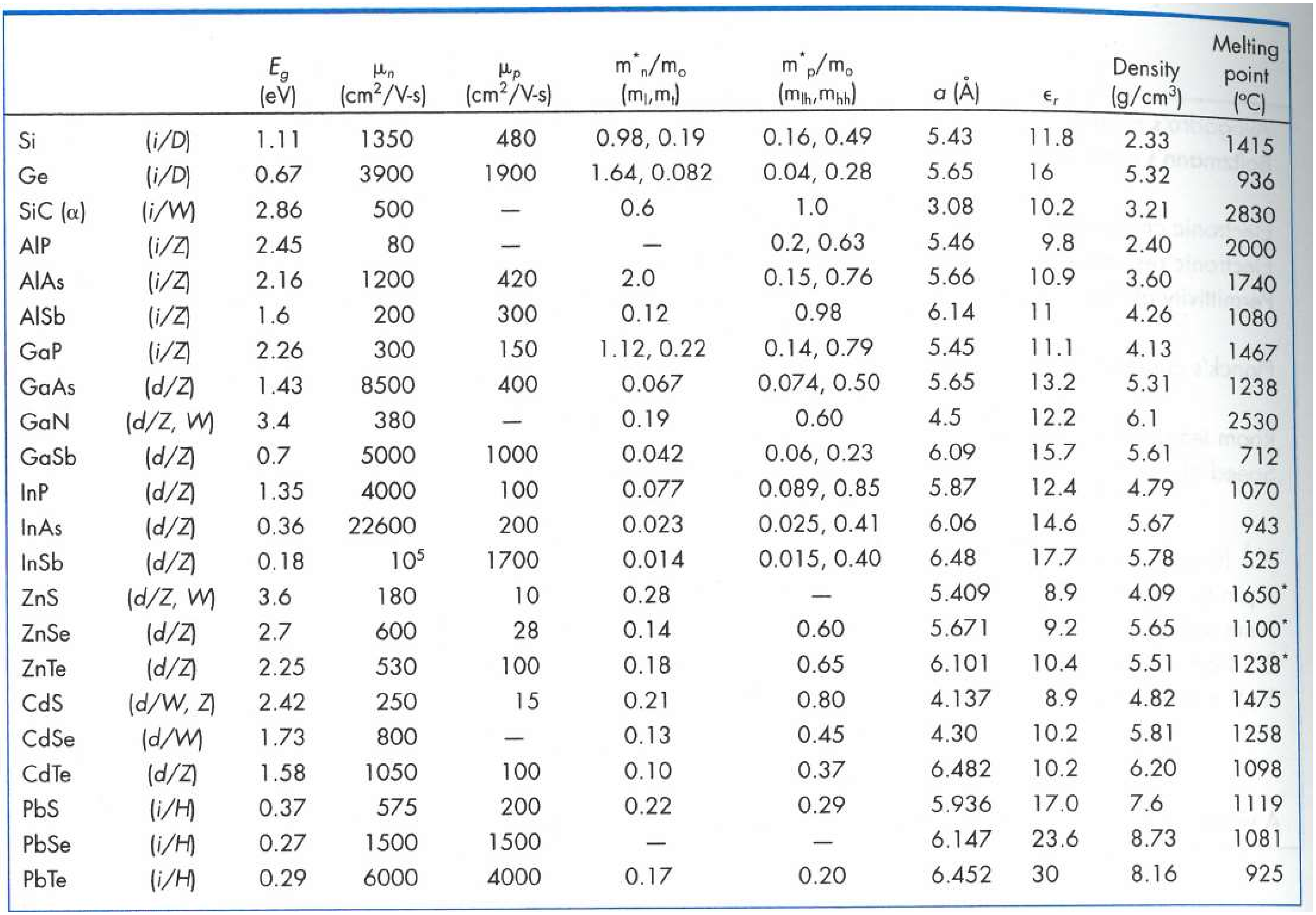
Carrier transport

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = \left(ne\mu_n + pe\mu_p\right)\vec{E} = \sigma\vec{E}$$

Drift



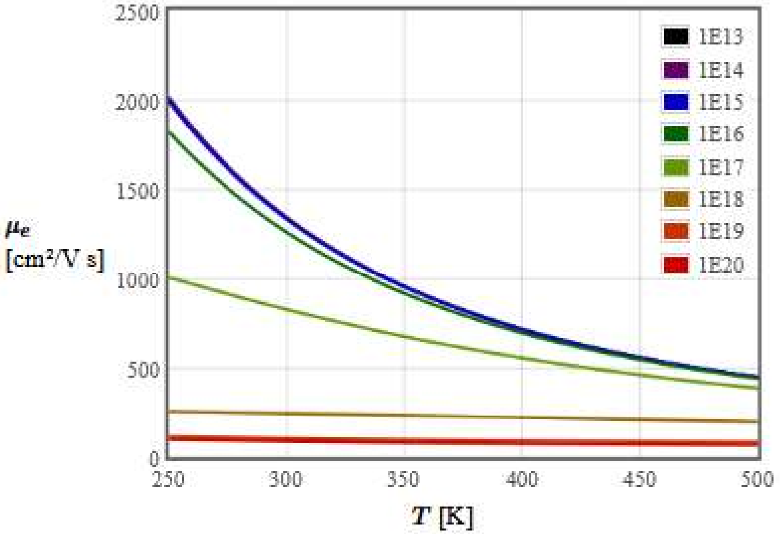
		E_g (eV)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)	m^*_n/m_o (m_l, m_t)	m^*_p/m_o (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm^3)	Melting point (°C)
Si	(*i*/*D*)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(*i*/*D*)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(*i*/*W*)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(*i*/*Z*)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(*i*/*Z*)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(*i*/*Z*)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(*i*/*Z*)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(*d*/*Z*)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(*d*/*Z*, *W*)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(*d*/*Z*)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(*d*/*Z*)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(*d*/*Z*)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(*d*/*Z*)	0.18	10^5	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(*d*/*Z*, *W*)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(*d*/*Z*)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(*d*/*Z*)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(*d*/*W*, *Z*)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(*d*/*W*)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(*d*/*Z*)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(*i*/*H*)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(*i*/*H*)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(*i*/*H*)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

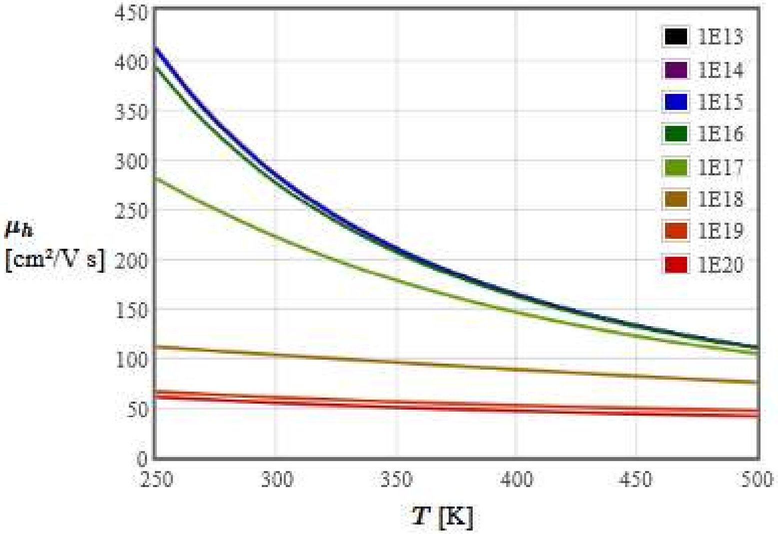
Solid state electronic devices, Streetman and Banerjee

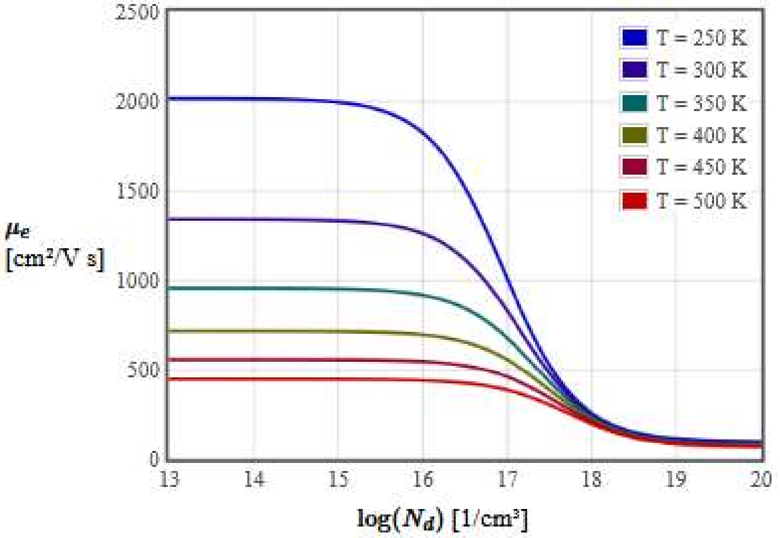
$$\vec{v}_{d,n} = -\mu_n \vec{E} \qquad \vec{v}_{d,p} = \mu_p \vec{E} \qquad \vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = \left(ne\mu_n + pe\mu_p\right)\vec{E} = \sigma\vec{E}$$

