6

Carrier transport

Ballistic transport
Drift
Diffusion
Tunneling

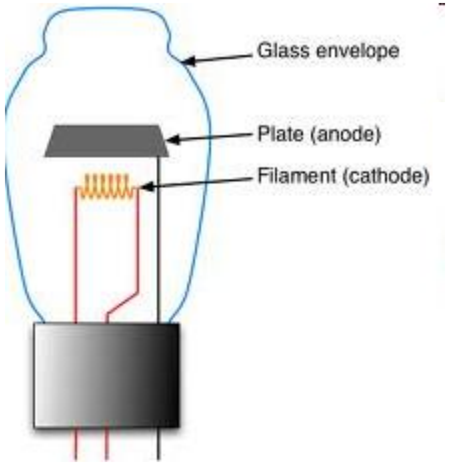
October 29th, 2025

Ballistic transport

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0 t + \vec{x}_0$$



Electrons moving in an electric field follow parabolic trajectories like a ball in a gravitational field.

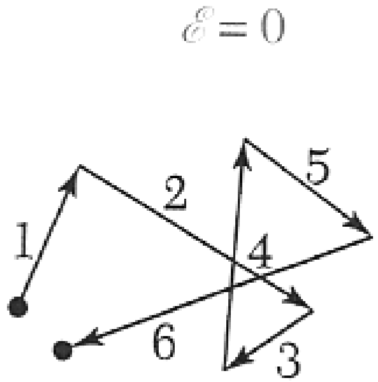
Drift

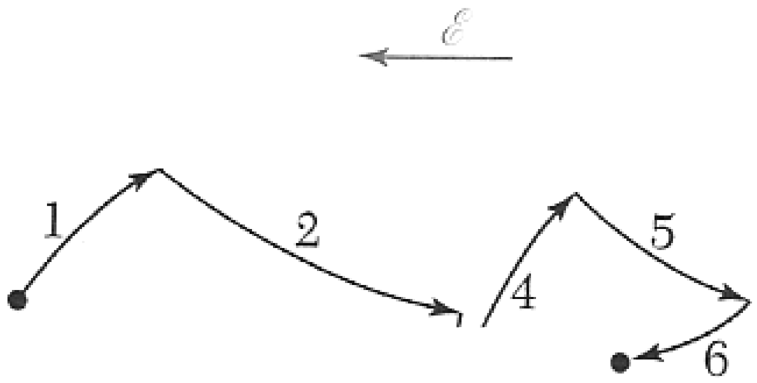
The electrons scatter and change direction after a time τ_{sc}.

thermal velocity $$\frac{1}{2}mv_{th}^2 = \frac{3}{2}k_BT$$ (classical equipartition)

at 300 K, v_{th} ~ 10^7 cm/s

mean free path: $\ell = v_{th}\tau_{sc}$ ~ 10 nm ~ 200 atoms



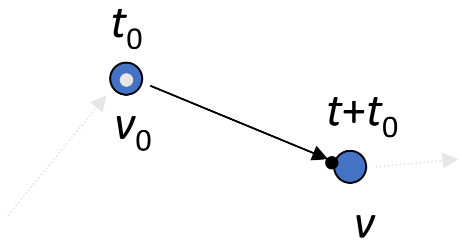


In a metal, the fermi velocity dominates over the thermal velocity. In a semiconductor, the thermal velocity dominates

Drift (diffusive transport)

$$\vec{F} = -e\vec{E} = m^*\vec{a} = m^*\frac{d\vec{v}}{dt}$$

time between two collisions



$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*}(t - t_0)$$

$$<v_0> = 0 \qquad <t - t_0> = \tau_{sc}$$

$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v}$$

drift velocity: $\vec{v}_{d,n} = -\mu_n \vec{E}$ $\qquad$ $\vec{v}_{d,p} = \mu_p \vec{E}$

Drift

drift velocity: $\vec{v}_{d,n} = -\mu_n \vec{E}$ $\vec{v}_{d,p} = \mu_p \vec{E}$

for Si: μ_n = 1350 cm^2/Vs
μ_p = 450 cm^2/Vs

current density:

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p}$$

$$= (ne\mu_n + pe\mu_p)\vec{E}$$

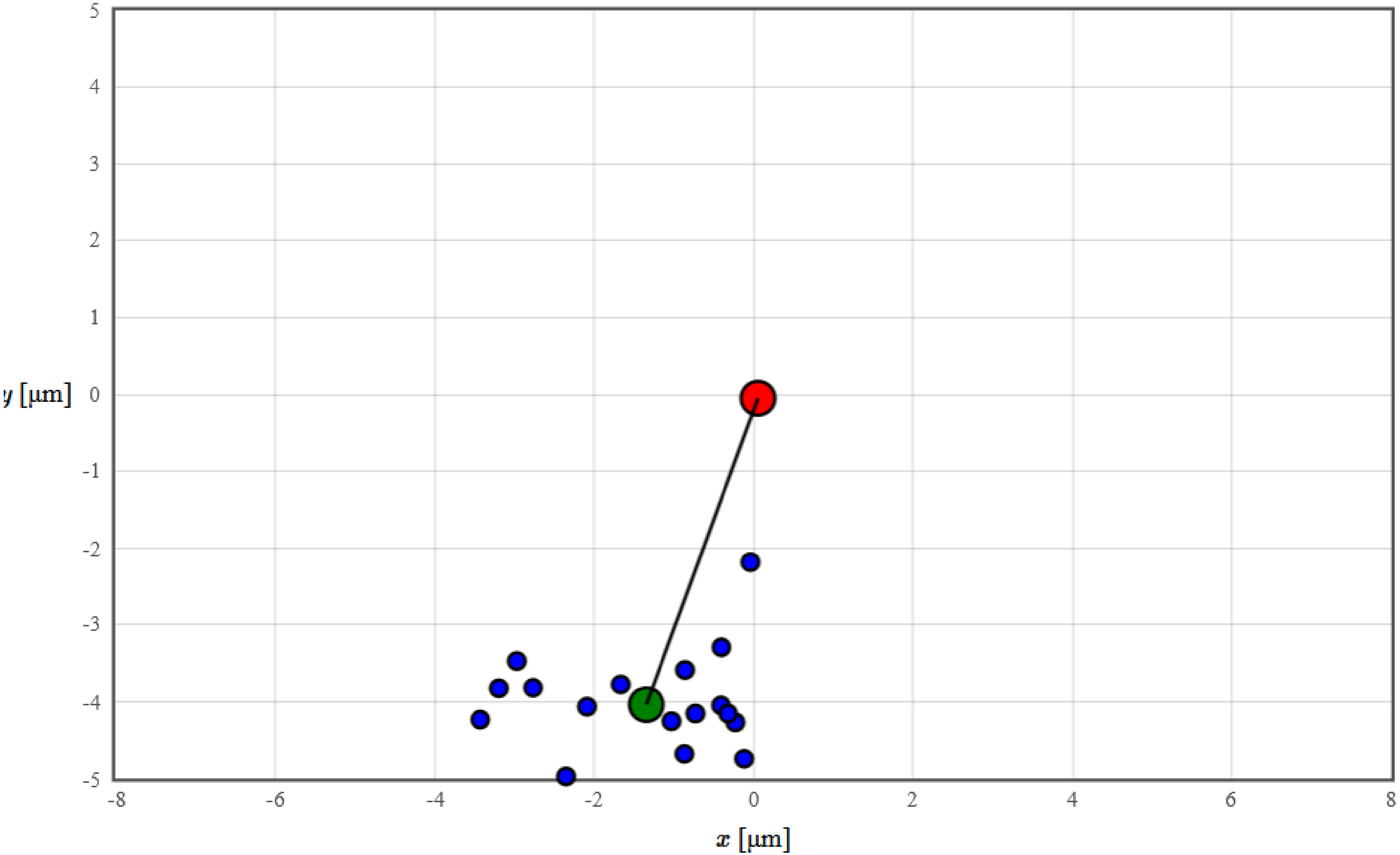
$$= \sigma\vec{E}$$

(Ohm's law)

