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Extrinsic Semiconductors

doping: small admixture of impurities

doping: impurity atoms that can add electrons or holes

n-doping: donor atoms add electrons in the conduction band

P, As in Si

p-doping: acceptor atoms add electrons in the conduction band

Al, B, Ga in Si

➤ change in macroscopic properties (electric conductivity)

Ionization of dopants

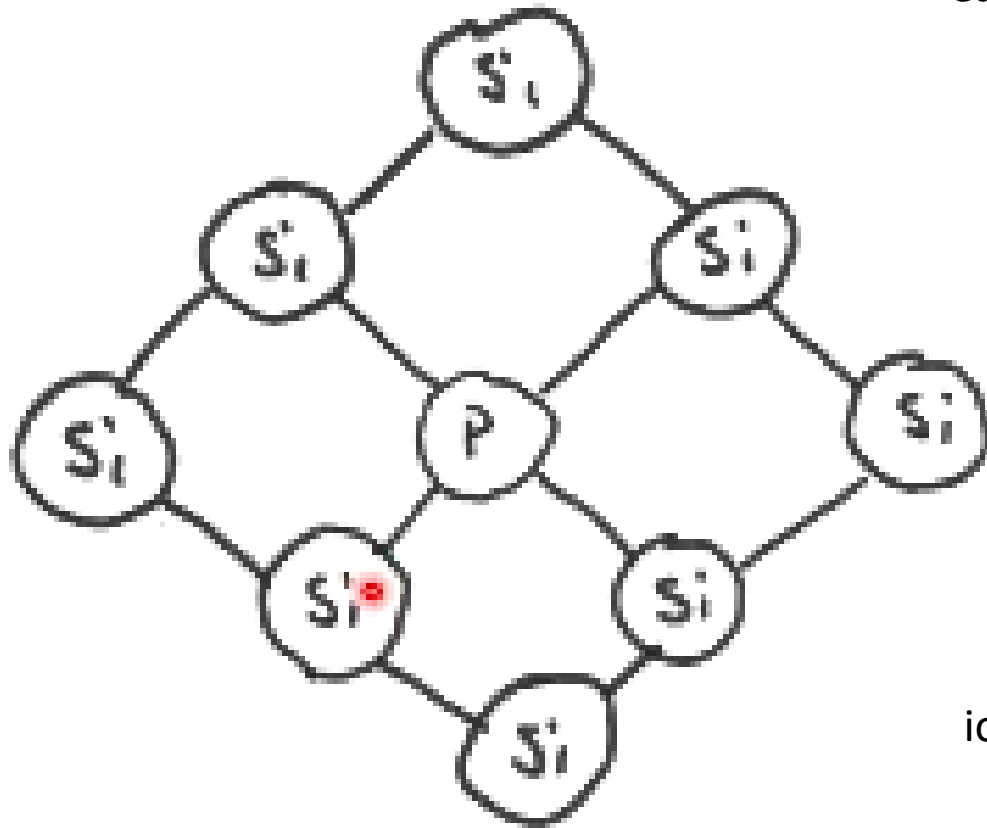
easier to ionize a P atom in Si than a free P atom

remove electron from ground state of a hydrogen-like impurity

$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2 n^2}$$

ionization in H-atom

ionization energy is smaller by factor $\frac{m^*}{m} \left(\frac{\epsilon_0}{\epsilon_r \epsilon_0} \right)^2$



ionization energy ~25 meV

n and p?

$$n = N_c(T) \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v(T) \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

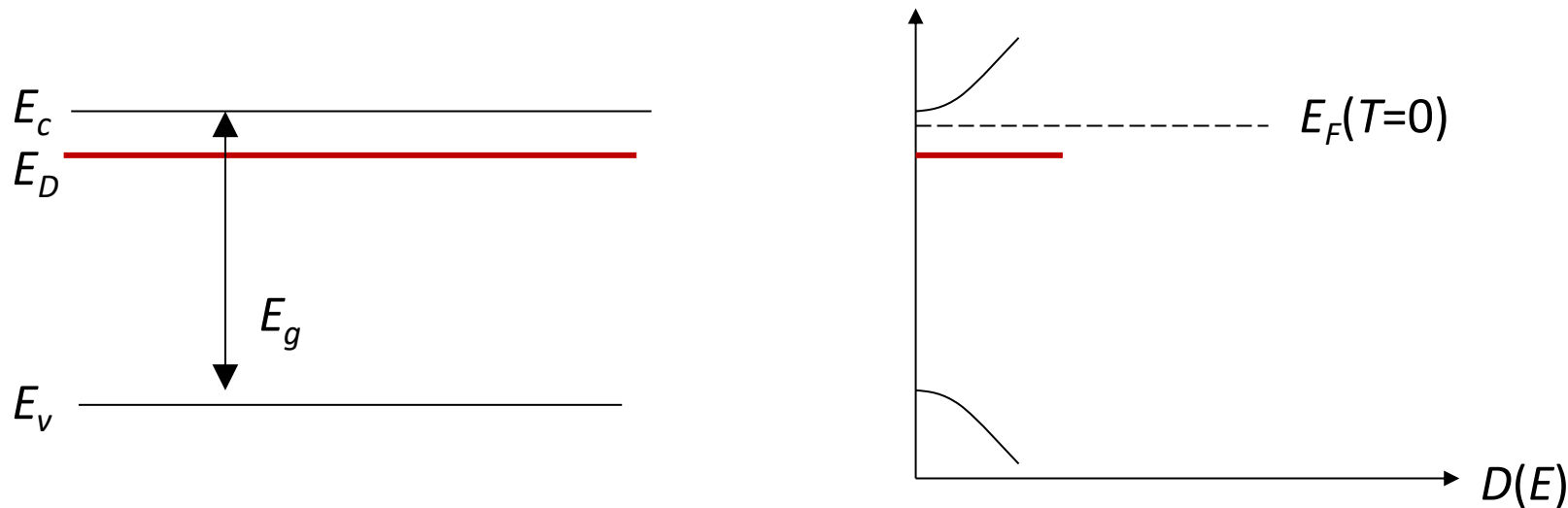
law of mass action

$$np = n_i^2 = N_c N_v \exp\left(-\frac{E_g}{k_B T}\right)$$

n-doping: increase number of electrons

Five valence electrons: P, As

States are added in the band gap just below the conduction band



n-type: $n \sim N_D$

many more electrons in the conduction band than holes in the valence band.

majority carriers: electrons; minority carriers: holes

Donors for Si

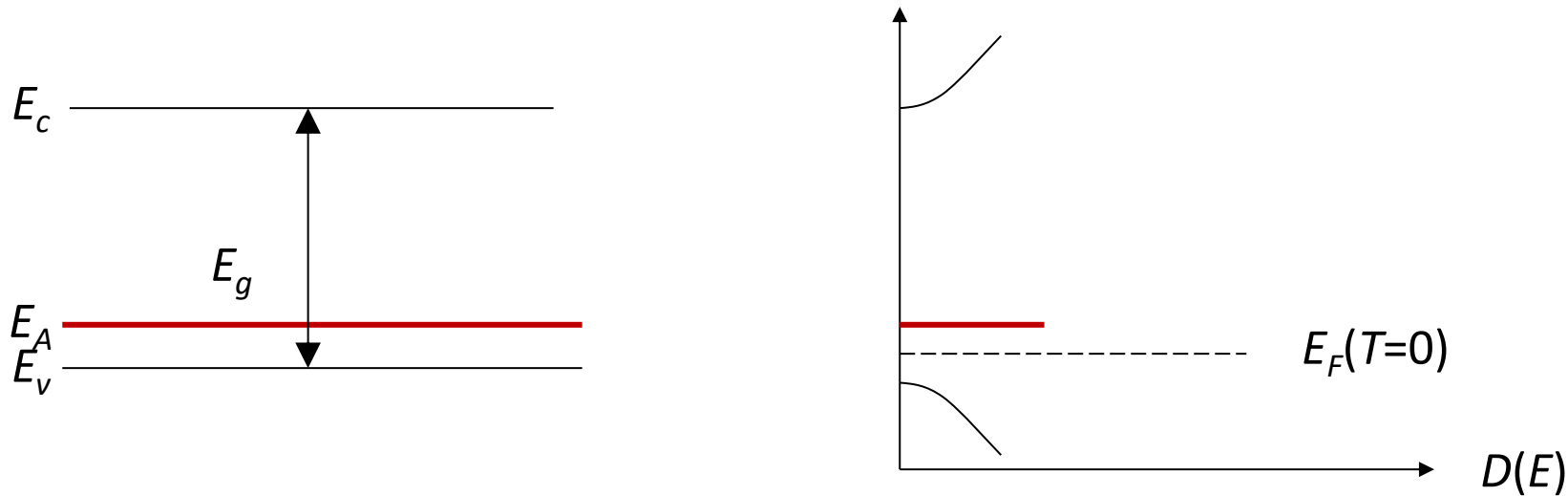
donors in Si

	5 B Boron	6 C Carbon	7 N Nitrogen	8 O Oxygen	
	13 Al Aluminium	14 Si Silicon	15 P Phosphorus	16 S Sulfur	
30 Zn Zinc	31 Ga Gallium	32 Ge Germanium	33 As Arsenic	34 Se Selenium	
48 Cd Cadmium	49 In Indium	50 Sn Tin	51 Sb Antimony	52 Te Tellurium	
IIb	IIIa	IVa	Va	VIa	

p-doping: increase number of holes

Three valence electrons: B, Al, Ga

States are added in the band gap just above the valence band



p-type: $p \sim N_A$

many more holes in the valence band than electrons in the conduction band

majority carriers: holes; minority carriers: electrons

Donors and Acceptors for Si

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	acceptors in Si		donors in Si	
	5	6	7	8
	B Boron	C Carbon	N Nitrogen	O Oxygen
	13	14	15	16
	Al Aluminium	Si Silicon	P Phosphorus	S Sulfur
30	31	32	33	34
Zn Zinc	Ga Gallium	Ge Germanium	As Arsenic	Se Selenium
48	49	50	51	52
Cd Cadmium	In Indium	Sn Tin	Sb Antimony	Te Tellurium
IIb	IIIa	IVa	Va	VIa

Donor and Acceptor Energies

Semiconductor	Donor	Energy (meV)
Si	Li	33
	Sb	39
	P	45
	As	54
Ge	Li	9.3
	Sb	9.6
	P	12
	As	13
GaAs	Si	5.8
	Ge	6.0
	S	6.0
	Sn	6.0

Energy below the conduction band



Semiconductor	Acceptor	Energy (meV)
Si	B	45
	Al	67
	Ga	72
	In	160
Ge	B	10
	Al	10
	Ga	11
	In	11
GaAs	C	26
	Be	28
	Mg	28
	Si	35

Energy above the valence band



Si as Donor and Acceptor for GaAs

acceptors in Si donors in Si

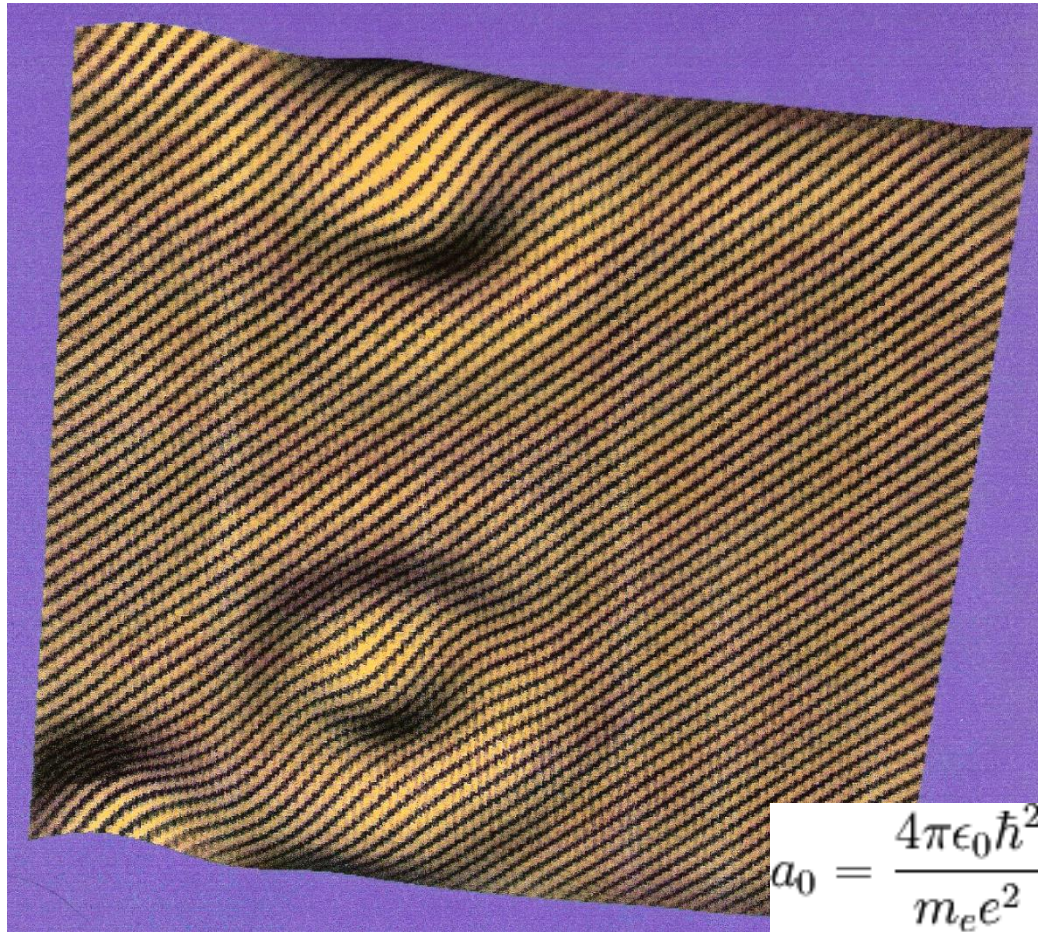
5 B Boron	6 C Carbon	7 N Nitrogen	8 O Oxygen	
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IIb	IIIa	IVa	Va	VIa

Direct Observation of Friedel Oscillations around Incorporated Si_{Ga} Dopants in GaAs by Low-Temperature Scanning Tunneling Microscopy

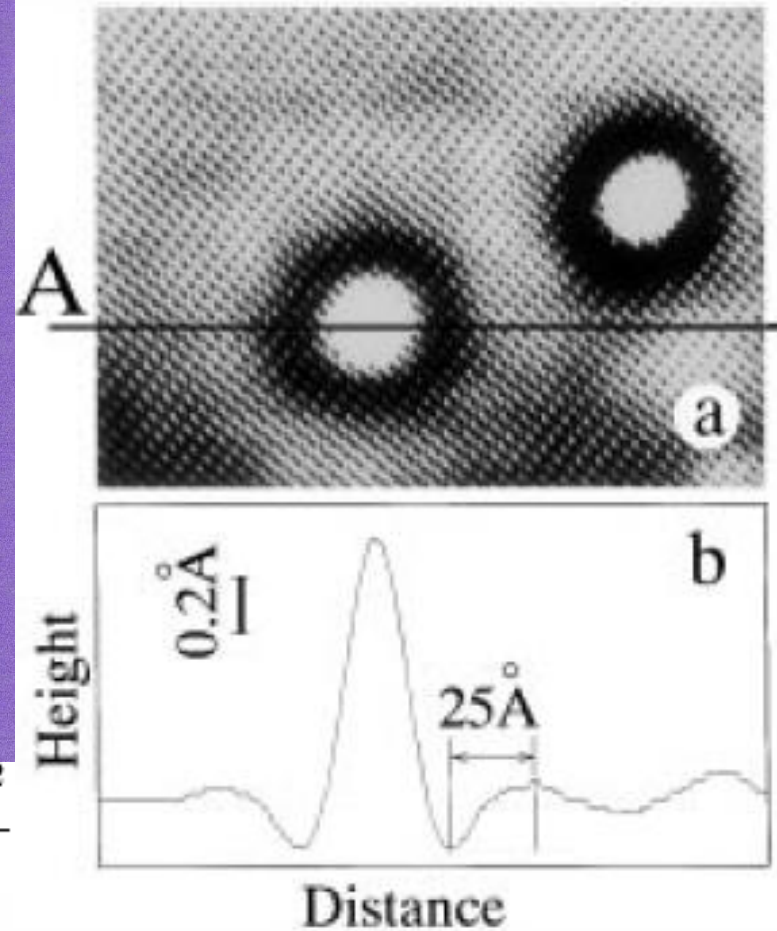
M. C. M. M. van der Wielen, A. J. A. van Roij, and H. van Kempen