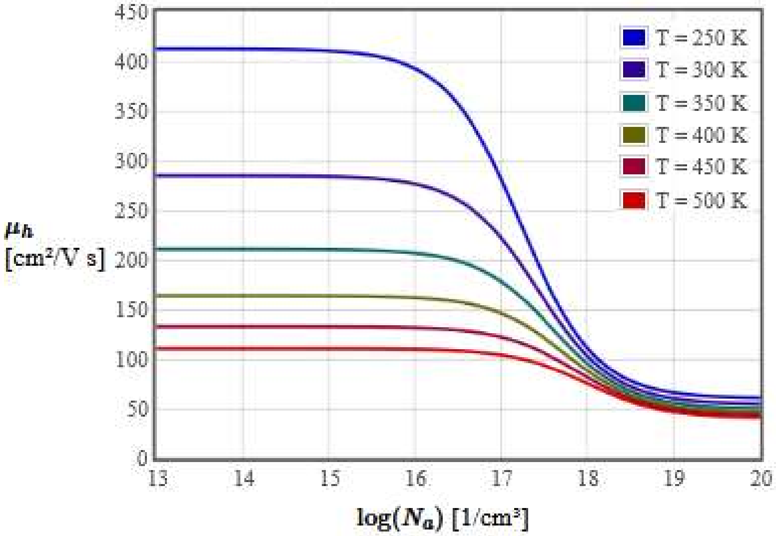
$$\mu_e = 88\left(\frac{T}{300}\right)^{-0.57} + \frac{7.4\times10^{8}T^{-2.33}}{1+0.88\left[\frac{N_d}{1.26\times10^{17}\left(\frac{T}{300}\right)^{2.4}}\right]\left(\frac{T}{300}\right)^{-0.146}} \quad \mathrm{cm^2/V\ s},$$

$$\mu_h = 54.3\left(\frac{T}{300}\right)^{-0.57} + \frac{1.36\times10^{8}T^{-2.33}}{1+0.88\left[\frac{N_a}{2.35\times10^{17}\left(\frac{T}{300}\right)^{2.4}}\right]\left(\frac{T}{300}\right)^{-0.146}} \quad \mathrm{cm^2/V\ s}.$$

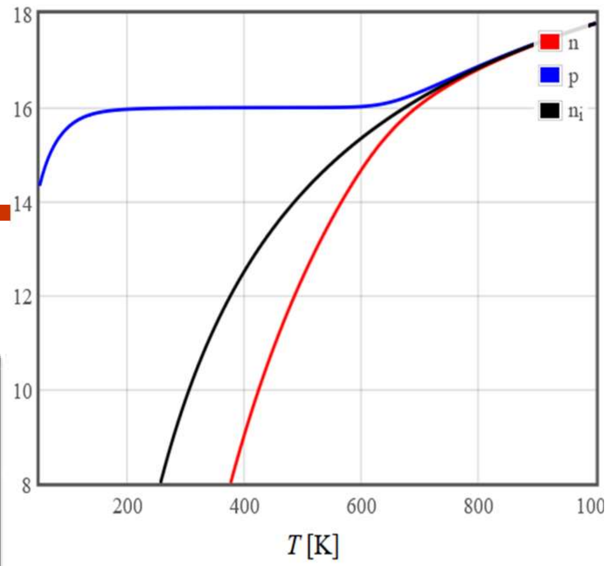


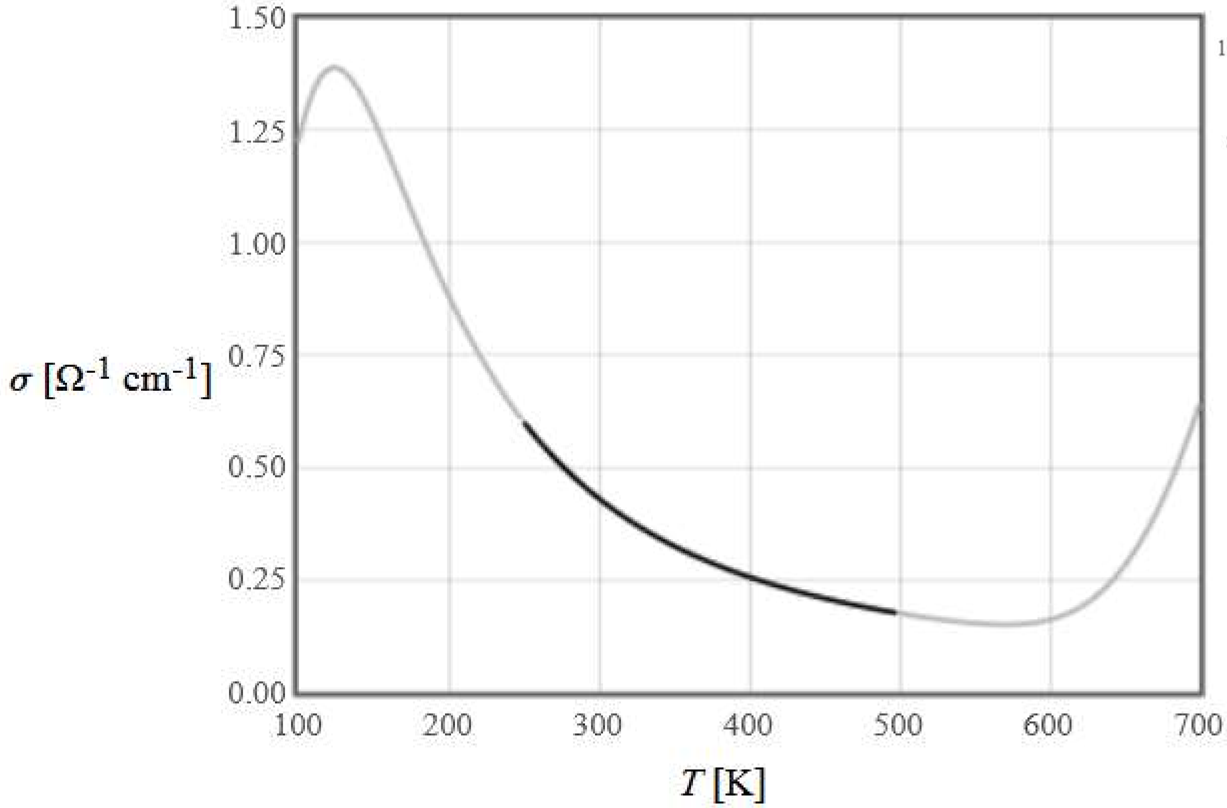






Conductivity of Silicon





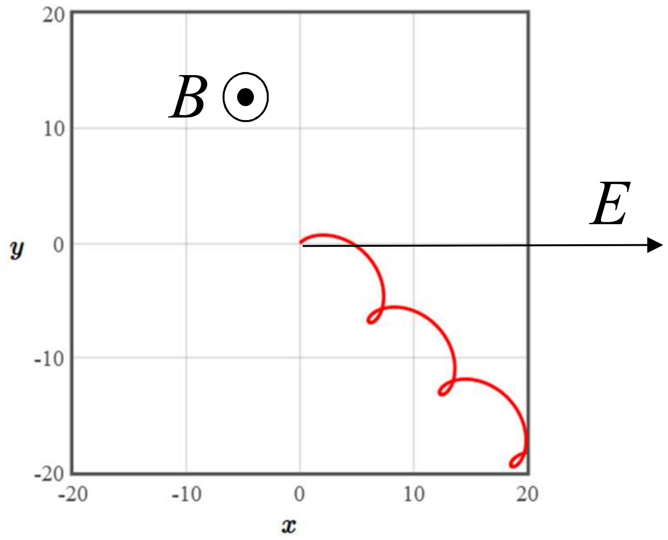
$N_A = 10^{16}$ 1/cm^3

http://lampx.tugraz.at/~hadley/psd/L4/conductivity.php

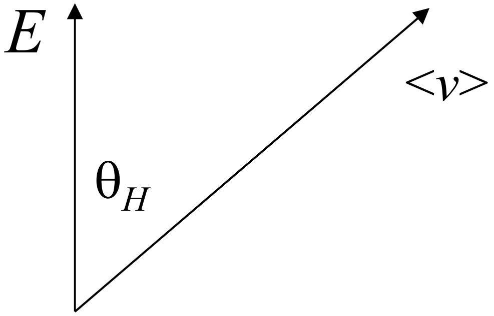
Crossed *E* and *B* fields

Ballistic transport

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right)$$



Diffusive transport



Hall angle: $\theta_H = \tan^{-1}\left(-\frac{eB_z\tau_{sc}}{m^*}\right)$

http://lampx.tugraz.at/~hadley/physikm/script/magnetism/constant_EB.en.php

Magnetic field (diffusive transport)

$$\vec{F} = m\vec{a} = -e\vec{E} = e\frac{\vec{v}_d}{\mu}$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right) = e\frac{\vec{v}_d}{\mu}$$