For E = 1000 V/cm:

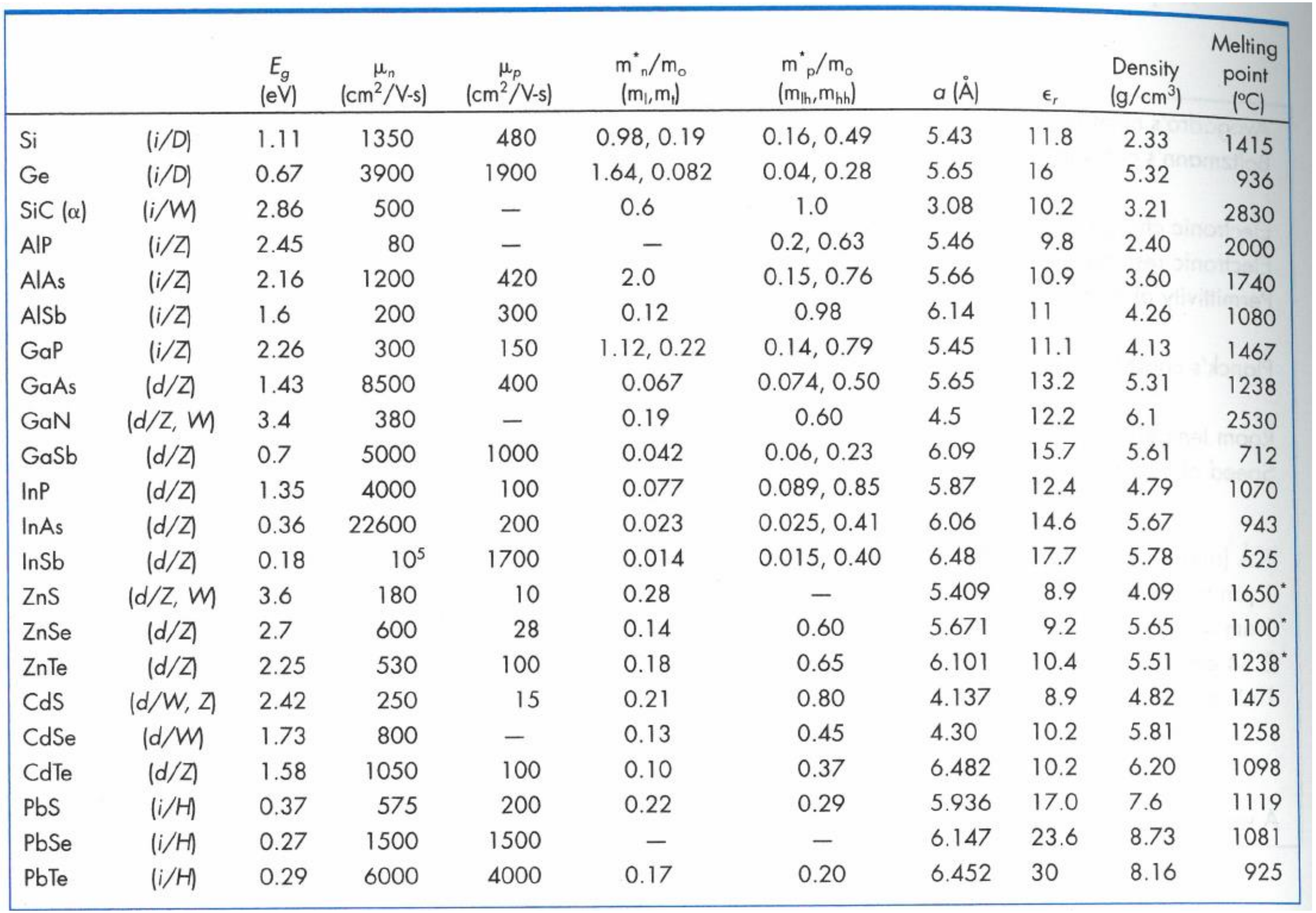
$v_d = 10^6$ cm/s

$$\mu = \frac{-e\tau_{sc}}{m^*} = \frac{-e\ell}{m^* v}$$



https://lampx.tugraz.at/~hadley/psd/L5/drude.php

Drift



		E_g (eV)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)	m^*_n/m_o (m_l, m_t)	m^*_p/m_o (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm^3)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10^5	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

Solid state electronic devices, Streetman and Banerjee

$$\vec{v}_{d,n} = -\mu_n \vec{E} \qquad \vec{v}_{d,p} = \mu_p \vec{E}$$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = \left(ne\mu_n + pe\mu_p\right)\vec{E} = \sigma\vec{E}$$

Matthiessen’s rule

rate

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

mostly temperature independent

phonons, temperature dependent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

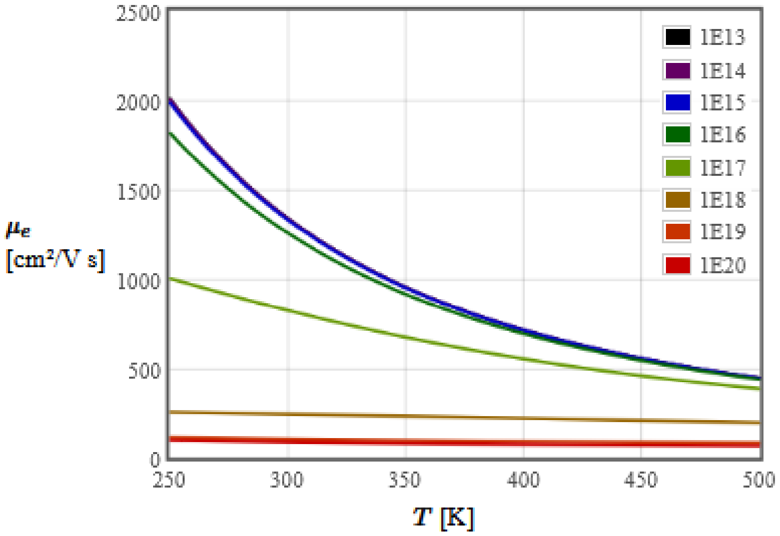
doping increases the conductivity
by increasing the carrier density
but decreases the mobility

