eA_{be} D_p \frac{p_{e0} \left(e^{eV_{be}/k_B T} - 1 \right)}{W_e - x_e}$$

$$I_{En} = -eA_{be} D_n \frac{n_{b0} \left(e^{eV_{be}/k_B T} - e^{eV_{bc}/k_B T} \right)}{W_{bc} - W_{be}}$$

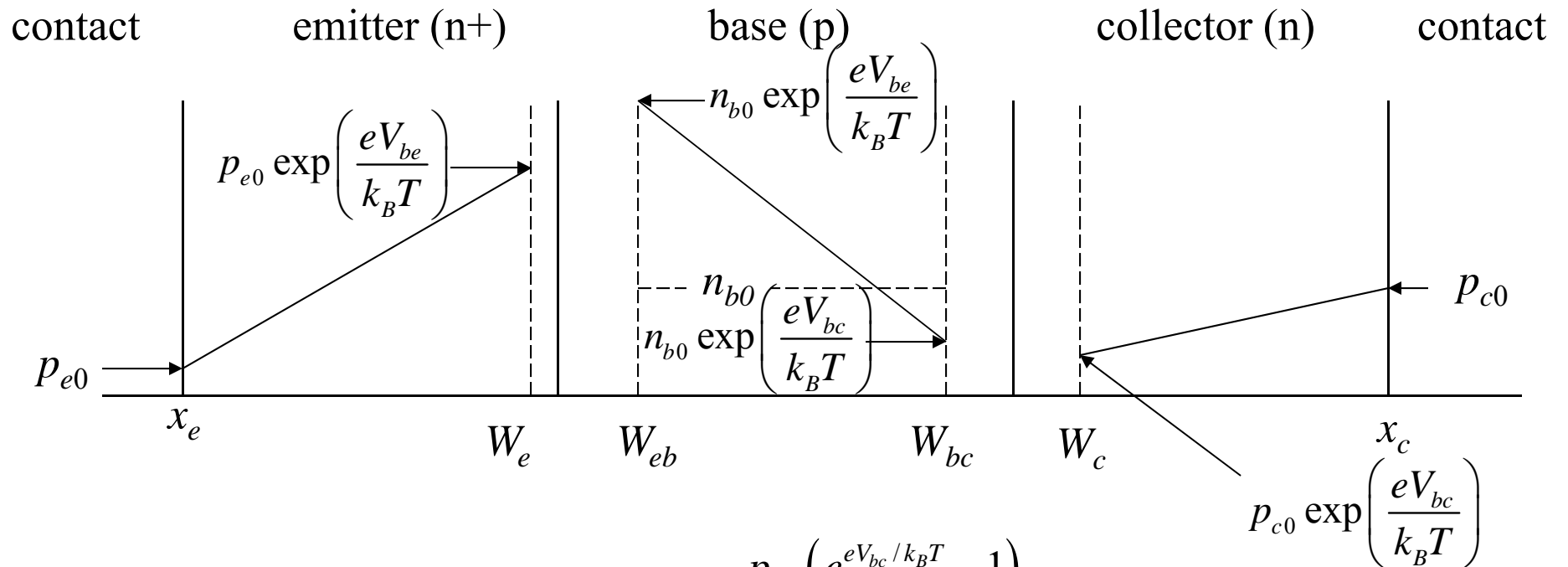
Emitter current

$$I_E = I_{En} + I_{Ep} = \left[\frac{eA_{be} D_p p_{e0}}{W_{eb} - x_e} + \frac{eA_{be} D_n n_{b0}}{W_{bc} - W_{be}} \right] \left(e^{eV_{be}/k_B T} - 1 \right) - \frac{eA_{be} D_n n_{b0}}{W_{bc} - W_{be}} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$



Collector current



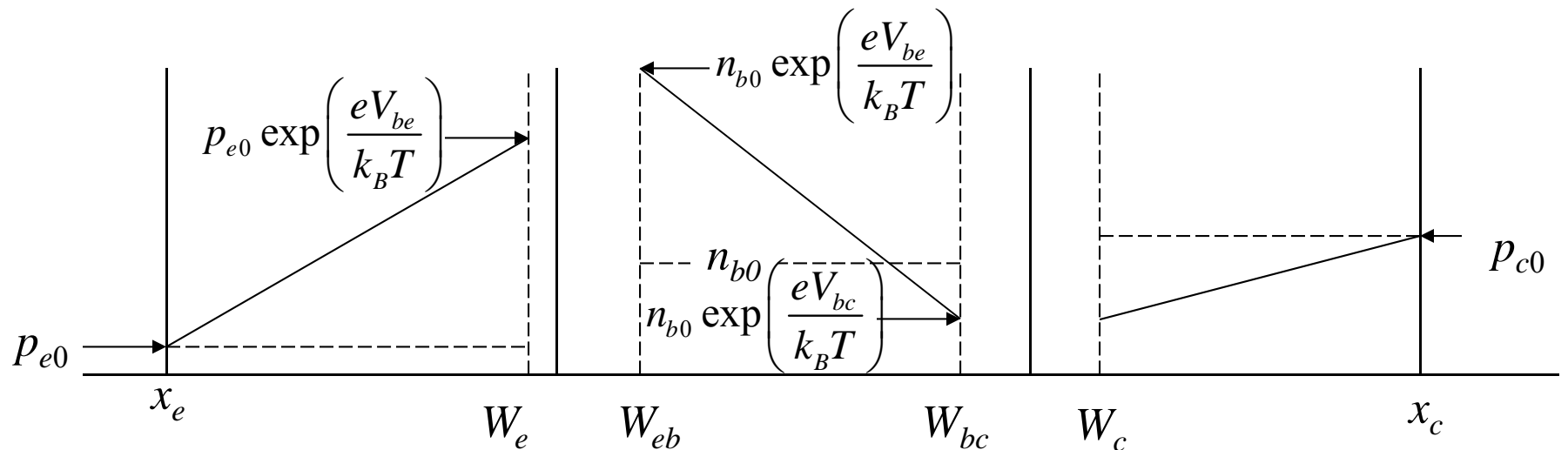
$$I_{cp} = -eA_{bc} D_p \frac{p_{c0} \left(e^{eV_{bc}/k_B T} - 1 \right)}{x_c - W_c}$$

$$I_{cn} = -eA_{bc} D_n \frac{n_{b0} \left(e^{eV_{be}/k_B T} - e^{eV_{bc}/k_B T} \right)}{W_{bc} - W_{eb}}$$

Collector current

$$I_c = I_{cp} + I_{cn} = \frac{eA_{bc}D_n n_{b0}}{W_{bc} - W_{be}} \left(e^{eV_{be}/k_B T} - 1 \right) - \left[\frac{eA_{bc}D_p p_{c0}}{x_c - W_c} + \frac{eA_{bc}D_n n_{b0}}{W_{bc} - W_{be}} \right] \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_c = I_{cp} + I_{cn} = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