If B is in the z-direction, the three components of the force are

$$-\mu\left(E_x + v_{dy}B_z\right) = v_{dx}$$

$$-\mu\left(E_y - v_{dx}B_z\right) = v_{dy}$$

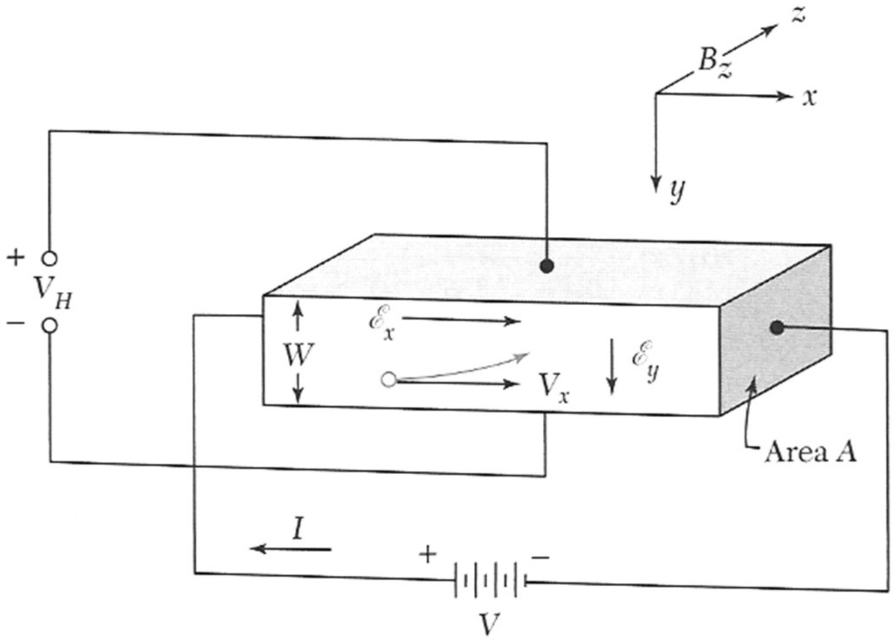
$$-\mu E_z = v_{dz}$$

If $E_y = 0$,

$$v_{d,y} = -\mu B_z v_{d,x}$$

$$\tan\theta_H = -\mu B_z$$

The Hall Effect (diffusive regime)



$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

$$v_{d,z} = -\mu E_z$$

$E_y = v_x B_z = V_H/W = R_H j_x B_z$ V_H = Hall voltage, R_H = Hall Constant

$j_x = I/A$

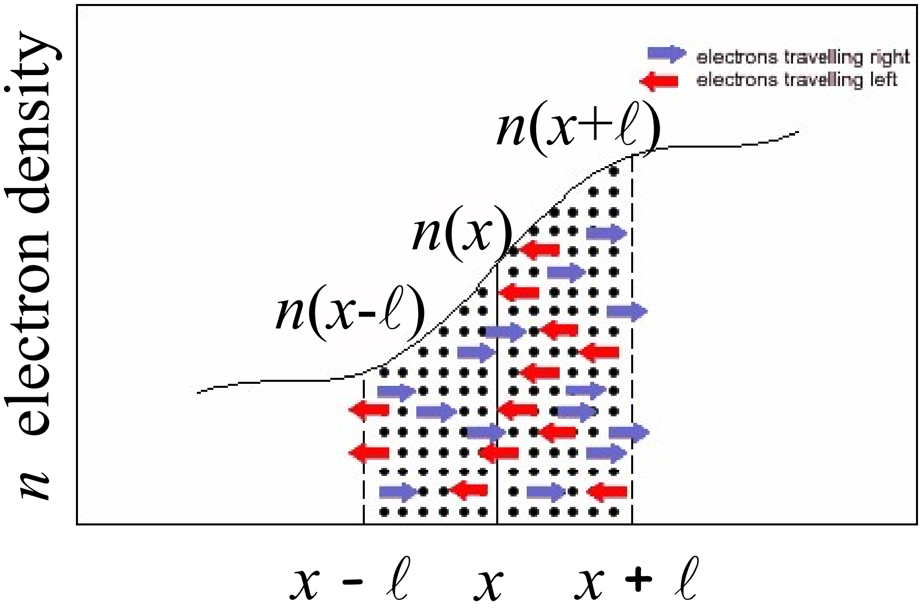
$v_x = -j_x/ne$ for n-type $v_x = j_x/pe$ for p-type

$R_H = -1/ne$ for n-type $R_H = 1/pe$ for p-type

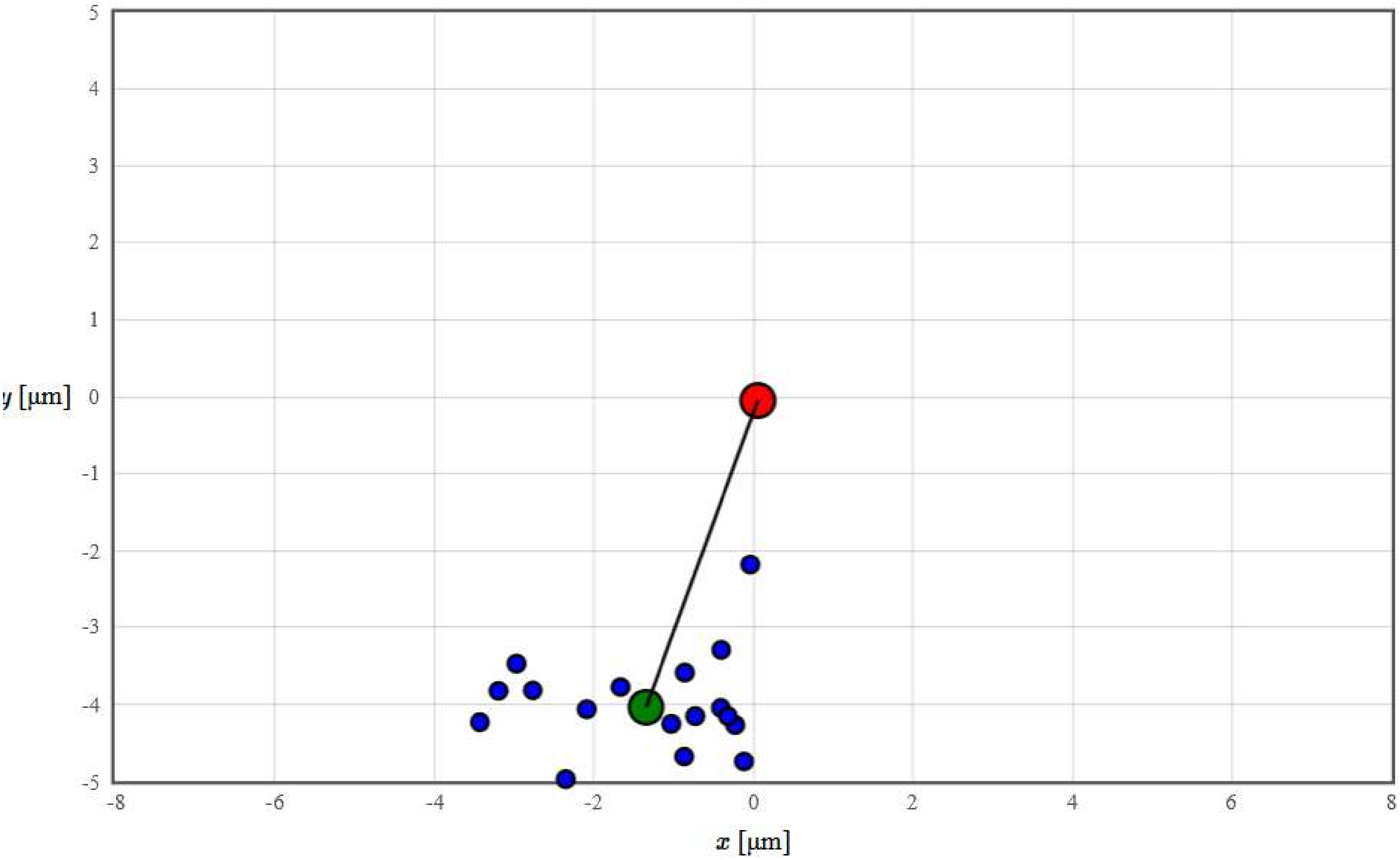
Diffusion

$$\overrightarrow{j_n} = |e|D\nabla n$$

$$\overrightarrow{j_p} = -|e|D\nabla p$$

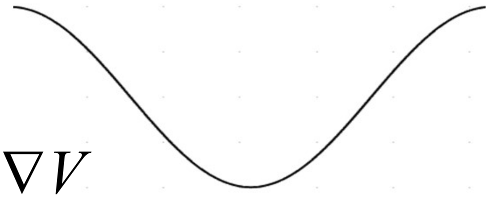


Diffusion is from high concentration to low concentration.