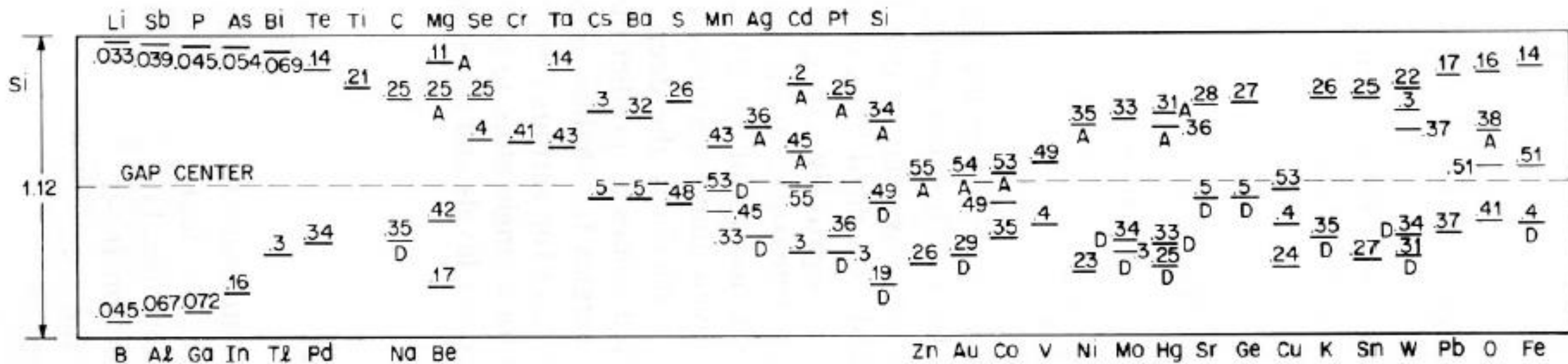
Research Institute for Materials, University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands

(Received 25 July 1995)



$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$$





Source: Semiconductor Devices Physics and Technology, S.M. Sze, 1985

Dopants

Donors

Examples: P, As in Si.

Mobile negative electrons

Fixed positive donors

Acceptors

Examples: B, Ga in Si.

Mobile positive holes

Fixed negative acceptors

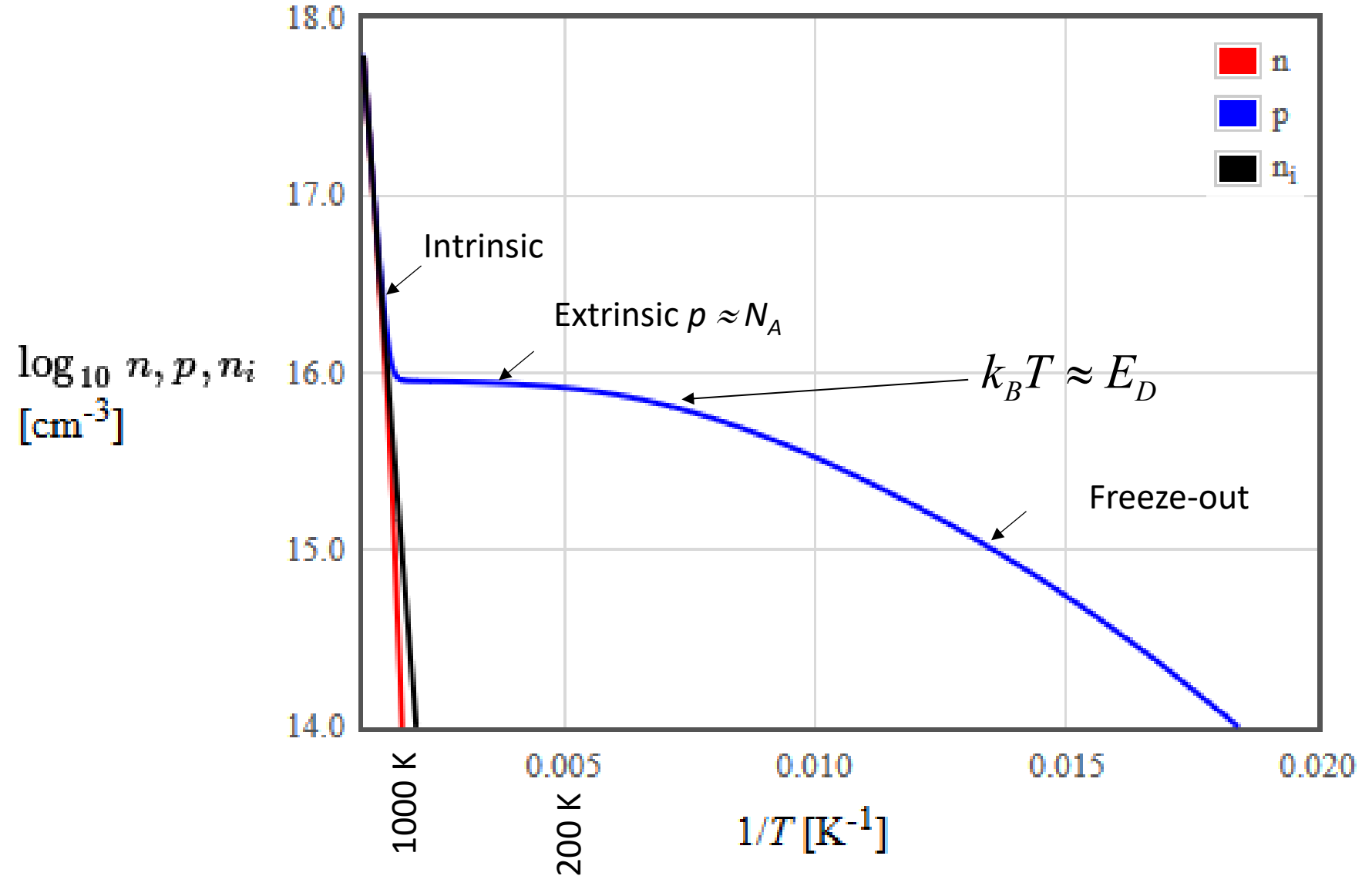
Charge Neutrality

$$n + N_A^- = p + N_D^+$$

$$n = N_c(T) \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v(T) \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

Temperature dependence



Ionized donors and acceptors

For $E_v + 3k_B T < E_F < E_c - 3k_B T$ Boltzmann approximation

$$N_D^+ = \frac{N_D}{1 + 2 \exp\left(\frac{E_F - E_D}{k_B T}\right)}$$

$$N_A^- = \frac{N_A}{1 + 4 \exp\left(\frac{E_A - E_F}{k_B T}\right)}$$

4 for materials with light
holes and heavy holes (Si)
2 otherwise

N_D = donor density cm^{-3}

N_D^+ = ionized donor density cm^{-3}

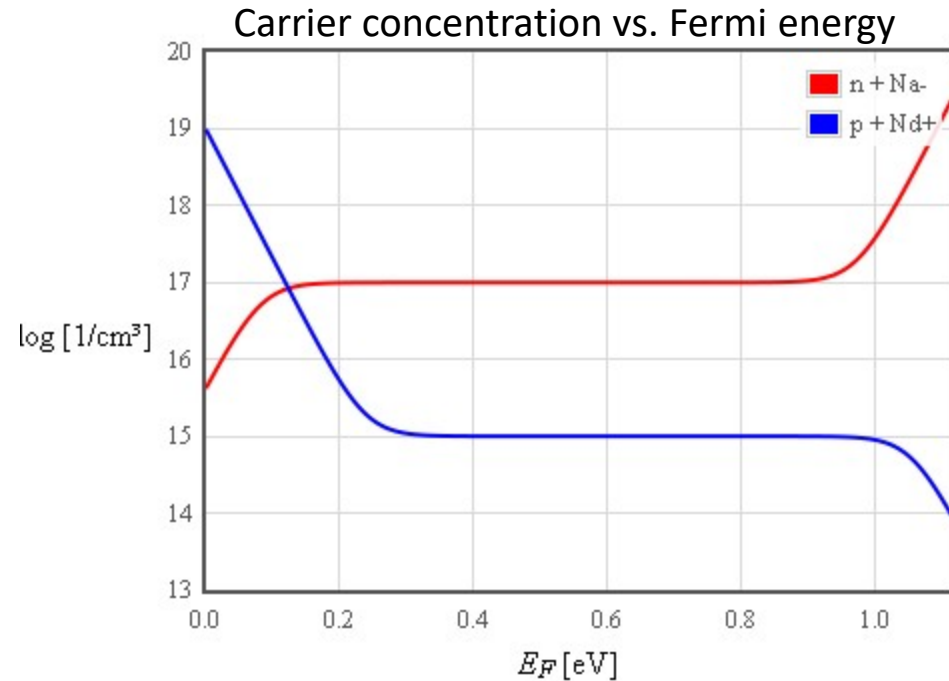
N_A = acceptor density cm^{-3}

N_A^- = ionized acceptor density cm^{-3}

Mostly, $N_D^+ = N_D$ and $N_A^- = N_A$

Ionized donors and acceptors

$$n + N_A^- = p + N_D^+$$

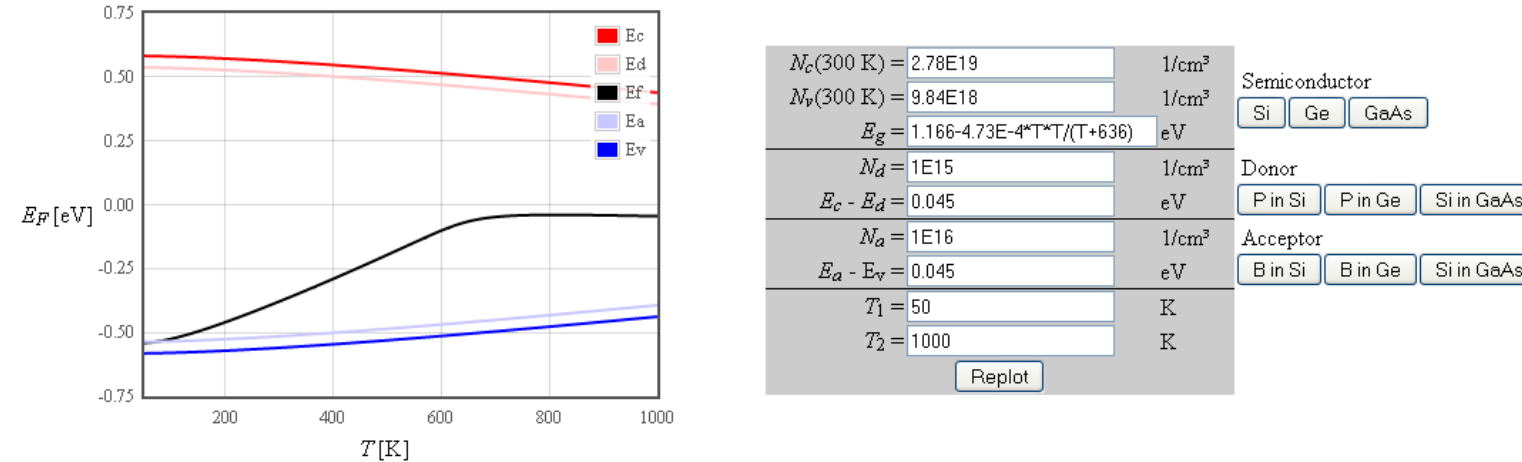


```
for ($i=0; $i<500; $i++) {
    $Ef = $i*$Eg/500;
    $n=$Nc*pow($T/300,1.5)*exp(1.6022E-19*($Ef-$Eg)/(1.38E-23*$T));
    $p=$Nv*pow($T/300,1.5)*exp(1.6022E-19*(-$Ef)/(1.38E-23*$T));
    $Namin = $Na/(1+4*exp(1.6022E-19*($Ea-$Ef)/(1.38E-23*$T)));
    $Ndplus = $Nd/(1+2*exp(1.6022E-19*($Ef-$Ed)/(1.38E-23*$T)));
}
```

E_f	n	p	N_d^+	N_a^-	$\log(n+N_a^-)$	$\log(p+N_d^+)$
0	4.16629283405	9.84E+18	1E+15	4.19743393218E+15	15.622983869	18.9930392318
0.00224	4.54358211887	9.0229075682E+18	1E+15	4.56020949614E+15	15.6589847946	18.9553946382
0.00448	4.95503779816	8.27366473417E+18	1E+15	4.95271809535E+15	15.694843609	18.9177504064
0.00672	5.40375389699	7.58663741327E+18	1E+15	5.37710747619E+15	15.7305487171	18.8801065693
0.00896	5.80210462781	6.95665026215E+18	1E+15	5.8256202025E+15	15.7660876057	18.8424621625

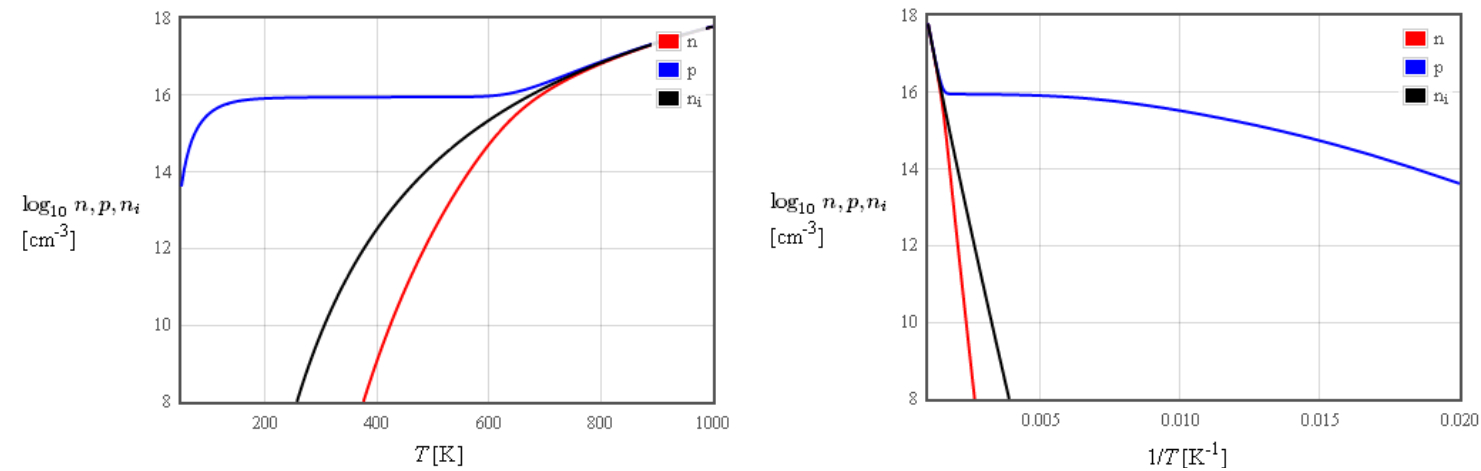
Fermi energy vs. temperature

Fermi energy of an extrinsic semiconductor is plotted as a function of temperature. At each temperature the Fermi energy was calculated by requiring that [charge neutrality](#) be satisfied.



