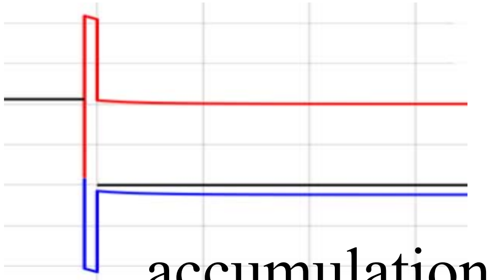
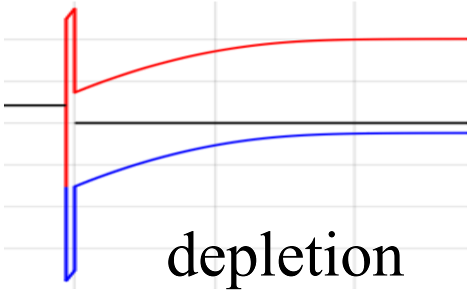
MOSFETs: Gradual Channel Approximation

Gradual channel approximation

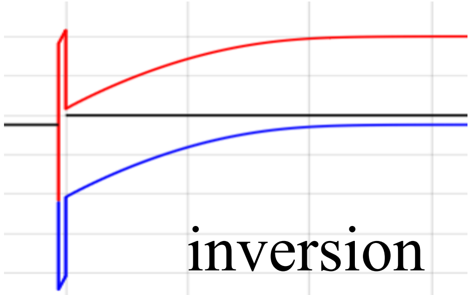


accumulation

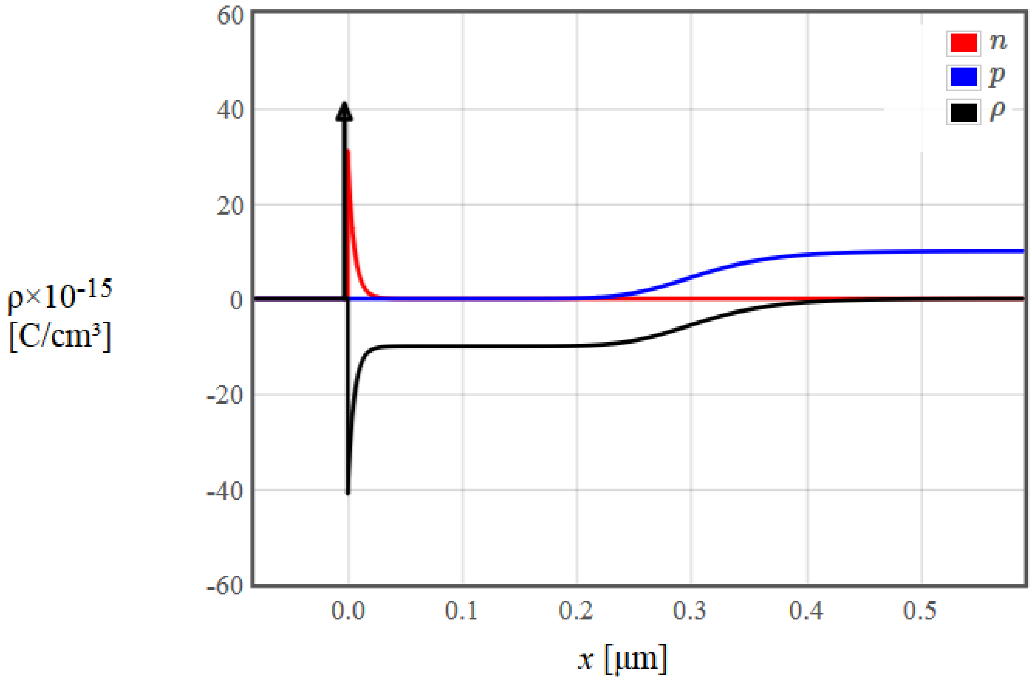
$$Q_{\text{mobile}} = \begin{cases} 0, & \text{for } V_G - V_B < V_T \\ -C_{\text{ox}}(V_G - V_B - V_T) & \text{for } V_G - V_B > V_T \end{cases}$$



depletion



inversion

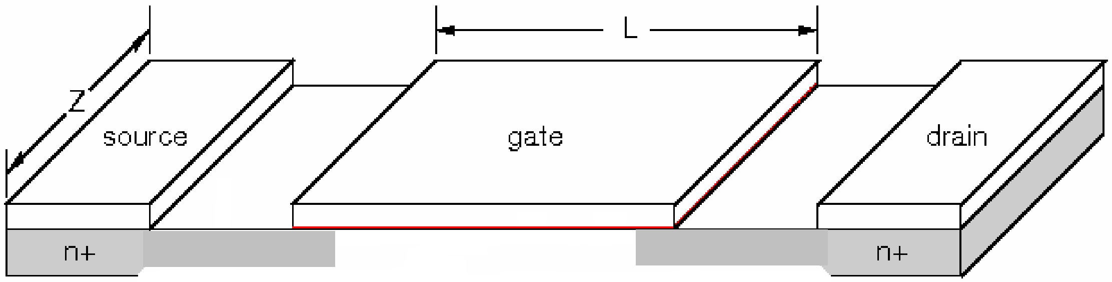


http://lampx.tugraz.at/~hadley/psd/L10/gradualchannelapprox.php

Gradual channel approximation

Ohm's law ⟶ $j = -nev_d = ne\mu_n E_y$

$$I = Ztj = Ztne\mu_n E_y = Ze\mu_n n_s E_y$$



$n_s = nt$ is the sheet charge at the interface.

