

# Carrier Transport

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# Carrier Transport

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Ballistic transport

Drift

Diffusion

Generation and recombination

The continuity equation

High field effects

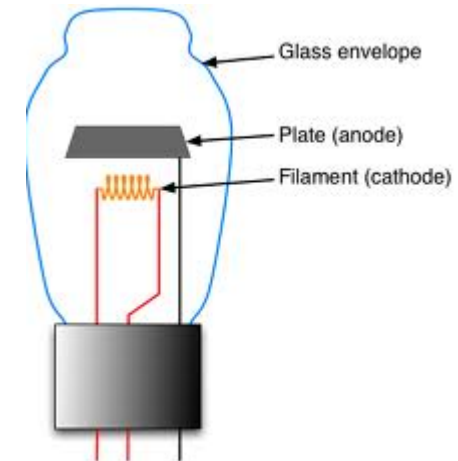
# Ballistic transport

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$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0t + \vec{x}_0$$



Electrons moving in an electric field follow parabolic trajectories like a ball in a gravitational field.

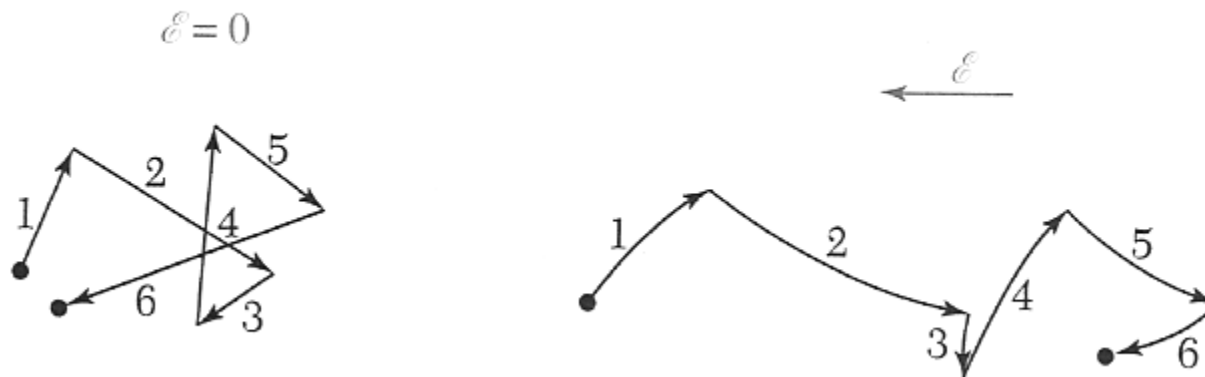
# Drift

The electrons scatter and change direction after a time  $\tau_{sc}$ .

Classical equipartition:  $\frac{1}{2} m v_{th}^2 = \frac{3}{2} k_B T$

At 300 K,  $v_{th} \sim 10^7$  cm/s.

mean free path:  $\ell = v_{th} \tau_{sc} \sim 10$  nm  $\sim 200$  atoms



# Drift (diffusive transport)

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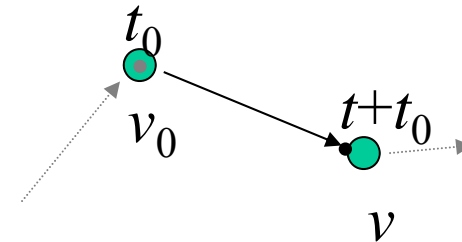
$$\vec{F} = -e\vec{E} = m^* \vec{a} = m^* \frac{d\vec{v}}{dt}$$

$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*} (t - t_0)$$

$$\langle \vec{v}_0 \rangle = 0$$

$$\langle t - t_0 \rangle = \tau_{sc}$$

*time between two collisions*



$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v}$$

drift velocity:  $\vec{v}_{d,n} = -\mu_n \vec{E}$

$$\vec{v}_{d,p} = \mu_p \vec{E}$$

# Drift

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drift velocity:  $\vec{v}_{d,n} = -\mu_n \vec{E}$        $\vec{v}_{d,p} = \mu_p \vec{E}$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = (ne\mu_n + pe\mu_p) \vec{E} = \sigma \vec{E}$$

$$\mu = \frac{-e\tau_{sc}}{m^*} = \frac{-e\ell}{m^* v}$$

for Si:  $\mu_n = 1500 \text{ cm}^2/\text{Vs}$   
 $\mu_p = 450 \text{ cm}^2/\text{Vs}$

For  $E = 1000 \text{ V/cm}$        $v_d = 10^6 \text{ cm/s}$

C:\Program Files\Cornell\SSS\winbin\drude.exe



quit

display:

large

configure...

presets

help...

show graph

show

average

run

initialize

E\_x (10<sup>4</sup> V/m): 10

E\_y (10<sup>4</sup> V/m): 10

B\_z (T): 2

tau (ps): 1.00e+00

temperature (K): 300

omega (10<sup>12</sup>/sec): 0

phase (radians): 0.0

speed 2

show graph

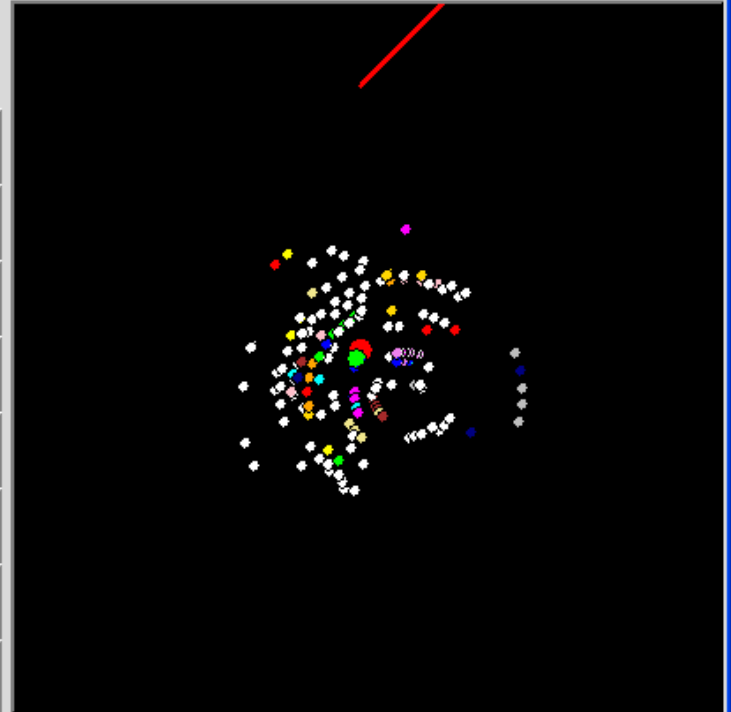
show

average

time (ps) 32.3



position: (4.14, -0.66) 10<sup>-6</sup> m



velocity: (0,0) 10<sup>4</sup> m/s

# Drift

		$E_g$ (eV)	$\mu_n$ (cm <sup>2</sup> /V-s)	$\mu_p$ (cm <sup>2</sup> /V-s)	$m_n^*/m_0$ ( $m_l, m_t$ )	$m_p^*/m_0$ ( $m_{lh}, m_{hh}$ )	$a$ (Å)	$\epsilon_r$	Density (g/cm <sup>3</sup> )	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC ( $\alpha$ )	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
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InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
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InSb	(d/Z)	0.18	10 <sup>5</sup>	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
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PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

Solid state electronic devices, Streetman and Banerjee

$$\vec{v}_{d,n} = -\mu_n \vec{E}$$

$$\vec{v}_{d,p} = \mu_p \vec{E}$$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = (ne\mu_n + pe\mu_p) \vec{E} = \sigma \vec{E}$$



# Matthiessen's rule

---

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

↑  
phonons, temperature dependent

↑ mostly temperature independent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

↑  
doping increases the conductivity  
by increasing the carrier density  
but decreases the mobility

# Mobility calculator

**INPUTS**

Semiconductor material	c-silicon	Excess electron conc. $\Delta n$	1	cm <sup>-3</sup>
Dopant atom	boron	Excess hole conc. $\Delta p$	1	cm <sup>-3</sup>
Ionised dopant conc. $N_{dop}$	1E+16	Electron eff. lifetime $\tau_{eff\ e}$	1E-4	s
Temperature $T$	300	Hole eff. lifetime $\tau_{eff\ h}$	1E-4	s