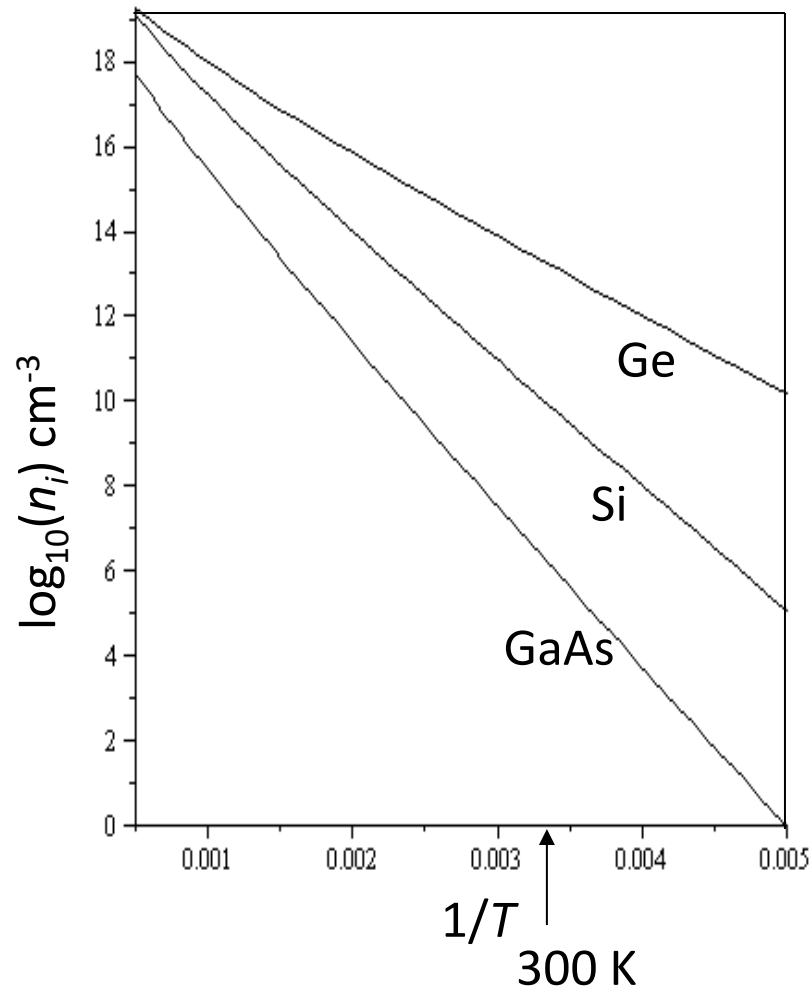
Once the Fermi energy is known, the carrier densities n and p can be calculated from the formulas, $n = N_c \left(\frac{T}{300} \right)^{3/2} \exp\left(\frac{E_F - E_c}{k_B T}\right)$ and $p = N_v \left(\frac{T}{300} \right)^{3/2} \exp\left(\frac{E_v - E_F}{k_B T}\right)$.

The intrinsic carrier density is $n_i = \sqrt{N_c \left(\frac{T}{300} \right)^{3/2} N_v \left(\frac{T}{300} \right)^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right)}$.



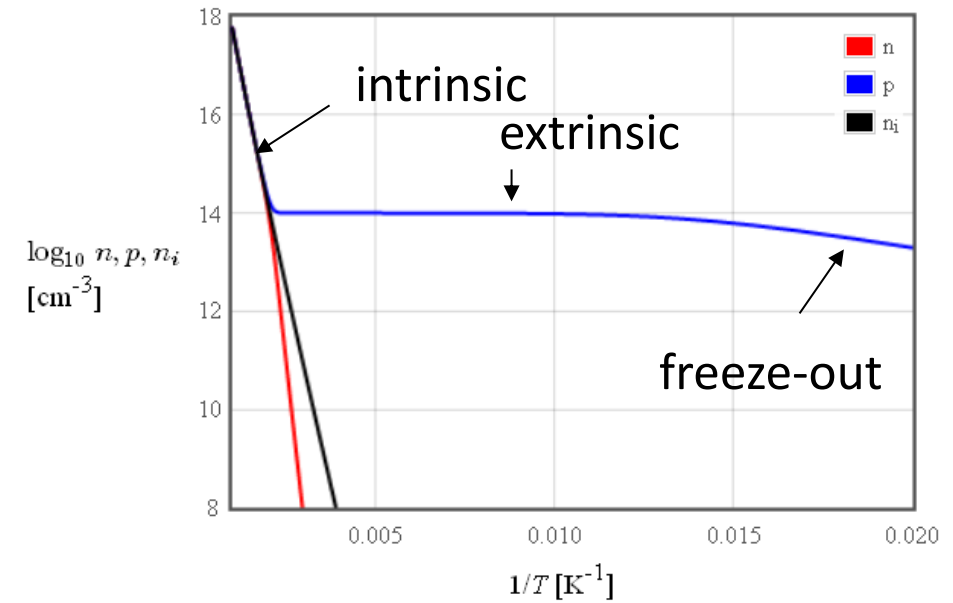
<https://lampz.tugraz.at/~hadley/psd/L4/eftplot.html>

Intrinsic semiconductors



$$n_i = \sqrt{N_v N_c} \exp\left(-\frac{E_g}{2k_B T}\right)$$

Extrinsic semiconductors



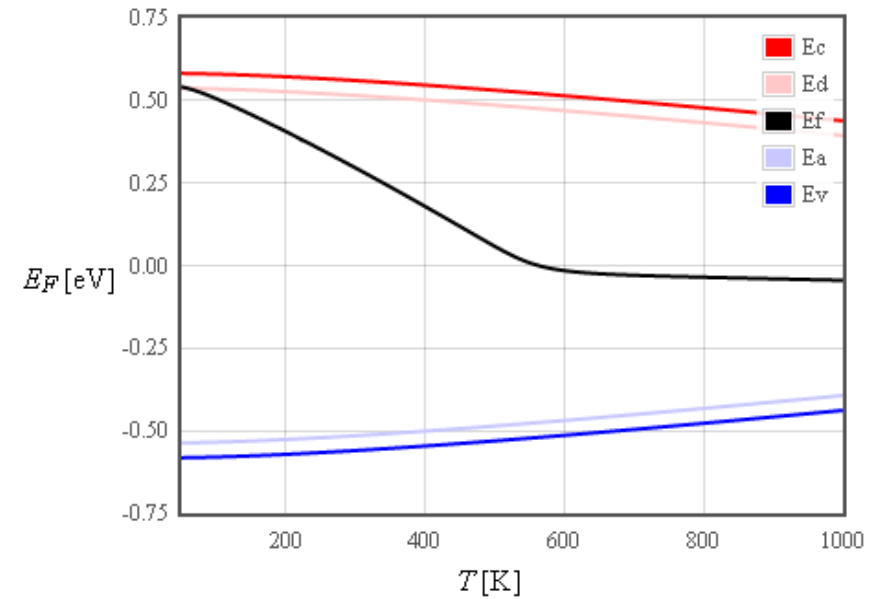
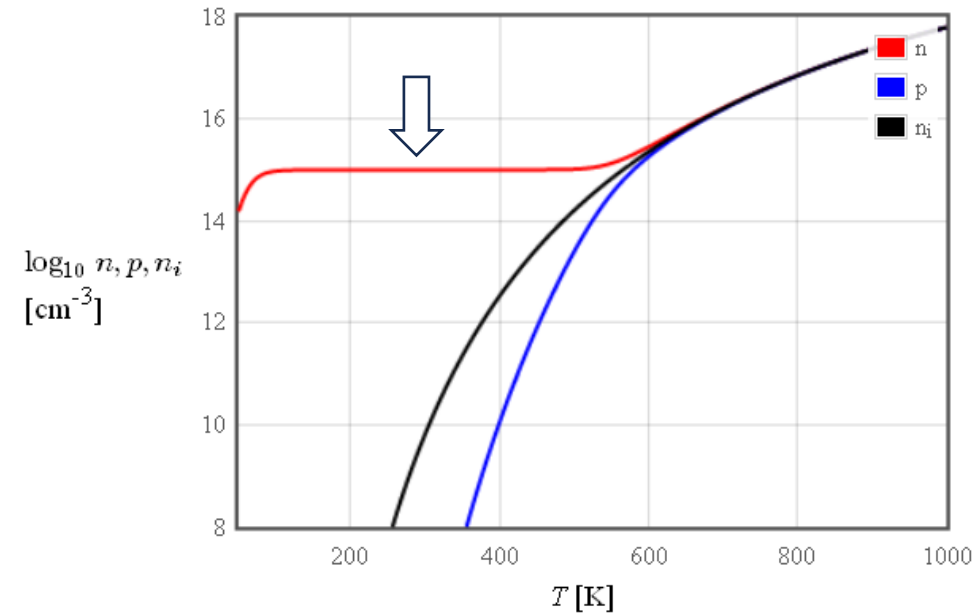
At high temperatures, extrinsic semiconductors have the same temperature dependence as intrinsic semiconductors.

n-type (extrinsic)

$$n = N_D = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)$$

For n-type:
 $n \sim$ density of donors,
 $p = n_i^2 / N_D$

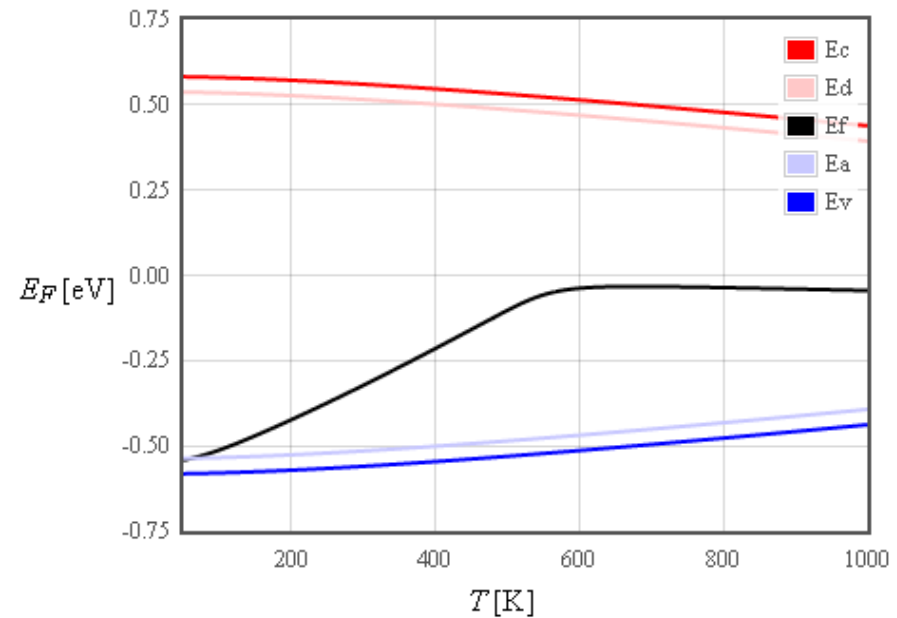
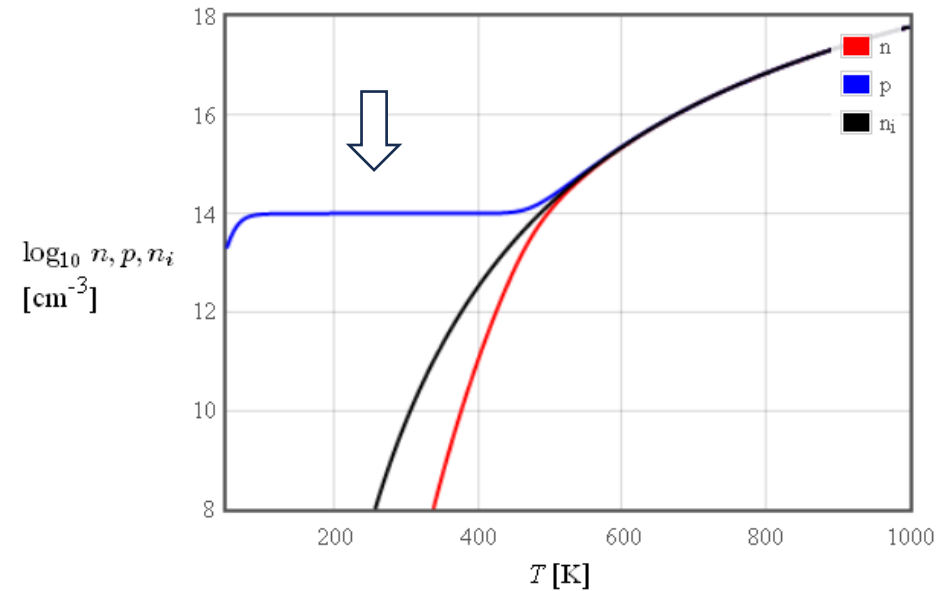


p-type (extrinsic)

$$p = N_A = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)$$

For p-type,
 $p \sim$ density of acceptors,
 $n = n_i^2 / N_A$



Intrinsic / Extrinsic

Intrinsic: $n = p$

Conductivity **strongly temperature dependent**

Conductivity is poor

Extrinsic: $n \neq p$

Conductivity **almost temperature independent** at room temperature

Conductivity can be strongly increased (up to a factor of 10^6)