$$n_s(y) = -\frac{Q}{e} = \frac{C_{ox}\left(V_G - V_{ch}(y) - V_T\right)}{e}$$

Gradual channel approximation

$$n_s(y) = -\frac{Q(y)}{e} = \frac{C_{ox}(V_G - V_{ch}(y) - V_T)}{e}$$

$$I = Ztj = Ztnev_d = Zen_s\mu_n E_y$$

$$I_D = -Z\mu_n C_{ox}(V_G - V_{ch}(y) - V_T)\frac{dV_{ch}}{dy}$$

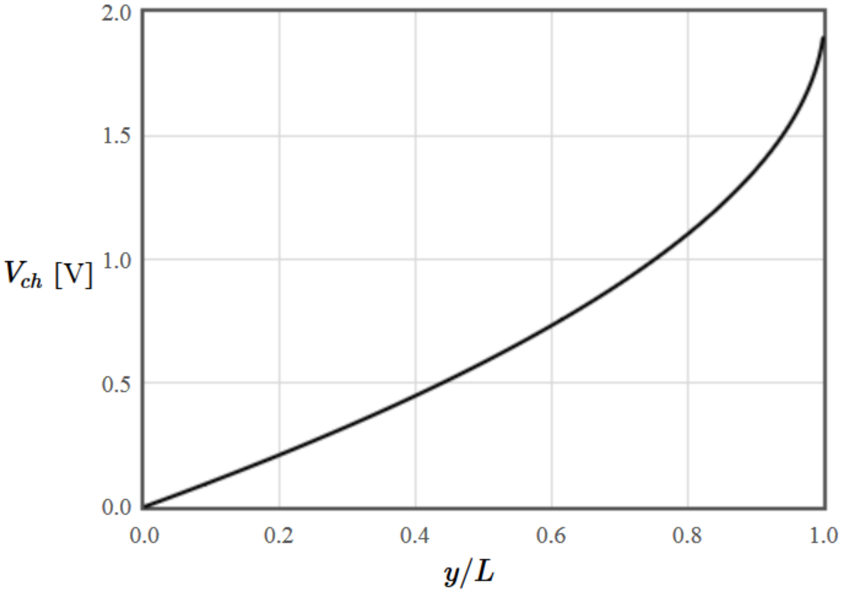
differential equation for V_{ch}

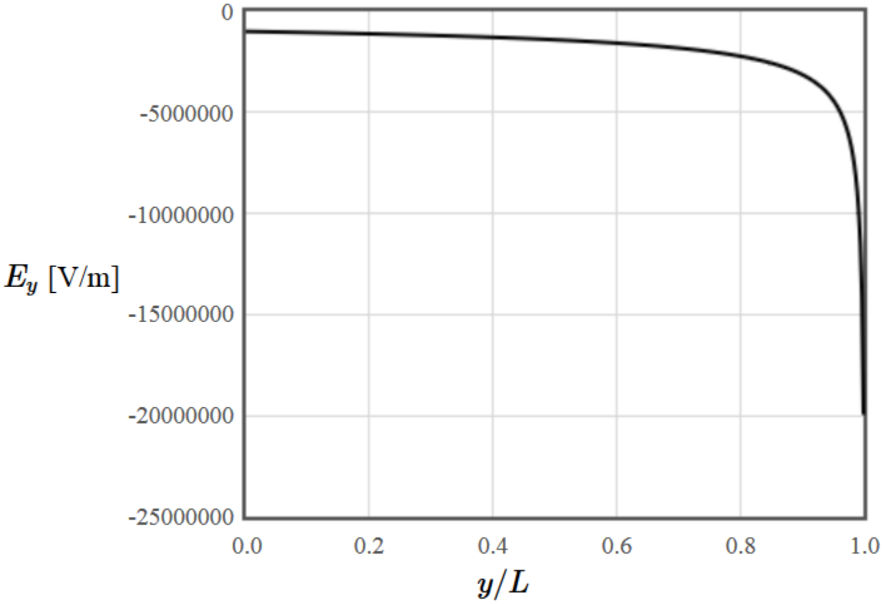
http://lampx.tugraz.at/~hadley/psd/L10/gradualchannelapprox.php

Gradual channel approximation

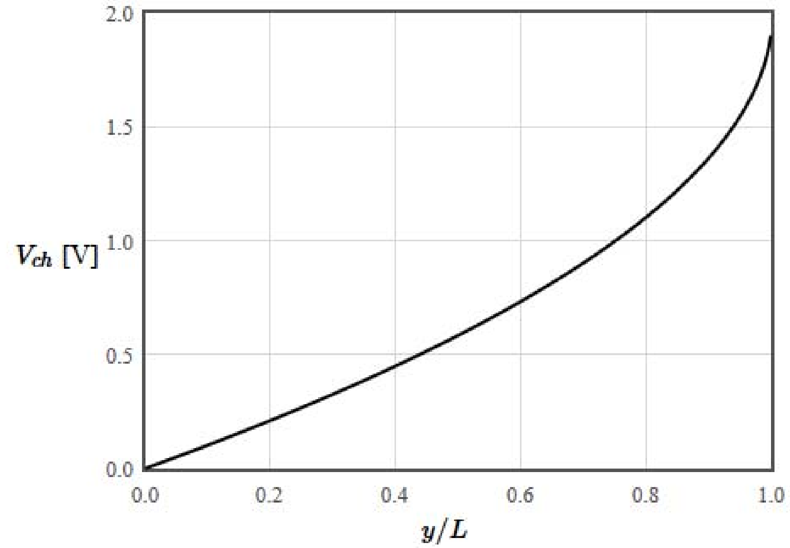
$$V_{ch}(y) = V_G - V_T - \sqrt{(V_G - V_T)^2 - \frac{2I_D y}{Z\mu_n C_{ox}}}$$