**OUTPUTS**

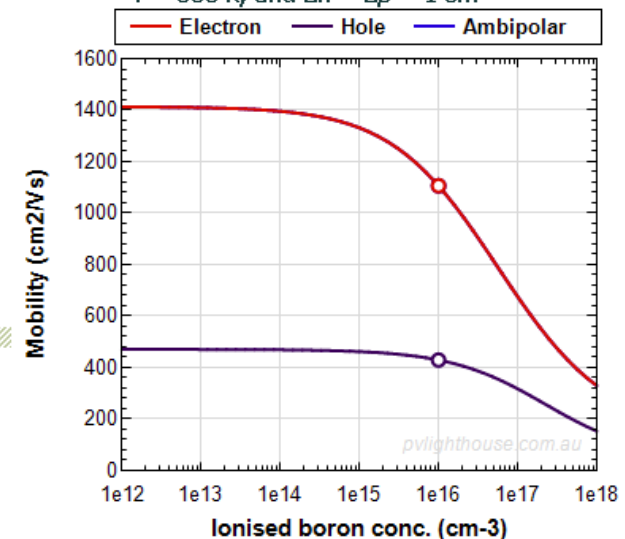
	Carrier concentrations			Carrier mobility etc.		
	Equilibrium $n_0, p_0$ (cm <sup>-3</sup> )	Excess $\Delta n, \Delta p$ (cm <sup>-3</sup> )	Net $n, p$ (cm <sup>-3</sup> )	Mobility $\mu_e, \mu_h, \mu_a$ (cm <sup>2</sup> V <sup>-1</sup> s <sup>-1</sup> )	Diffusivity $D_e, D_h, D_a$ (cm <sup>2</sup> s <sup>-1</sup> )	Diff Length $L_e, L_h, L_a$ (cm)
Intrinsic	9300	1.0	9300	1107	28.61	5.349E-2
Majority	1.0E+16	1.0	1.0E+16	429.3	11.10	3.331E-2
Minority				1107	28.61	5.349E-2

	Resistivity ( $\Omega$ -cm)	
Intrinsic	$\rho_0$	1.454
Majority	$\rho$	1.454

**More inputs**

Mobility	
Ionised dopant conc.	
	1E+12 cm <sup>-3</sup>
	1E+18 cm <sup>-3</sup>
Points	50

Mobility vs ionised dopant concentration for boron-doped c-silicon with  $T = 300\text{ K}$ , and  $\Delta n = \Delta p = 1\text{ cm}^{-3}$



$$\mu = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + (N/N_{ref})^\gamma}$$

For Electrons:

$$\mu_{min} = 47 \left(\frac{T}{300}\right)^{-1,23} \frac{\text{cm}^2}{\text{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 1373 \left(\frac{T}{300}\right)^{-2,38} \frac{\text{cm}^2}{\text{Vs}}$$

$$N_{ref} = 1,05 \cdot 10^{17} \left(\frac{T}{300}\right)^{3,65} \text{cm}^{-3}; \gamma = 0,68 \left(\frac{T}{300}\right)^{-0,32}$$

For Holes:

$$\mu_{min} = 36 \left(\frac{T}{300}\right)^{-0,87} \frac{\text{cm}^2}{\text{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 438 \left(\frac{T}{300}\right)^{-2,01} \frac{\text{cm}^2}{\text{Vs}}$$

$$N_{ref} = 2,85 \cdot 10^{17} \left(\frac{T}{300}\right)^{2,93} \text{cm}^{-3}; \gamma = 0,65 \left(\frac{T}{300}\right)^{0,26}$$

# Resistivity calculator

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

**Calculator**

Advanced inputs

About

EN 中文

RELATED LINKS

Mobility calculator

PV Education: 4 Point Probe

PV Education: Base Resistance

**MATERIAL INPUTS**

Semiconductor: c-silicon

Dopant species: Phosphorus

Temperature T: 300 K

**PLOT**

Plot: Resistivity

against: Dopant concentration

Sweep limits and steps:

$N_D$  min: 1E+12 cm<sup>-3</sup>

$N_D$  max: 1E+20 cm<sup>-3</sup>

Steps: 100

**Resistivity vs dopant concentration**

Resistivity (Ω·cm)

Dopant concentration (cm<sup>-3</sup>)

1E+12 1E+13 1E+14 1E+15 1E+16 1E+17 1E+18 1E+19 1E+20

1e4 1e3 1e2 1e1 1e0 1e-1 1e-2 1e-3 1e-4

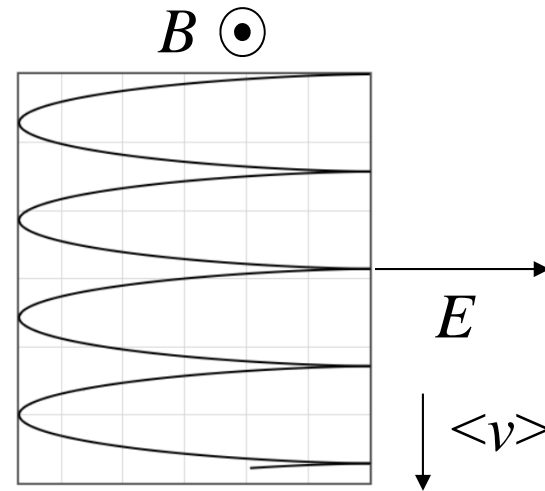
<http://www.pvlighthouse.com.au/calculators/Resistivity%20calculator/Resistivity%20calculator.aspx>

# Crossed $E$ and $B$ fields

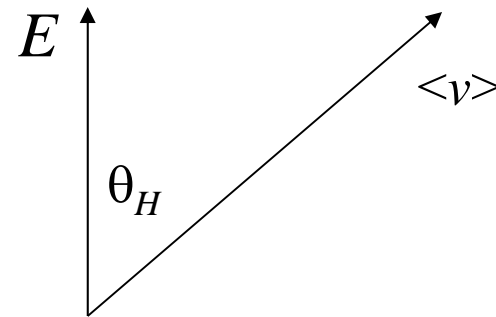
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Ballistic transport

$$\vec{F} = m\vec{a} = -e(\vec{E} + \vec{v} \times \vec{B})$$



Diffusive transport



Hall angle:

$$\theta_H = \tan^{-1} \left( -\frac{eB_z \tau_{sc}}{m^*} \right)$$

# Magnetic field (diffusive transport)

---

$$\vec{F} = m\vec{a} = -e\vec{E} = e\frac{\vec{v}_d}{\mu}$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right) = e\frac{\vec{v}_d}{\mu}$$

If  $B$  is in the  $z$ -direction, the three components of the force are

$$-\mu\left(E_x + v_{dy}B_z\right) = v_{dx}$$

$$-\mu\left(E_y - v_{dx}B_z\right) = v_{dy}$$

$$-\mu E_z = v_{dz}$$

# Magnetic field

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$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