include dopants

Czochralski process

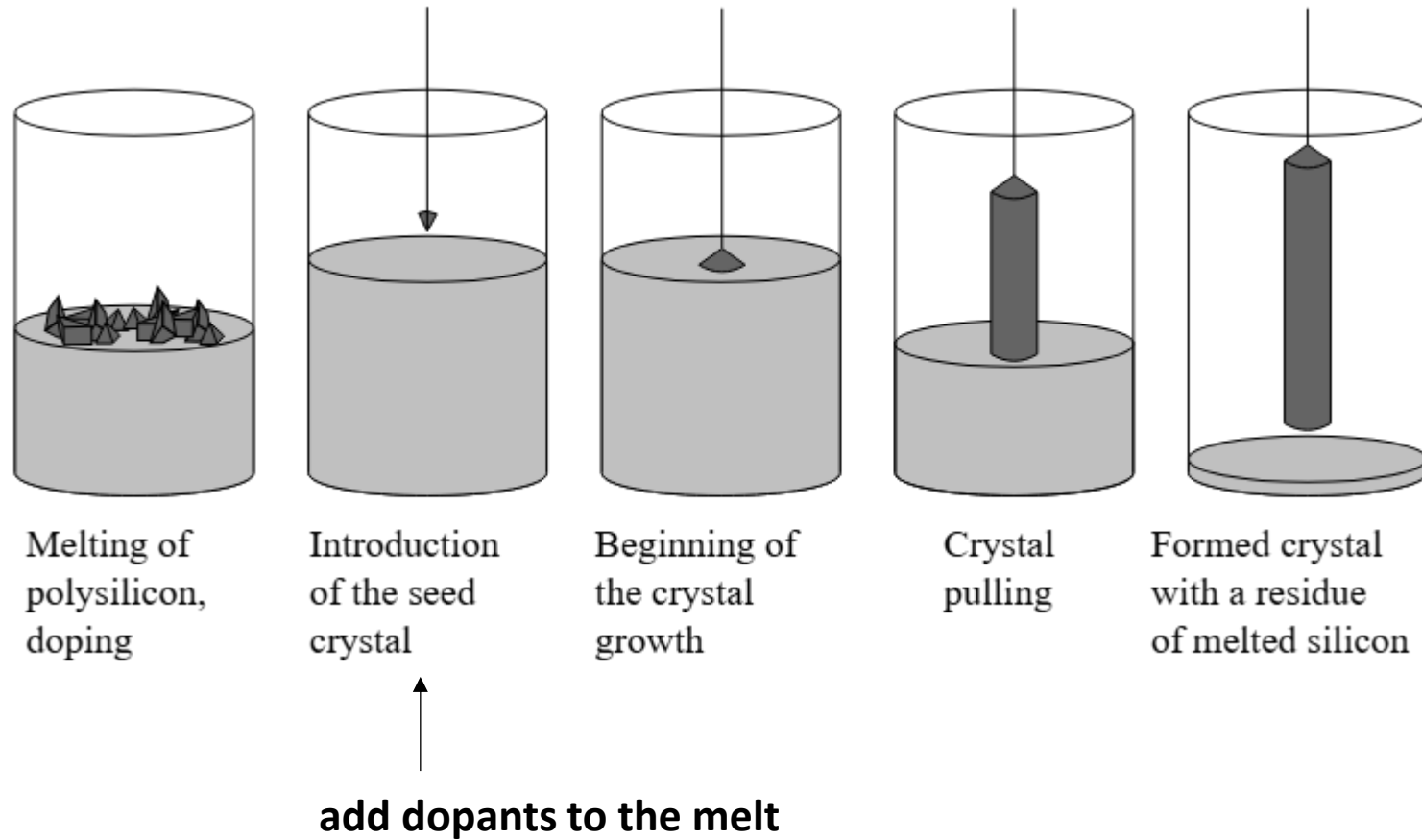


image from wikipedia

include dopants

float zone process

neutron transmutation

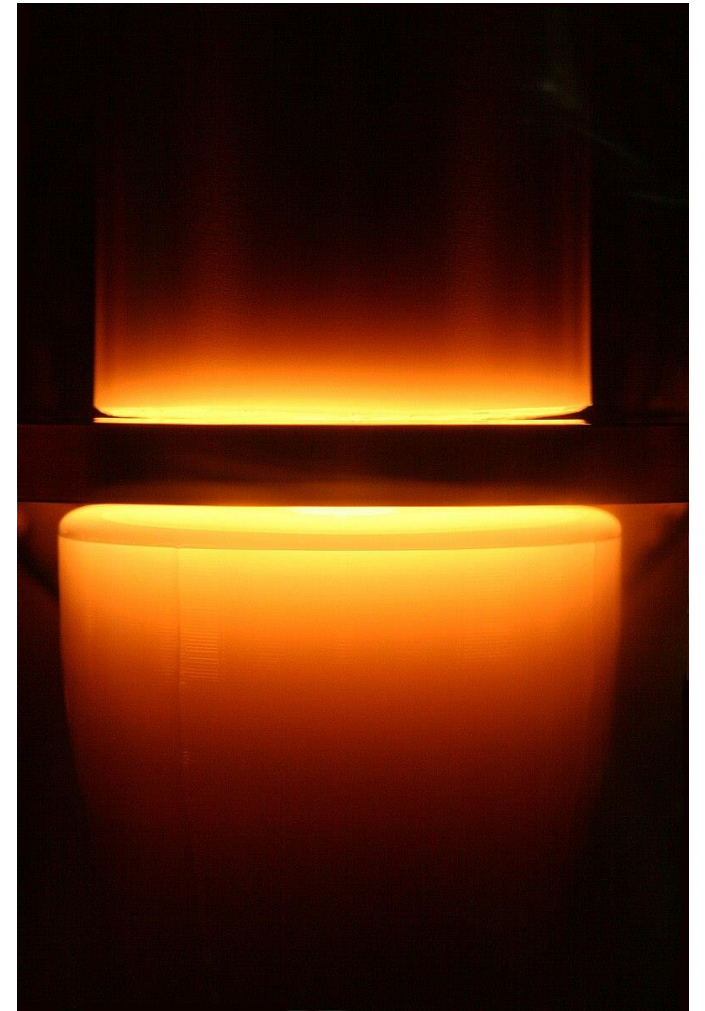
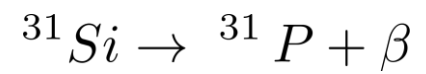
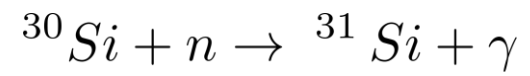


image from wikipedia

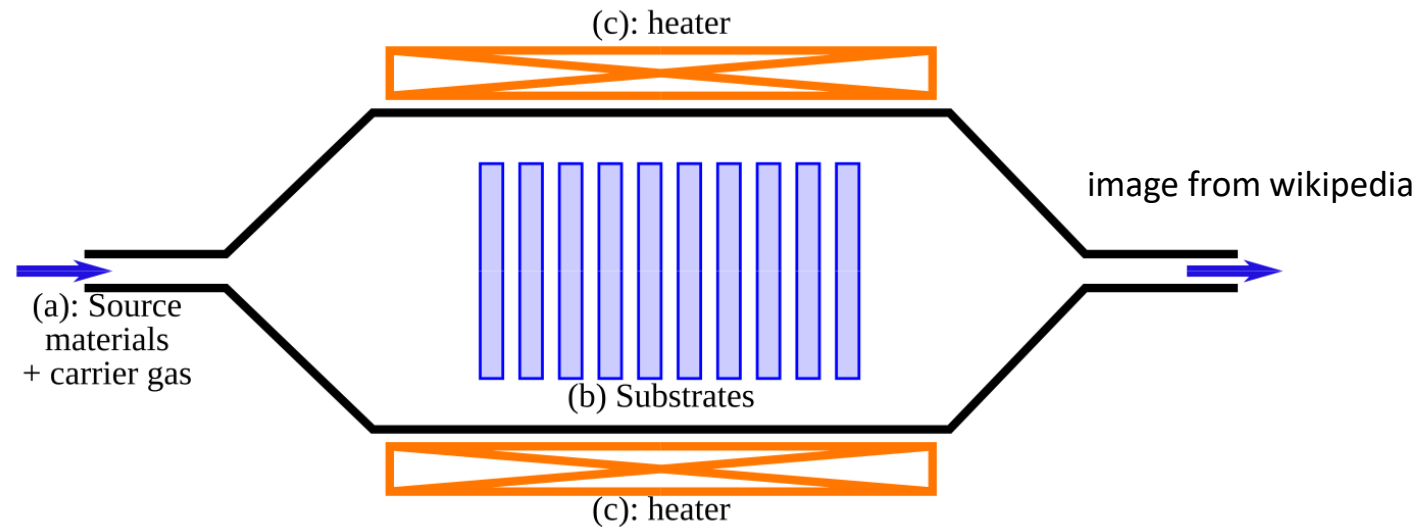
include dopants

gas phase diffusion

AsH_3 (arsine) or PH_3 (phosphine) for n-doping

B_2H_6 (diborane) for p-doping

chemical vapor deposition



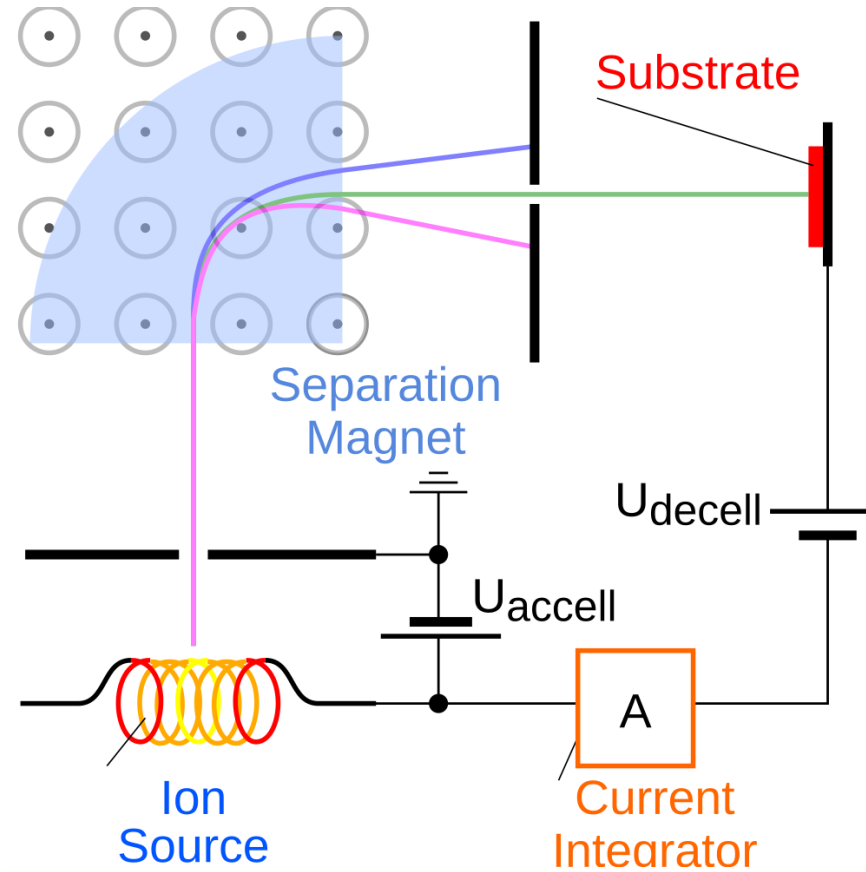
epitaxial silicon CVD SiH_4 (silane) or SiH_2Cl_2 (dichlorosilane)
 PH_3 (phosphine) for n-doping or B_2H_6 (diborane) for p-doping

include dopants

ion implantation



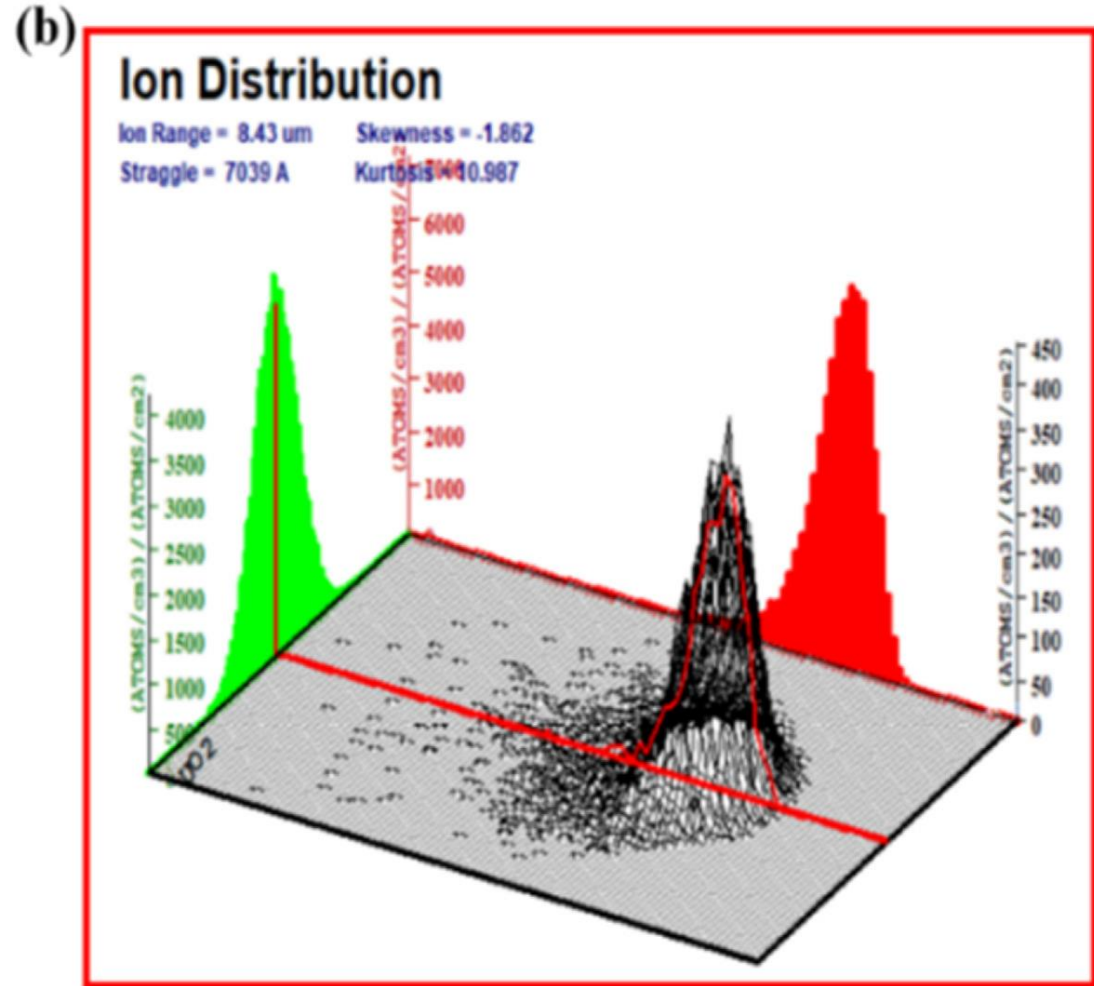
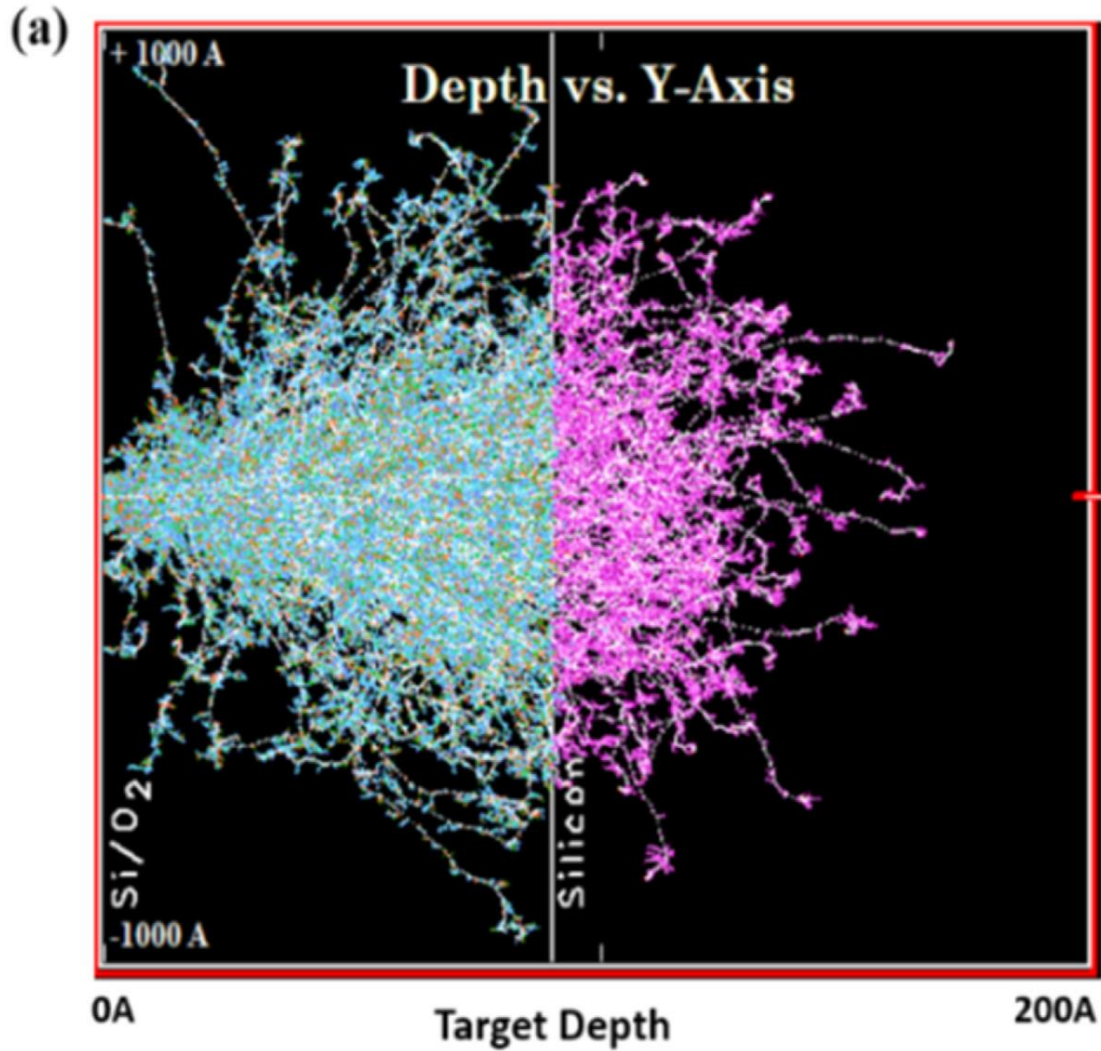
images from wikipedia



implant at 7 degrees to avoid channeling

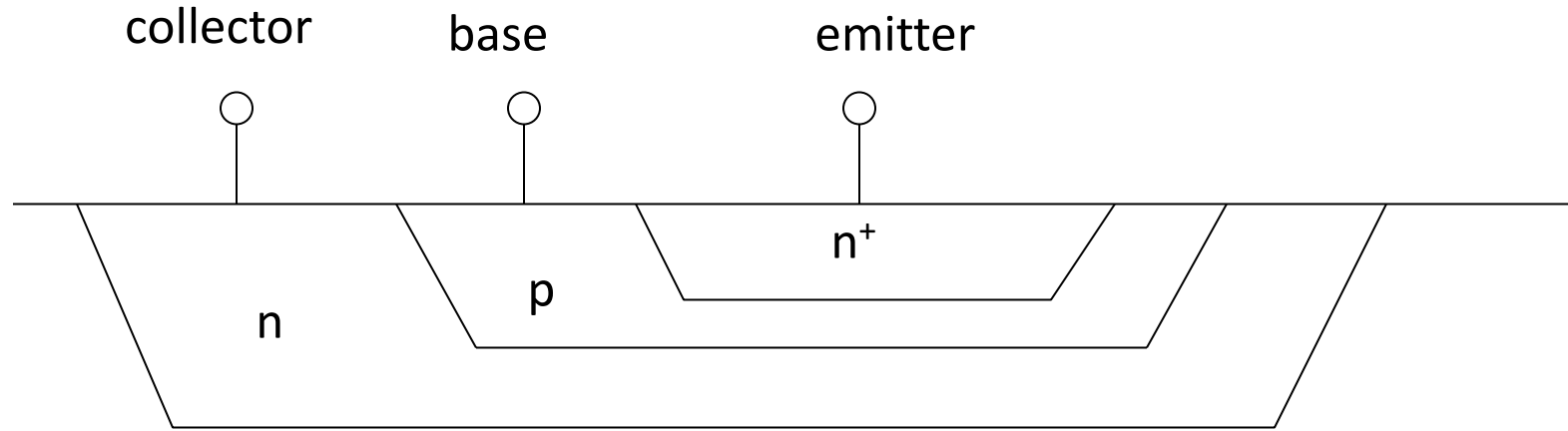
Simulate dopant distribution

the stopping and range of ions in matter



Why dope with donor and acceptors ?