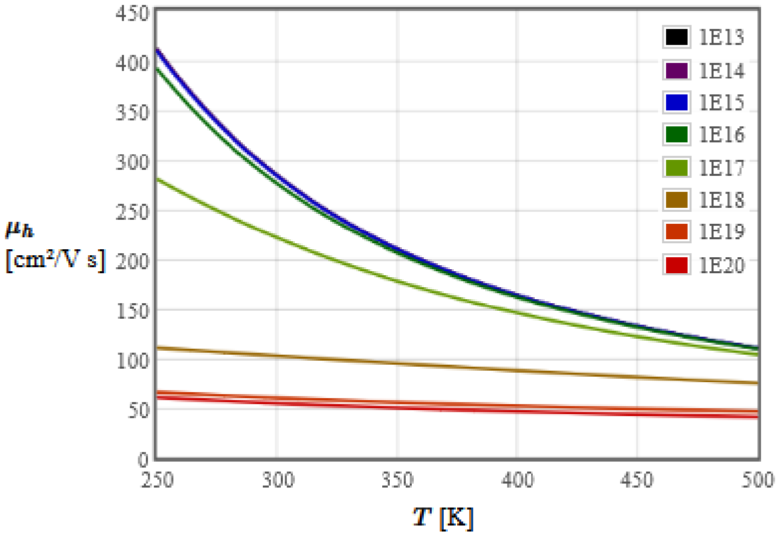
mobility model for Si

$$\mu_e = 88\left(\frac{T}{300}\right)^{-0.57} + \frac{7.4 \times 10^8 T^{-2.33}}{1 + 0.88\left[\frac{N_d}{1.26\times10^{17}\left(\frac{T}{300}\right)^{2.4}}\right]\left(\frac{T}{300}\right)^{-0.146}} \quad \mathrm{cm}^2/\mathrm{V\ s},$$

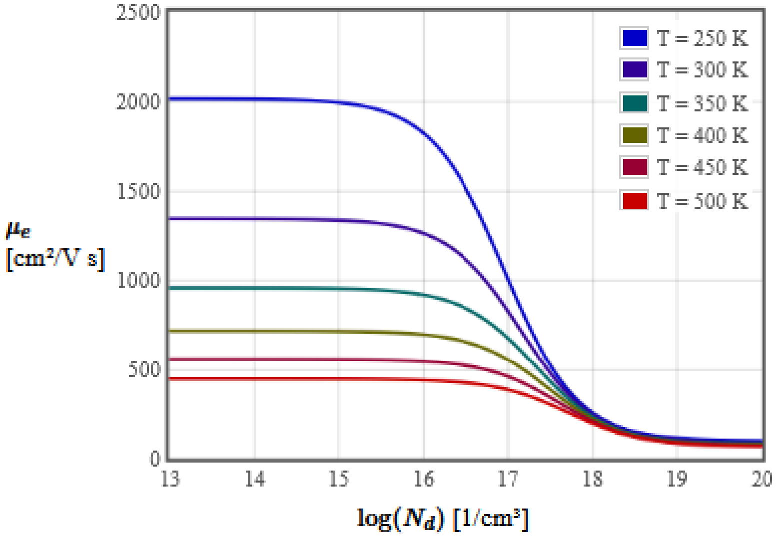
$$\mu_h = 54.3\left(\frac{T}{300}\right)^{-0.57} + \frac{1.36 \times 10^8 T^{-2.33}}{1 + 0.88\left[\frac{N_a}{2.35\times10^{17}\left(\frac{T}{300}\right)^{2.4}}\right]\left(\frac{T}{300}\right)^{-0.146}} \quad \mathrm{cm}^2/\mathrm{V\ s}.$$