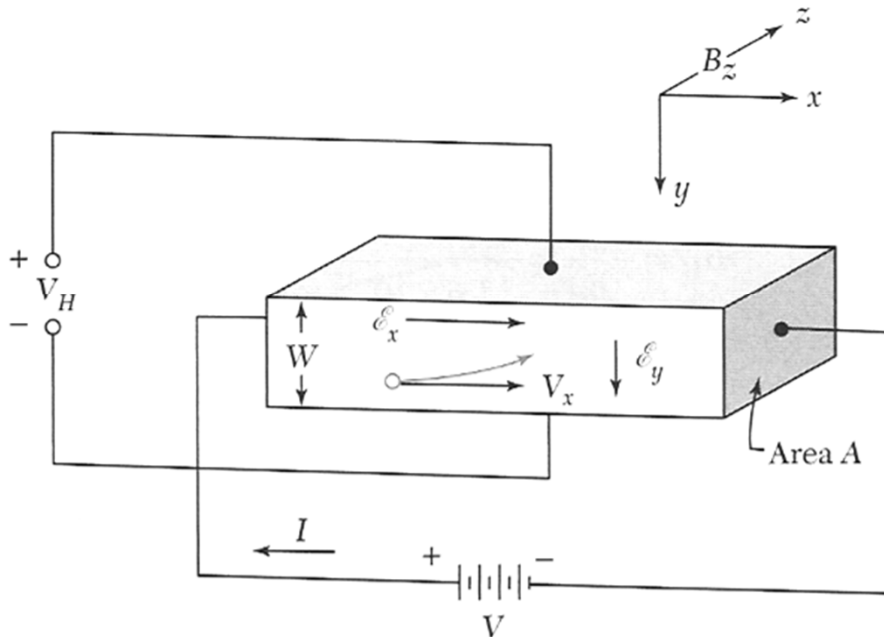
$$v_{d,z} = -\mu E_z$$

If  $E_y = 0$ ,

$$v_{d,y} = -\mu B_z v_{d,x}$$

$$\tan \theta_H = -\mu B_z$$

# The Hall Effect (diffusive regime)



$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

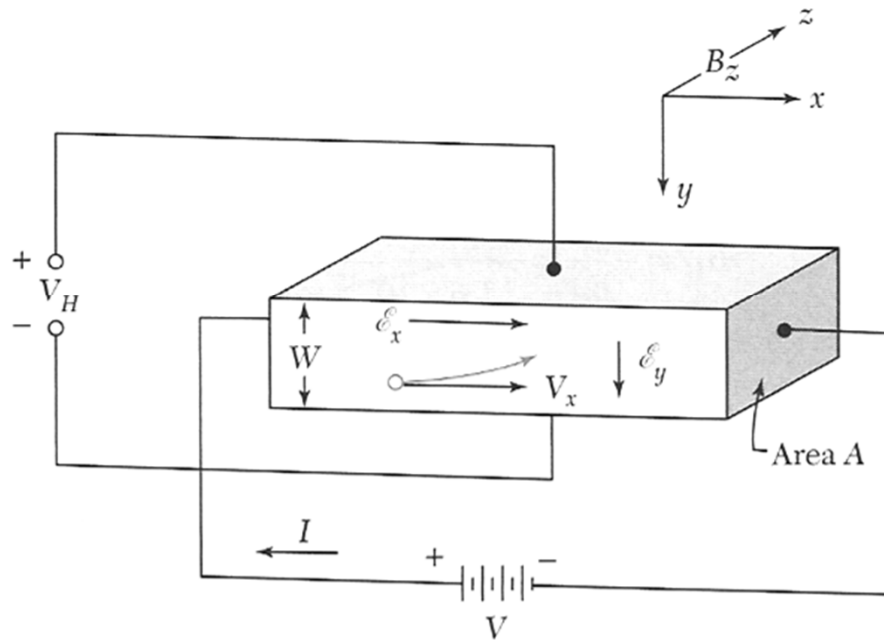
$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

$$v_{d,z} = -\mu E_z$$

If  $v_{d,y} = 0$ ,

$$E_y = v_x B_z = V_H / W = R_H j_x B_z \quad V_H = \text{Hall voltage}, R_H = \text{Hall Constant}$$

# The Hall Effect (diffusive regime)



$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$E_y = v_x B_z = V_H / W = R_H j_x B_z$$

$V_H$  = Hall voltage,  $R_H$  = Hall Constant

$$j_x = I / A$$

$$v_x = -j_x / ne \quad \text{for n-type}$$

$$v_x = j_x / pe \quad \text{for p-type}$$

$$R_H = -1/ne \quad \text{for n-type}$$

$$R_H = 1/pe \quad \text{for p-type}$$



# Ballistic transport in transistors

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The mean free path  $\sim 100$  nm  $>$  gate length  $\sim 20$  nm

$v$  not proportional to  $E$

$$\vec{v} = \mu \vec{E}$$

$j$  not proportional to  $E$

$$\vec{j} = \sigma \vec{E}$$

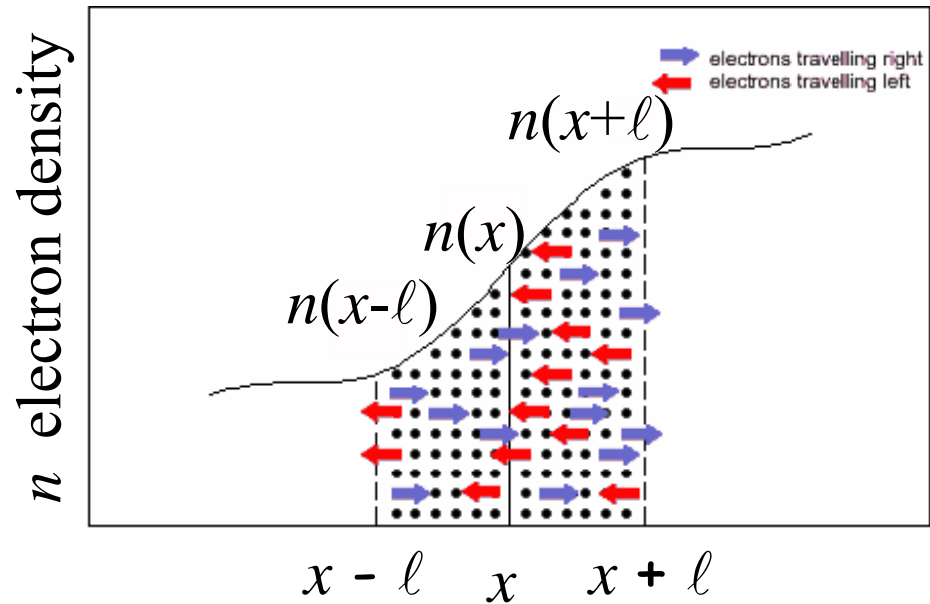
nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

# Diffusion

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -|e| D_p \frac{dp}{dx}$$



Diffusion is from high concentration to low concentration.

C:\Program Files\Cornell\SSS\winbin\drude.exe



quit

display:

large

configure...

presets

help...

show graph

show

average

run

show graph

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average

time (ps) 32.3



position: (4.14, -0.66)  $10^{-6}$  m

initialize

E\_x ( $10^4$  V/m): 10

E\_y ( $10^4$  V/m): 10

B\_z (T): 2

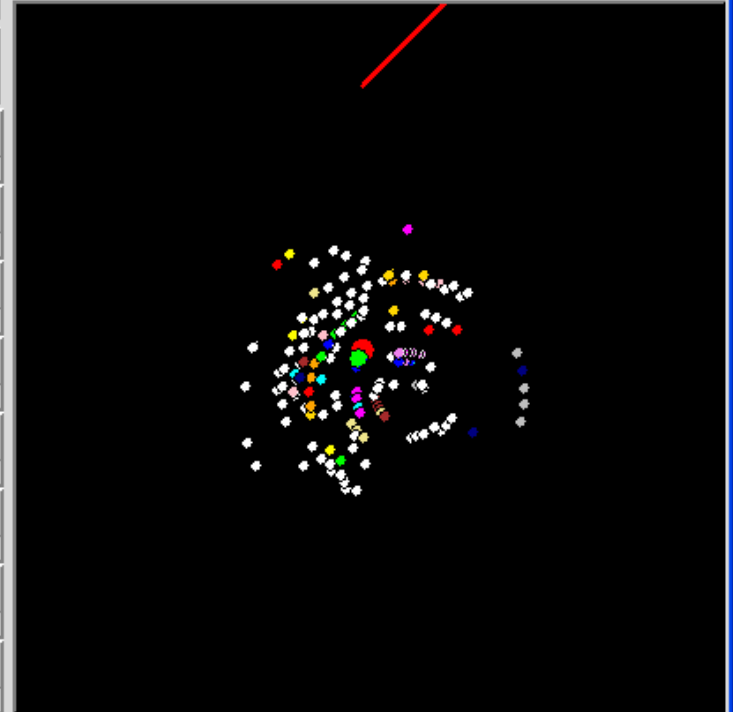
tau (ps): 1.00e+00

temperature (K): 300

omega ( $10^{12}$ /sec): 0

phase (radians): 0.0

speed 2



velocity: (0,0)  $10^4$  m/s

# Drift

		$E_g$ (eV)	$\mu_n$ (cm <sup>2</sup> /V-s)	$\mu_p$ (cm <sup>2</sup> /V-s)	$m_n^*/m_0$ ( $m_l, m_t$ )	$m_p^*/m_0$ ( $m_{lh}, m_{hh}$ )	$a$ (Å)	$\epsilon_r$	Density (g/cm <sup>3</sup> )	Melting point (°C)
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Solid state electronic devices, Streetman and Banerjee

$$\vec{v}_{d,n} = -\mu_n \vec{E} \quad \vec{v}_{d,p} = \mu_p \vec{E} \quad \vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = (ne\mu_n + pe\mu_p) \vec{E} = \sigma \vec{E}$$

# Matthiessen's rule

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$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