http://lampx.tugraz.at/~hadley/psd/L5/drude.php

Einstein relation



$$\vec{E} = -\nabla V$$

$$n = A\exp\left(\frac{-eV}{k_B T}\right)$$ Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{e}{k_B T} A\exp\left(\frac{-eV_{pot}}{k_B T}\right)\nabla V = -\frac{ne}{k_B T}\nabla V = \frac{ne\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + eD\frac{ne\vec{E}}{k_B T} = 0$$

$$\boxed{D = \frac{\mu k_B T}{e}}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen

Current Density Equations

Drift

Diffusion

$$\vec{j}_n = -ne\mu_n\vec{E} + eD_n\nabla n$$

$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

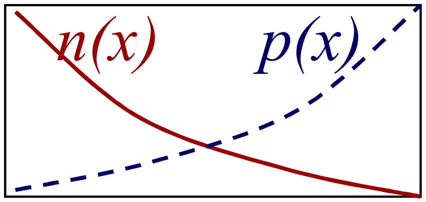
Current Density Equations

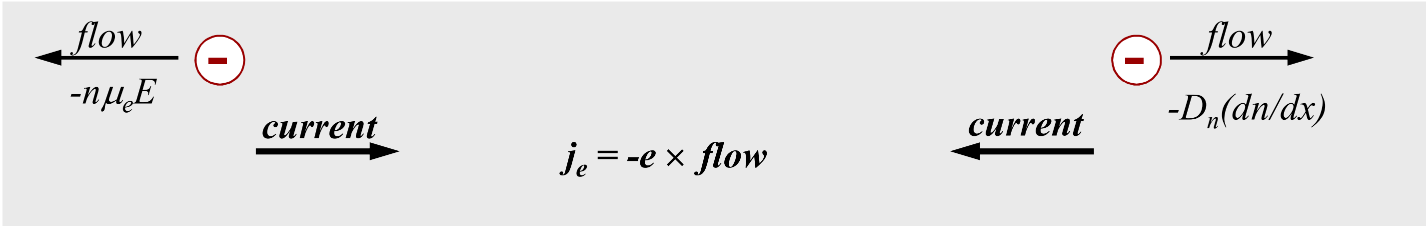
note: electron and hole currents have same direction

electric current = charge × particle flow

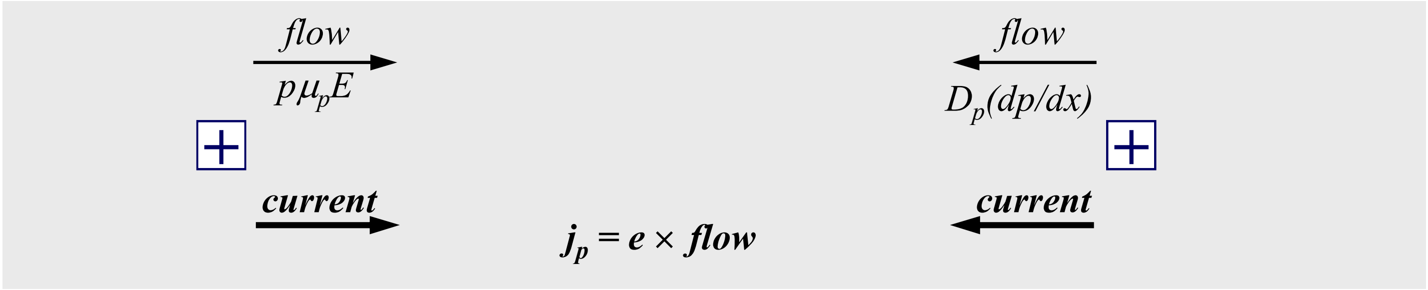
drift

diffusion



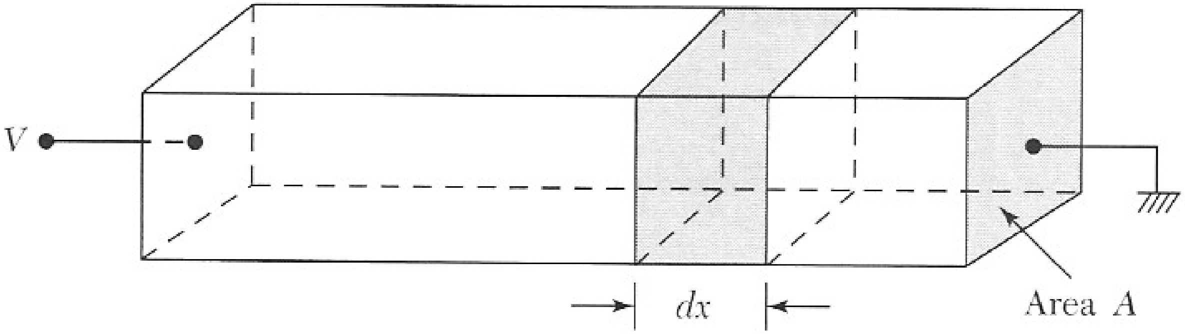


$j_e = -e \times flow$



$j_p = e \times flow$

Continuity equations

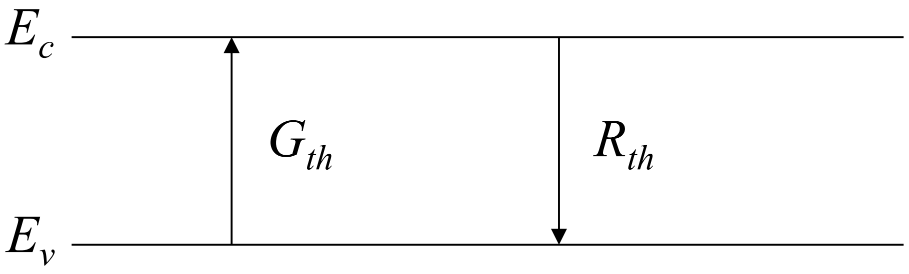


$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{e}\nabla \cdot \vec{j}_p + G_p - R_p$$

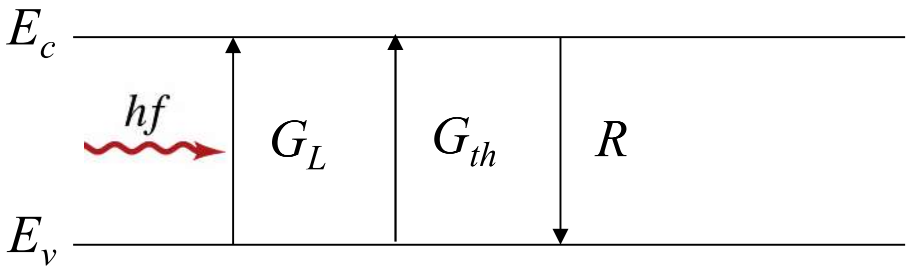
j_n and j_p consist of drift and diffusion terms