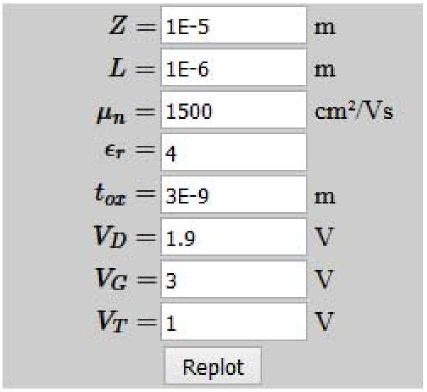
$$E_y = -\frac{dV_{ch}}{dy} = -\frac{I_D}{Z\mu_n C_{ox}\sqrt{(V_G - V_T)^2 - \frac{2I_D y}{Z\mu_n C_{ox}}}}$$





MOSFET Gradual Channel Approximation




$Z =$ 1E-5 m

$L =$ 1E-6 m

$\mu_n =$ 1500 cm²/Vs

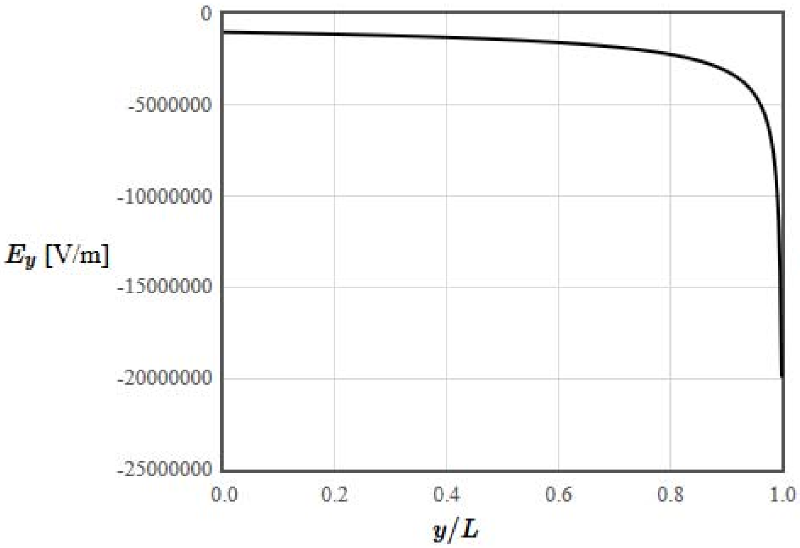
$\epsilon_r =$ 4

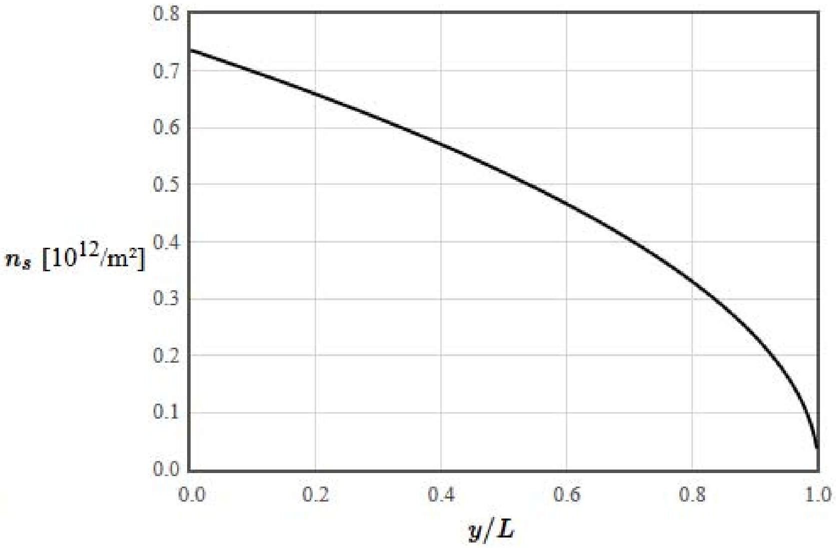
$t_{ox} =$ 3E-9 m

$V_D =$ 1.9 V

$V_G =$ 3 V

$V_T =$ 1 V





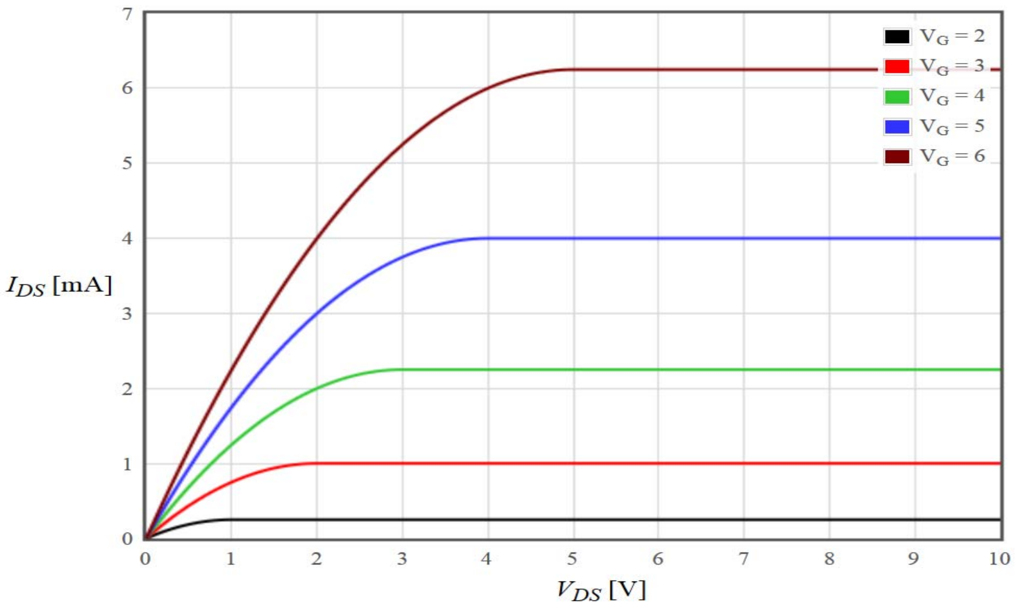
http://lampx.tugraz.at/~hadley/psd/L10/gradualchannelapprox.php

Gradual channel approximation