↑  
phonons, temperature dependent

↑ mostly temperature independent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

↑  
doping increases the conductivity  
by increasing the carrier density  
but decreases the mobility

# Mobility calculator

**INPUTS**

Semiconductor material	c-silicon	Excess electron conc. $\Delta n$	1	cm <sup>-3</sup>
Dopant atom	boron	Excess hole conc. $\Delta p$	1	cm <sup>-3</sup>
Ionised dopant conc. $N_{dop}$	1E+16	Electron eff. lifetime $\tau_{eff\ e}$	1E-4	s
Temperature $T$	300	Hole eff. lifetime $\tau_{eff\ h}$	1E-4	s

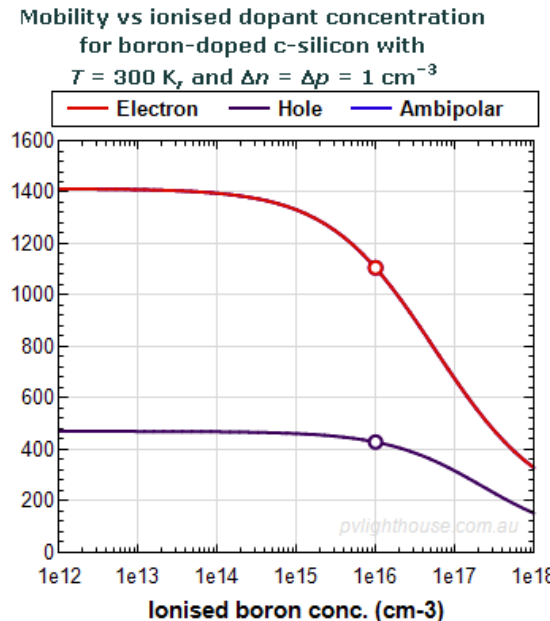
**OUTPUTS**

	Carrier concentrations			Carrier mobility etc.		
	Equilibrium $n_0, p_0$ (cm <sup>-3</sup> )	Excess $\Delta n, \Delta p$ (cm <sup>-3</sup> )	Net $n, p$ (cm <sup>-3</sup> )	Mobility $\mu_e, \mu_h, \mu_a$ (cm <sup>2</sup> V <sup>-1</sup> s <sup>-1</sup> )	Diffusivity $D_e, D_h, D_a$ (cm <sup>2</sup> s <sup>-1</sup> )	Diff Length $L_e, L_h, L_a$ (cm)
Intrinsic	9300	1.0	9300	1107	28.61	5.349E-2
Majority	1.0E+16	1.0	1.0E+16	429.3	11.10	3.331E-2
Minority				1107	28.61	5.349E-2

	Resistivity ( $\Omega$ -cm)	
Intrinsic	$\rho_0$	1.454
Majority	$\rho$	1.454

**More inputs**

Mobility	
Ionised dopant conc.	
	1E+12 cm <sup>-3</sup>
	1E+18 cm <sup>-3</sup>
Points	50



$$\mu = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + (N/N_{ref})^\gamma}$$

**For Electrons:**

$$\mu_{min} = 47 \left(\frac{T}{300}\right)^{-1,23} \frac{\text{cm}^2}{\text{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 1373 \left(\frac{T}{300}\right)^{-2,38} \frac{\text{cm}^2}{\text{Vs}}$$

$$N_{ref} = 1,05 \cdot 10^{17} \left(\frac{T}{300}\right)^{3,65} \text{cm}^{-3}; \gamma = 0,68 \left(\frac{T}{300}\right)^{-0,32}$$

**For Holes:**

$$\mu_{min} = 36 \left(\frac{T}{300}\right)^{-0,87} \frac{\text{cm}^2}{\text{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 438 \left(\frac{T}{300}\right)^{-2,01} \frac{\text{cm}^2}{\text{Vs}}$$

$$N_{ref} = 2,85 \cdot 10^{17} \left(\frac{T}{300}\right)^{2,93} \text{cm}^{-3}; \gamma = 0,65 \left(\frac{T}{300}\right)^{0,26}$$

# Resistivity calculator

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

Calculator

Advanced inputs

About

EN 中文

RELATED LINKS

Mobility calculator

PV Education: 4 Point Probe

PV Education: Base Resistance

Dopant concentration,  $N_D$

1E+15  $\text{cm}^{-3}$

Resistivity,  $\rho$

4.58  $\Omega \cdot \text{cm}$

MATERIAL INPUTS

Semiconductor: c-silicon

Dopant species: Phosphorus

Temperature  $T$ : 300 K

PLOT

Plot: Resistivity

against: Dopant concentration

Sweep limits and steps:

$N_D \text{ min}$ : 1E+12  $\text{cm}^{-3}$

$N_D \text{ max}$ : 1E+20  $\text{cm}^{-3}$

Steps: 100

Resistivity vs dopant concentration

Resistivity ( $\Omega \cdot \text{cm}$ )

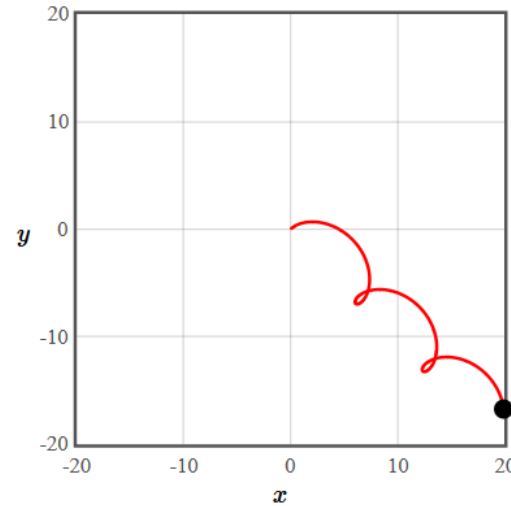
Dopant concentration ( $\text{cm}^{-3}$ )

<http://www.pvlighthouse.com.au/calculators/Resistivity%20calculator/Resistivity%20calculator.aspx>

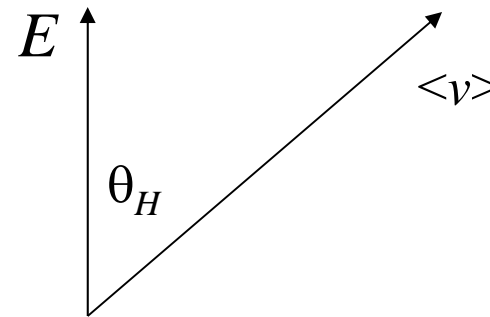
# Crossed $E$ and $B$ fields

Ballistic transport

$$\vec{F} = m\vec{a} = -e(\vec{E} + \vec{v} \times \vec{B})$$



Diffusive transport



Hall angle:

$$\theta_H = \tan^{-1} \left( -\frac{eB_z \tau_{sc}}{m^*} \right)$$



# Magnetic field (diffusive transport)

---

$$\vec{F} = m\vec{a} = -e\vec{E} = e\frac{\vec{v}_d}{\mu}$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right) = e\frac{\vec{v}_d}{\mu}$$

If  $B$  is in the  $z$ -direction, the three components of the force are

$$-\mu\left(E_x + v_{dy}B_z\right) = v_{dx}$$

$$-\mu\left(E_y - v_{dx}B_z\right) = v_{dy}$$

$$-\mu E_z = v_{dz}$$

# Magnetic field

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$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