Generation and Recombination



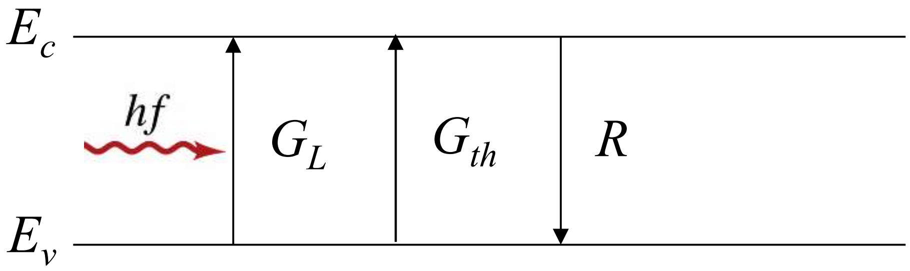
thermal
equilibrium

Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution $np \neq n_i^2$



non equilibrium

Recombination



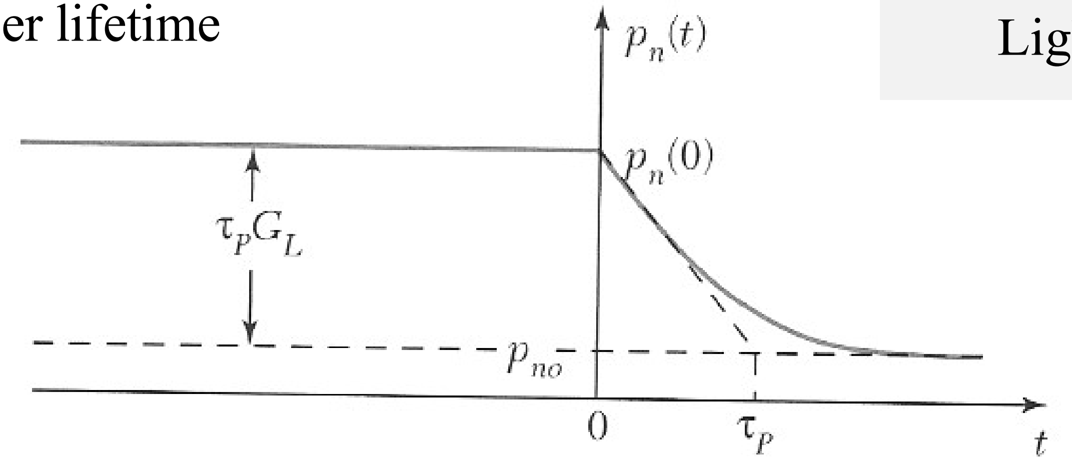
$$R - R_{th} = \frac{p_n - p_{n0}}{\tau_p}$$

Recombination rate is limit by the density of minority carriers.
The majority carriers have to find a minority carrier to recombine.

p_n (or n_p) = minority carrier concentration
p_{n0} (or n_{p0}) = equilibrium minority carrier concentration
τ_p = minority carrier lifetime

$$\frac{\partial p}{\partial t} = \frac{p_n - p_{n0}}{\tau_p}$$

Light off



minority carrier lifetimes

p-type

$$n_p(t) = n_{excess} \exp(-t/\tau_n) + n_{p0}$$

minority carrier lifetimes

n-type

$$p_n(t) = p_{excess} \exp(-t/\tau_p) + p_{n0}$$

$$np = n_i^2$$

Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

drift: $\vec{j}_n = -ne\mu_n \vec{E}$ $\nabla \cdot \vec{j}_n = -en\mu_n \nabla \cdot \vec{E} - e\nabla n \mu_n \vec{E}$

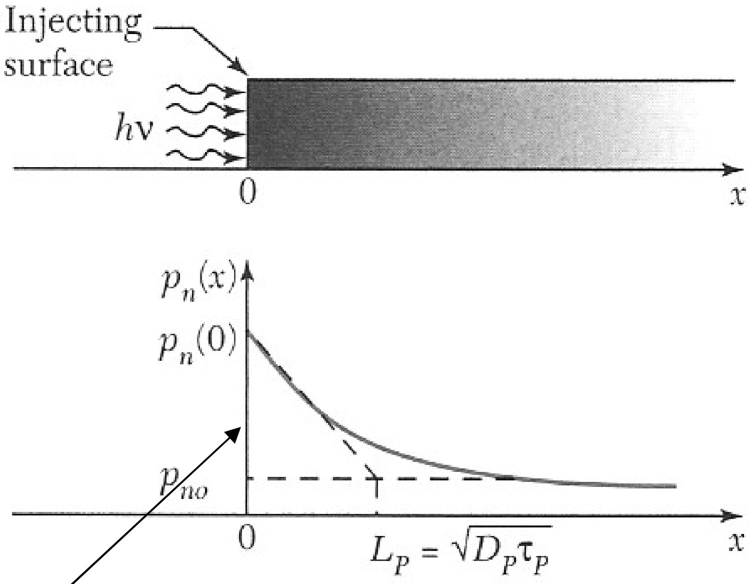
diffusion: $\vec{j}_{n,diff} = |e| D_n \nabla n$ $\nabla \cdot \vec{j}_{n,diff} = |e| D_n \nabla^2 n$

$$\frac{\partial n}{\partial t} = n\mu_n \nabla \cdot \vec{E} + \nabla n \mu_n \vec{E} + D_n \nabla^2 n + G_n - \frac{n - n_0}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p\mu_p \nabla \cdot \vec{E} - \nabla p \mu_p \vec{E} + D_p \nabla^2 p + G_p - \frac{p - p_0}{\tau_p}$$

Diffusion Length

n-type



Steady state

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

Generation only occurs at the surface. There the minority carrier density is $p_n(0)$.

Diffusion Length

$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \quad \Leftarrow \quad p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

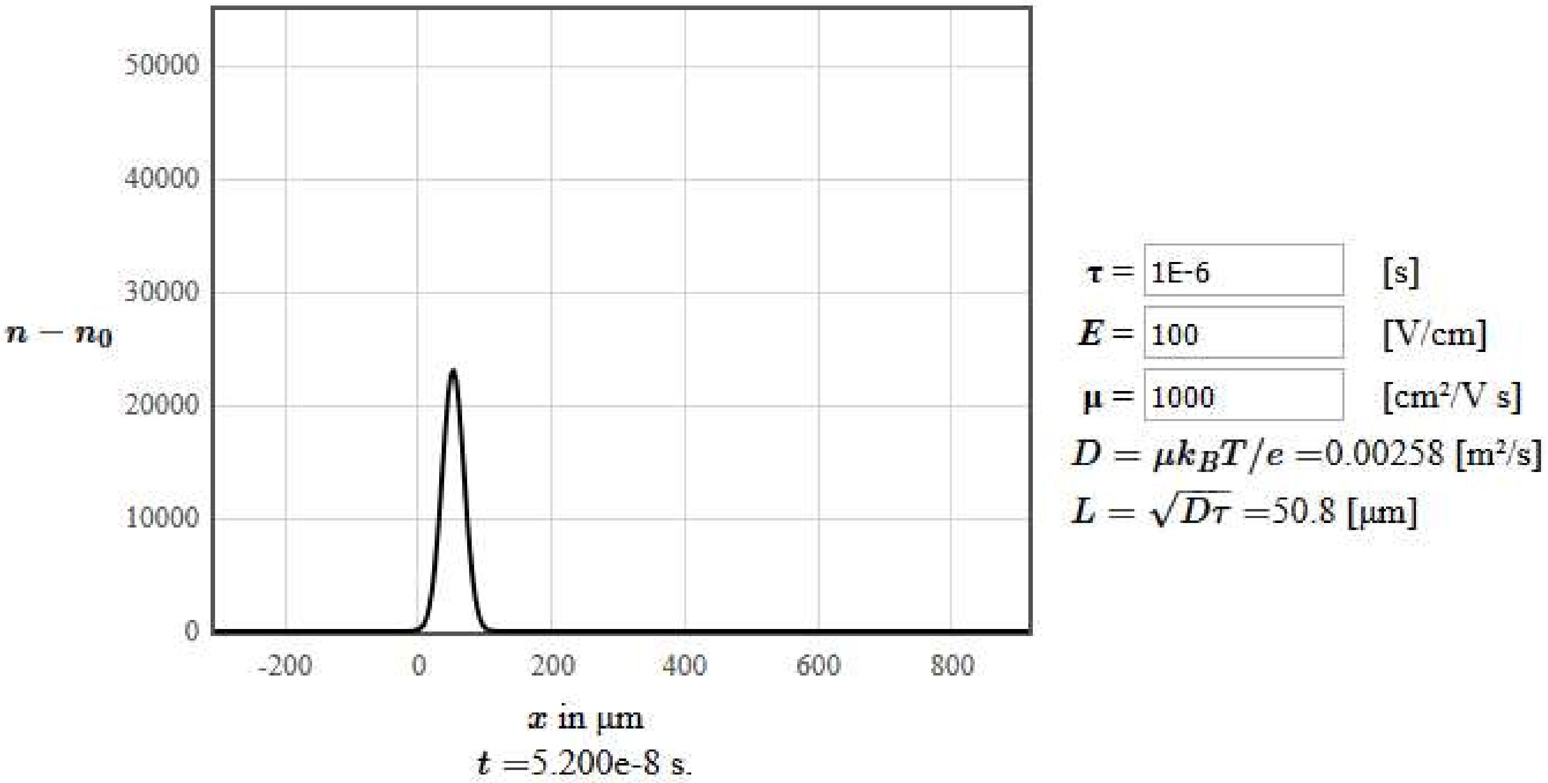
$$0 = \frac{D_p\left(p_n(0) - p_{n0}\right)}{L_p^2}\exp\left(\frac{-x}{L_p}\right) - \frac{\left(p_n(0) - p_{n0}\right)}{\tau_p}\exp\left(\frac{-x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

diffusion length,
typically microns

Haynes Shockley experiment

$$n_p(x,t) = \frac{n_{generated}}{\sqrt{4\pi D_n t}} \exp\left(-\frac{(x-\mu_n E t)^2}{4 D_n t}\right) \exp\left(-\frac{t}{\tau_n}\right) + n_{p0}$$



t =5.200e-8 s.