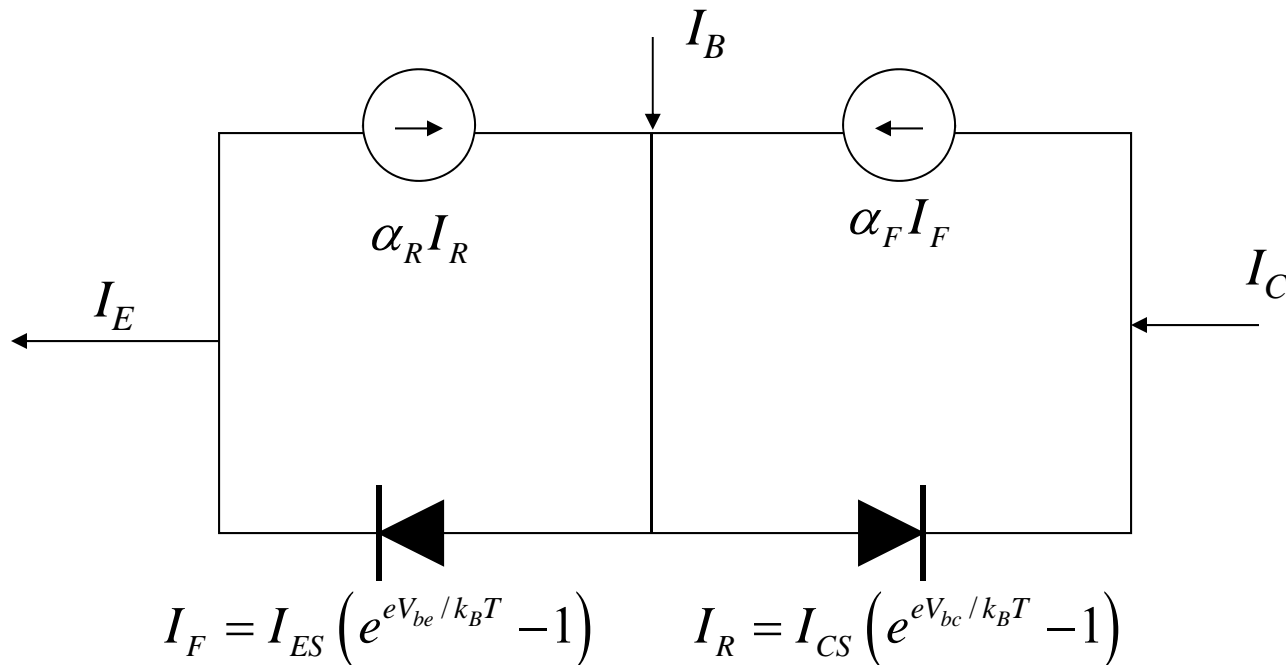


Ebers-Moll model

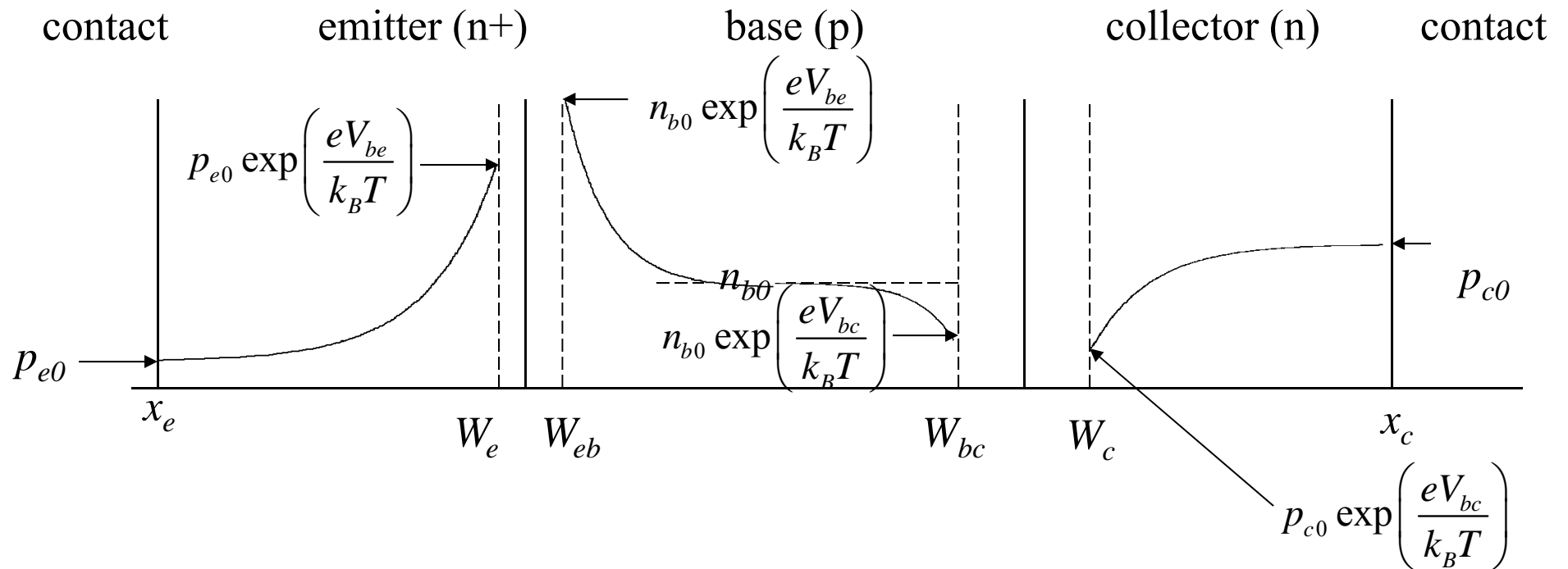
$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_C = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_B = I_E - I_C$$



Not an npn transistor



Emitter efficiency

$$\gamma_e = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{1}{1 + I_{Ep} / I_{En}} \quad \leftarrow \text{for npn}$$

$$I_{Ep} = eA_{be} D_p \frac{p_{e0} (e^{eV_{be}/k_B T} - 1)}{W_{eb} - x_e}$$

$$I_{En} = -eA_{be} D_n \frac{n_{b0} (e^{eV_{be}/k_B T} - e^{eV_{bc}/k_B T})}{W_{bc} - W_{be}}$$

For $\gamma_e \sim 1$, $W_{bc} - W_{be} \ll L_b$, $W_{eb} - x_e$ and $n_{b0} \gg p_{e0}$