$$\int_0^L I_D \, \mathrm{d}y = \int_0^{V_D} Z\mu_n C_{ox}(V_G - V_{ch}(y) - V_T)\, \mathrm{d}V$$

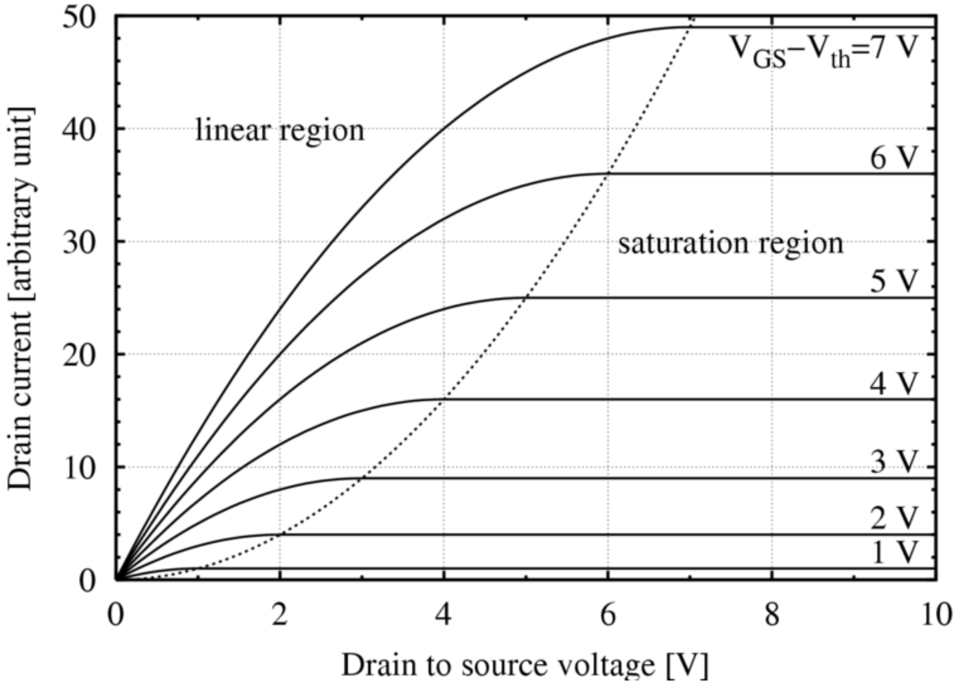
$$\boxed{I_D = \frac{Z}{L}\mu_n C_{ox}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]}$$

Valid in the linear regime (until pinch-off occurs at the drain).



MOSFET-saturation voltage

$$I=\frac{Z}{L}\mu_n C_{ox}\left[\left(V_G-V_T\right)V_D-\frac{V_D^2}{2}\right]$$



At pinch-off, $dI_{ds}/dV_{ds} = 0$

$$\frac{dI}{dV_D}=\frac{Z}{L}\mu_n C_{ox}\left[\left(V_G-V_T\right)-V_D\right]=0$$

$$V_{sat}=\left(V_G-V_T\right)$$

A MOSFET in saturation is a voltage controlled current source.