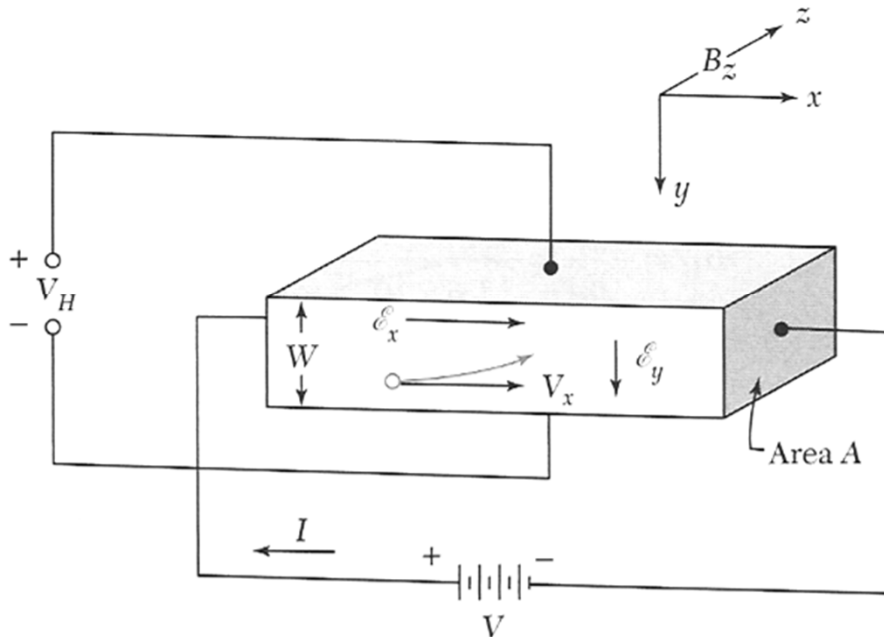
$$v_{d,z} = -\mu E_z$$

If  $E_y = 0$ ,

$$v_{d,y} = -\mu B_z v_{d,x}$$

$$\tan \theta_H = -\mu B_z$$

# The Hall Effect (diffusive regime)



$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

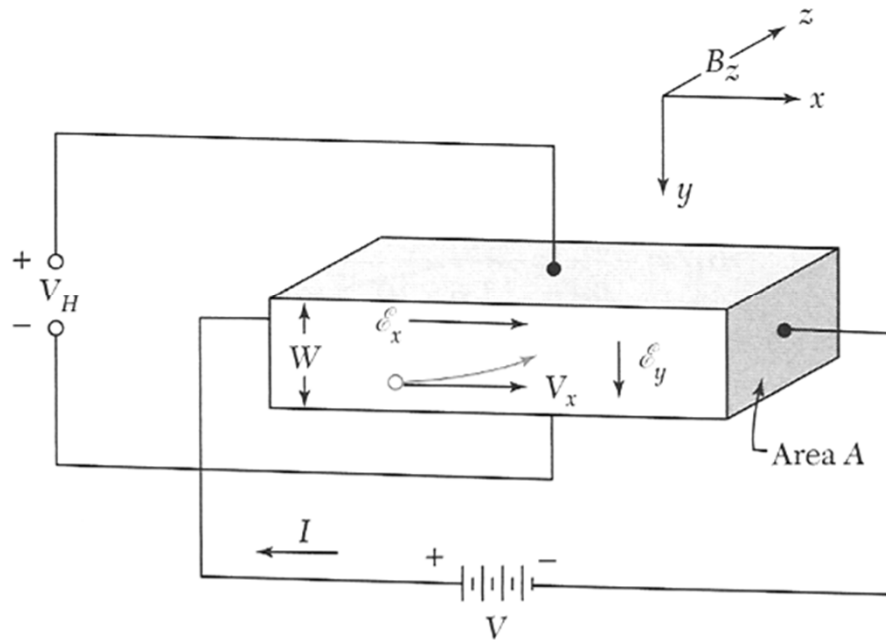
$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

$$v_{d,z} = -\mu E_z$$

If  $v_{d,y} = 0$ ,

$$E_y = v_x B_z = V_H / W = R_H j_x B_z \quad V_H = \text{Hall voltage}, R_H = \text{Hall Constant}$$

# The Hall Effect (diffusive regime)



$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$E_y = v_x B_z = V_H / W = R_H j_x B_z$$

$V_H$  = Hall voltage,  $R_H$  = Hall Constant

$$j_x = I / A$$

$$v_x = -j_x / ne \quad \text{for n-type}$$

$$v_x = j_x / pe \quad \text{for p-type}$$

$$R_H = -1 / ne \quad \text{for n-type}$$

$$R_H = 1 / pe \quad \text{for p-type}$$

# Ballistic transport in transistors

---

The mean free path  $\sim 100$  nm  $>$  gate length  $\sim 20$  nm

$v$  not proportional to  $E$

$$\vec{v} = \mu \vec{E}$$

$j$  not proportional to  $E$

$$\vec{j} = \sigma \vec{E}$$

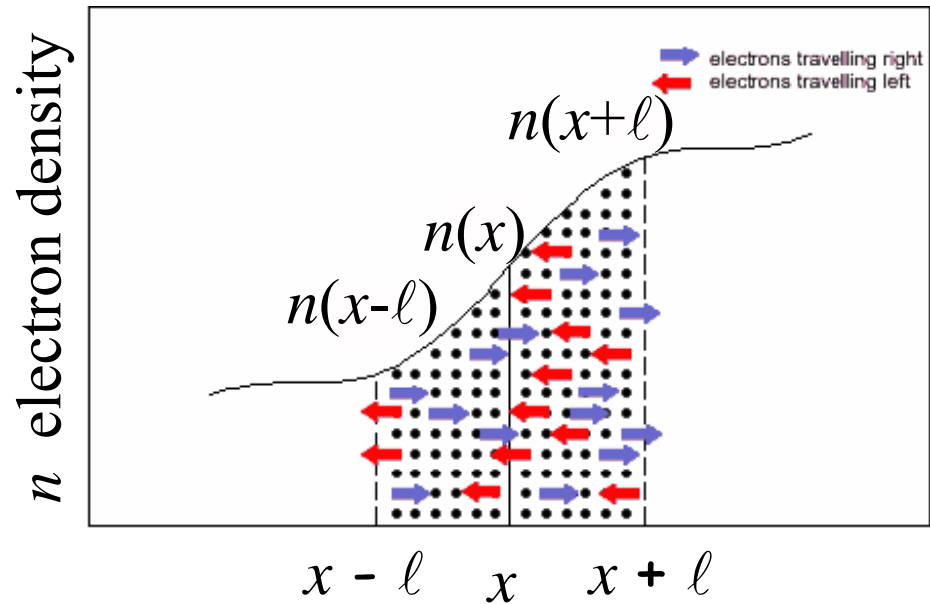
nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