neutral base width

$$\frac{n_i^2}{N_{Ab}}$$

$$\frac{n_i^2}{N_{De}}$$

Small base width and heavy emitter doping

Base transport factor

$$B = \frac{I_c}{I_{En}}$$

ratio of the injected current to the collected current

recombination in the base would reduce the base transport factor

A thin base with low doping results in a base transport factor ~ 1

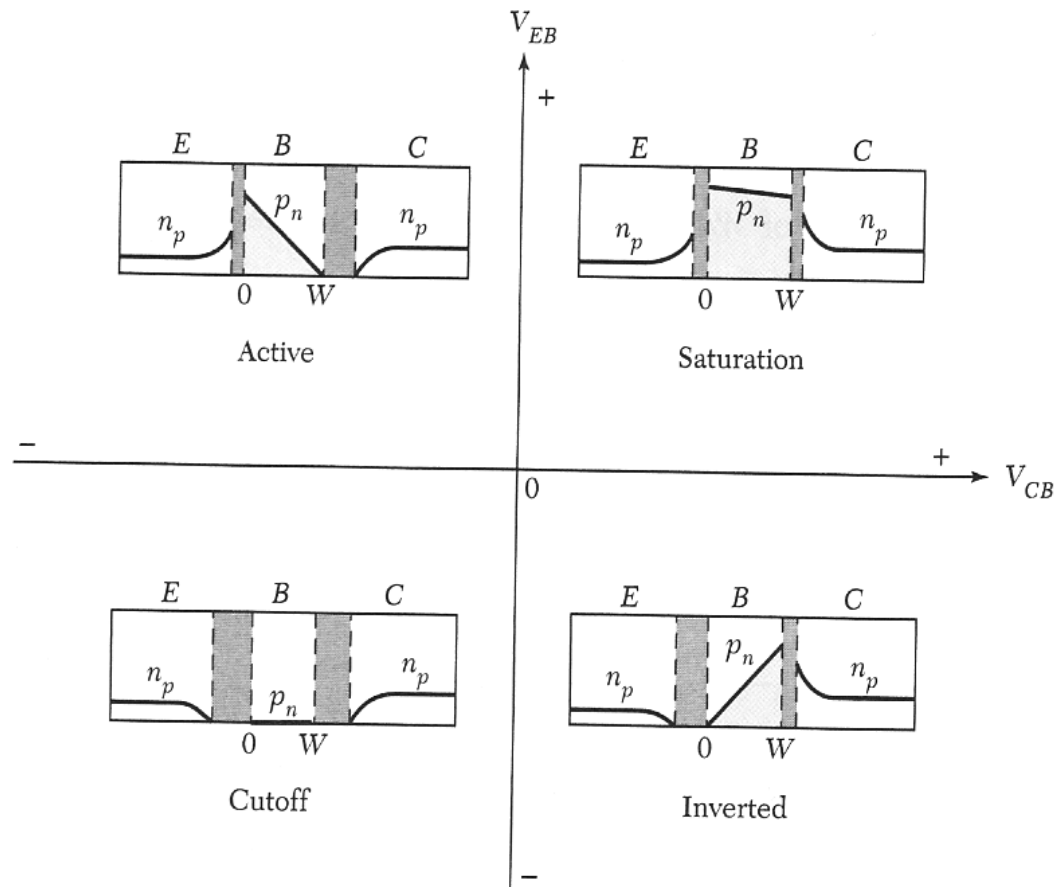
Current transfer ratio

$$\alpha = \frac{I_C}{I_E} = B\gamma_e$$

$\alpha \sim 1$ for a good BJT

Transistor modes

1. Forward active: emitter-base **forward**, base-collector **reverse**
2. Saturation: emitter-base **forward**, base-collector **forward**
3. Reverse active: emitter-base **reverse**, base-collector **forward**
4. Cut-off: emitter-base **reverse**, base-collector **reverse**

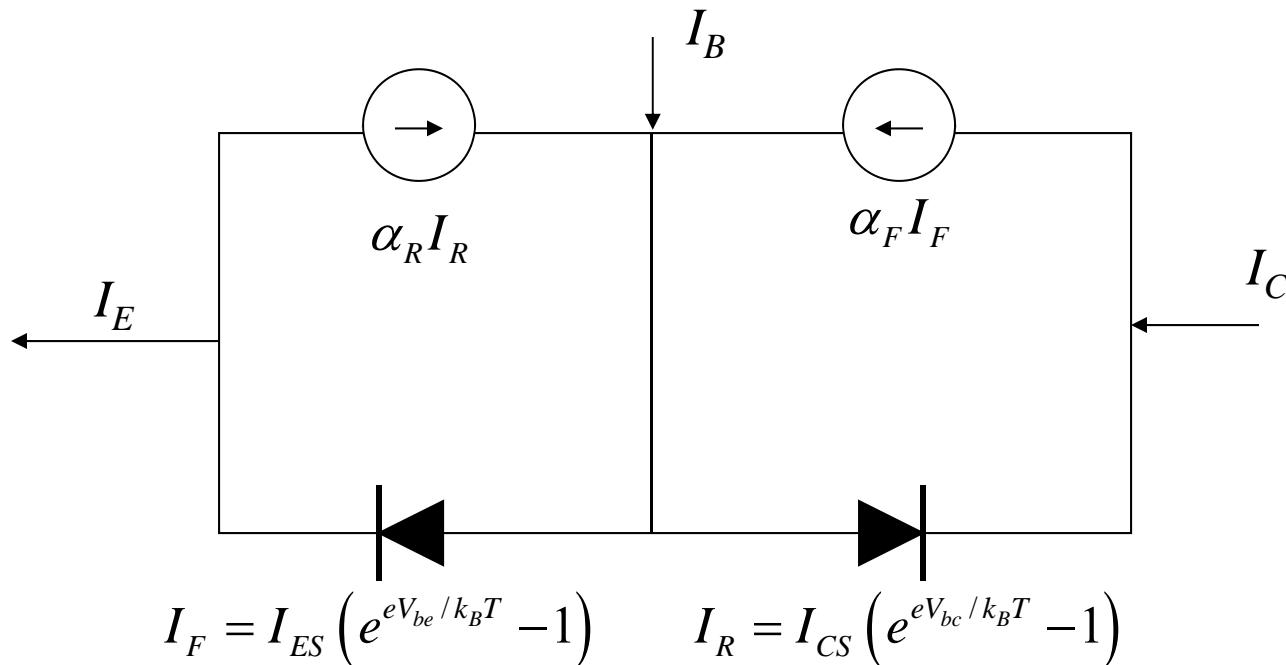


Ebers-Moll model

$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_C = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_B = I_E - I_C$$

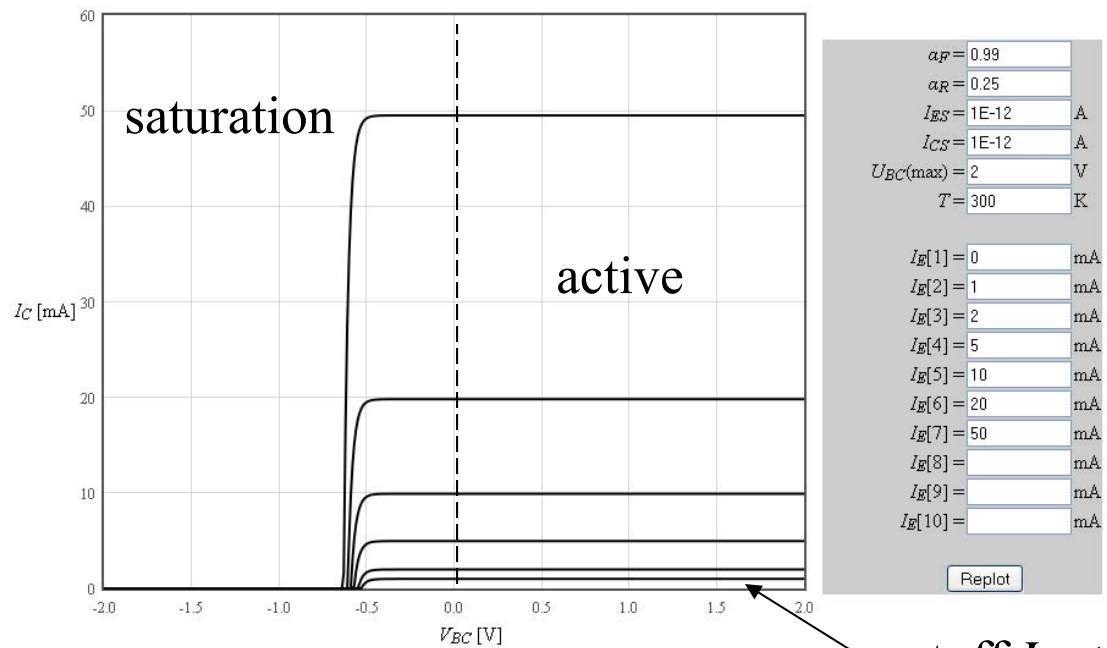
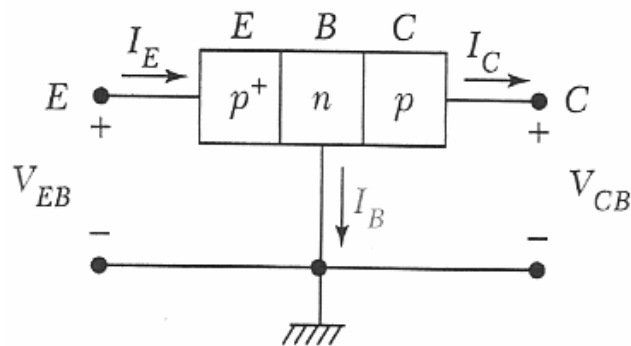