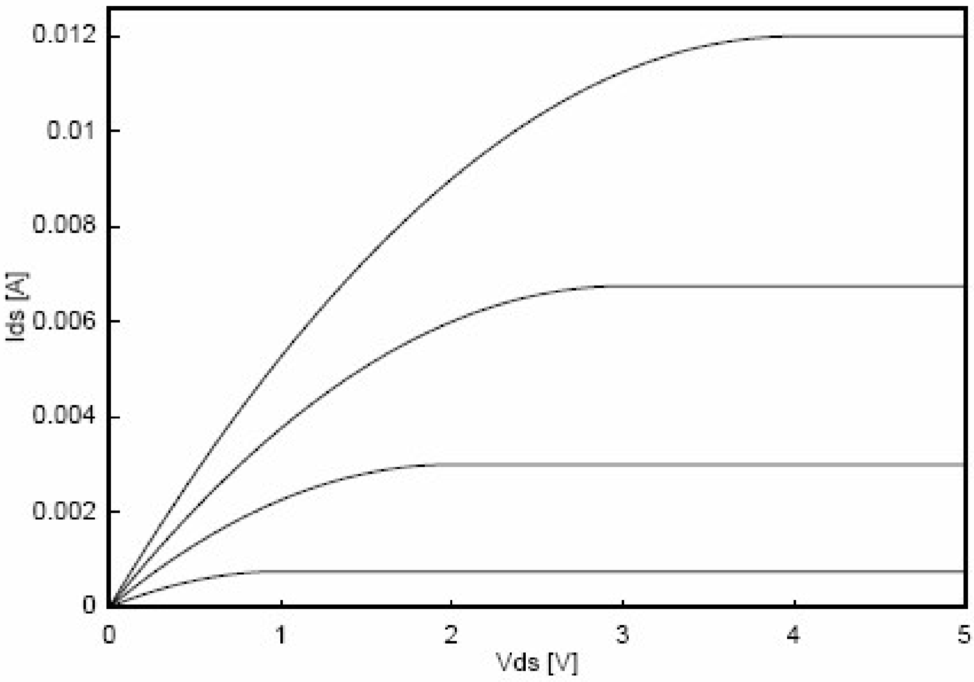
MOSFET - saturation current

Use the saturation voltage at pinch-off to determine the saturation current

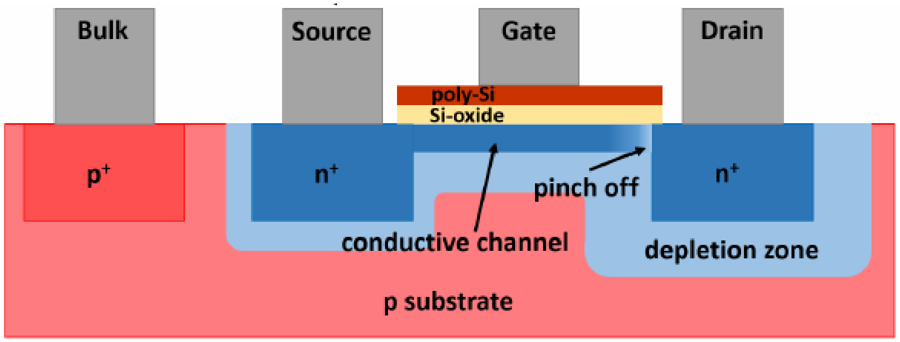
$$V_{sat} = \left(V_G - V_T\right)$$

$$I = \frac{Z}{L}\mu_n C_{ox}\left[\left(V_G - V_T\right)V_D - \frac{V_D^2}{2}\right]$$

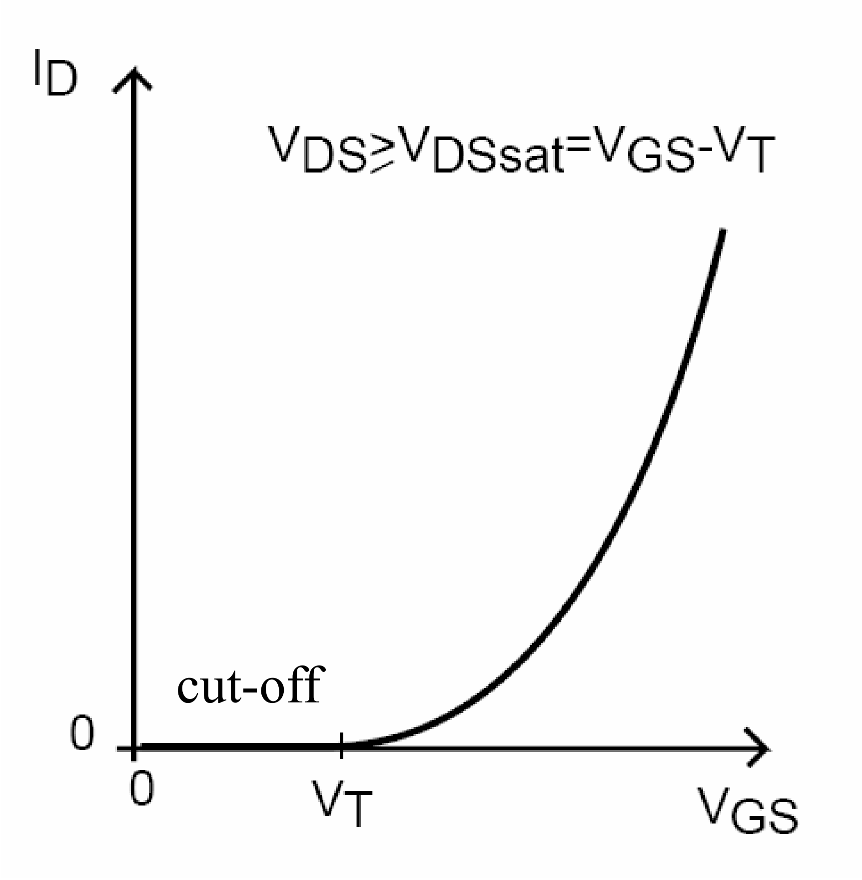
$$I_{sat} = \frac{Z}{2L}\mu_n C_{ox}\left(V_G - V_T\right)^2$$