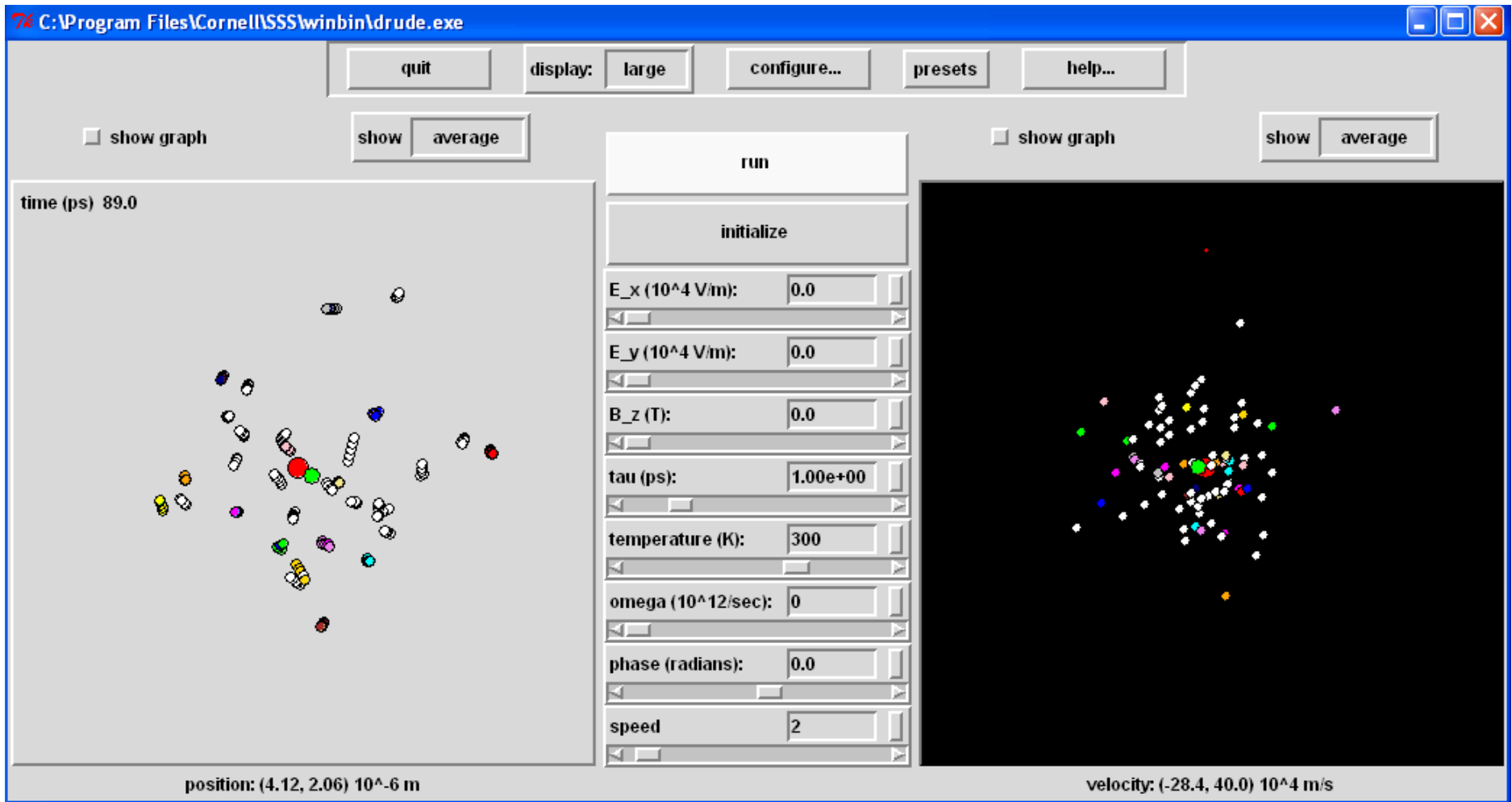
# Diffusion

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -|e| D_p \frac{dp}{dx}$$



Diffusion is from high concentration to low concentration.



If no forces are applied, the electrons diffuse.  
 The average velocity moves against an electric field.  
 In just a magnetic field, the average velocity is zero.  
 In an electric and magnetic field, the electrons move in a straight line at the Hall angle.

# Einstein relation

---

$$\vec{E} = -\nabla V$$

$$n = A \exp\left(\frac{-eV}{k_B T}\right)$$

Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{e}{k_B T} A \exp\left(\frac{-eV_{pot}}{k_B T}\right) \nabla V = -\frac{ne}{k_B T} \nabla V = \frac{ne\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + eD \frac{ne\vec{E}}{k_B T} = 0$$

$$D = \frac{\mu k_B T}{e}$$

Über die von der molekular-kinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen



# Current Density Equations

---

Drift



Diffusion



$$\vec{j}_n = -ne\mu_n\vec{E} + eD_n\nabla n$$

$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

# Current Density Equations

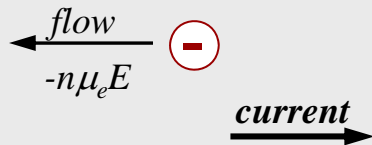
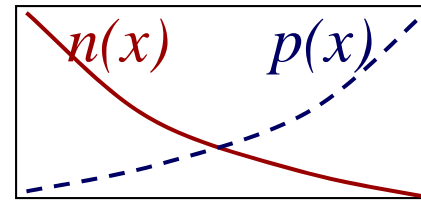
note: electron and hole currents have same direction

electric current = charge  $\times$  particle flow

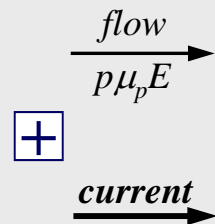
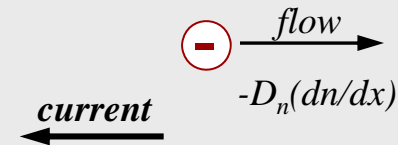
**drift**



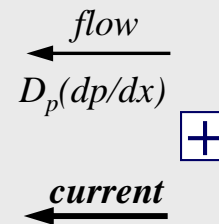
**diffusion**



$$j_e = -e \times \text{flow}$$

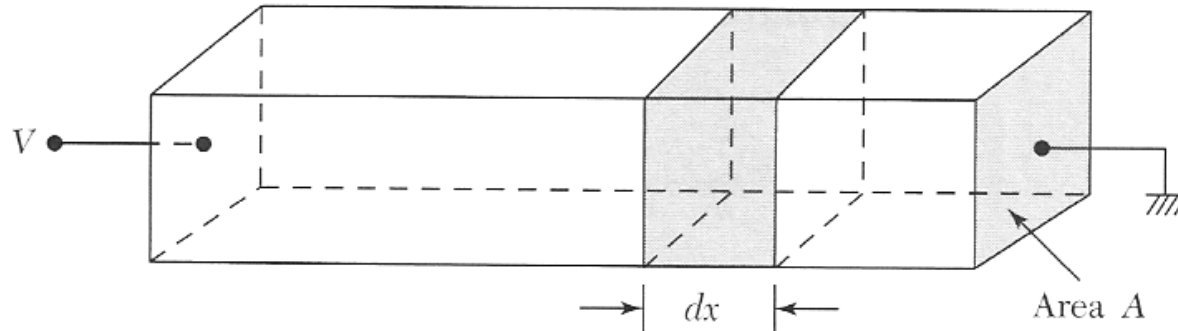


$$j_p = e \times \text{flow}$$



# Continuity equations

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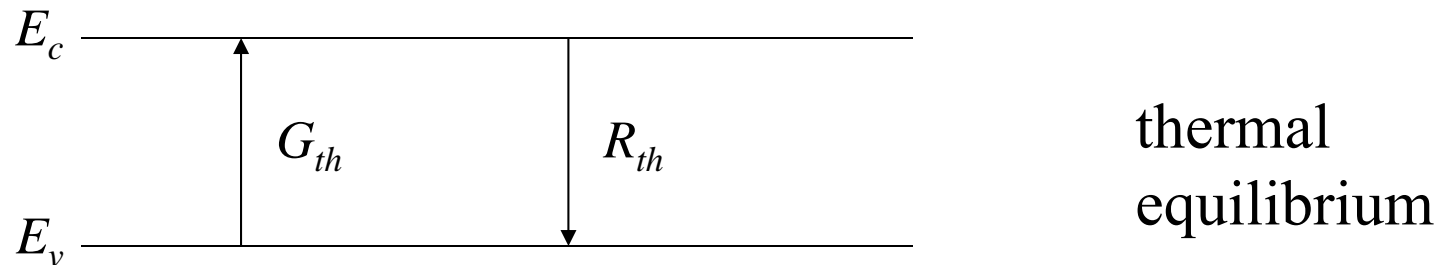
$$\frac{\partial n}{\partial t} = \frac{1}{e} \nabla \cdot \vec{j}_n + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{e} \nabla \cdot \vec{j}_p + G_p - R_p$$

$j_n$  and  $j_p$  consist of drift and diffusion terms

# Generation and Recombination

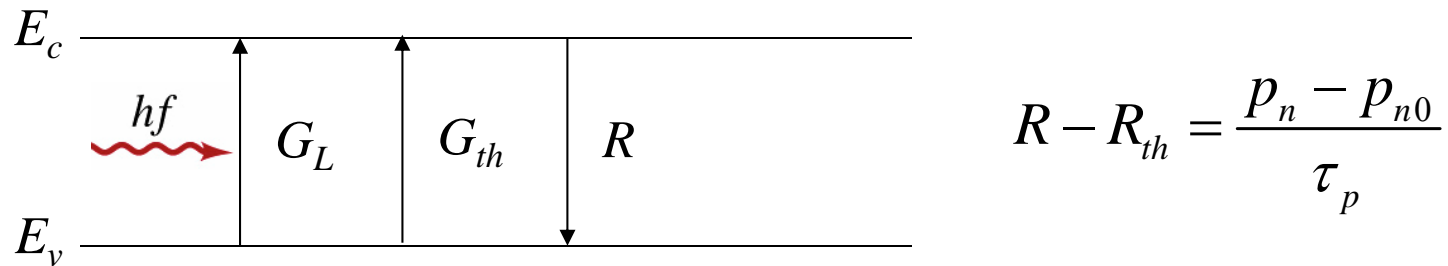
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Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution  $np \neq n_i^2$



# Recombination

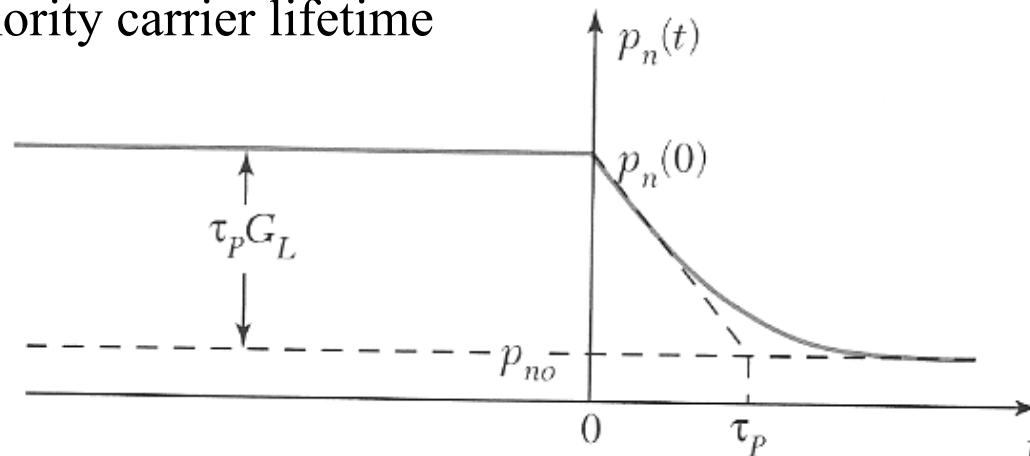


Recombination rate is limit by the density of minority carriers.  
The majority carriers have to find a minority carrier to recombine.