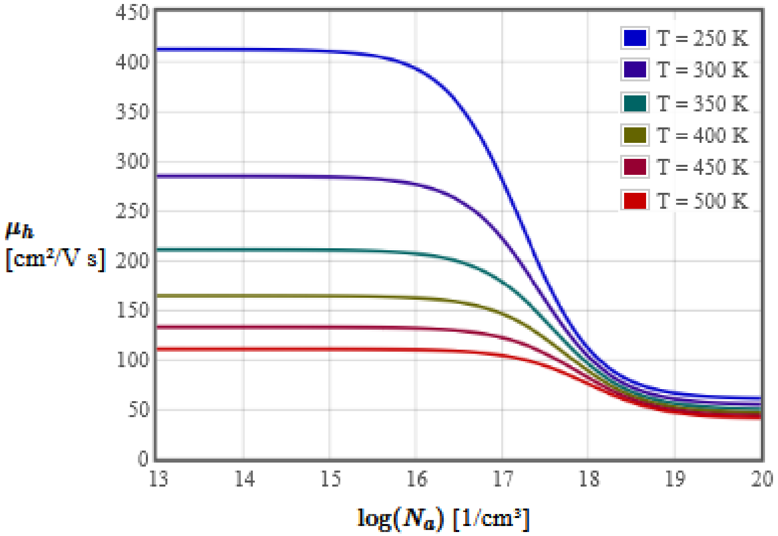
temperature





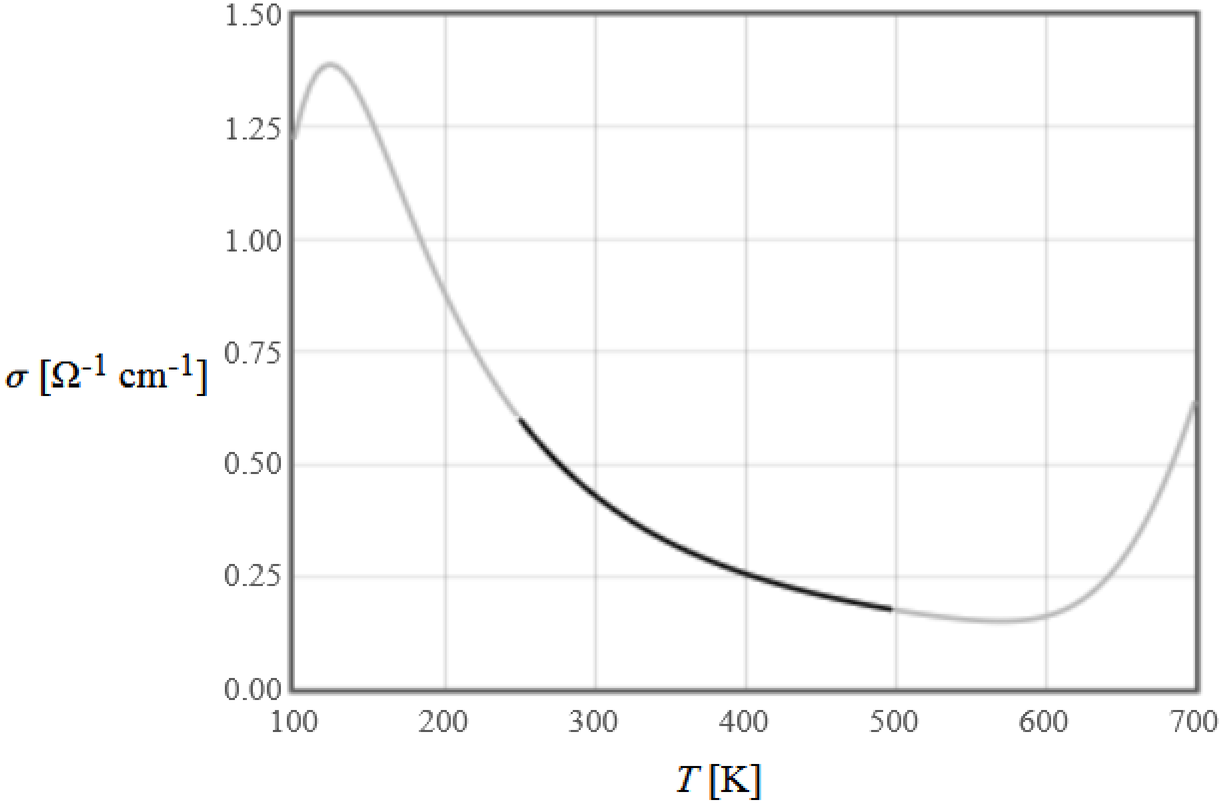
doping density





conductivity in Si

$$\sigma = ne\mu_n + pe\mu_p$$



$N_A = 10^{16}\ 1/\text{cm}^3$

https://lampz.tugraz.at/~hadley/psd/L4/conductivity.php

Ballistic transport in transistors

The mean free path ~100 nm > gate length ~ 20 nm

enables motion without significant collisions and scattering

v not proportional to *E*

$$\xcancel{\vec{v} = \mu \vec{E}}$$

j not proportional to *E*

$$\xcancel{\vec{j} = \sigma \vec{E}}$$

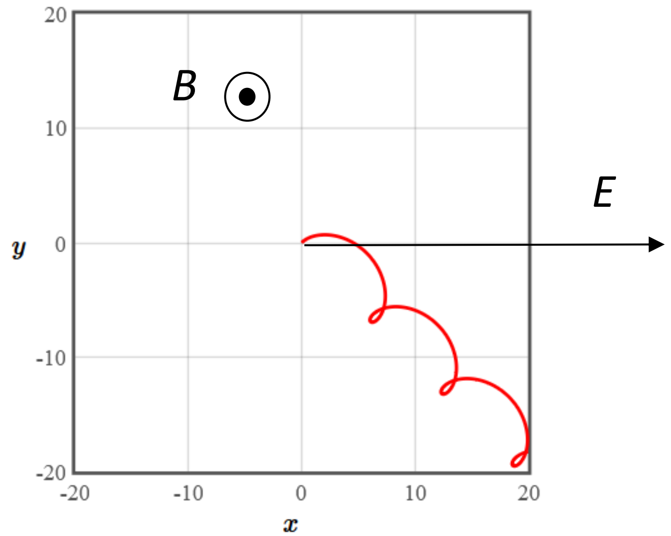
nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

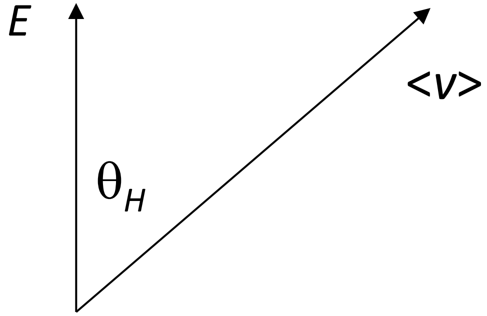
Crossed E and B fields

Ballistic transport

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



Diffusive transport



Hall angle:

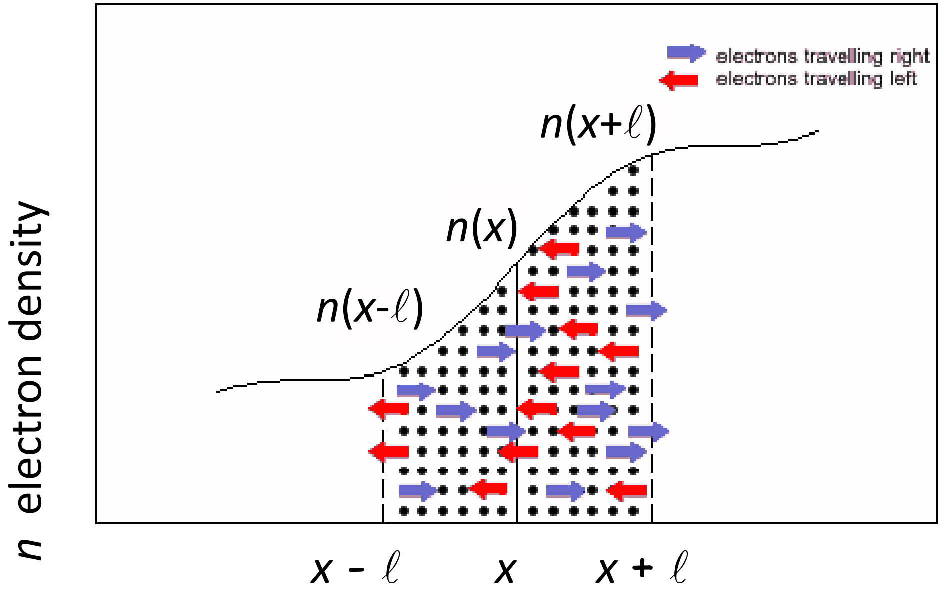
$$\theta_H = \tan^{-1}\left(-\frac{eB_z\tau_{sc}}{m^*}\right)$$

