7

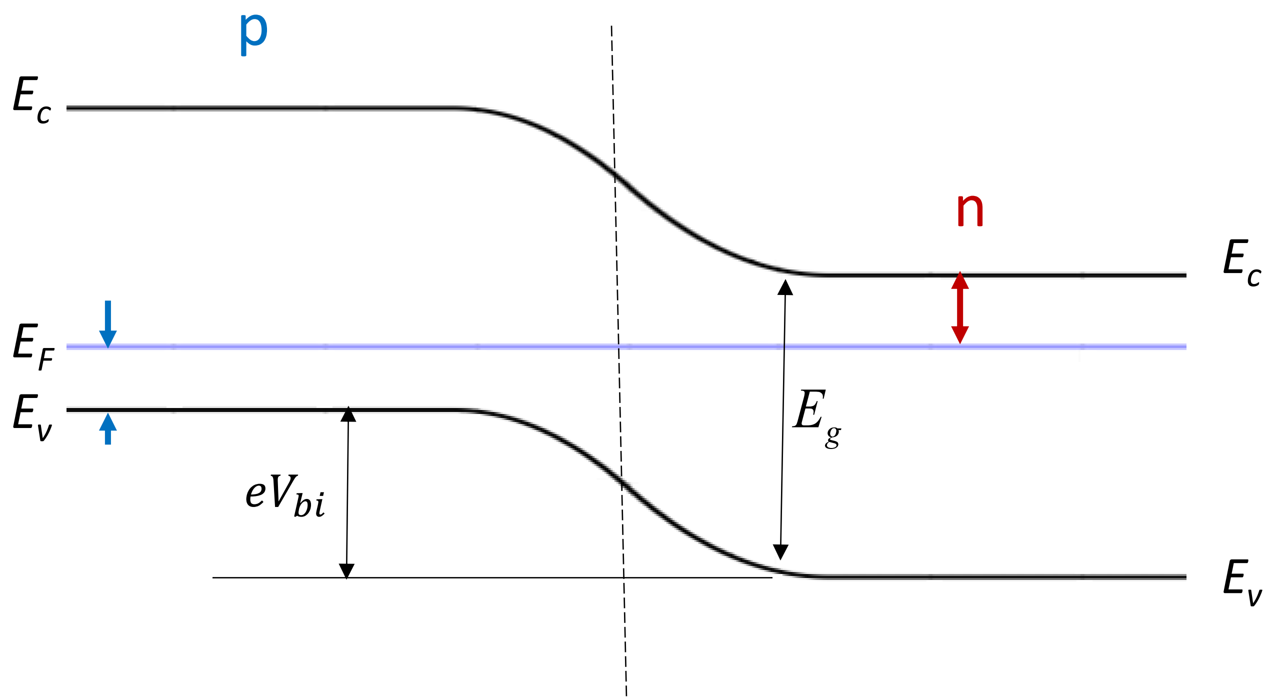
pn junctions

November 12th, 2025

Built-in voltage V_{bi}

E_v in p-region

$$E_F - E_{vp} = k_B T \ln\left(\frac{N_v}{N_A}\right)$$



E_c in n-region

$$E_{cn} - E_F = k_B T \ln\left(\frac{N_c}{N_D}\right)$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c}{N_{D,n} - N_{A,n}}\right) - k_B T \ln\left(\frac{N_v}{N_{A,p} - N_{D,p}}\right)$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{(N_{D,n} - N_{A,n})(N_{A,p} - N_{D,p})}\right)$$

V_{bi}

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{(N_{D,n} - N_{A,n})(N_{A,p} - N_{D,p})}\right)$$

$$n_i^2 = N_v N_c \exp\left(\frac{-E_g}{k_B T}\right) \qquad E_g = -k_B T \ln\left(\frac{n_i^2}{N_v N_c}\right)$$