Common base configuration

$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

solve for V_{be}

$$I_c = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$



cutoff $I_E < 0$

Ebers - Moll Model

$$I_{ES} = \left[\frac{eA_{be}D_p p_{e0}}{W_{eb} - x_e} + \frac{eA_{be}D_n n_{b0}}{W_{bc} - W_{be}} \right]$$

$$\alpha_R I_{CS} = \frac{eA_{be}D_n n_{b0}}{W_{bc} - W_{be}}$$

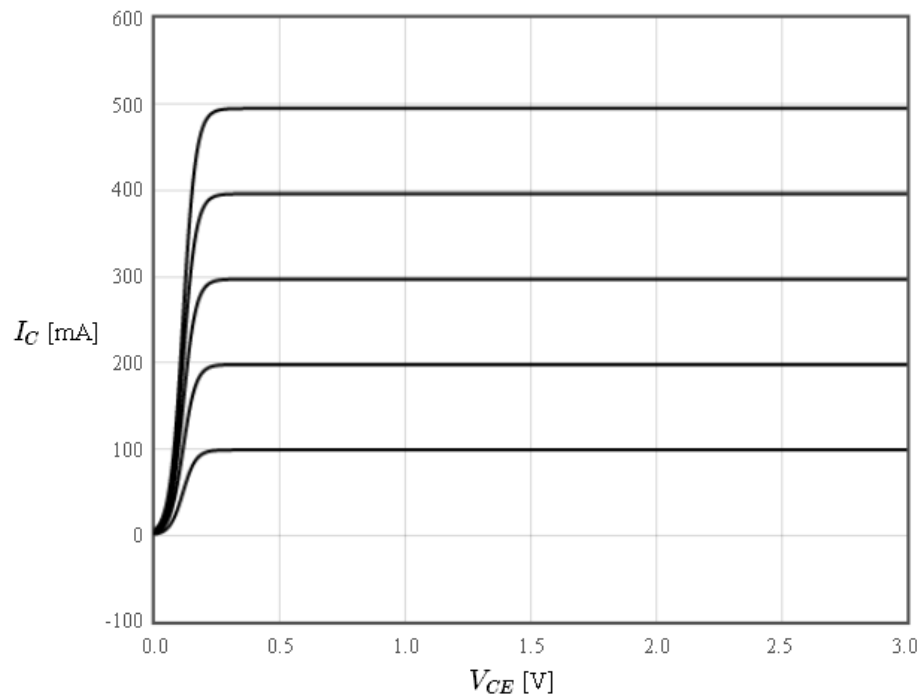
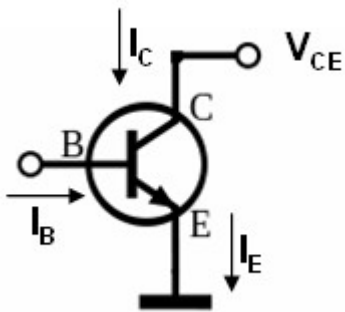
$$\alpha_F I_{ES} = \frac{eA_{bc}D_n n_{b0}}{W_{bc} - W_{be}}$$

$$I_{CS} = \left[\frac{eA_{bc}D_p p_{c0}}{x_c - W_c} + \frac{eA_{bc}D_n n_{b0}}{W_{bc} - W_{be}} \right]$$

Common emitter configuration

$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right) \quad I_B = I_E - I_C$$

$$I_C = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$



$\alpha_F = 0.99$
 $\alpha_R = 0.25$
 $I_{ES} = 1\text{E-}12$ A
 $I_{CS} = 1\text{E-}12$ A
 $V_{CE(\text{max})} = 3$ V
 $T = 300$ K

$I_B[1] = 1$ mA
 $I_B[2] = 2$ mA
 $I_B[3] = 3$ mA
 $I_B[4] = 4$ mA
 $I_B[5] = 5$ mA
 $I_B[6] =$ mA
 $I_B[7] =$ mA
 $I_B[8] =$ mA
 $I_B[9] =$ mA
 $I_B[10] =$ mA

Replot

current amplification ~ 100

Current amplification factor

$$\beta = h_{fe} = \frac{I_C}{I_B}$$

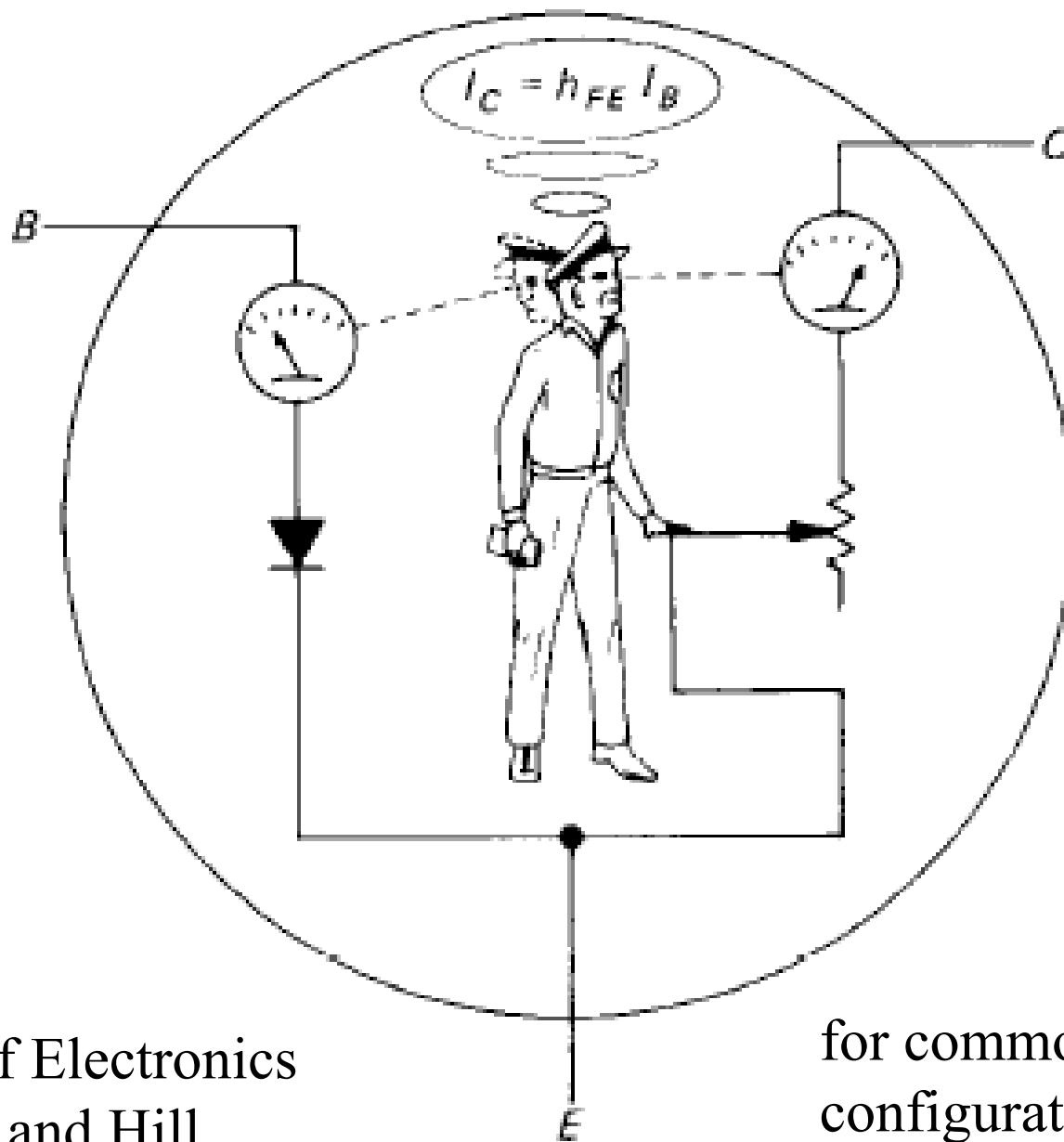
$$I_B = I_E - I_C$$

$$I_C = \alpha I_E$$

$$I_B = \left(\frac{1}{\alpha} - 1 \right) I_C$$

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1 - \alpha} = \frac{B\gamma_e}{1 - B\gamma_e}$$

$$\beta \sim 50 - 500$$



The Art of Electronics
Horowitz and Hill

for common emitter
configuration

“Transistor man”