http://lampz.tugraz.at/~hadley/physikm/script/magnetism/constant_EB.en.php

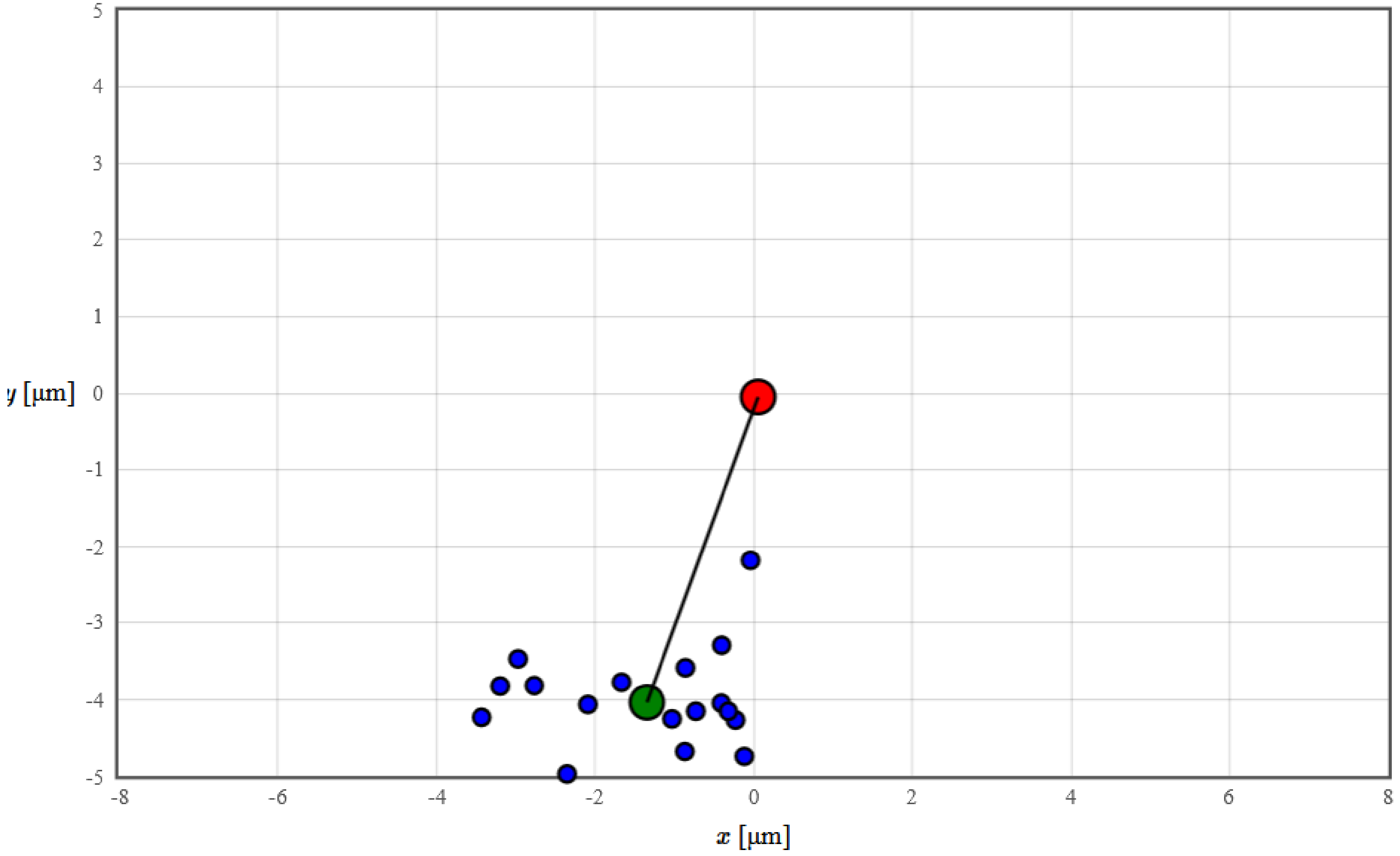
Diffusion

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -|e| D_p \frac{dp}{dx}$$



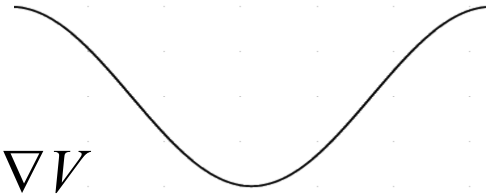
Diffusion is from high concentration to low concentration.



http://lampz.tugraz.at/~hadley/psd/L5/drude.php

Einstein relation

$$\vec{E} = -\nabla V$$



$$n = A\exp\left(\frac{-eV}{k_B T}\right)$$

Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{e}{k_B T}A\exp\left(\frac{-eV_{pot}}{k_B T}\right)\nabla V = -\frac{ne}{k_B T}\nabla V = \frac{ne\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + eD\frac{ne\vec{E}}{k_B T} = 0$$

$$\boxed{D = \frac{\mu k_B T}{e}}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen

Current density equations

Drift

Diffusion

$$\vec{j}_n = -ne\mu_n\vec{E} + eD_n\nabla n$$

$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

in equilibrium ?

$$\vec{j}_n = en\mu_n\vec{E} + eD_n\nabla n$$

The electric field is proportional to the gradient of the conduction band edge

$$e\vec{E} = \nabla E_c$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

If the Fermi energy is constant,

$$\nabla n = -\frac{\nabla E_c}{k_B T} N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = -\frac{\nabla E_c}{k_B T} n$$

$$\vec{j}_n = n\nabla E_c\left(\mu_n - \frac{eD_n}{k_B T}\right)$$

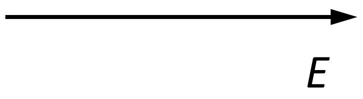
This means that the current density in a semiconductor where the Fermi energy is constant is zero.

Current density equations

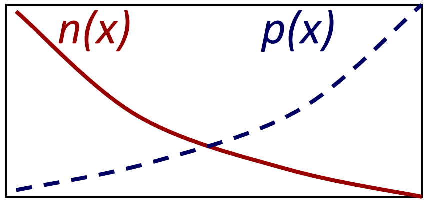
note: electron and hole currents have same direction

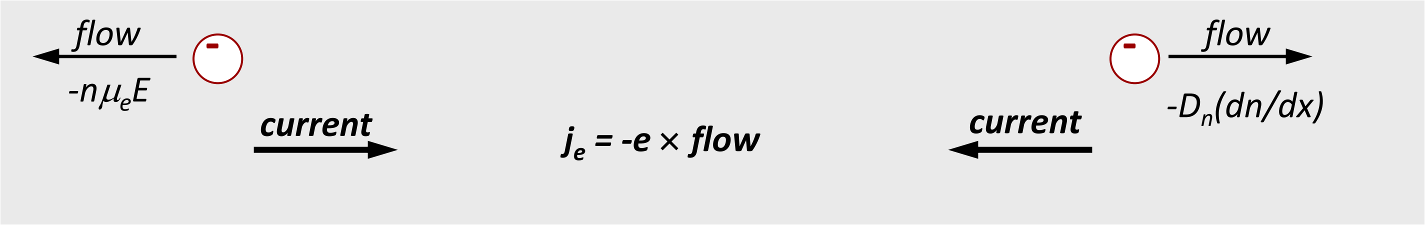
electric current = charge × particle flow

drift

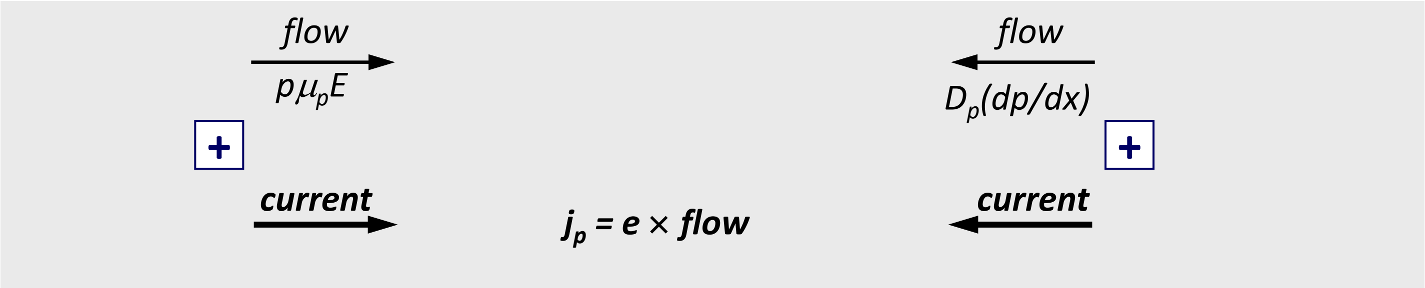


diffusion



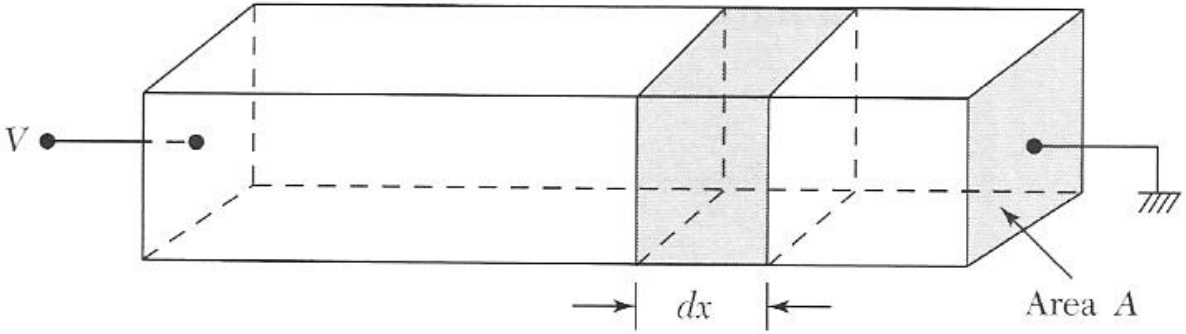


$j_e = -e \times \text{flow}$



$j_p = e \times \text{flow}$

Continuity equations

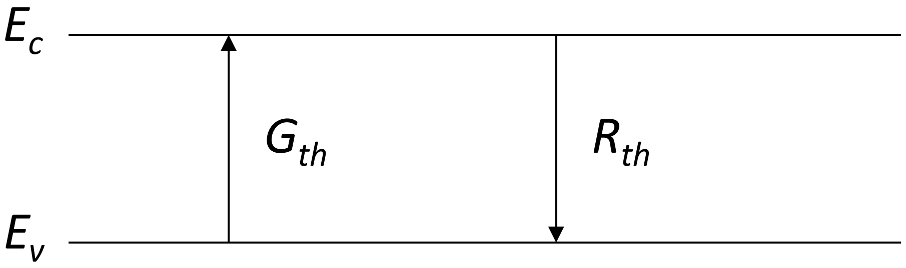


$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{e}\nabla \cdot \vec{j}_p + G_p - R_p$$