τ = 1E-6 [s]

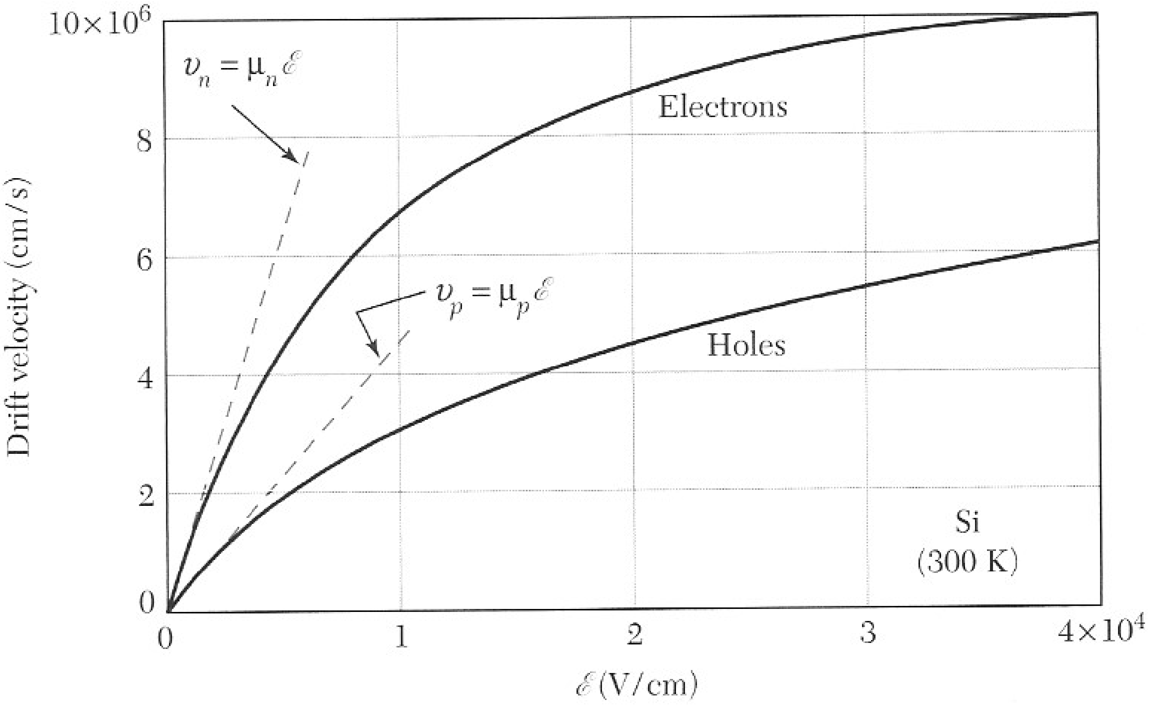
E = 100 [V/cm]

μ = 1000 [cm²/V s]

$D = \mu k_B T / e$ =0.00258 [m²/s]

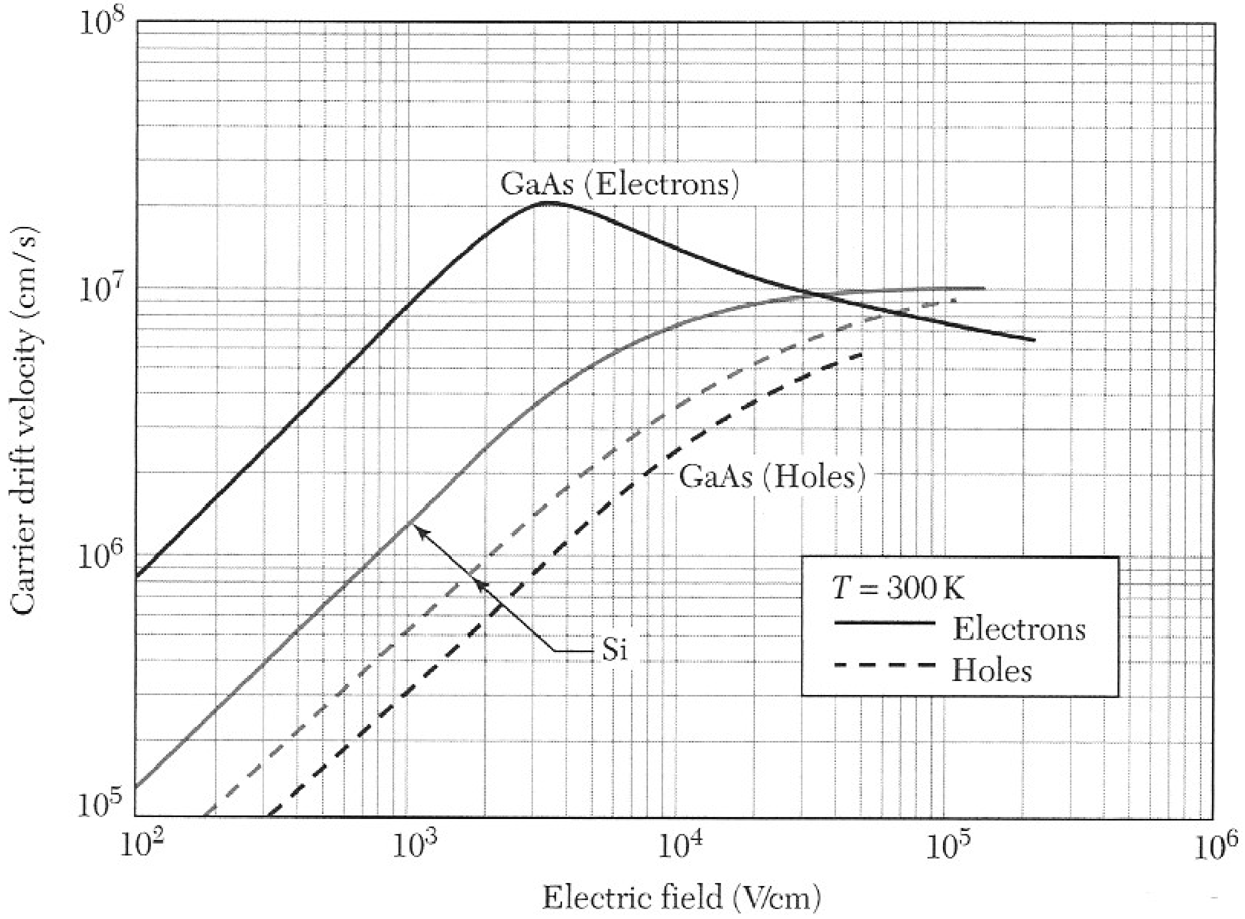
$L = \sqrt{D\tau}$ =50.8 [µm]

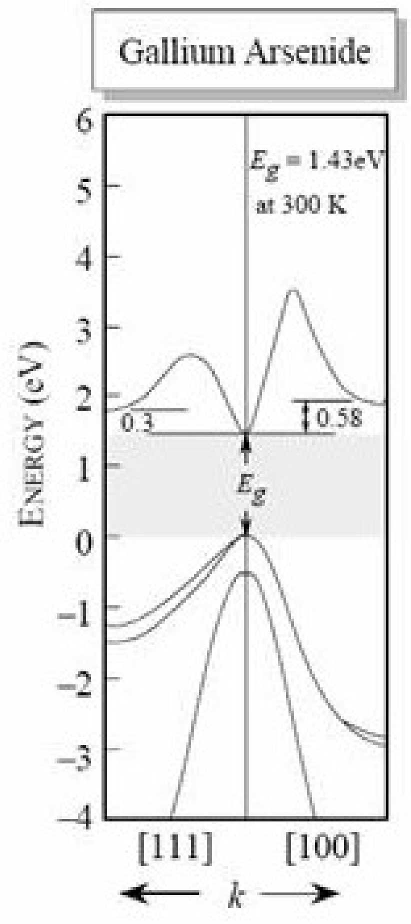
High Fields



Silicon

High Fields



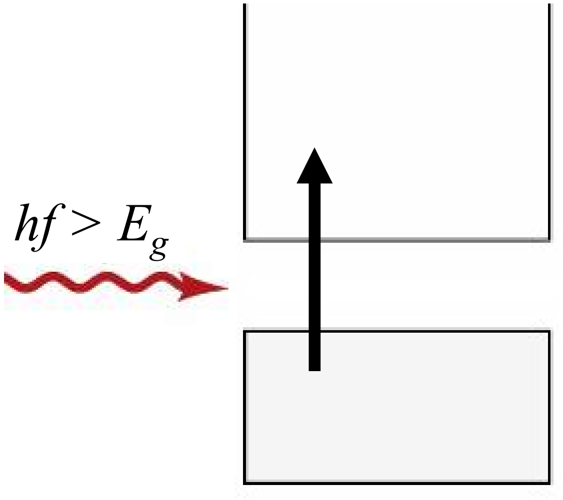


GaAs

Impact ionization

Carriers are accelerated to an energy above the gap before they scatter. They generate more electron-hole pairs. This results in an avalanche breakdown of the device.