Transconductance

$$g_m = \frac{\partial I_C}{\partial V_{be}}$$

$$I_c = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

The first term depends on V_{be}

$$g_m = \frac{e\alpha_F I_{ES}}{k_B T} e^{eV_{be}/k_B T} \approx \frac{eI_C}{k_B T} = \frac{e\beta I_B}{k_B T}$$

The transconductance can be very high.

Early effect

Ebers - Moll:

$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

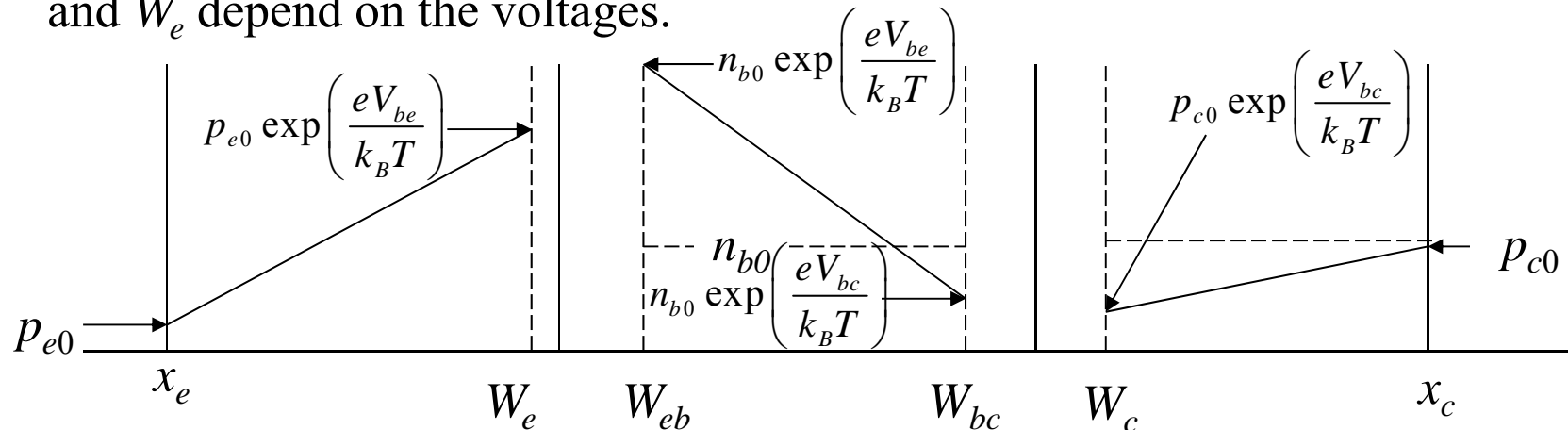
$$I_C = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_B = I_E - I_C$$

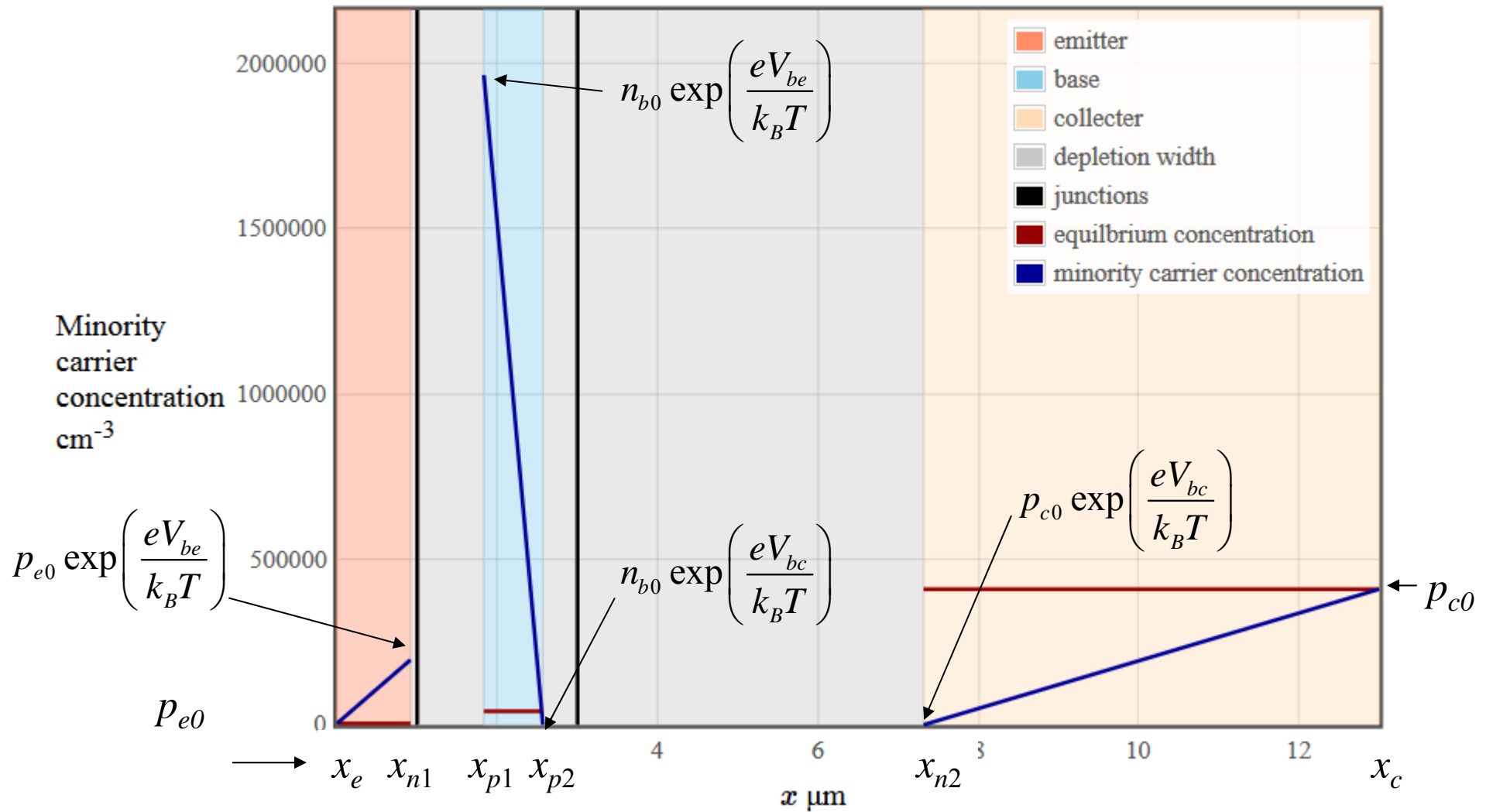
$$I_{ES} = \left[\frac{eA_{be} D_p p_{e0}}{W_{eb} - x_e} + \frac{eA_{be} D_n n_{b0}}{W_{bc} - W_{be}} \right]$$

$$I_{CS} = \left[\frac{eA_{bc} D_p p_{c0}}{x_c - W_c} + \frac{eA_{bc} D_n n_{b0}}{W_{bc} - W_{be}} \right]$$