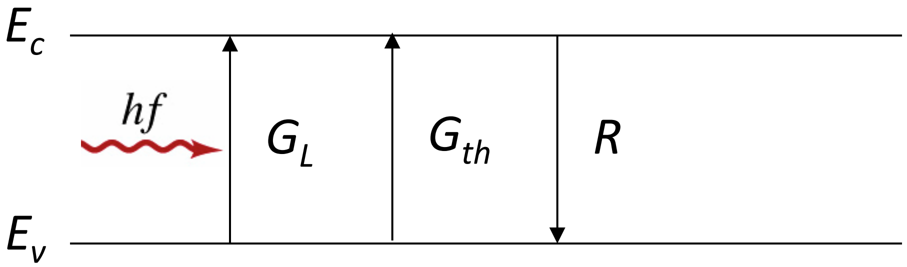
j_n and j_p consist of drift and diffusion terms

Generation and recombination



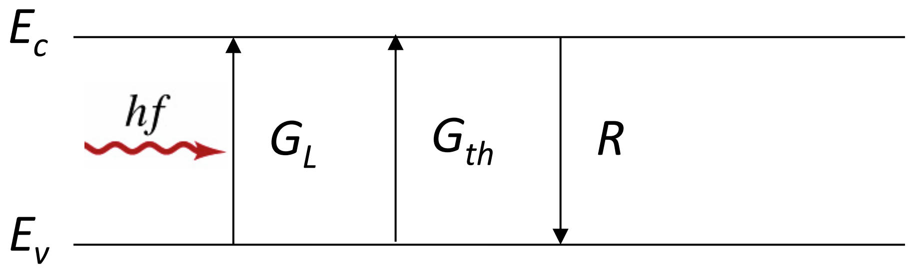
thermal equilibrium

Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution $np \neq n_i^2$



non equilibrium

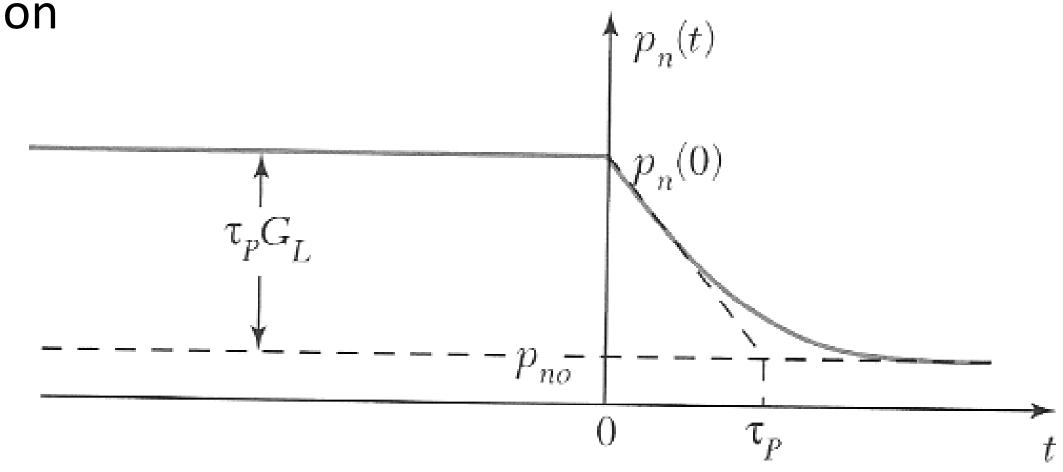
Recombination



$$R - R_{th} = \frac{p_n - p_{n0}}{\tau_p}$$

Recombination rate is limited by the density of minority carriers.
The majority carriers have to find a minority carrier to recombine.

p_n (or n_p) = minority carrier concentration
p_{n0} (or n_{p0}) = equilibrium minority carrier concentration
τ_p = minority carrier lifetime



minority carrier lifetimes

p-type

$$n_p(t) = n_{excess} \exp(-t/\tau_n) + n_{p0}$$

minority carrier lifetimes

n-type

$$p_n(t) = p_{excess} \exp(-t/\tau_p) + p_{n0}$$

$$np = n_i^2$$

continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

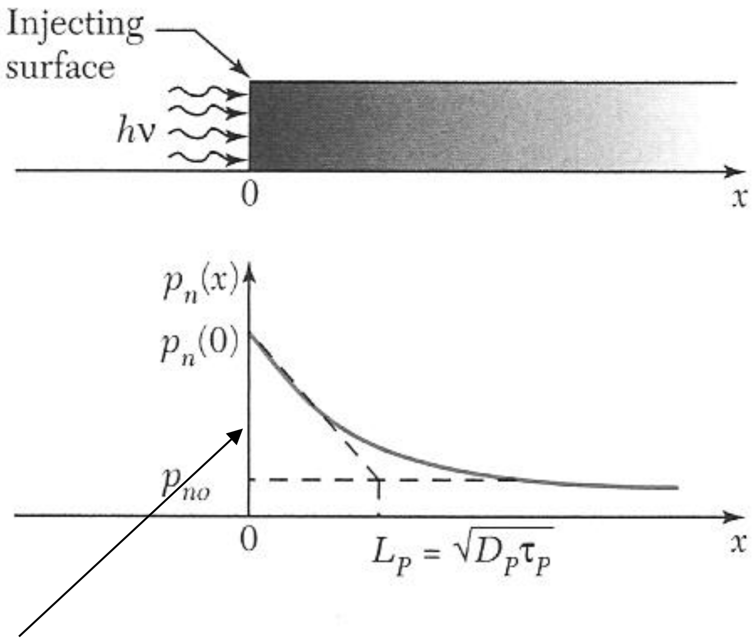
drift: $\vec{j}_n = -ne\mu_n \vec{E}$ $\qquad \nabla \cdot \vec{j}_n = -en\mu_n \nabla \cdot \vec{E} - e\nabla n \mu_n \vec{E}$

diffusion: $\vec{j}_{n,diff} = |e| D_n \nabla n$ $\qquad \nabla \cdot \vec{j}_{n,diff} = |e| D_n \nabla^2 n$

$$\boxed{\begin{aligned} \frac{\partial n}{\partial t} &= n\mu_n \nabla \cdot \vec{E} + \nabla n \mu_n \vec{E} + D_n \nabla^2 n + G_n - \frac{n - n_0}{\tau_n} \\ \frac{\partial p}{\partial t} &= -p\mu_p \nabla \cdot \vec{E} - \nabla p \mu_p \vec{E} + D_p \nabla^2 p + G_p - \frac{p - p_0}{\tau_p} \end{aligned}}$$

Diffusion length

n-type



$$\frac{\partial p_n}{\partial t} = \overset{\text{Steady state}}{\overset{\downarrow}{0}} = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

Generation only occurs at the surface. There the minority carrier density is $p_n(0)$.