Bipolar transistor



lightly doped p substrate

body

source

drain

polysilicon gate

gate

n⁺

p⁺

n⁺

p

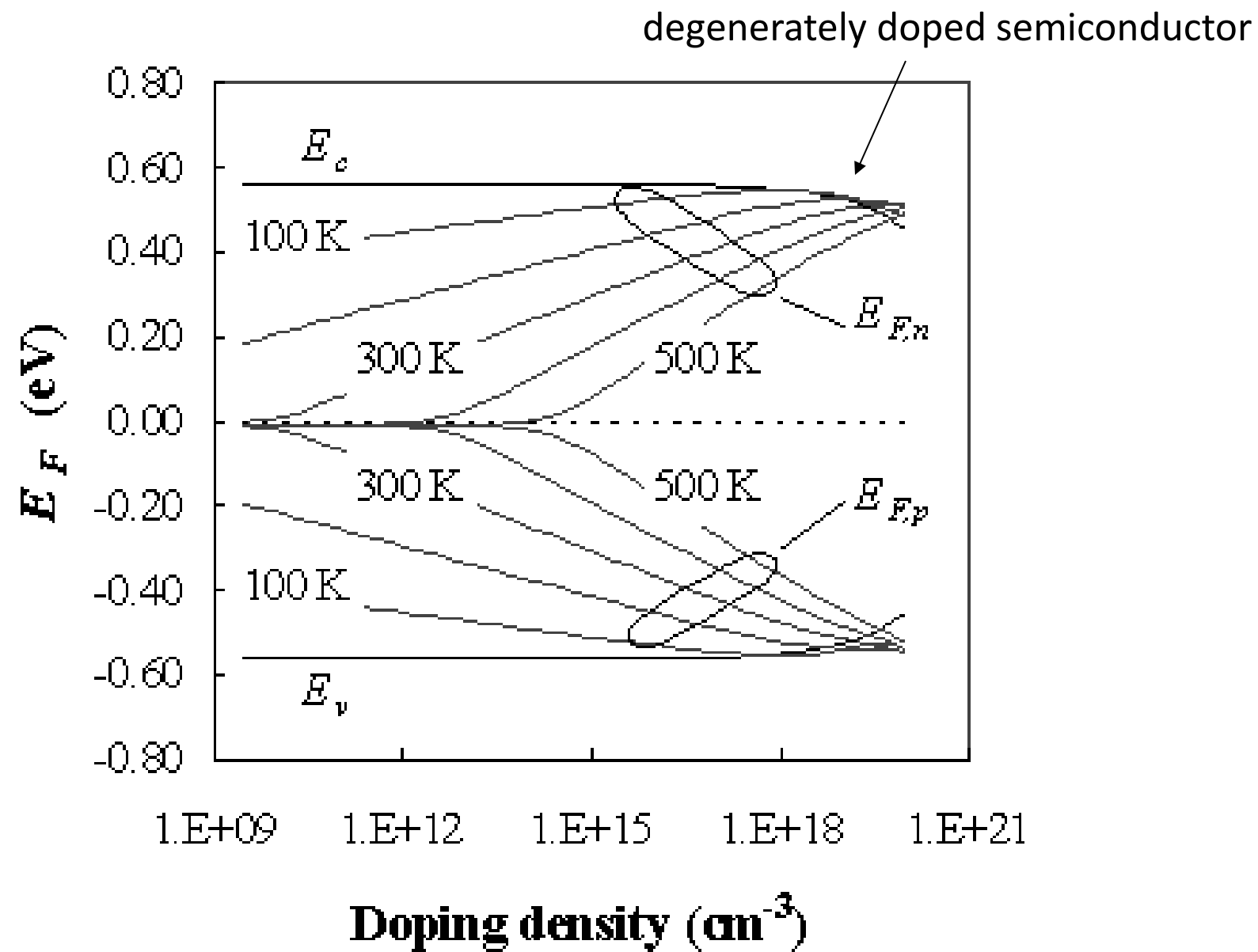
n

inversion layer channel

gate oxide

A 3D schematic diagram of a semiconductor device structure. The structure consists of several layers and regions. The top layer is a brown, stepped structure. Below it is a white layer with a dashed line indicating a boundary. The main body of the device is a white layer with a dashed line indicating a boundary. The bottom layer is a white layer with a dashed line indicating a boundary. The regions are labeled with 'n', 'p', 'n+', 'n++', and 'p-'. The 'n' regions are the top and bottom layers. The 'p' region is the middle layer. The 'n+' and 'n++' regions are the regions immediately above and below the 'p' region. The 'p-' region is the bottom-most layer.

Oxide isolated integrated BJT - a modern process



degenerate semiconductor

Heavily doped semiconductors are called **degenerately doped**

$N_D > 0.1 N_c \rightarrow E_F$ in the conduction band

$N_A > 0.1 N_v \rightarrow E_F$ in the valence band

- ☐ heavy doping narrows the band gap
- ☐ Boltzmann approximation is not valid
- ☐ degenerate semiconductors = metal

6

Carrier transport

Ballistic transport

Drift

Diffusion

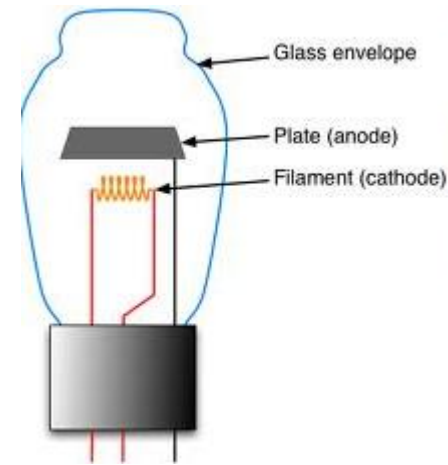
Tunneling

Ballistic transport

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0t + \vec{x}_0$$



Electrons moving in an electric field follow parabolic trajectories like a ball in a gravitational field.

Drift

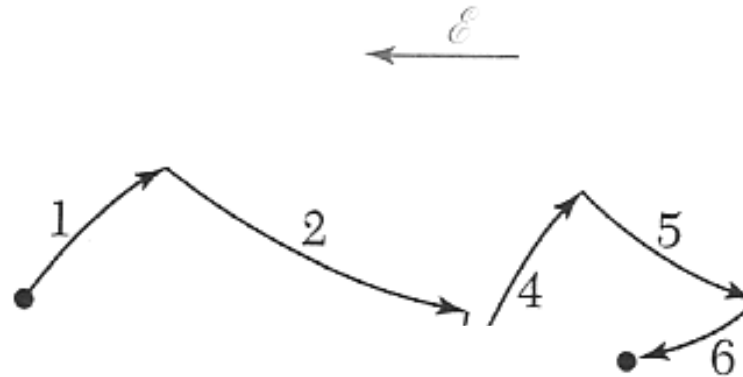
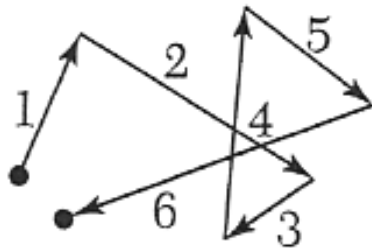
The electrons scatter and change direction after a time τ_{sc} .

thermal velocity $\frac{1}{2} m \mathbf{v}_{th}^2 = \frac{3}{2} k_B T$ (classical equipartition)

at 300 K, $v_{th} \sim 10^7$ cm/s

mean free path: $\ell = v_{th} \tau_{sc} \sim 10$ nm ~ 200 atoms

$\mathcal{E} = 0$



In a metal, the fermi velocity dominates over the thermal velocity. In a semiconductor, the thermal velocity dominates

Drift (diffusive transport)

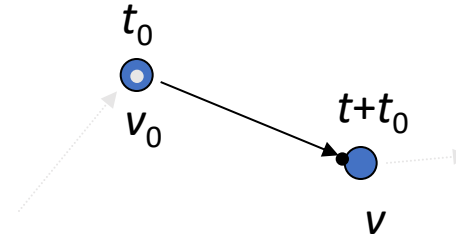
$$\vec{F} = -e\vec{E} = m^*\vec{a} = m^*\frac{d\vec{v}}{dt}$$

$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*}(t - t_0)$$

$$\langle v_0 \rangle = 0$$

$$\langle t - t_0 \rangle = \tau_{sc}$$

time between two collisions



$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^*v}$$

drift velocity: $\vec{v}_{d,n} = -\mu_n\vec{E}$

$$\vec{v}_{d,p} = \mu_p\vec{E}$$

Drift

drift velocity:

$$\vec{v}_{d,n} = -\mu_n \vec{E}$$

$$\vec{v}_{d,p} = \mu_p \vec{E}$$

for Si: $\mu_n = 1350 \text{ cm}^2/\text{Vs}$
 $\mu_p = 450 \text{ cm}^2/\text{Vs}$

current density:

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p}$$

$$= (ne\mu_n + pe\mu_p)\vec{E}$$

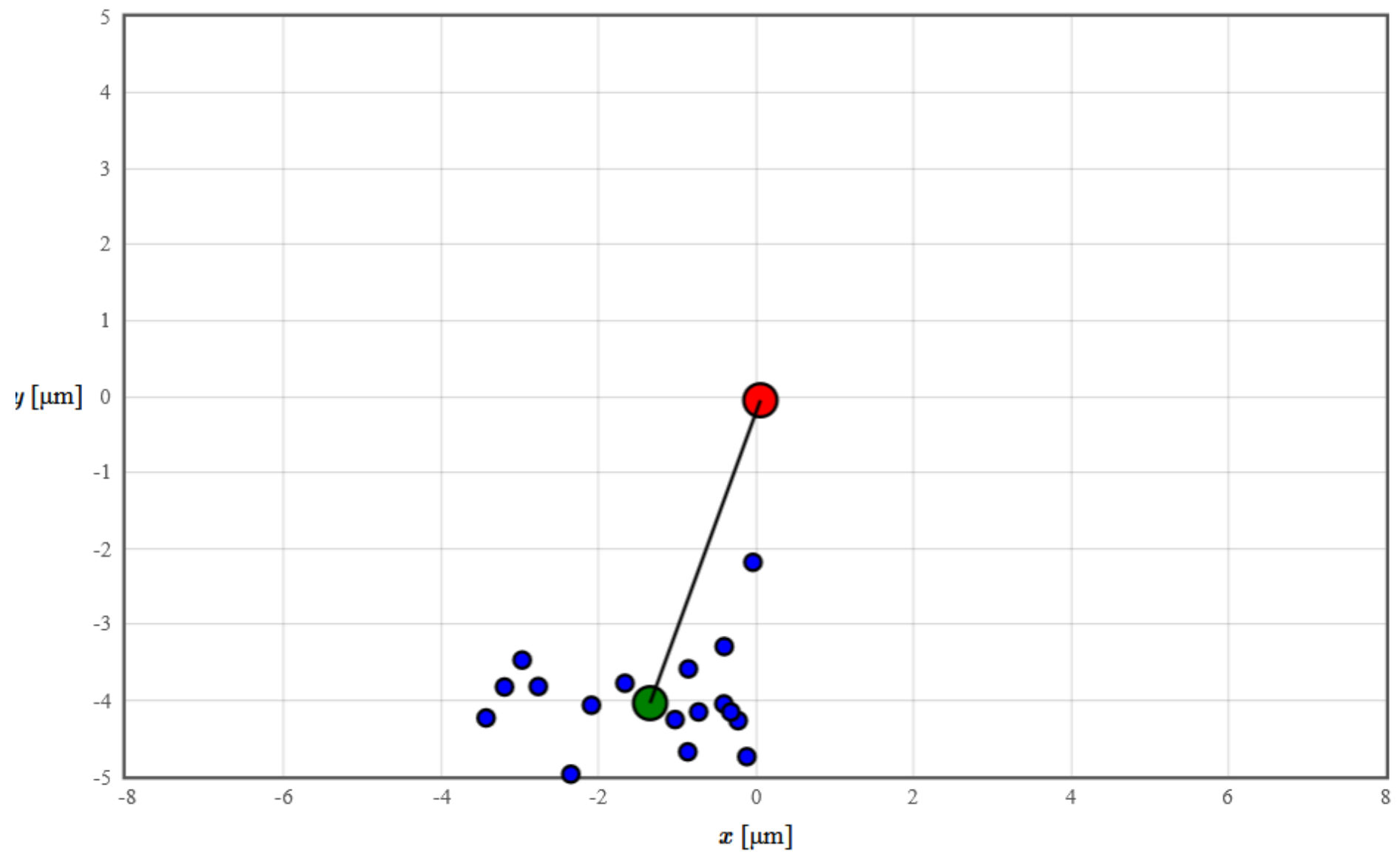
$$= \sigma \vec{E}$$

(Ohm's law)

For $E = 1000 \text{ V/cm}$:

$$v_d = 10^6 \text{ cm/s}$$

$$\mu = \frac{-e\tau_{sc}}{m^*} = \frac{-e\ell}{m^*v}$$



<https://lampx.tugraz.at/~hadley/psd/L5/drude.php>

Drift

Solid state electronic devices, Streetman and Banerjee

		E_g (eV)	μ_n (cm ² /V-s)	μ_p (cm ² /V-s)	m_n^*/m_0 (m_l, m_t)	m_p^*/m_0 (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm ³)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10 ⁵	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

$$\vec{v}_{d,n} = -\mu_n \vec{E}$$

$$\vec{v}_{d,p} = \mu_p \vec{E}$$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = (ne\mu_n + pe\mu_p)\vec{E} = \sigma\vec{E}$$

Matthiessen's rule

rate

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

phonons, temperature dependent

mostly temperature independent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$
$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