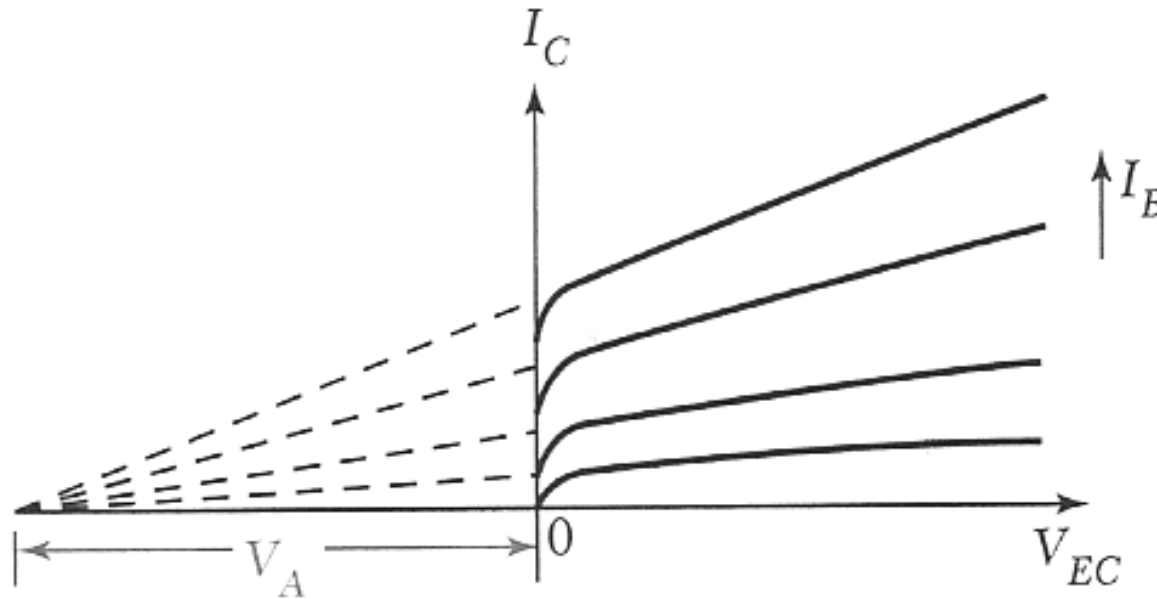
I_{ES} and I_{CS} are treated as constants but the depletion widths W_{bc} , W_{be} , W_c and W_e depend on the voltages.



Minority carrier concentration



Early effect



Common emitter configuration

Base width modulation: smaller width increases the diffusion current and increases the gain.

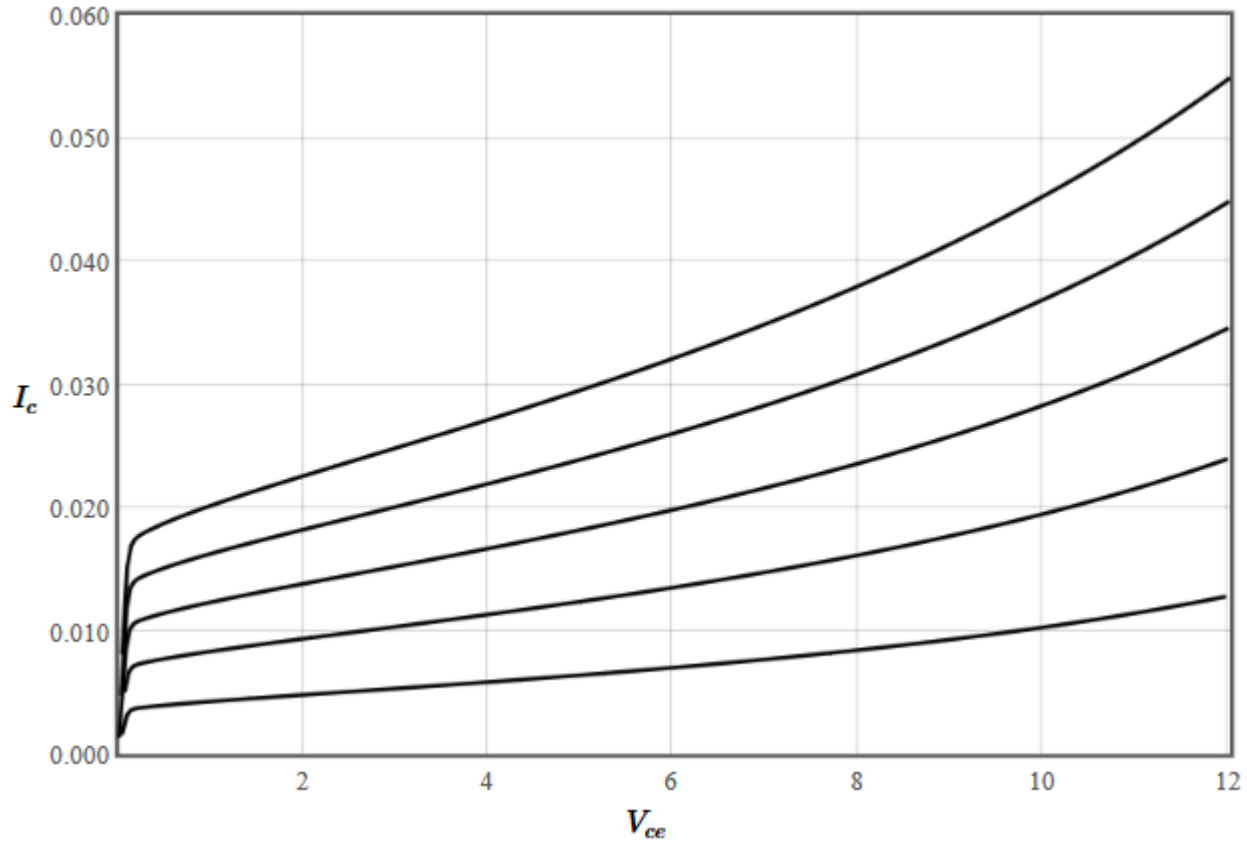
Punchthrough: The neutral base width goes to zero and all gain is lost.

Lightly dope the collector -> voltage drops in collector. Makes circuit slower.