Diffusion length

$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \quad \Leftarrow \quad p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

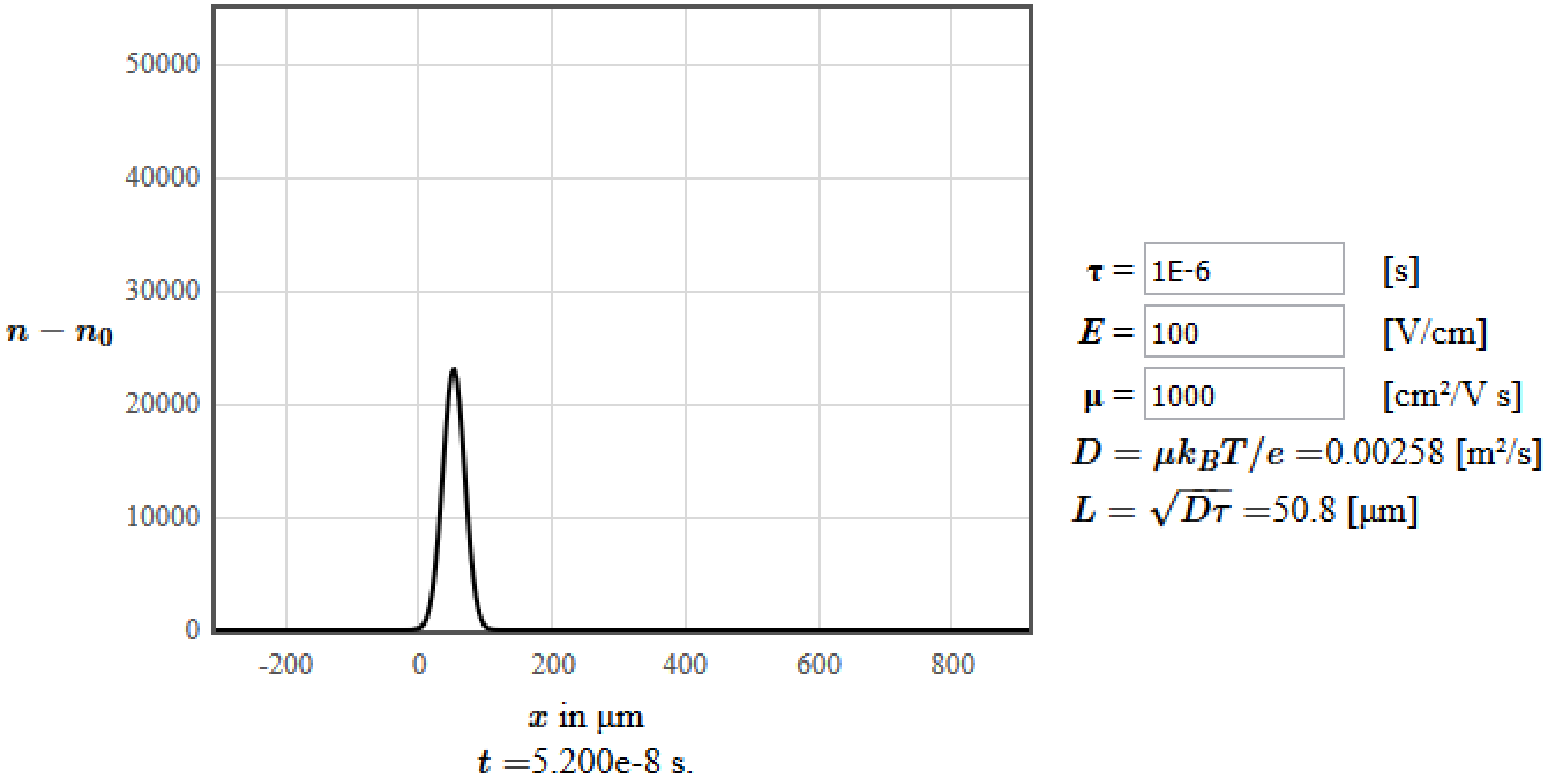
$$0 = \frac{D_p\left(p_n(0) - p_{n0}\right)}{L_p^2}\exp\left(\frac{-x}{L_p}\right) - \frac{\left(p_n(0) - p_{n0}\right)}{\tau_p}\exp\left(\frac{-x}{L_p}\right)$$

$$\underset{\nwarrow}{L_p} = \sqrt{D_p \tau_p}$$

diffusion length,
typically microns

Haynes Shockley experiment

$$n_p(x,t) = \frac{n_{generated}}{\sqrt{4\pi D_n t}} \exp\left(-\frac{\left(x-\mu_n E t\right)^2}{4D_n t}\right)\exp\left(-\frac{t}{\tau_n}\right) + n_{p0}$$



t =5.200e-8 s.

τ = 1E-6 [s]

E = 100 [V/cm]

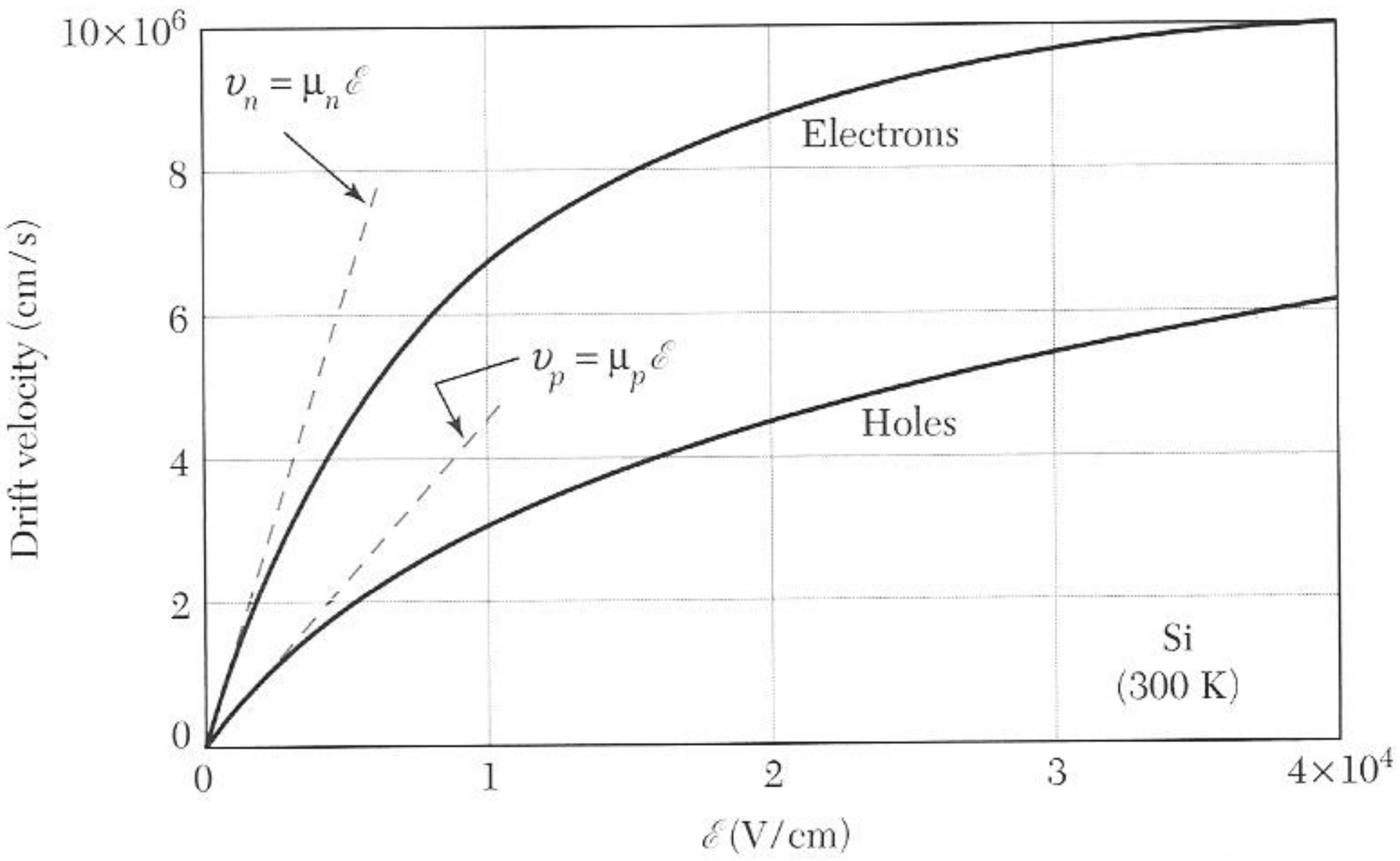
μ = 1000 [cm^2/V s]