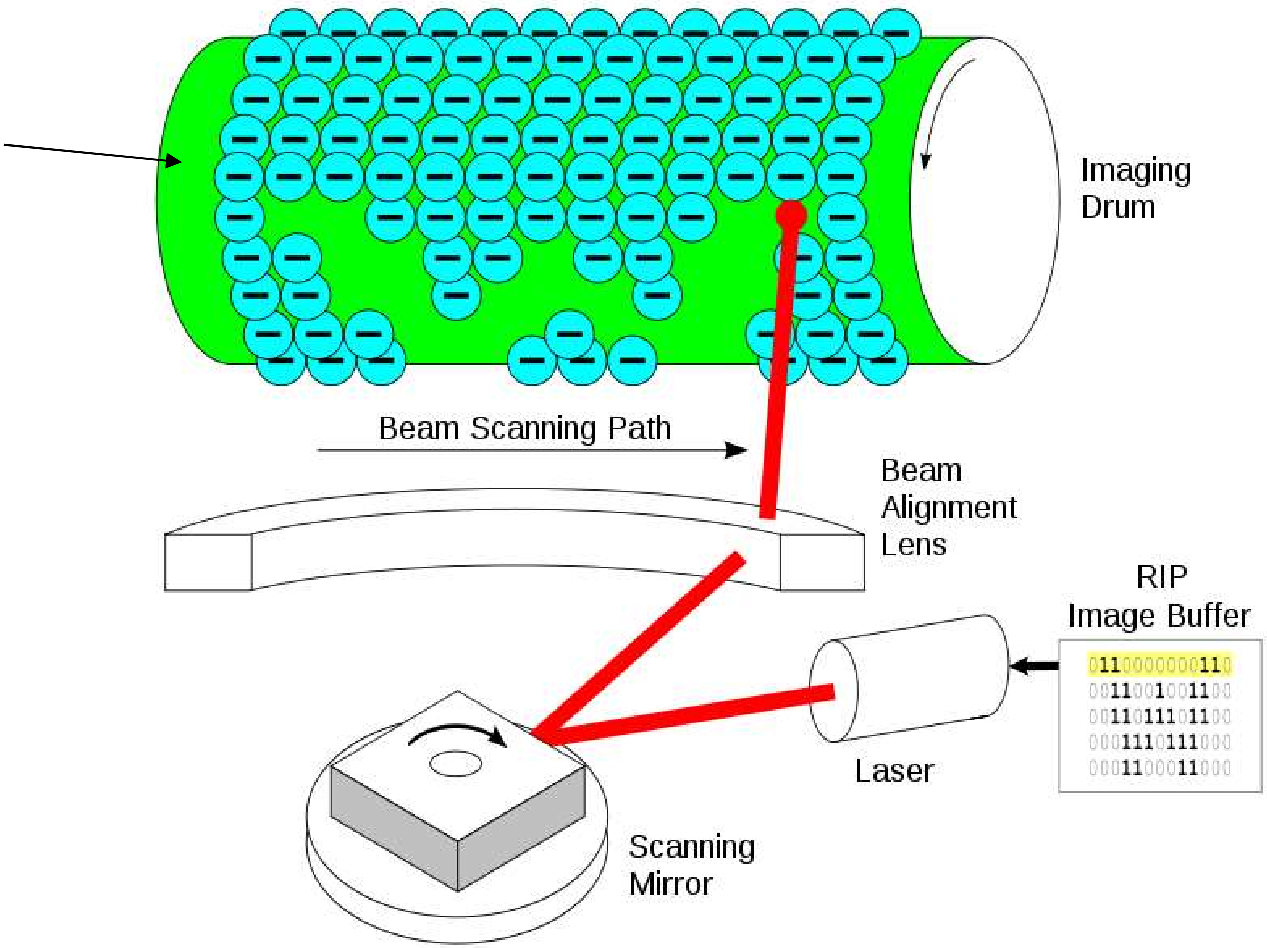
Photoconductivity



$$\sigma = ne\mu_n + pe\mu_p$$

Light increases the conductivity of a semiconductor.

Laser printer



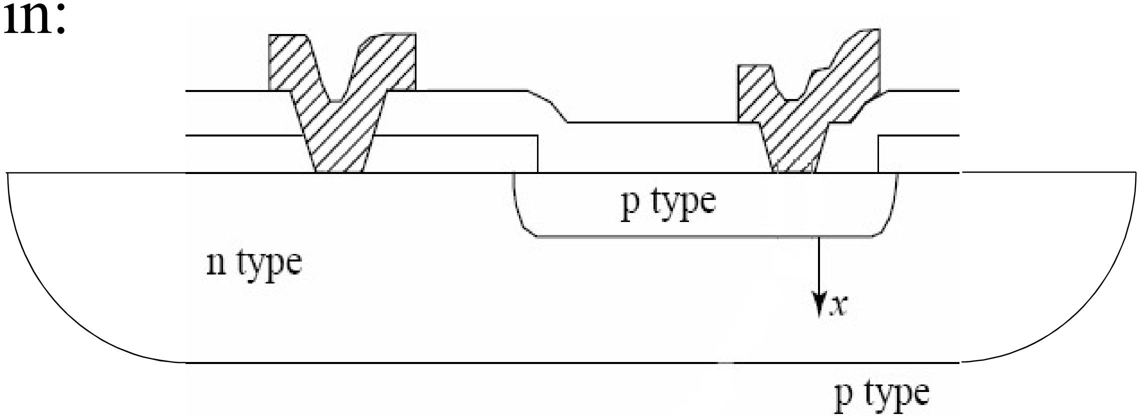
semiconducting
photoconductor

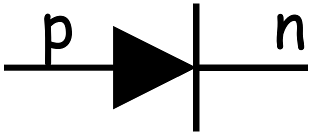
pn - Junctions

pn junctions

pn junctions are found in:

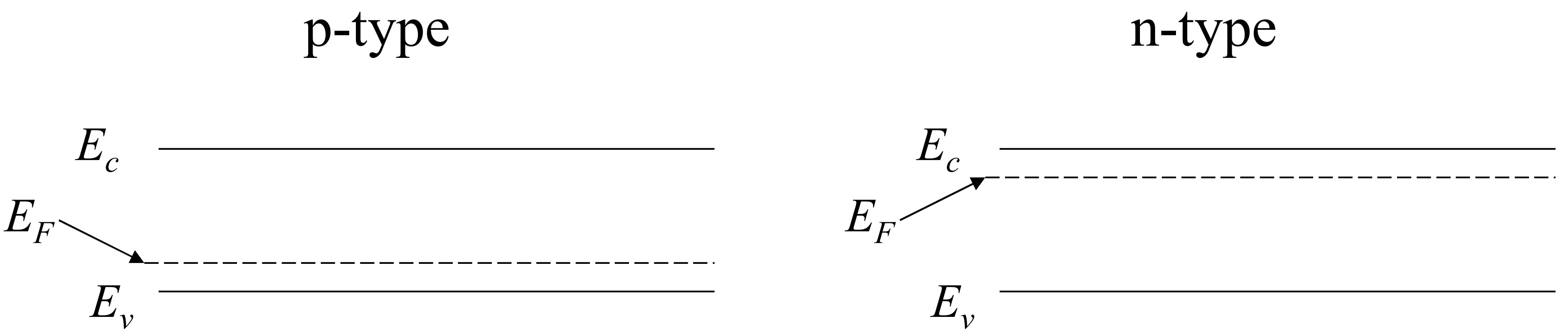
- diodes
- solar cells
- LEDs
- isolation
- JFETs
- bipolar transistors
- MOSFETs
- solid state lasers