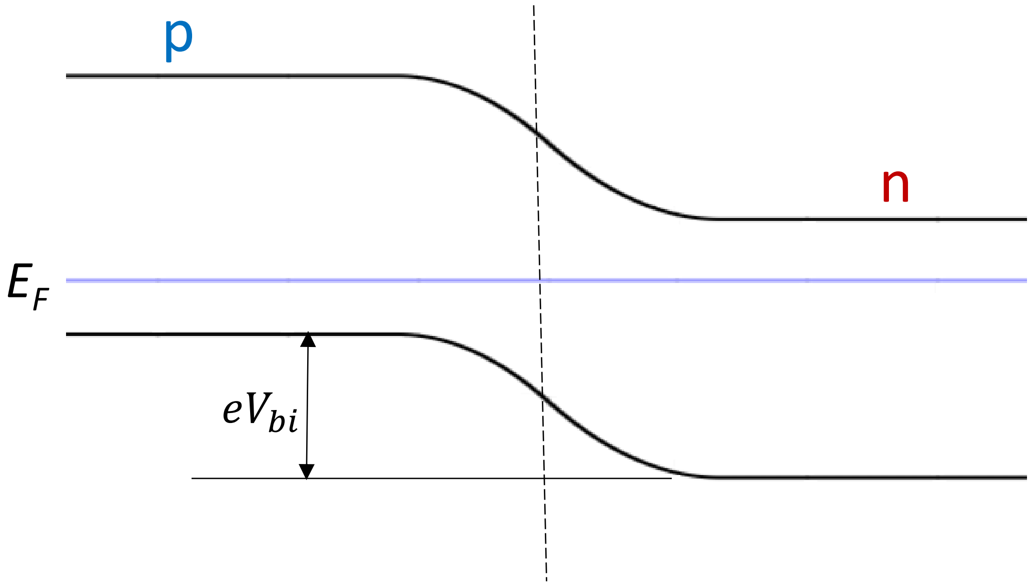
$$eV_{bi} = k_B T \ln\left(\frac{(N_{D,n} - N_{A,n})(N_{A,p} - N_{D,p})}{n_i^2}\right)$$

for $N_{D,n} - N_{A,n} = N_D$ and $N_{A,p} - N_{D,p} = N_A$

$$\boxed{eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

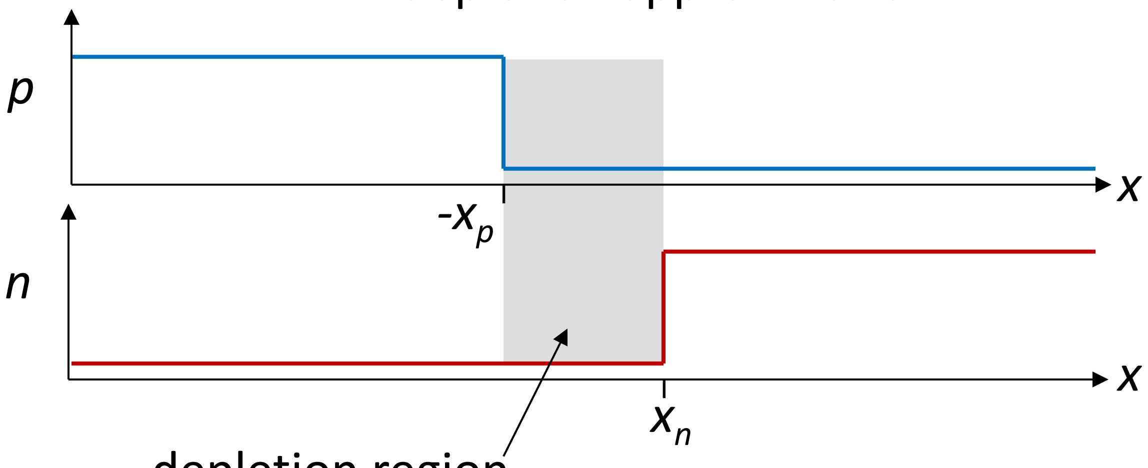
https://lampz.tugraz.at/~hadley/psd/L6/Vbi.html