$D = \mu k_B T/e$ =0.00258 [m^2/s]

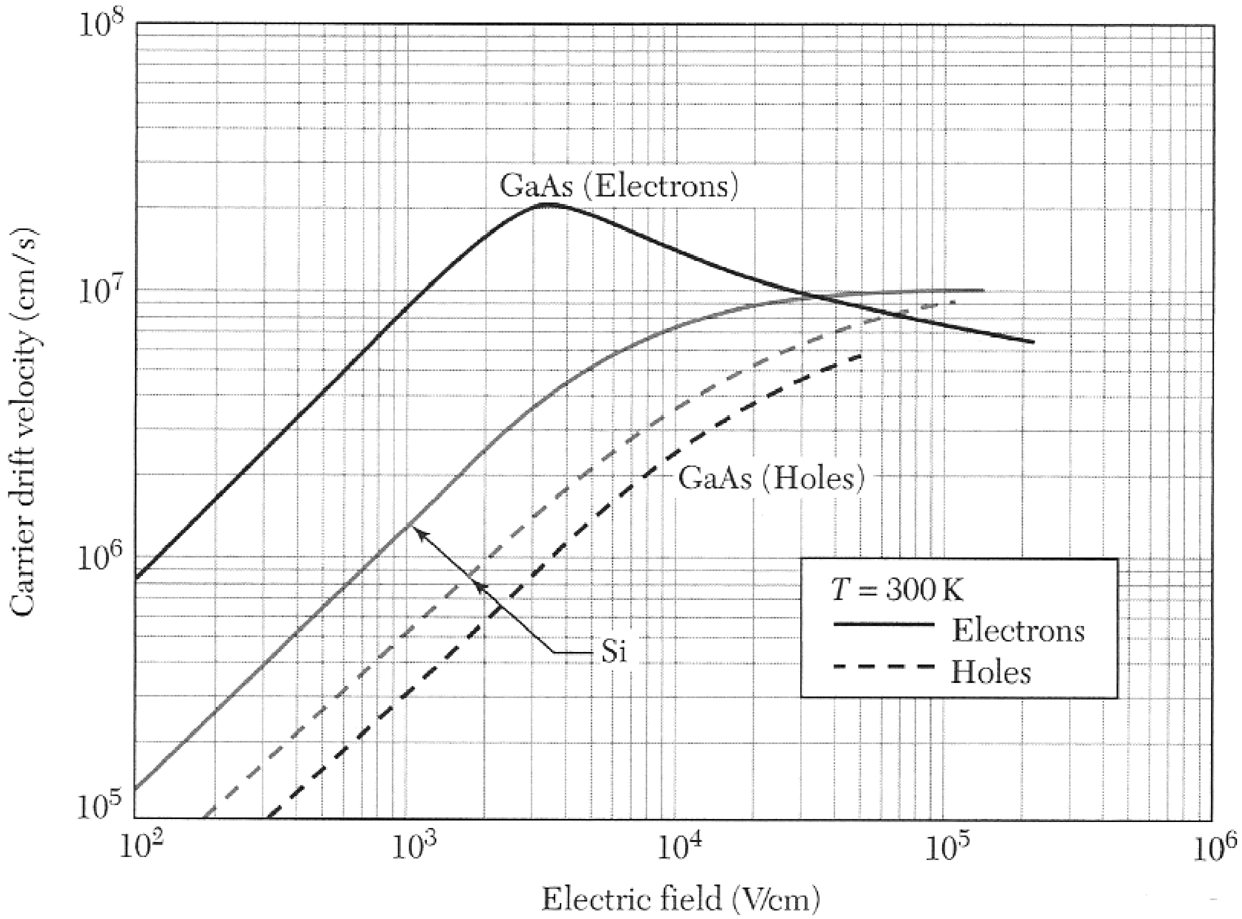
$L = \sqrt{D\tau}$ =50.8 [μm]

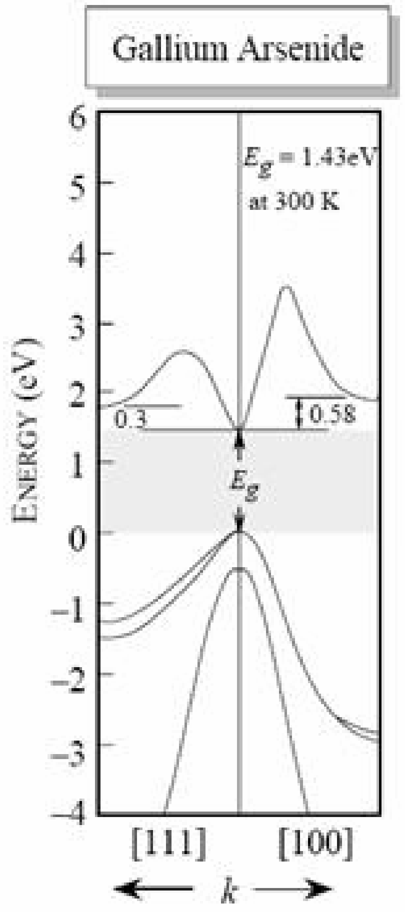
High fields

Silicon



High fields





GaAs

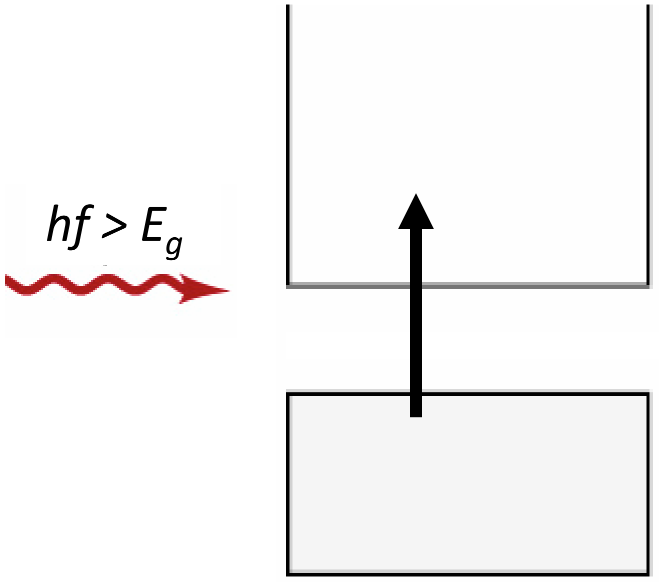
Impact ionization

Carriers are accelerated to an energy above the gap before they scatter.

They generate more electron-hole pairs.

This results in an **avalanche** breakdown of the device.

Photoconductivity



$$\sigma = ne\mu_n + pe\mu_p$$

Light increases the conductivity of a semiconductor.

Laser printer

semiconducting photoconductor