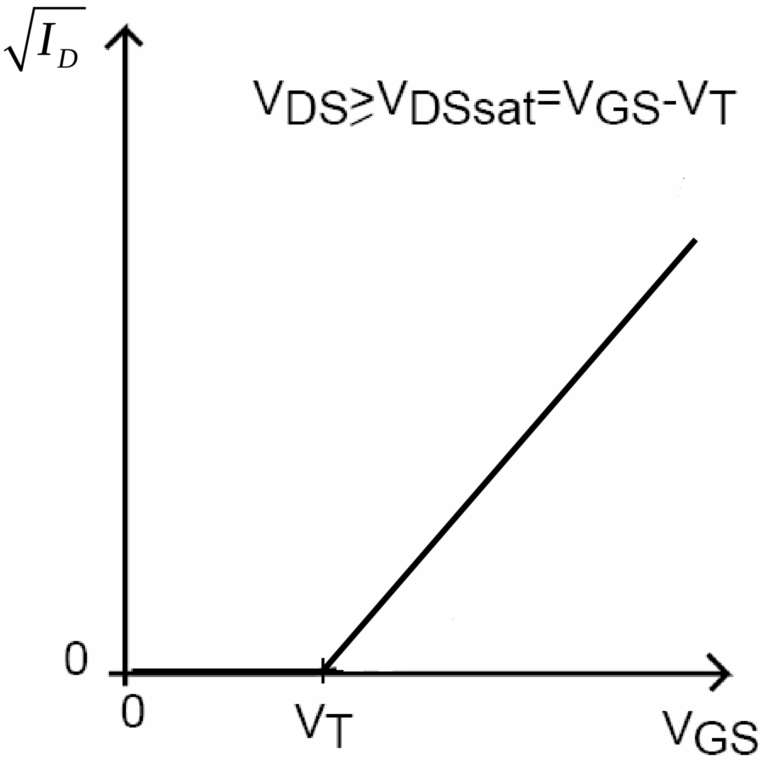


MOSFET (saturation regime)



$$I_{sat} = \frac{Z}{2L}\mu_n C_{ox}\left(V_{GS} - V_T\right)^2$$





MOSFET (linear regime)

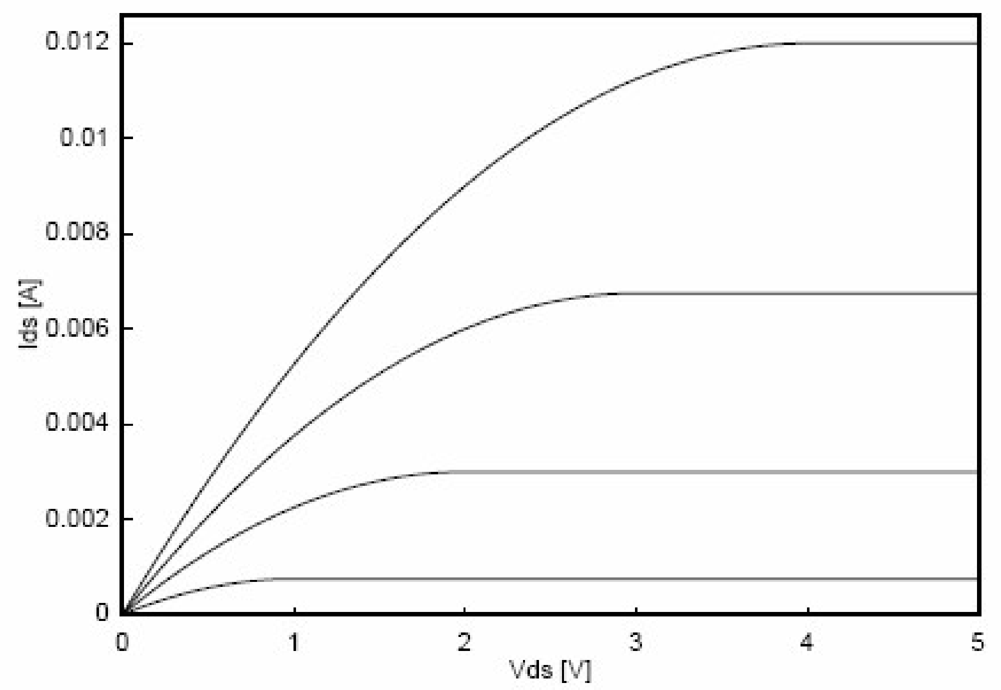
Channel conductance in the linear regime. For small V_D

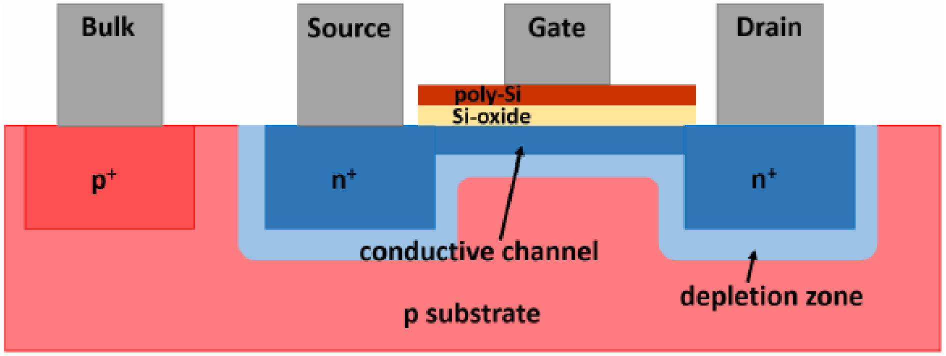
$$I \approx \frac{Z}{L}\mu_n C_{ox}\left[(V_G - V_T)V_D\right]$$