pn junction

isolated semiconductors



$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

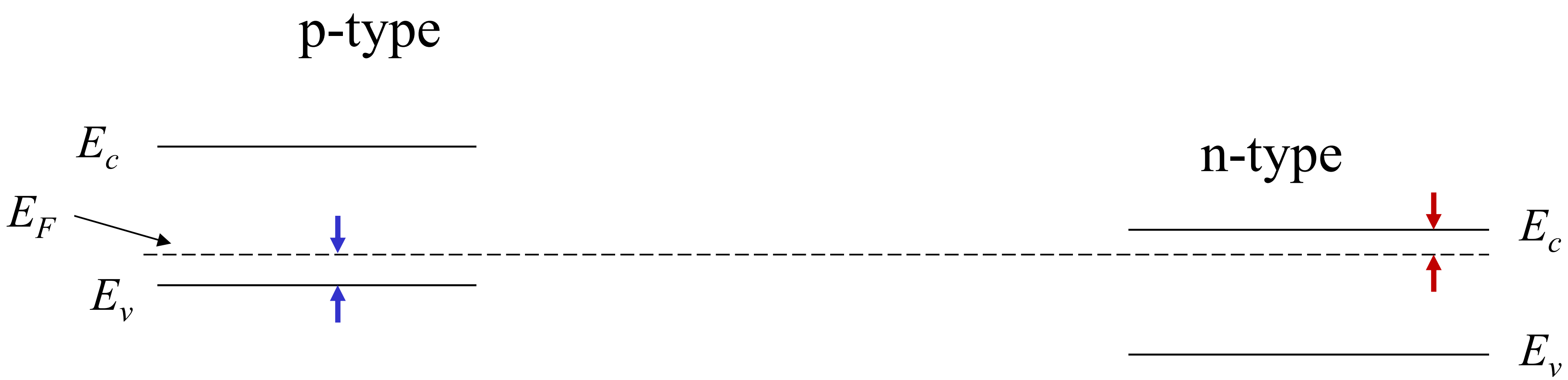
$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

valid for both n and p doping

$$E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)$$

pn junction

semiconductors in contact
electrons flow from n to p

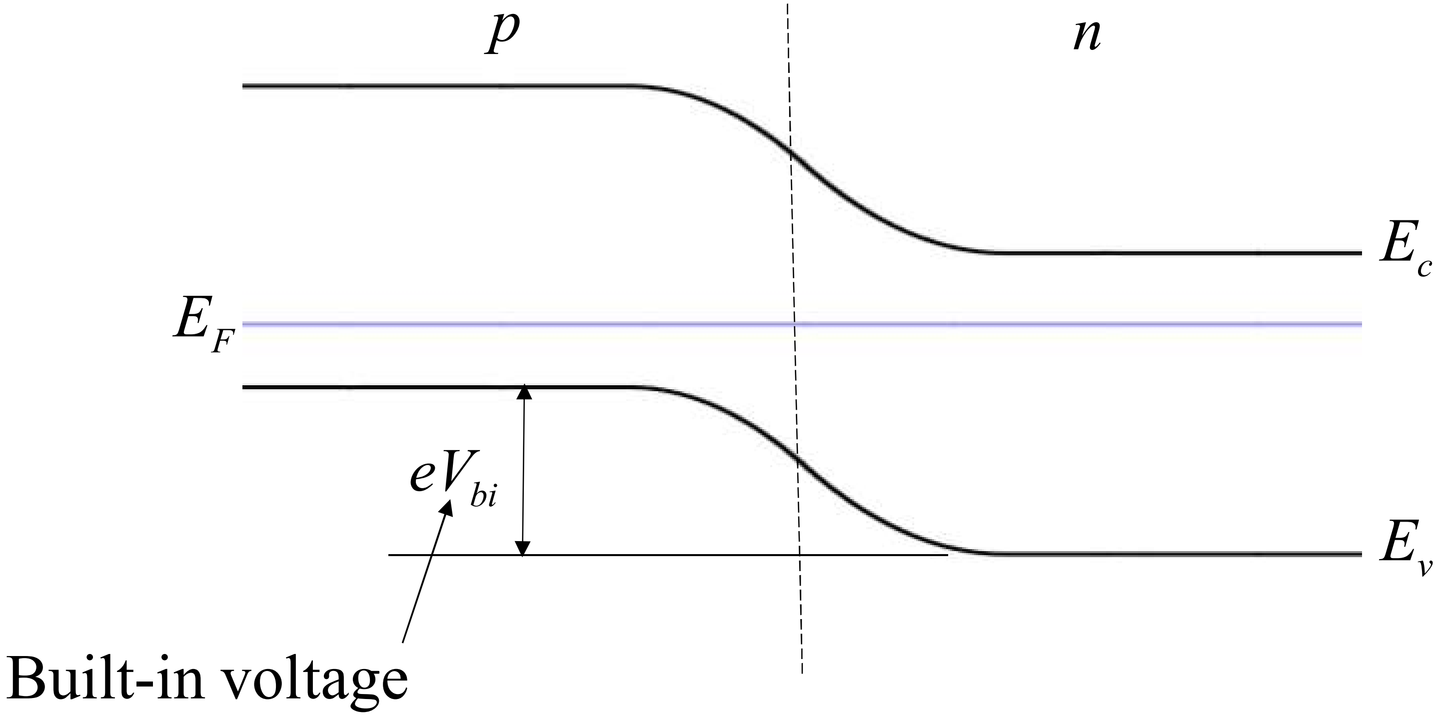


$$n = N_c \exp\left(\frac{\boxed{E_F - E_c}}{k_B T}\right) \approx N_D$$

$$p = N_v \exp\left(\frac{\boxed{E_v - E_F}}{k_B T}\right) \approx N_A$$

pn junction

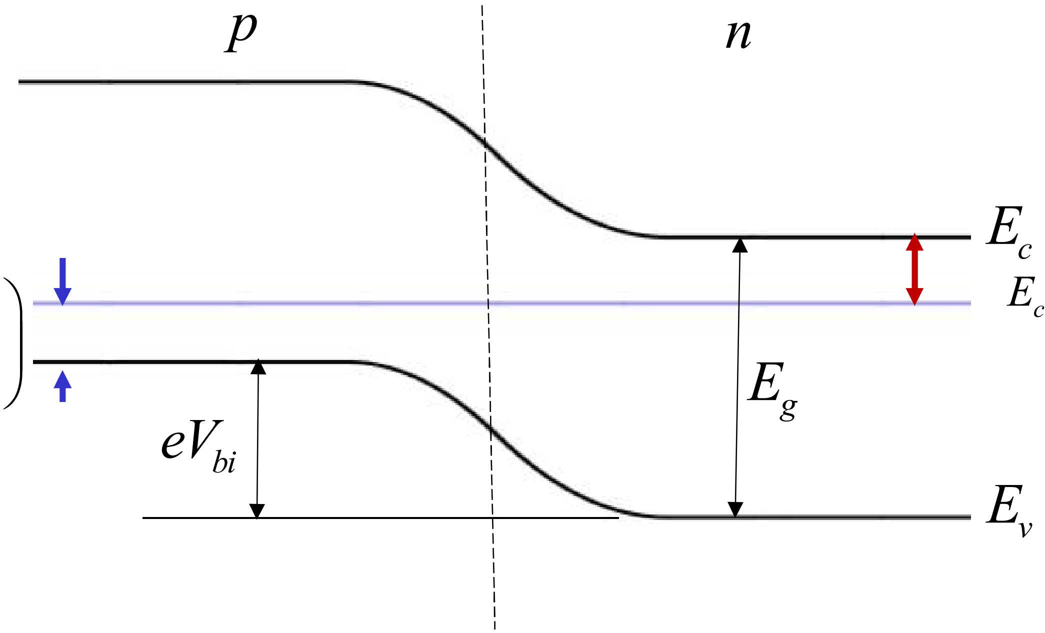
semiconductors in contact



Abrupt junction: the doping changes abruptly from p to n

Built-in voltage V_{bi}

$$E_F - E_{vp} = k_B T \ln\left(\frac{N_v}{N_A}\right)$$



$$E_{cn} - E_F = k_B T \ln\left(\frac{N_c}{N_D}\right)$$

$$eV_{bi} = E_g - \boxed{k_B T \ln\left(\frac{N_c}{N_{D,n} - N_{A,n}}\right)} - \boxed{k_B T \ln\left(\frac{N_v}{N_{A,p} - N_{D,p}}\right)}$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}\right)$$

V_{bi}

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)} \right)$$

$$n_i^2 = N_v N_c \exp\left(\frac{-E_g}{k_B T} \right) \qquad E_g = -k_B T \ln\left(\frac{n_i^2}{N_v N_c} \right)$$

$$eV_{bi} = k_B T \ln\left(\frac{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}{n_i^2} \right)$$

for $N_{D,n}$ - $N_{A,n} = N_D$ and $N_{A,p}$ - $N_{D,p} = N_A$

$$\boxed{eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2} \right)}$$

V_{bi}

Can V_{bi} perform work?