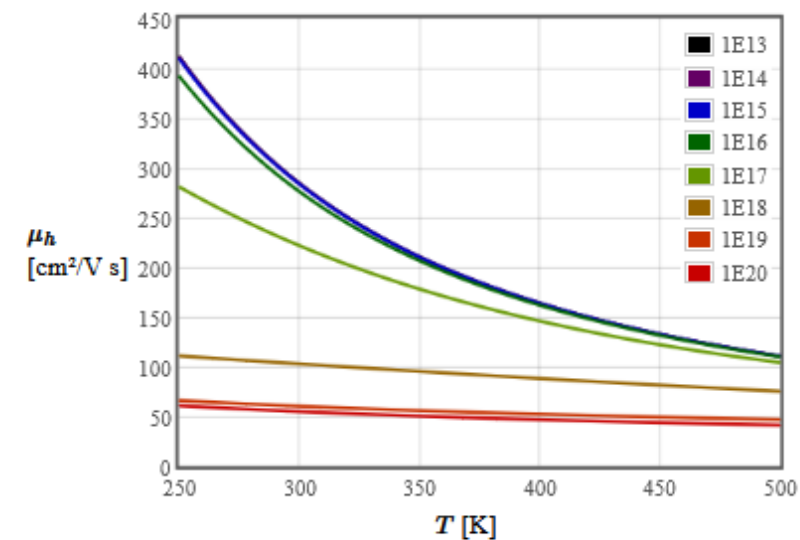
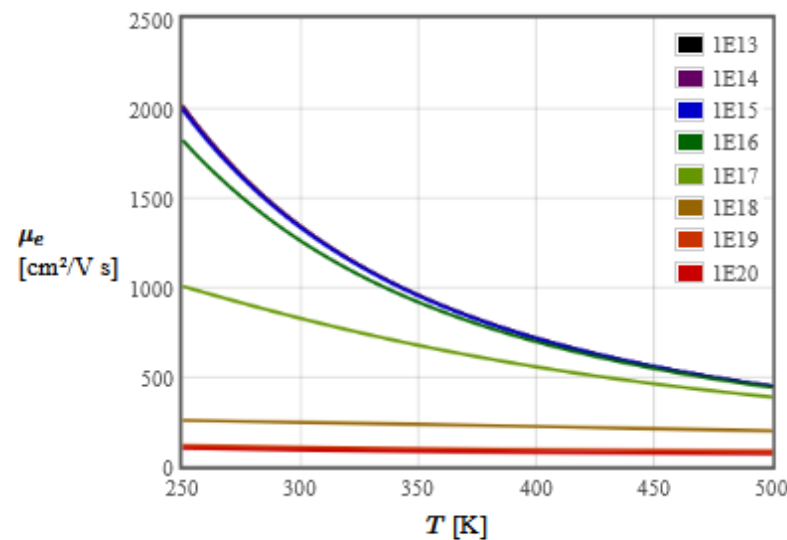
doping increases the conductivity by increasing the carrier density but decreases the mobility

mobility model for Si

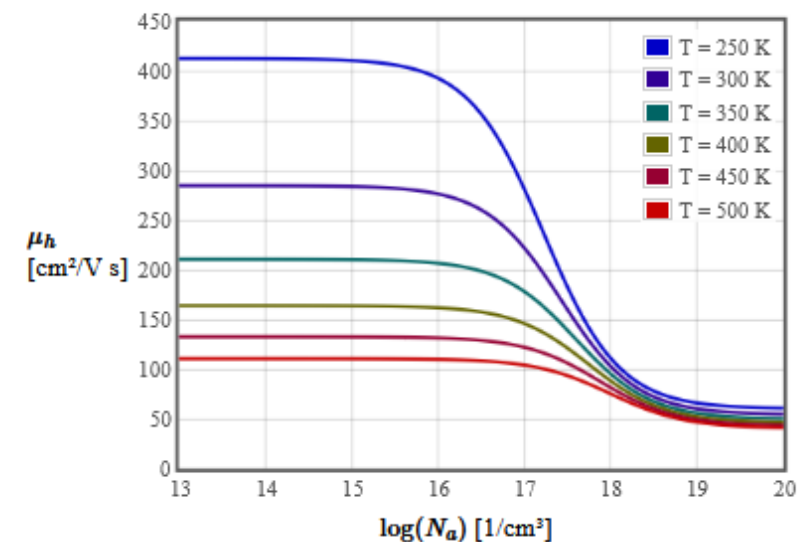
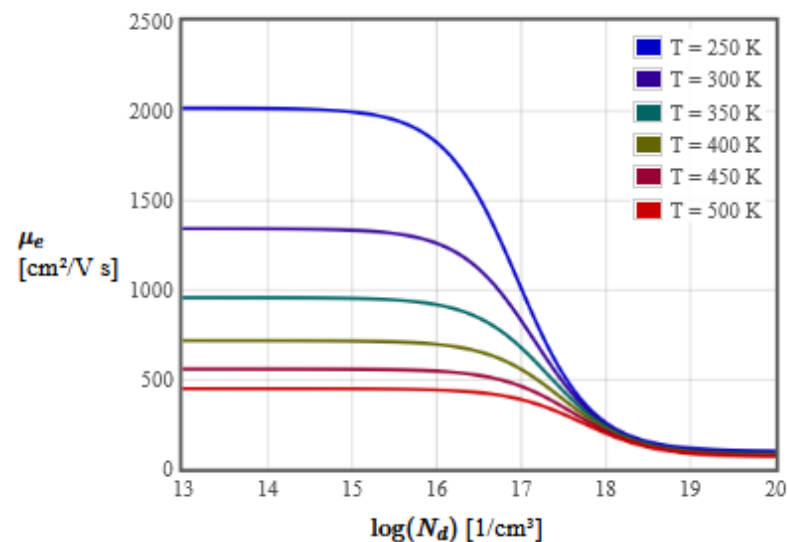
$$\mu_e = 88 \left(\frac{T}{300} \right)^{-0.57} + \frac{7.4 \times 10^8 T^{-2.33}}{1 + 0.88 \left[\frac{N_d}{1.26 \times 10^{17} \left(\frac{T}{300} \right)^{2.4}} \right]} \left(\frac{T}{300} \right)^{-0.146} \text{ cm}^2/\text{V s},$$

$$\mu_h = 54.3 \left(\frac{T}{300} \right)^{-0.57} + \frac{1.36 \times 10^8 T^{-2.33}}{1 + 0.88 \left[\frac{N_a}{2.35 \times 10^{17} \left(\frac{T}{300} \right)^{2.4}} \right]} \left(\frac{T}{300} \right)^{-0.146} \text{ cm}^2/\text{V s}.$$

temperature

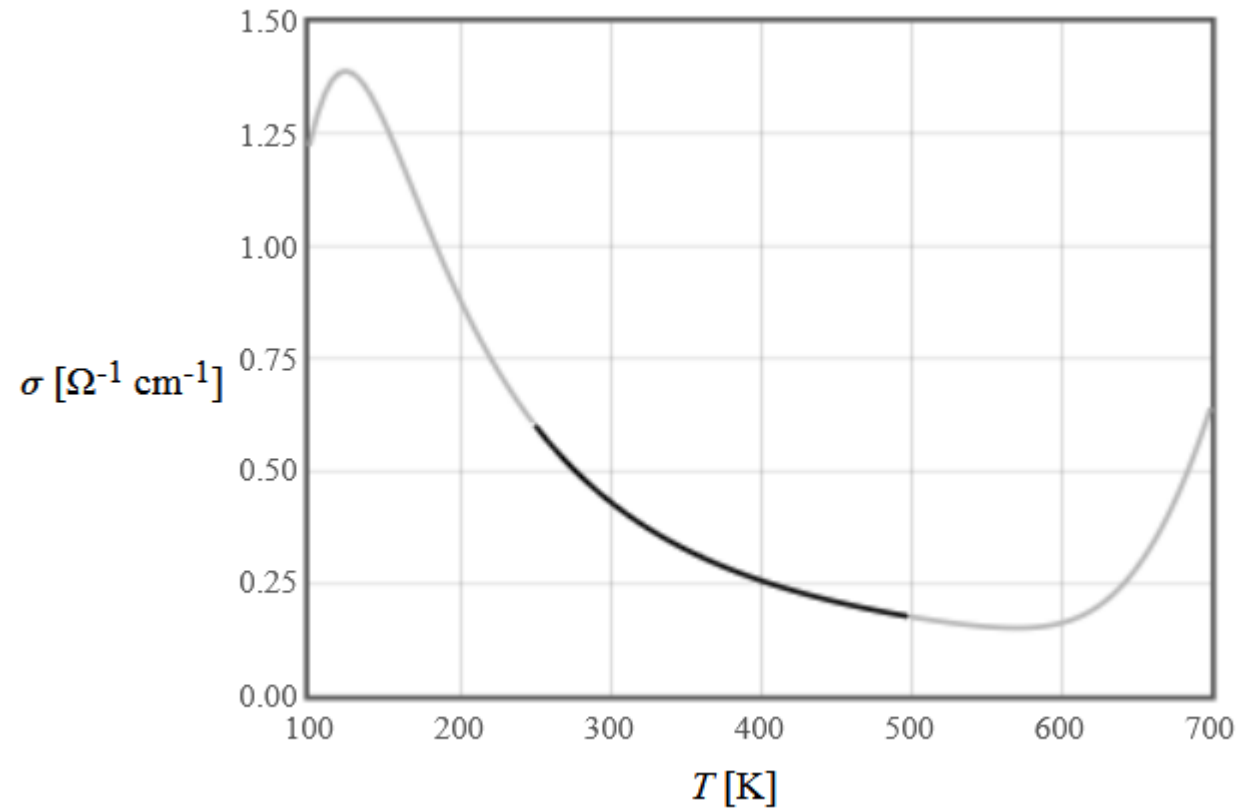


doping density



conductivity in Si

$$\sigma = ne\mu_n + pe\mu_p$$



$$N_A = 10^{16} \text{ 1/cm}^3$$

Ballistic transport in transistors

The mean free path ~ 100 nm $>$ gate length ~ 20 nm

enables motion without significant collisions and scattering

v not proportional to E

~~$$\vec{v} = \mu \vec{E}$$~~

j not proportional to E

~~$$\vec{j} = \sigma \vec{E}$$~~

nonlocal response

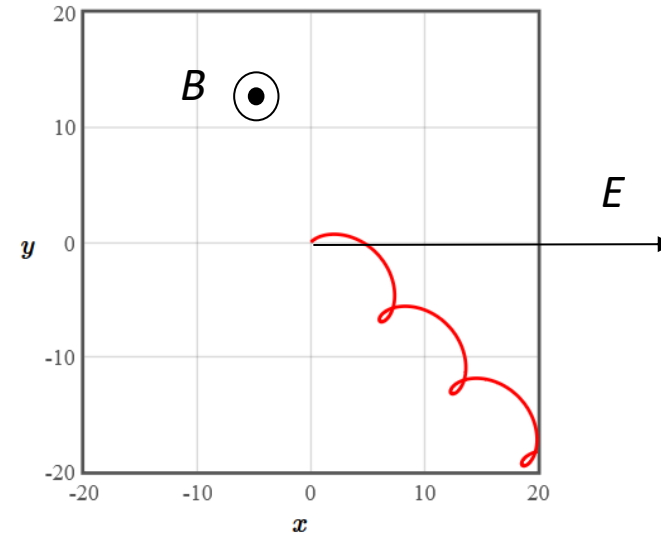
Electrons bend in a magnetic field like they do in vacuum.

The magnetic field slides could be skipped. They are not used later in the semester.

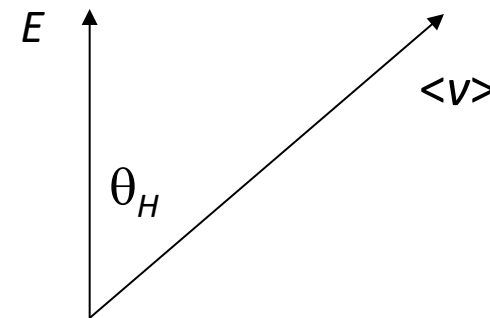
Crossed E and B fields

Ballistic transport

$$\vec{F} = m\vec{a} = -e(\vec{E} + \vec{v} \times \vec{B})$$



Diffusive transport



Hall angle:

$$\theta_H = \tan^{-1} \left(-\frac{eB_z \tau_{sc}}{m^*} \right)$$

Magnetic field (diffusive transport)

$$\vec{F} = m\vec{a} = -e\vec{E} = e\frac{\vec{v}_d}{\mu}$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right) = e\frac{\vec{v}_d}{\mu}$$

If B is in the z -direction, the three components of the force are

$$-\mu\left(E_x + v_{dy}B_z\right) = v_{dx}$$

$$-\mu\left(E_y - v_{dx}B_z\right) = v_{dy}$$

$$-\mu E_z = v_{dz}$$

Magnetic field

$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

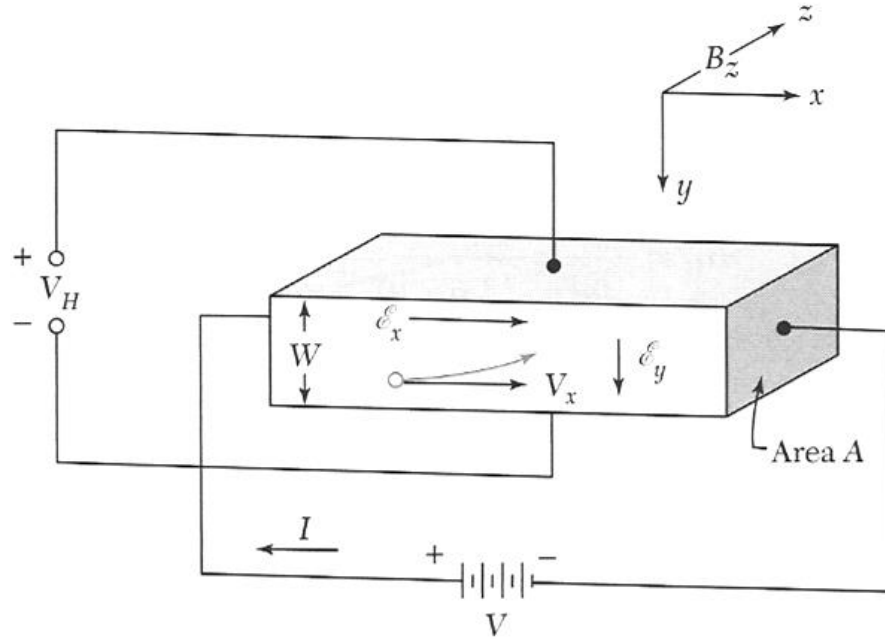
$$v_{d,z} = -\mu E_z$$

If $E_y = 0$,

$$v_{d,y} = -\mu B_z v_{d,x}$$

$$\tan \theta_H = -\mu B_z$$

The Hall Effect (diffusive regime)



$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

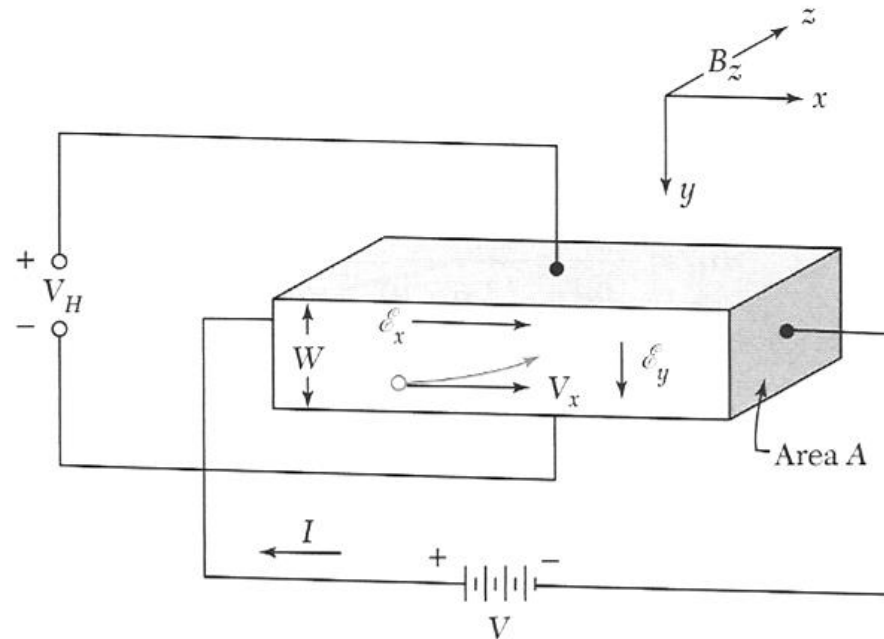
$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

$$v_{d,z} = -\mu E_z$$

If $v_{d,y} = 0$,

$$E_y = v_x B_z = V_H / W = R_H j_x B_z \quad V_H = \text{Hall voltage}, R_H = \text{Hall Constant}$$