$$g_D = \frac{dI_D}{dV_D} = \frac{Z}{L}\mu_n C_{ox}(V_G - V_T)$$

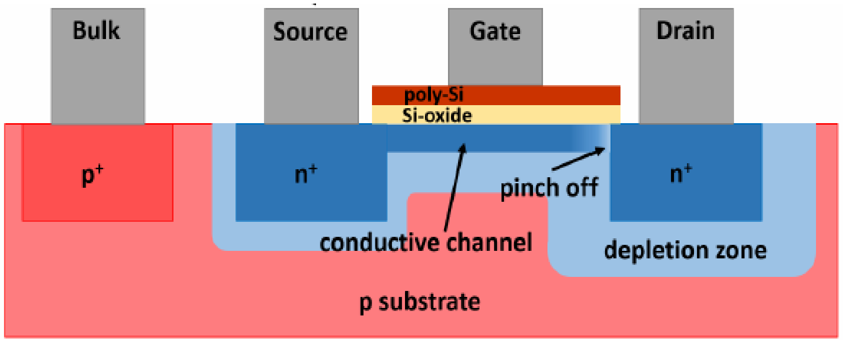
Transconductance

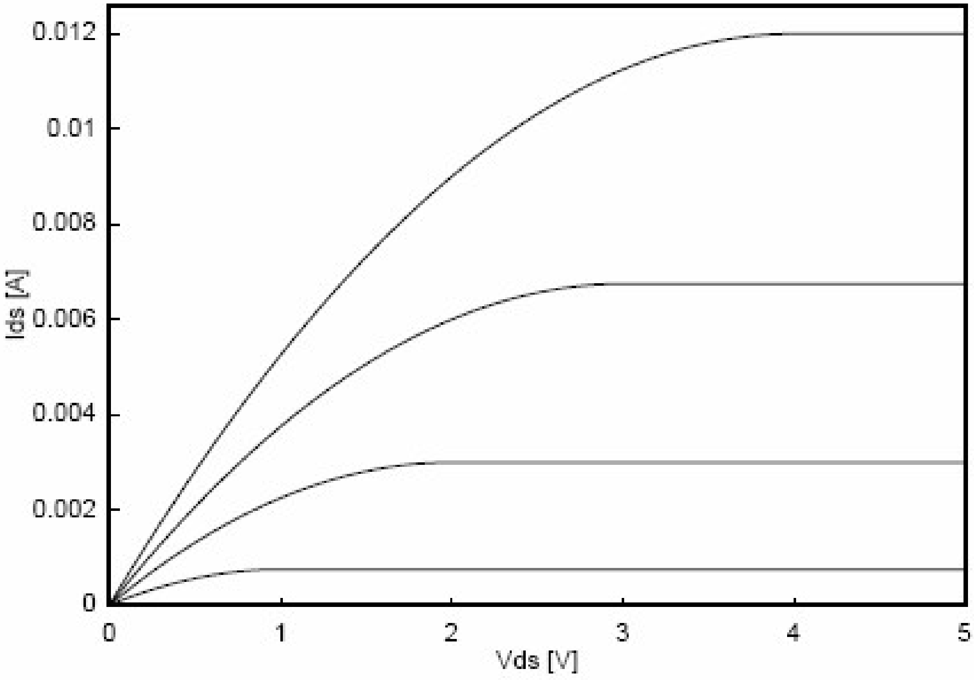
$$g_m = \frac{dI_D}{dV_G} = \frac{Z}{L}\mu_n C_{ox} V_D$$





MOSFET (saturation regime)





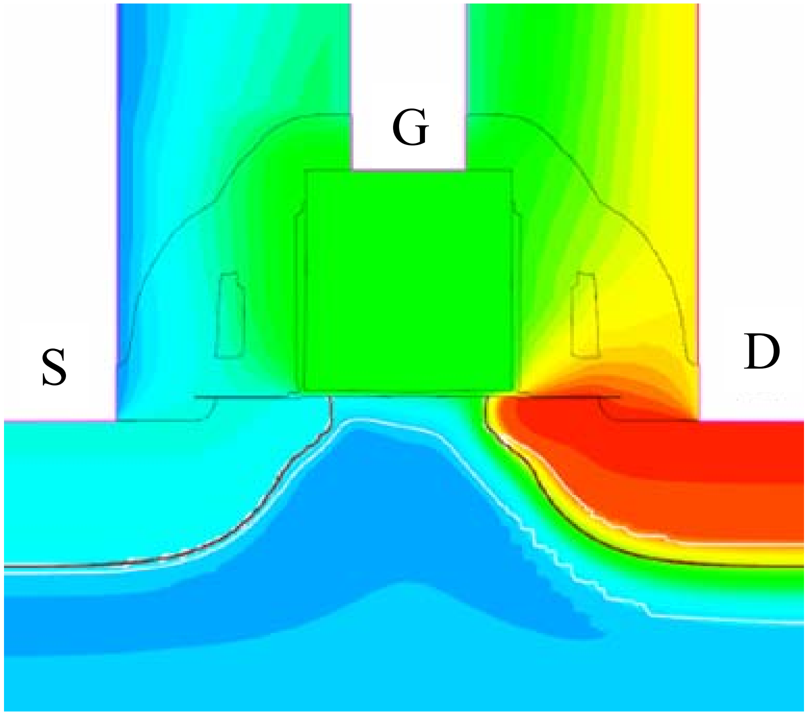
$$I_{sat} = \frac{Z}{2L}\mu_n C_{ox}\left(V_G - V_T\right)^2$$