p and n profiles



$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

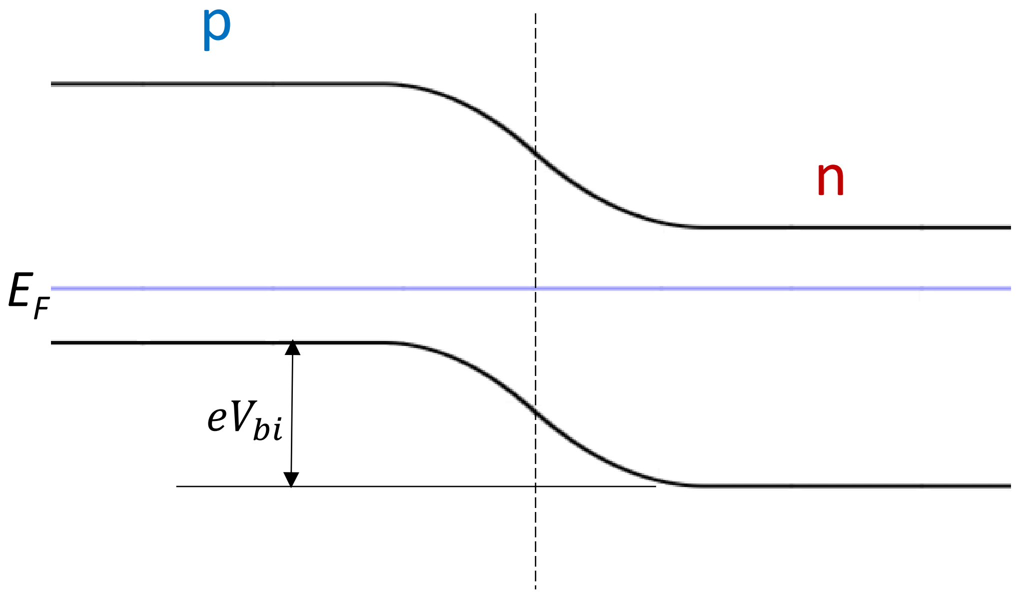
depletion approximation



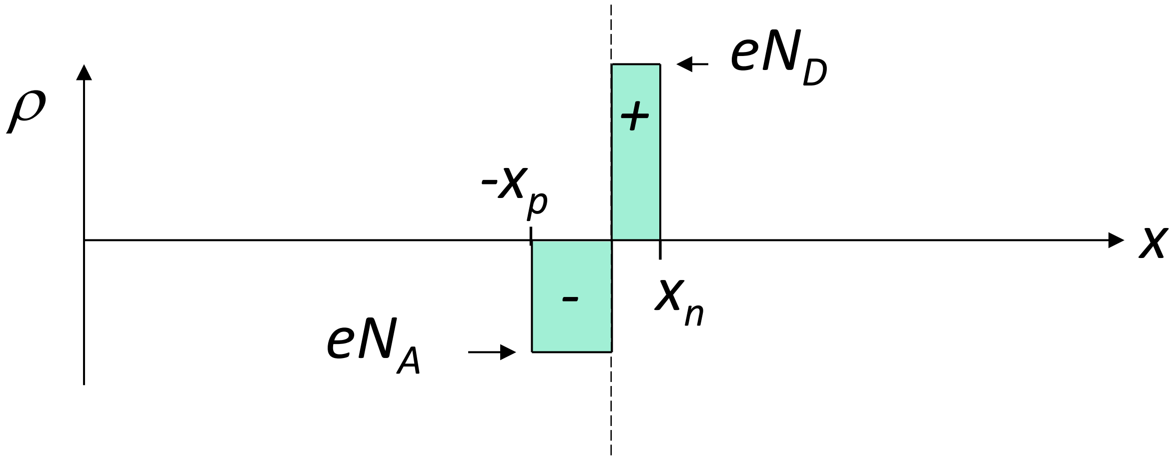
depletion region

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

space charge



exposed ionized dopants



$$N_A x_p = N_D x_n$$

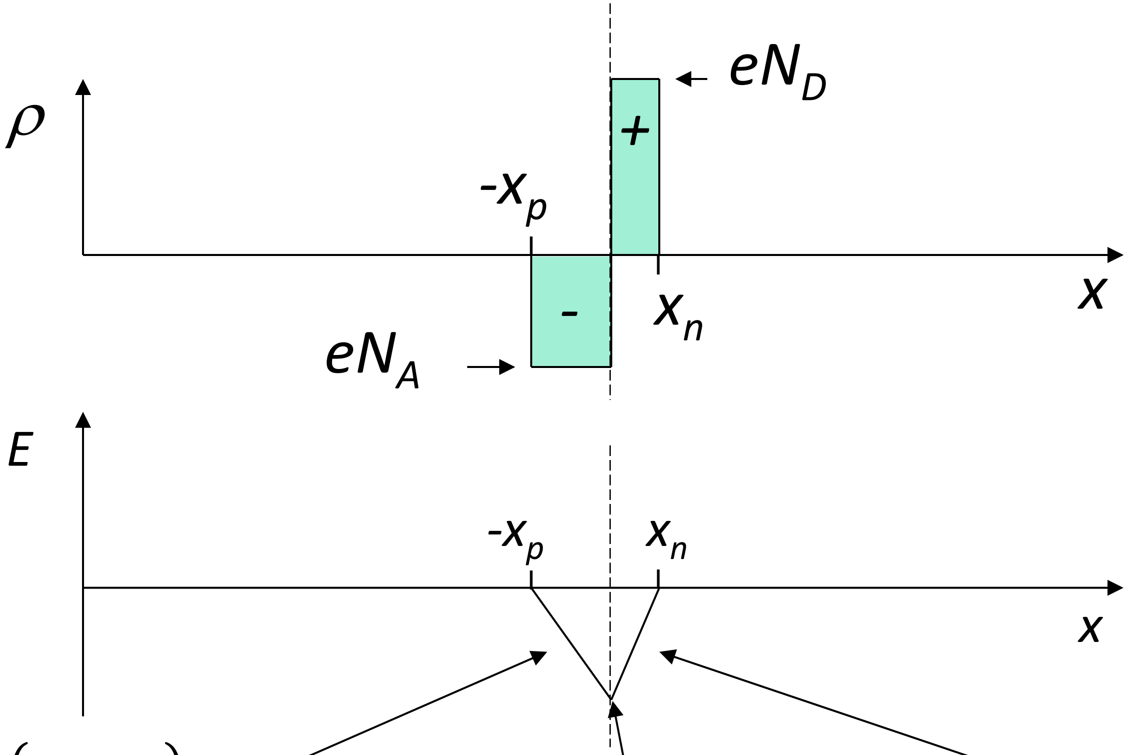
electric field

Gauss's law

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

in 1-D is

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$



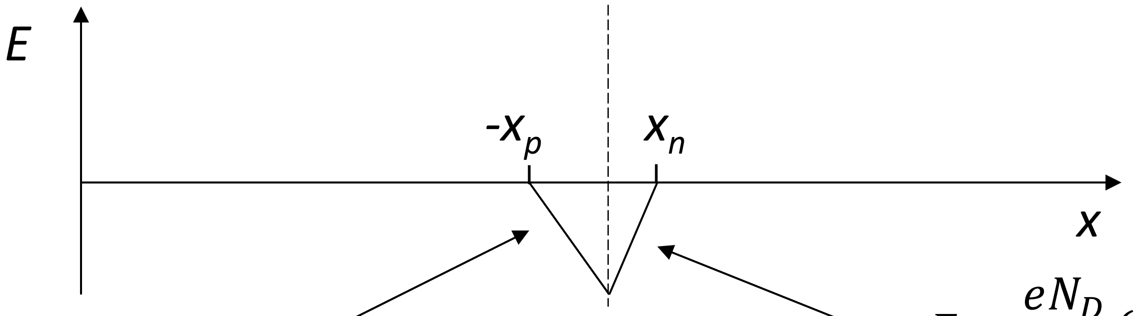
$$E = -\frac{eN_A}{\varepsilon}\left(x + x_p\right)$$

$$E = \frac{eN_D}{\varepsilon}(x - x_n)$$

$$|E_{\max}| = \frac{eN_D x_n}{\varepsilon} = \frac{eN_A x_p}{\varepsilon}$$

E pushes the holes towards *p* and the electrons towards *n*

electrostatic potential



$$E = -\frac{eN_A}{\varepsilon}(x + x_p)$$

$$E = \frac{eN_D}{\varepsilon}(x - x_n)$$

$$\frac{dV}{dx} = -E$$

$$-x_p > x > 0 \qquad 0 > x > x_n$$

$$V = \frac{eN_A}{\varepsilon}\left(\frac{x^2}{2} + x x_p\right) \qquad V = \frac{-eN_D}{\varepsilon}\left(\frac{x^2}{2} - x x_n\right)$$