$p_n$  (or  $n_p$ ) = minority carrier concentration

$p_{n0}$  (or  $n_{p0}$ ) = equilibrium minority carrier concentration

$\tau_p$  = minority carrier lifetime



# minority carrier lifetimes

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p-type

$$n_p(t) = n_{excess} \exp(-t / \tau_n) + n_{p0}$$

n-type

$$p_n(t) = p_{excess} \exp(-t / \tau_p) + p_{n0}$$

minority carrier  
lifetimes



$$np = n_i^2$$

# Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{e} \nabla \cdot \vec{j}_n + G_n - R_n$$

---

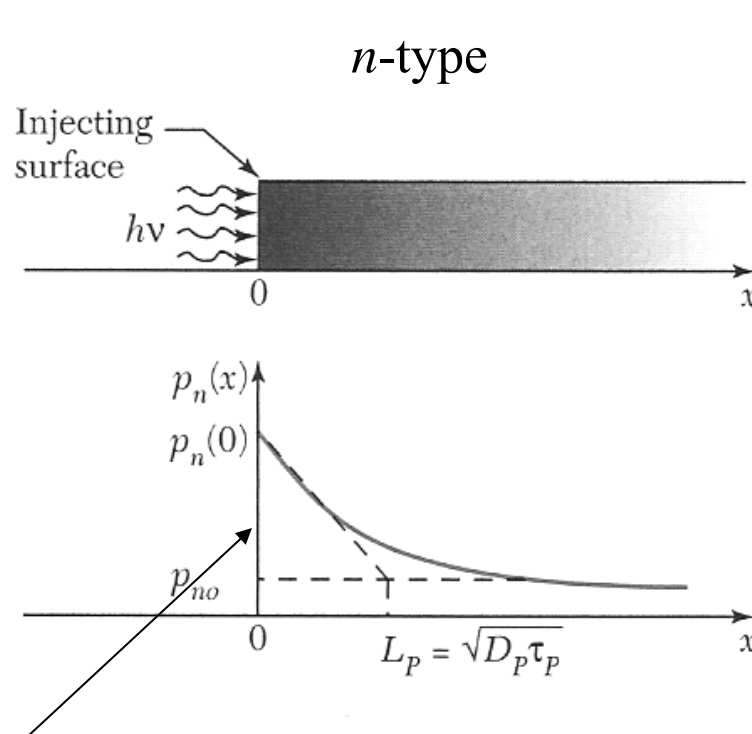
drift:  $\vec{j}_n = -ne\mu_n\vec{E}$        $\nabla \cdot \vec{j}_n = -en\mu_n\nabla \cdot \vec{E} - e\nabla n\mu_n\vec{E}$

diffusion:  $\vec{j}_{n,diff} = |e|D_n\nabla n$        $\nabla \cdot \vec{j}_{n,diff} = |e|D_n\nabla^2 n$

$$\frac{\partial n}{\partial t} = n\mu_n\nabla \cdot \vec{E} + \nabla n\mu_n\vec{E} + D_n\nabla^2 n + G_n - \frac{n - n_0}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p\mu_p\nabla \cdot \vec{E} - \nabla p\mu_p\vec{E} + D_p\nabla^2 p + G_p - \frac{p - p_0}{\tau_p}$$

# Diffusion Length



Steady state

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

Generation only occurs at the surface. There the minority carrier density is  $p_n(0)$ .



# Diffusion Length

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$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \quad \Leftrightarrow \quad p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

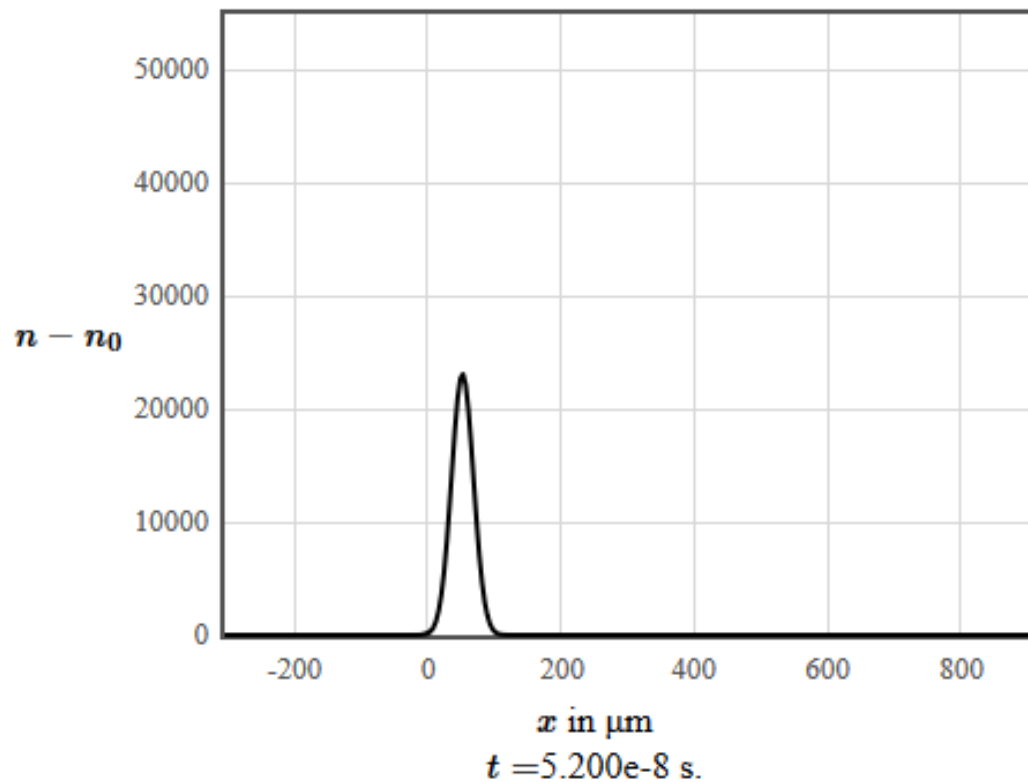
$$0 = \frac{D_p (p_n(0) - p_{n0})}{L_p^2} \exp\left(\frac{-x}{L_p}\right) - \frac{(p_n(0) - p_{n0})}{\tau_p} \exp\left(\frac{-x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