The Hall Effect (diffusive regime)



$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$E_y = v_x B_z = V_H / W = R_H j_x B_z$$

V_H = Hall voltage, R_H = Hall Constant

$$R_H = v_x / j_x$$

$$v_x = -j_x / ne \quad \text{for n-type}$$

$$v_x = j_x / pe \quad \text{for p-type}$$

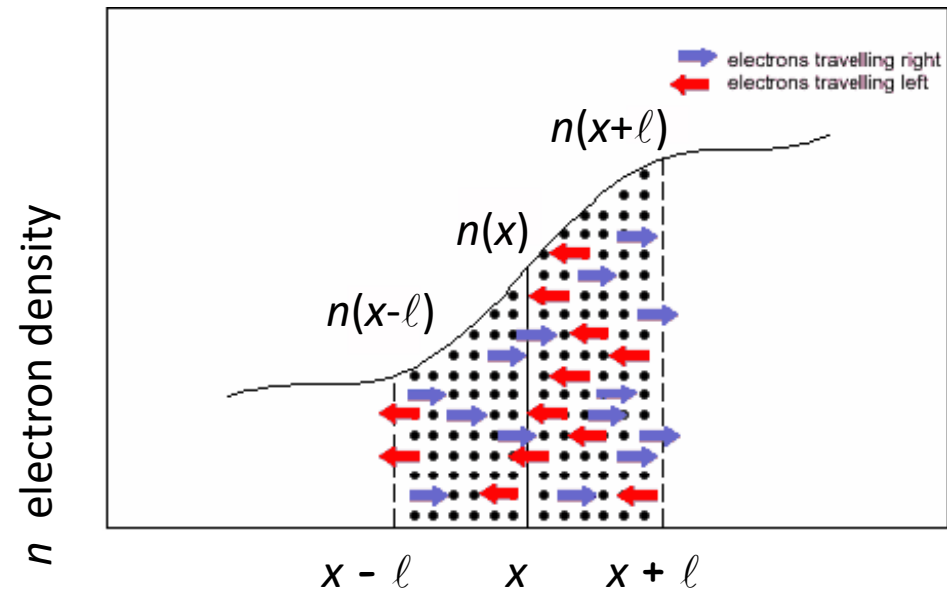
$$R_H = -1/ne \quad \text{for n-type}$$

$$R_H = 1/pe \quad \text{for p-type}$$

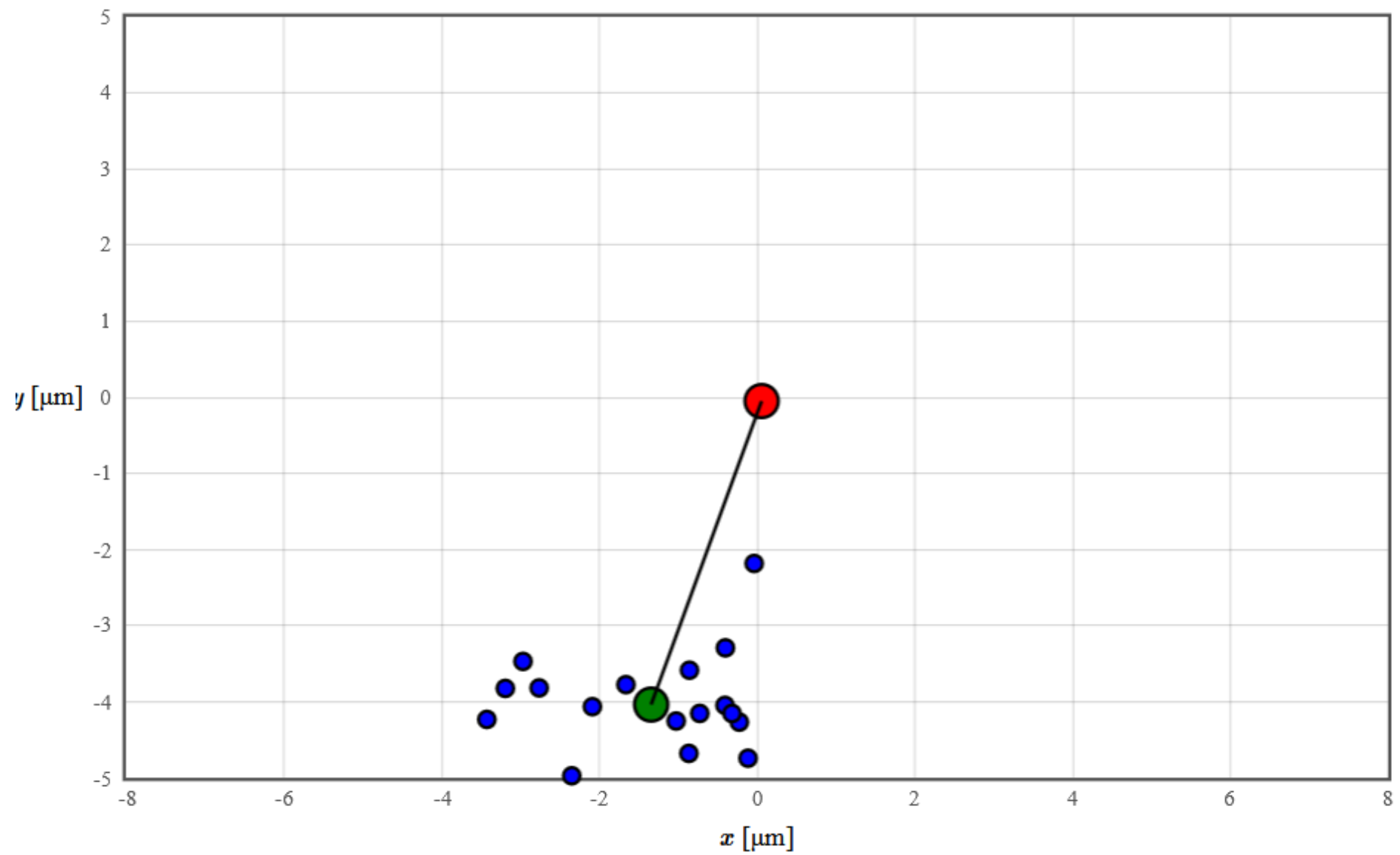
Diffusion

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -|e| D_p \frac{dp}{dx}$$

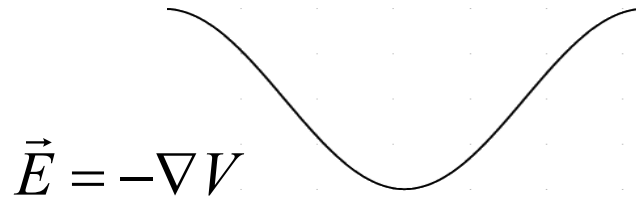


Diffusion is from high concentration to low concentration.



<http://lampx.tugraz.at/~hadley/psd/L5/drude.php>

Einstein relation



$$\vec{E} = -\nabla V$$

$$n = A \exp\left(\frac{-eV}{k_B T}\right)$$

Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{e}{k_B T} A \exp\left(\frac{-eV_{pot}}{k_B T}\right) \nabla V = -\frac{ne}{k_B T} \nabla V = \frac{ne\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + eD \frac{ne\vec{E}}{k_B T} = 0$$

$$D = \frac{\mu k_B T}{e}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen

Current Density Equations

Drift
↓

Diffusion
↙

$$\vec{j}_n = -ne\mu_n\vec{E} + eD_n\nabla n$$
$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

In Equilibrium

$$\vec{j}_n = en\mu_n\vec{E} + eD_n\nabla n$$

The electric field is proportional to the gradient of the conduction band edge

$$e\vec{E} = \nabla E_c$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

If the Fermi energy is constant,

$$\nabla n = -\frac{\nabla E_c}{k_B T} N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = -\frac{\nabla E_c}{k_B T} n$$

$$\vec{j}_n = n\nabla E_c \left(\mu_n - \frac{eD_n}{k_B T} \right)$$

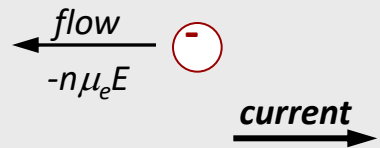
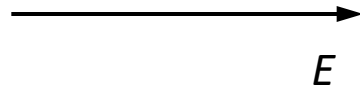
This means that the current density in a semiconductor where the Fermi energy is constant is zero.

Current Density Equations

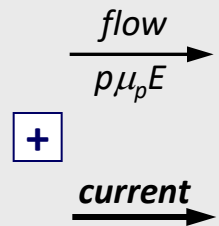
note: electron and hole currents have same direction

electric current = charge $\times$ particle flow

drift

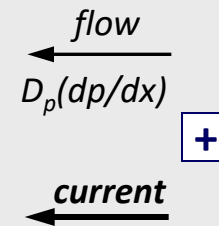
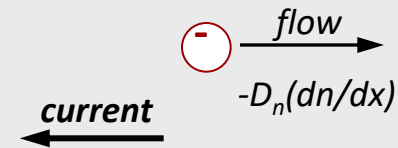
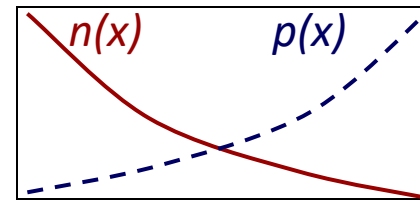


$$j_e = -e \times \text{flow}$$

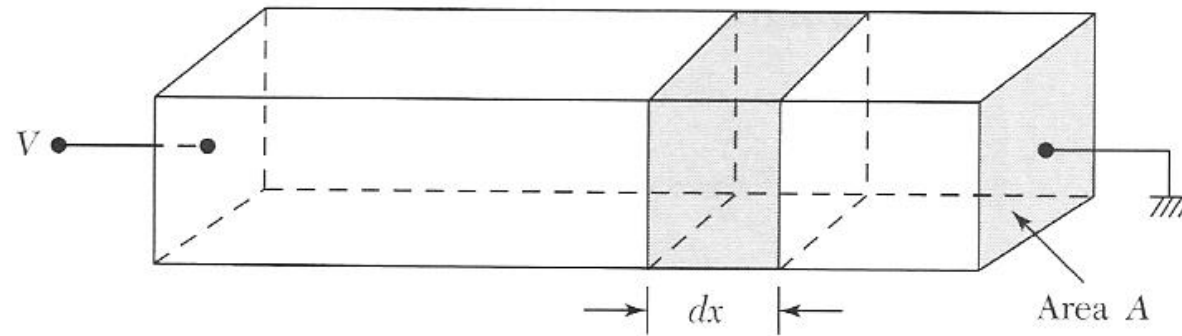


$$j_p = e \times \text{flow}$$

diffusion



Continuity equations

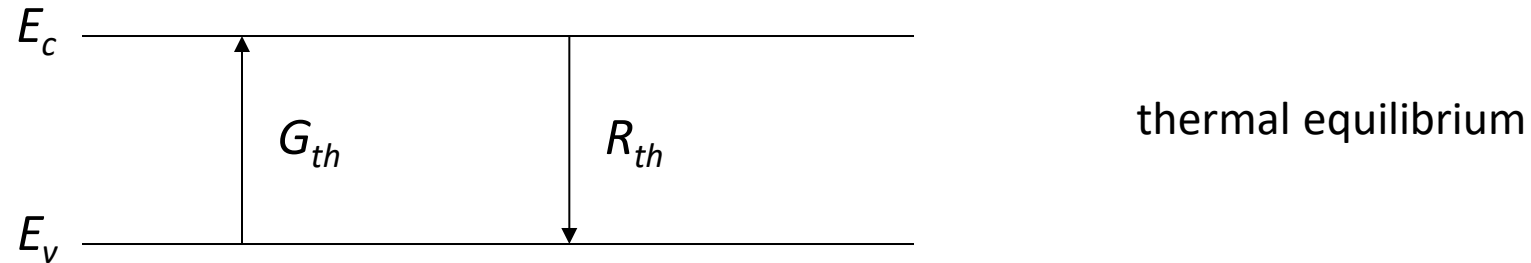


$$\frac{\partial n}{\partial t} = \frac{1}{e} \nabla \cdot \vec{j}_n + G_n - R_n$$

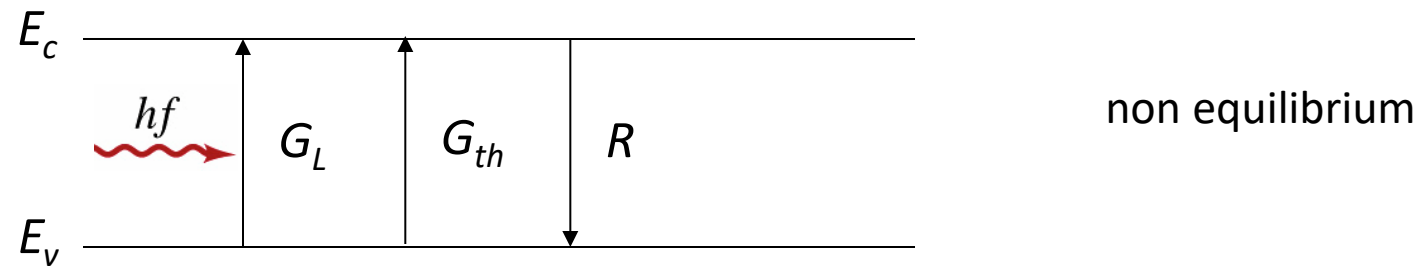
$$\frac{\partial p}{\partial t} = -\frac{1}{e} \nabla \cdot \vec{j}_p + G_p - R_p$$

j_n and j_p consist of drift and diffusion terms

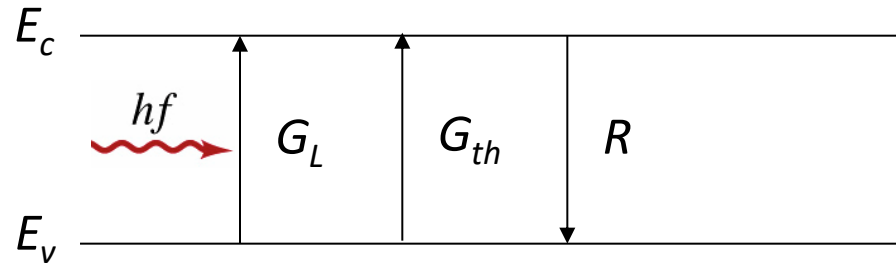
Generation and Recombination



Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution $np \neq n_i^2$



Recombination



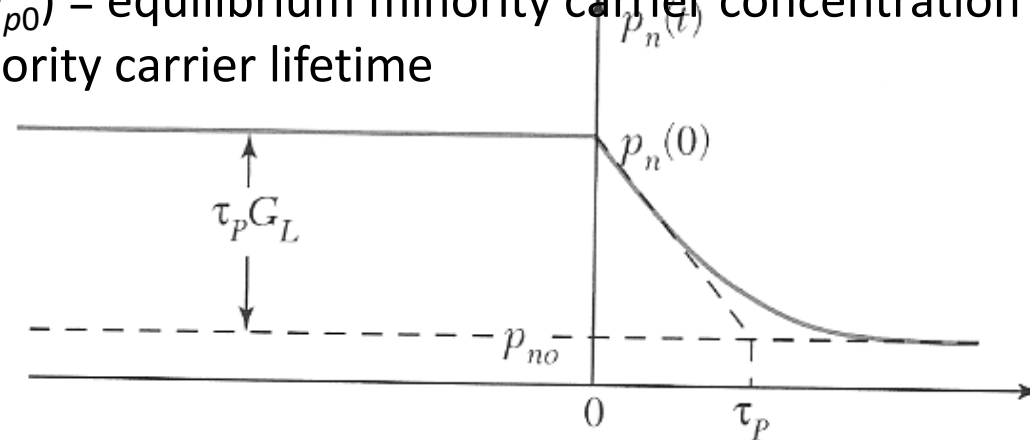
$$R - R_{th} = \frac{p_n - p_{n0}}{\tau_p}$$