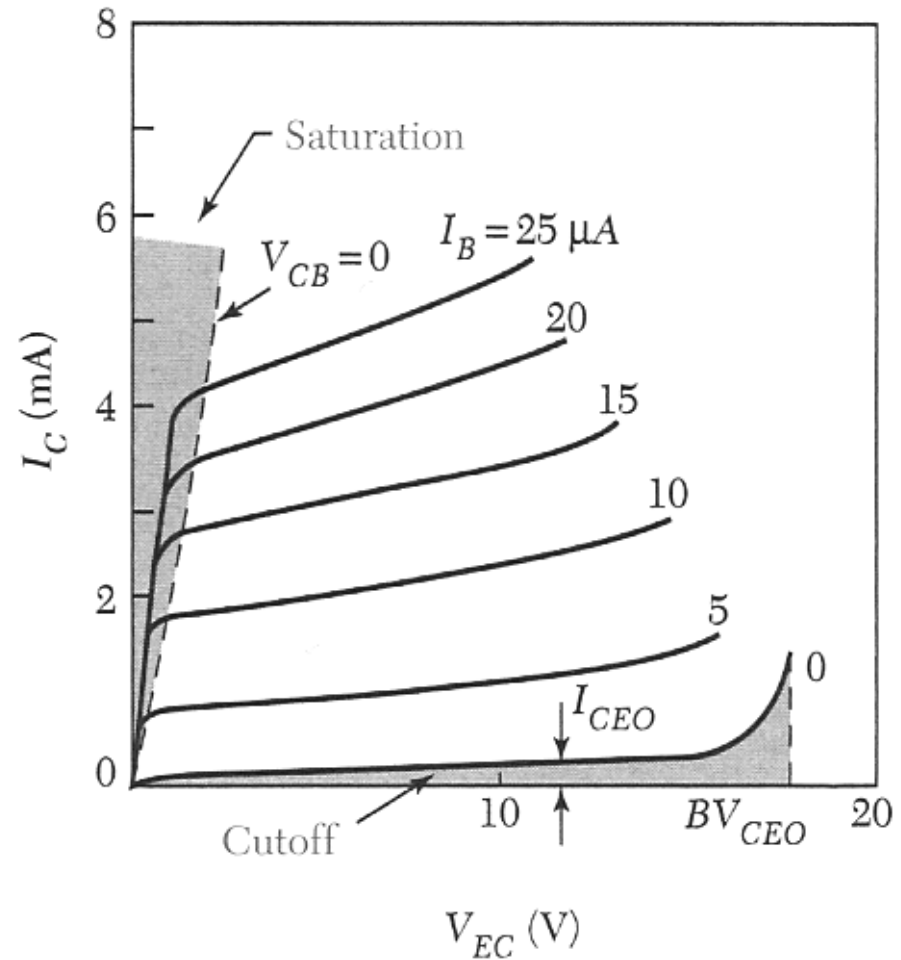
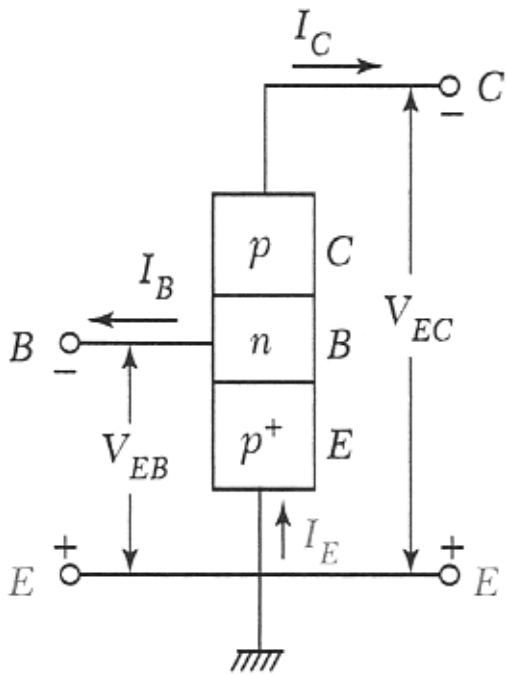
NPN common emitter configuration

n-Emitter		$A_{eb} = 1E-3$ cm ²
Minority $\mu_{pe} = 480$ cm ² /Vs	$N_{de} = 1E16$ cm ⁻³	$N_c(300K) = 2.78E19$ cm ⁻³
$\tau_{pe} = 1E-5$ s		$N_v(300K) = 9.84E18$ cm ⁻³
		$E_g = 1.166-4.73E-4*T*(T+636)$ eV
		$\epsilon_r = 11.9$
p-Base		$I_{bmax} = 0.001$ eV
Minority $\mu_{nb} = 1350$ cm ² /Vs	$N_{ab} = 1E15$ cm ⁻³	$V_{ce max} = 12$ eV
$\tau_{nb} = 1E-5$ s		$x_1 - x_e = 1$ μm
		$x_2 - x_1 = 2$ μm
		$x_c - x_2 = 10$ μm
		$T = 300$ K
n-Collector		<input type="button" value="Calculate"/>
Minority $\mu_{pc} = 480$ cm ² /Vs	$N_{dc} = 1E14$ cm ⁻³	
$\tau_{pc} = 1E-5$ s		

$$I_C \sim \beta I_B$$

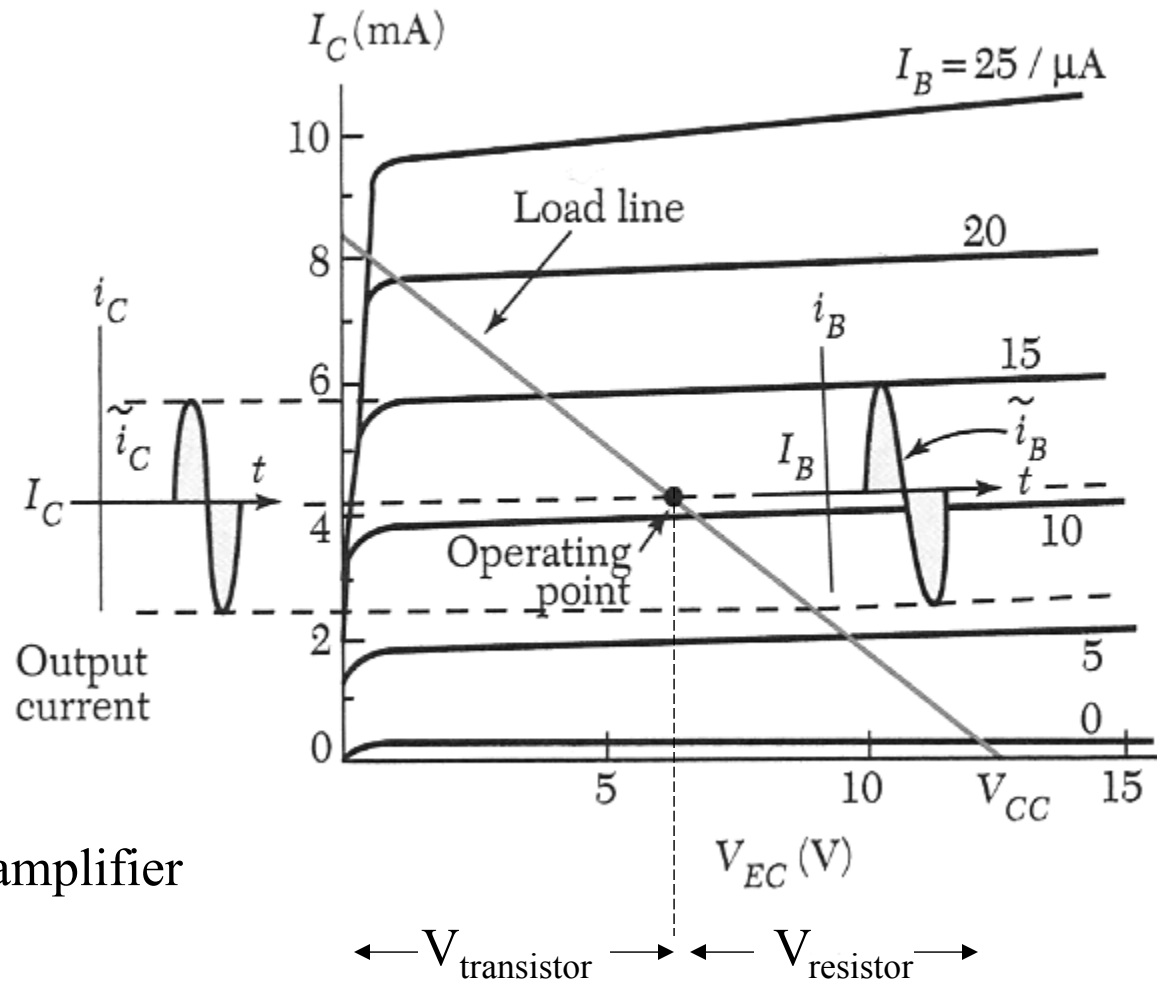
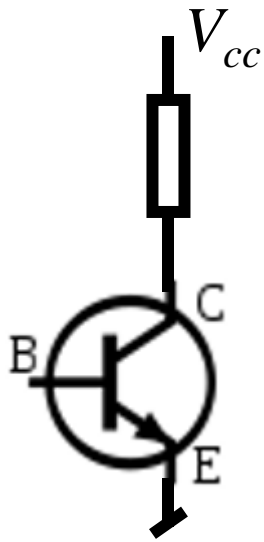