diffusion length,  
typically microns

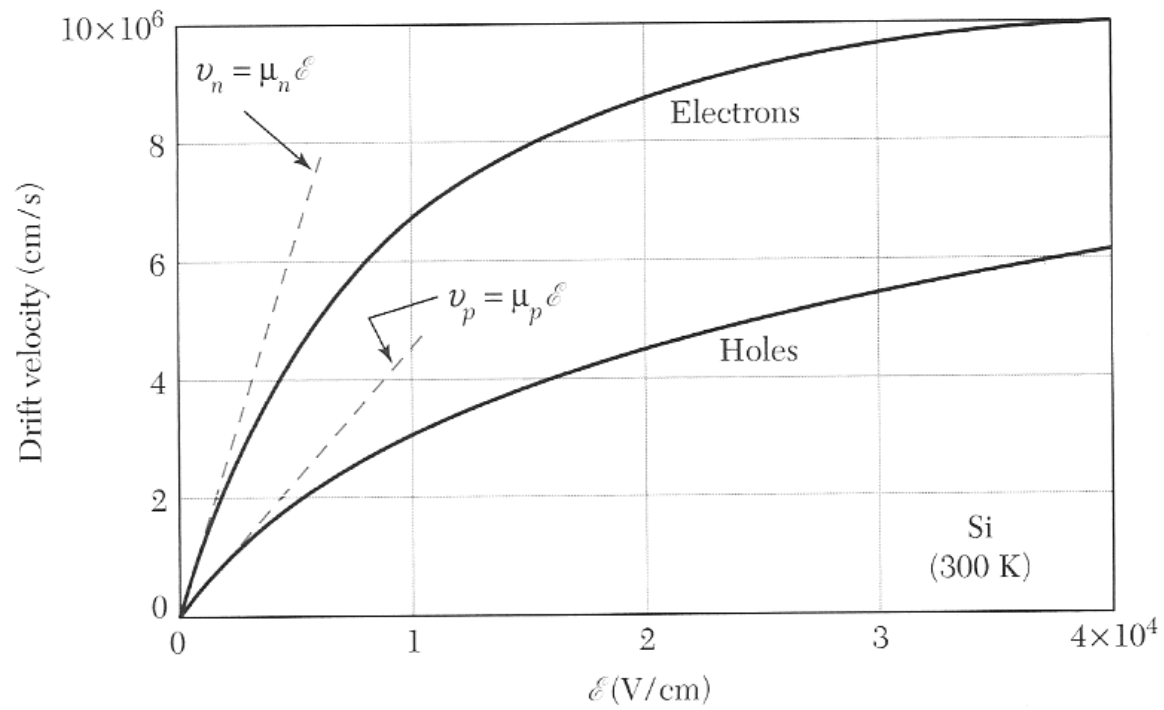
# Haynes Shockley experiment

$$n_p(x, t) = \frac{n_{\text{generated}}}{\sqrt{4\pi D_n t}} \exp\left(-\frac{(x - \mu_n E t)^2}{4D_n t}\right) \exp\left(-\frac{t}{\tau_n}\right) + n_{p0}$$



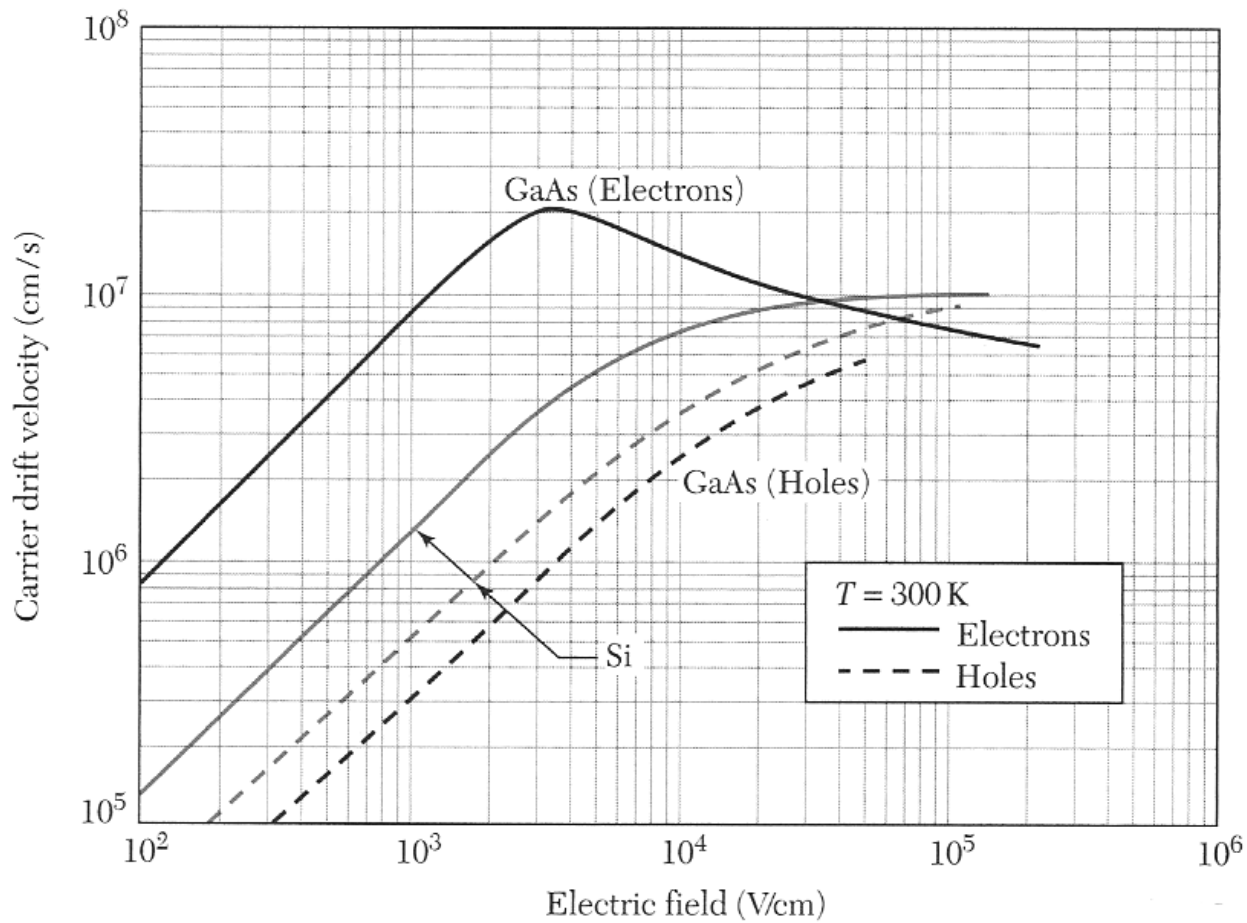
$\tau = 1\text{E-}6$  [s]  
 $E = 100$  [V/cm]  
 $\mu = 1000$  [ $\text{cm}^2/\text{V s}$ ]  
 $D = \mu k_B T / e = 0.00258$  [ $\text{m}^2/\text{s}$ ]  
 $L = \sqrt{D\tau} = 50.8$  [ $\mu\text{m}$ ]

# High Fields

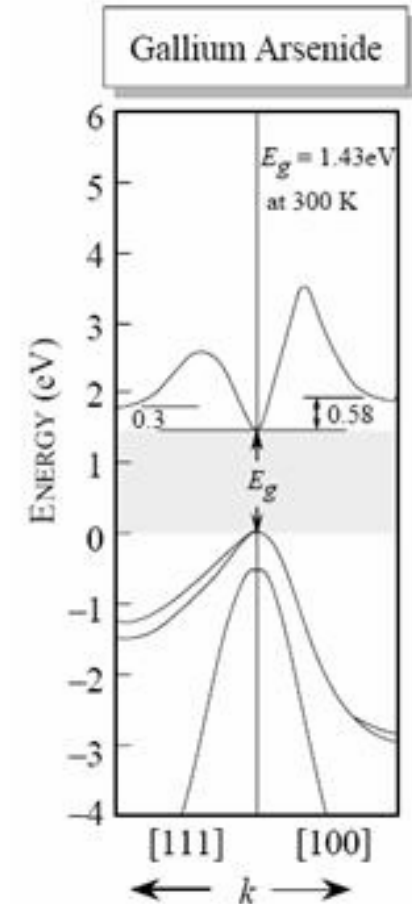


Silicon

# High Fields



GaAs



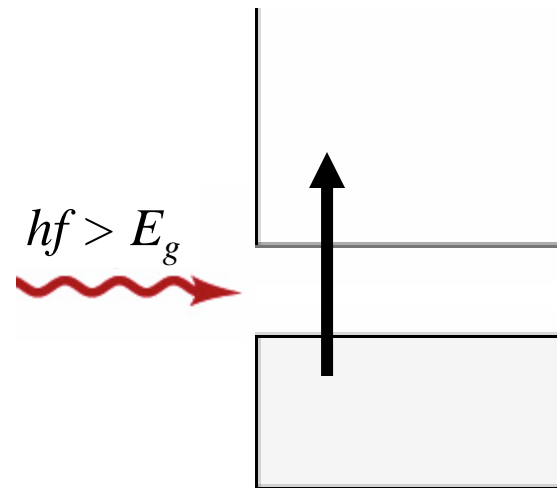
# Impact ionization

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Carriers are accelerated to an energy above the gap before they scatter. They generate more electron-hole pairs. This results in an avalanche breakdown of the device.

# Photoconductivity

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$$\sigma = ne\mu_n + pe\mu_p$$

Light increases the conductivity of a semiconductor.

# Laser printer

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