Recombination rate is limit by the density of minority carriers.
The majority carriers have to find a minority carrier to recombine.

p_n (or n_p) = minority carrier concentration

p_{n0} (or n_{p0}) = equilibrium minority carrier concentration

τ_p = minority carrier lifetime



minority carrier lifetimes

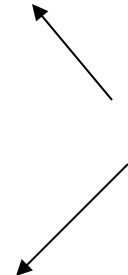
p-type

$$n_p(t) = n_{excess} \exp(-t / \tau_n) + n_{p0}$$

n-type

$$p_n(t) = p_{excess} \exp(-t / \tau_p) + p_{n0}$$

minority carrier lifetimes



$$np = n_i^2$$

Continuity equations $\frac{\partial n}{\partial t} = \frac{1}{e} \nabla \cdot \vec{j}_n + G_n - R_n$

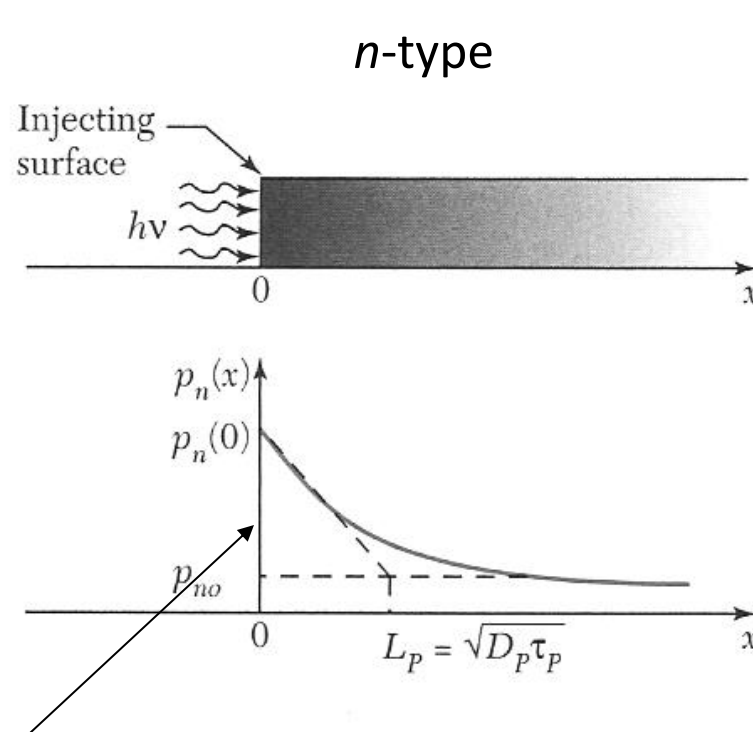
drift: $\vec{j}_n = -ne\mu_n \vec{E}$ $\nabla \cdot \vec{j}_n = -en\mu_n \nabla \cdot \vec{E} - e\nabla n \mu_n \vec{E}$

diffusion: $\vec{j}_{n,diff} = |e| D_n \nabla n$ $\nabla \cdot \vec{j}_{n,diff} = |e| D_n \nabla^2 n$

$$\frac{\partial n}{\partial t} = n\mu_n \nabla \cdot \vec{E} + \nabla n \mu_n \vec{E} + D_n \nabla^2 n + G_n - \frac{n - n_0}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p\mu_p \nabla \cdot \vec{E} - \nabla p \mu_p \vec{E} + D_p \nabla^2 p + G_p - \frac{p - p_0}{\tau_p}$$

Diffusion Length



Steady state

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

Generation only occurs at the surface. There the minority carrier density is $p_n(0)$.

Diffusion Length

$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \quad \Leftrightarrow \quad p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

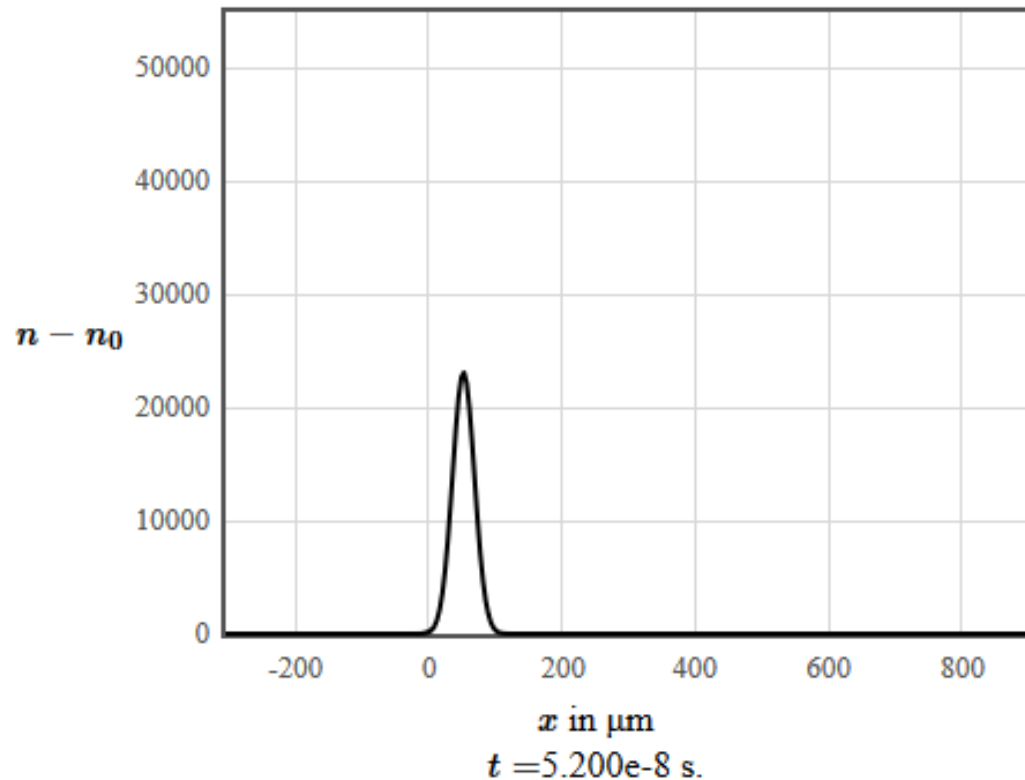
$$0 = \frac{D_p (p_n(0) - p_{n0})}{L_p^2} \exp\left(\frac{-x}{L_p}\right) - \frac{(p_n(0) - p_{n0})}{\tau_p} \exp\left(\frac{-x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

diffusion length,
typically microns

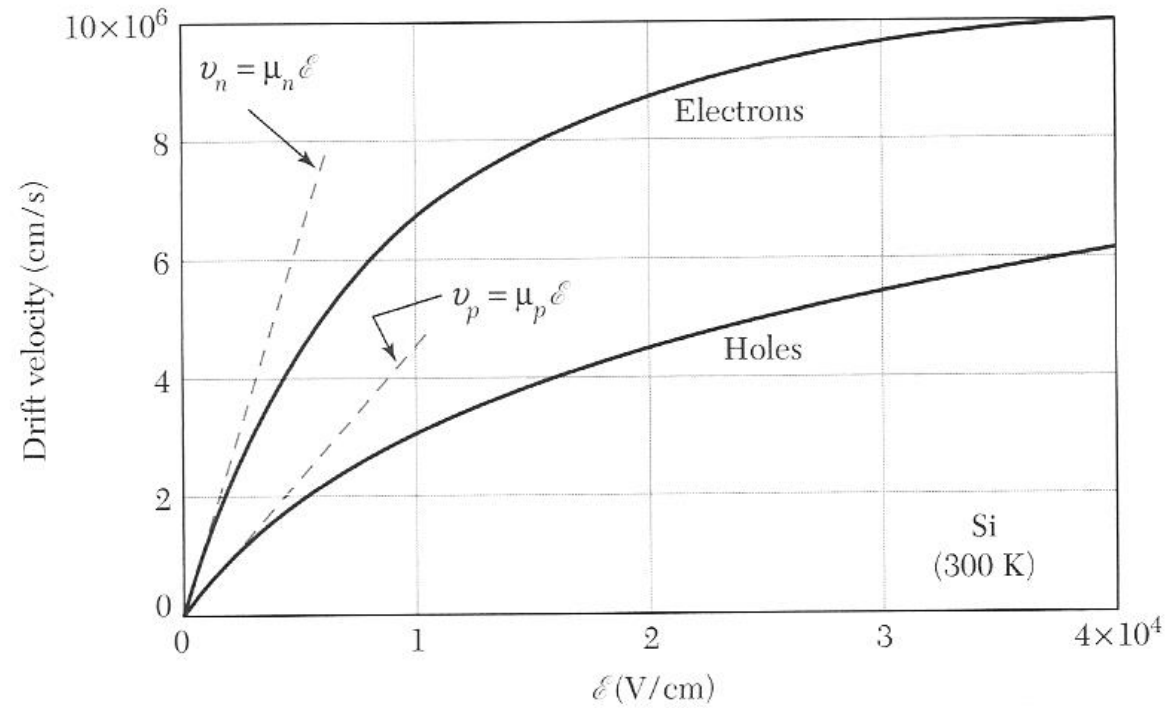
Haynes Shockley experiment

$$n_p(x,t) = \frac{n_{\text{generated}}}{\sqrt{4\pi D_n t}} \exp\left(-\frac{(x - \mu_n E t)^2}{4D_n t}\right) \exp\left(-\frac{t}{\tau_n}\right) + n_{p0}$$



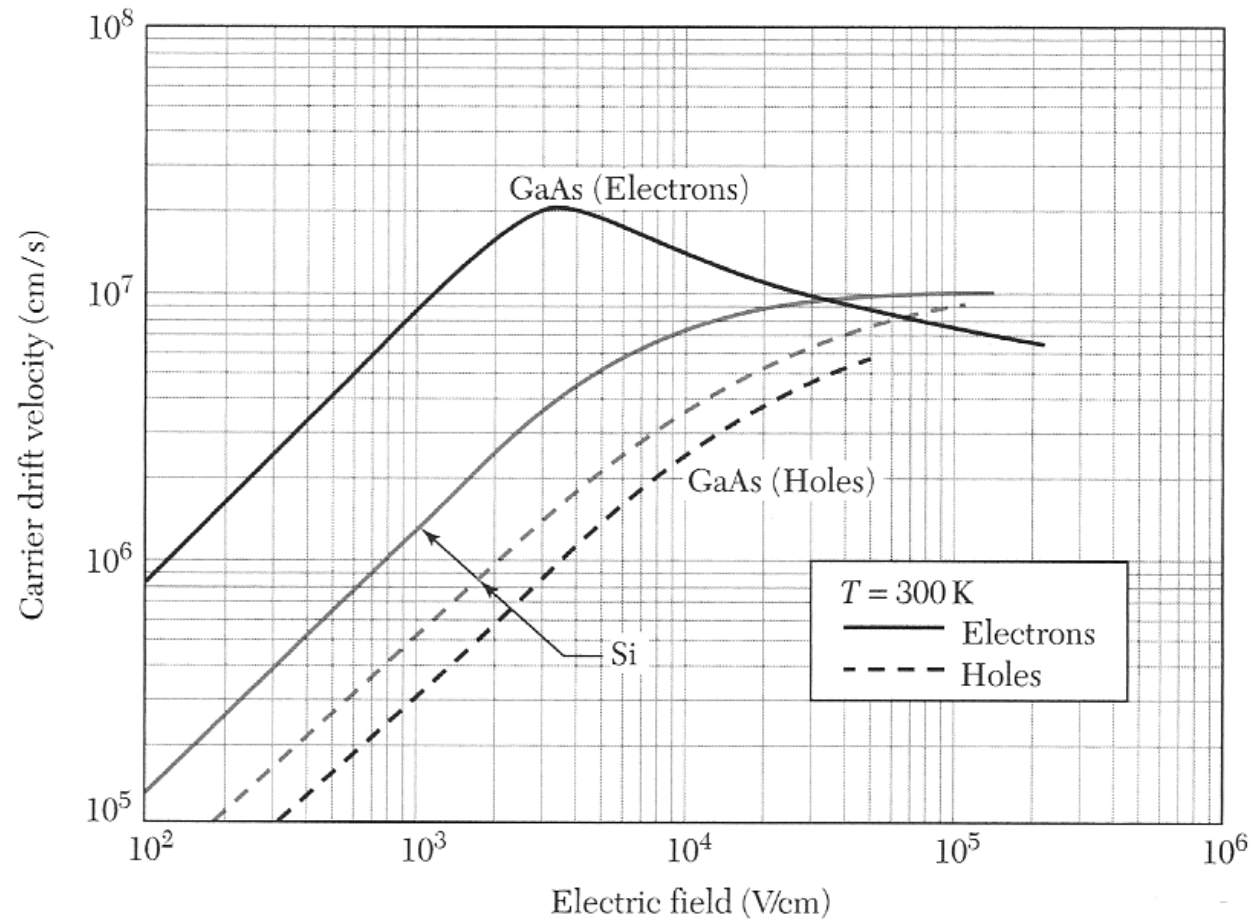
$\tau = 1\text{E-}6$ [s]
 $E = 100$ [V/cm]
 $\mu = 1000$ [$\text{cm}^2/\text{V s}$]
 $D = \mu k_B T / e = 0.00258$ [m^2/s]
 $L = \sqrt{D\tau} = 50.8$ [μm]

High Fields

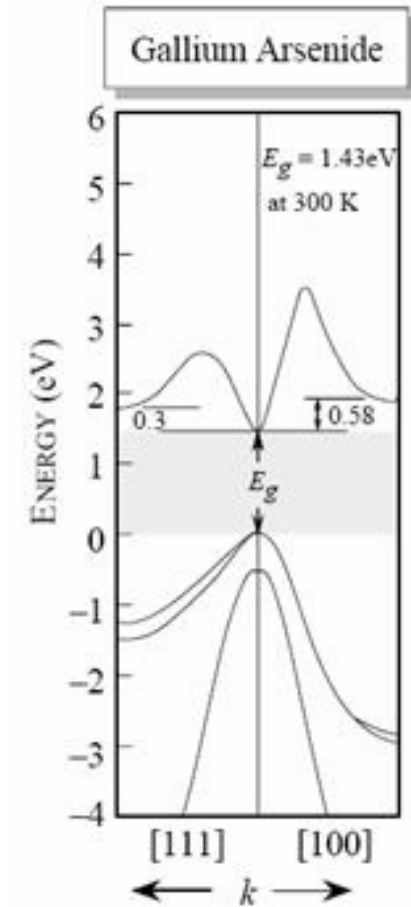


Silicon

High Fields



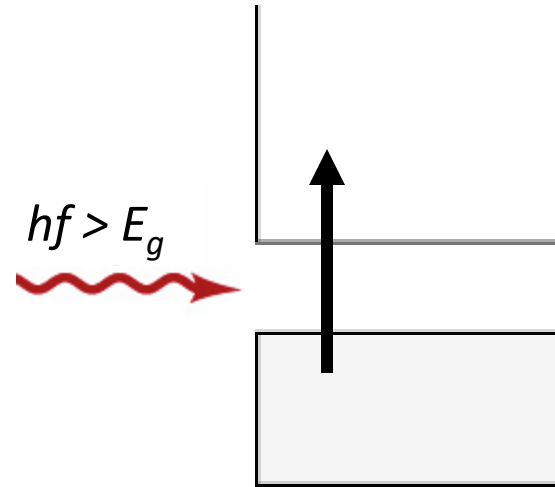
GaAs



Impact ionization

Carriers are accelerated to an energy above the gap before they scatter. They generate more electron-hole pairs. This results in an avalanche breakdown of the device.

Photoconductivity



$$\sigma = ne\mu_n + pe\mu_p$$

Light increases the conductivity of a semiconductor.

Laser printer