Transconductance

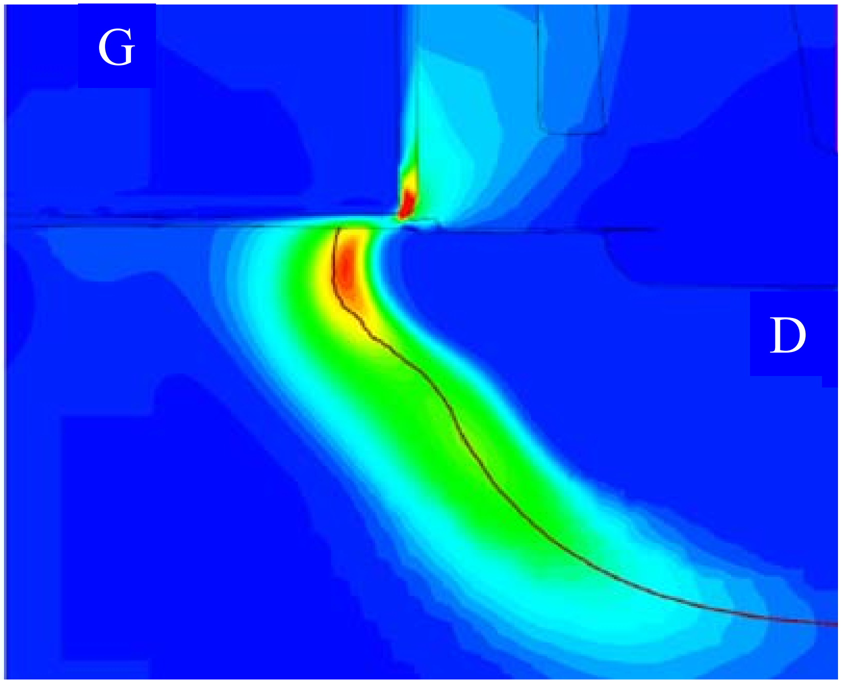
$$g_m = \frac{dI_D}{dV_G} = \frac{Z}{L}\mu_n C_{ox}\left(V_G - V_T\right)$$

A MOSFET in the saturation regime acts like a voltage controlled current source.

Saturation



Potential

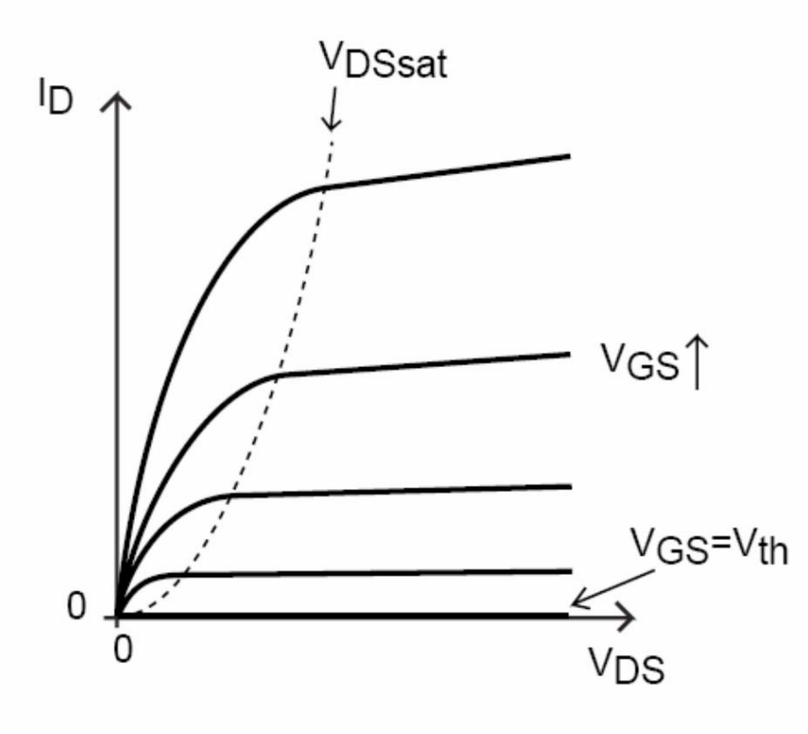


Electric field strength

Alexander Schiffmann, Master Thesis (2016)

MOSFET (saturation regime)

$$I_{sat} = \frac{Z}{2L}\mu_n C_{ox}\left(V_G - V_T\right)^2\left(1-\lambda\left(V_D - V_{sat}\right)\right)$$



Experimentally: channel length modulation $\quad \lambda \propto \frac{1}{L}$

High frequencies

$$\tilde{i}_{in} = 2\pi f C_G \tilde{v}_G$$

$$\tilde{i}_{out} = g_m \tilde{v}_G$$

$$\tilde{i}_{in} < \tilde{i}_{out}$$

$$f < \frac{g_m}{2\pi C_G} \propto \frac{1}{L^2} = f_T$$

For large E, Ohm's law ($j = ne\mu E$) is not valid. The electron velocity saturates. For velocity saturation:

$$f_T \approx \frac{v_s}{L}$$