Common emitter configuration



$$I_C \sim \beta I_B \text{ amplifier}$$

Small signal response



Low input impedance amplifier

Small signal response

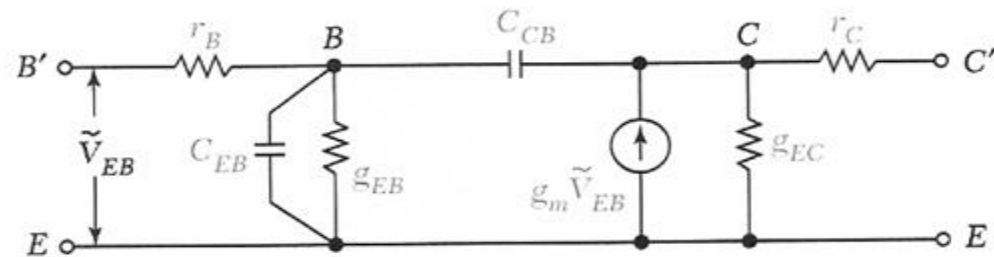
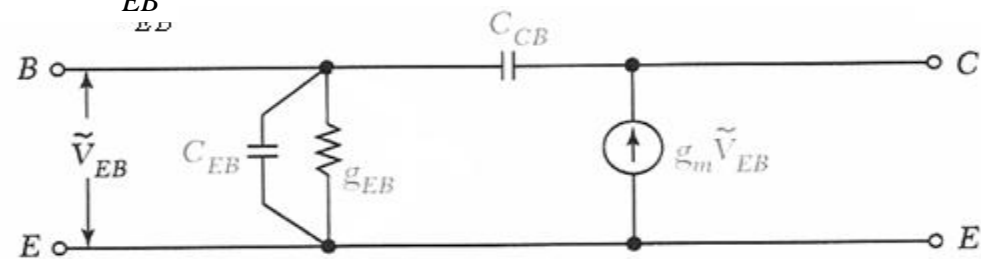
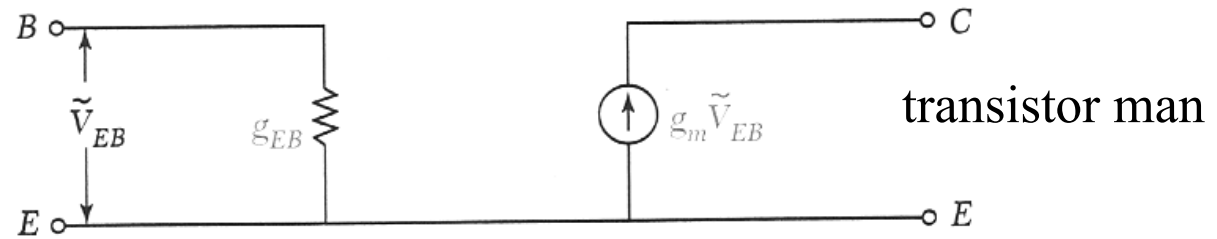
$$\tilde{i}_c = \beta \tilde{i}_B = \beta g_{EB} \tilde{v}_{EB}$$

input conductance:

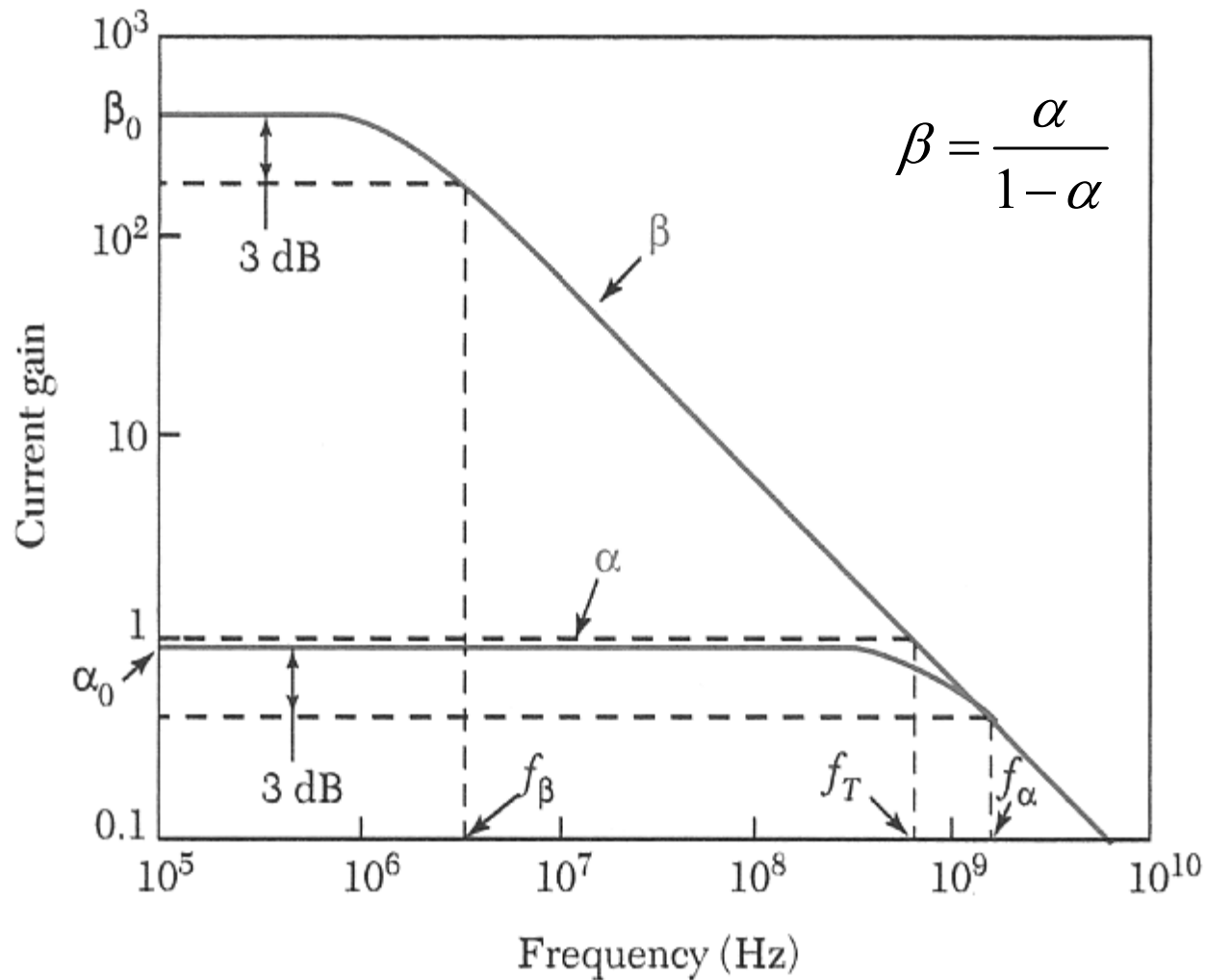
$$g_{EB} = \frac{\tilde{i}_B}{\tilde{v}_{EB}}$$

transconductance:

$$g_m = \frac{\tilde{i}_c}{\tilde{v}_{EB}}$$



Small signal response



$$f_\beta = (1 - \alpha_0) f_\alpha$$

$$f_T = \alpha_0 f_\alpha$$