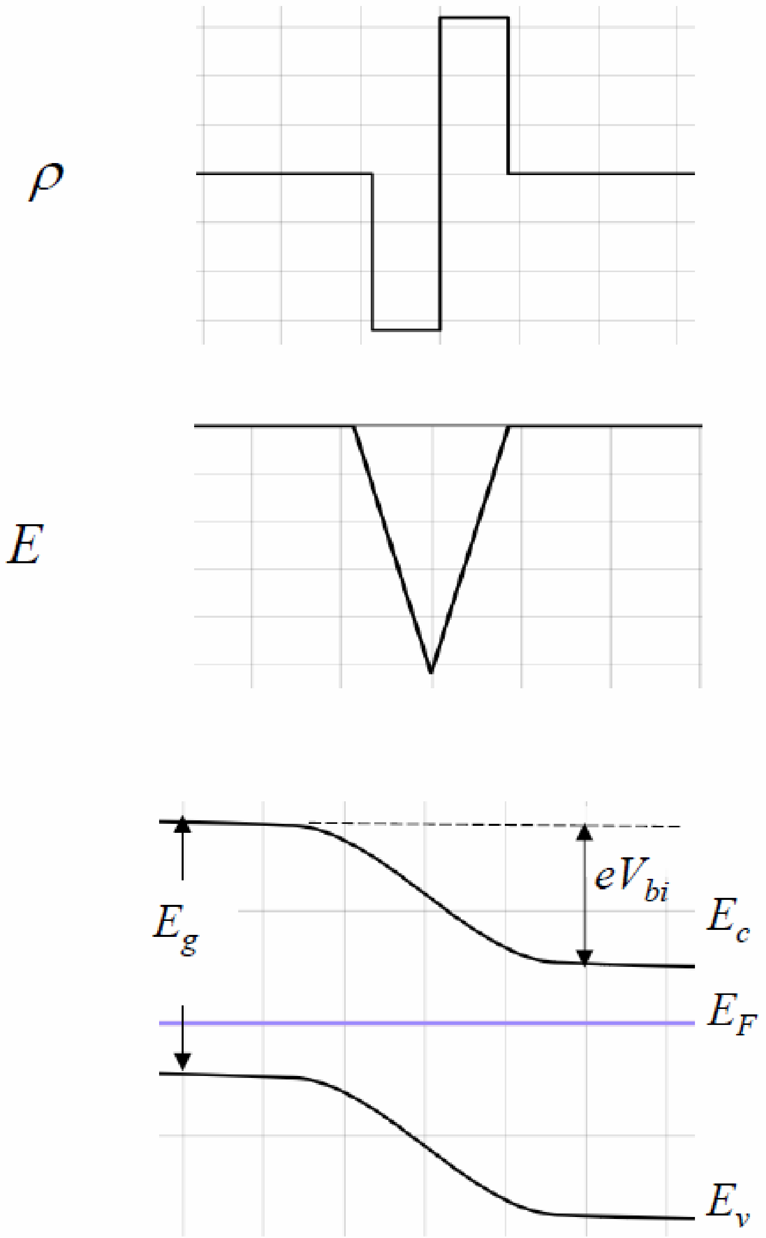
$$V(-x_p) = \frac{-eN_A}{2\varepsilon}x_p^2 \qquad V(0) = 0 \qquad V(x_n) = \frac{eN_D}{2\varepsilon}x_n^2$$

abrupt pn junction

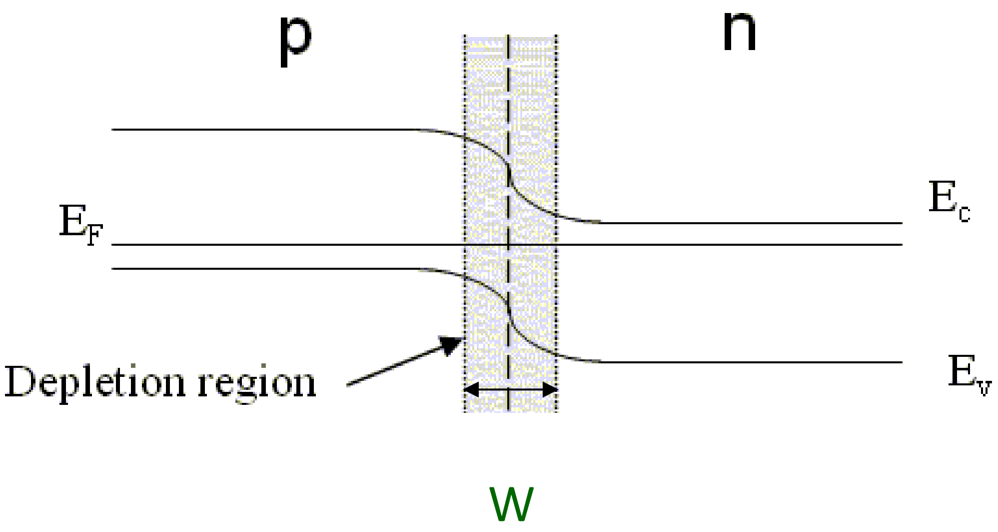
$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$= \frac{e N_A x_p^2}{2\varepsilon} + \frac{e N_D x_n^2}{2\varepsilon}$$

https://lampz.tugraz.at/~hadley/psd/L6/abrupt.html



Depletion width



$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{e N_A x_p^2}{2\varepsilon} + \frac{e N_D x_n^2}{2\varepsilon}$$

$$N_A x_p = N_D x_n = N_D(W - x_p) = N_A(W - x_n)$$

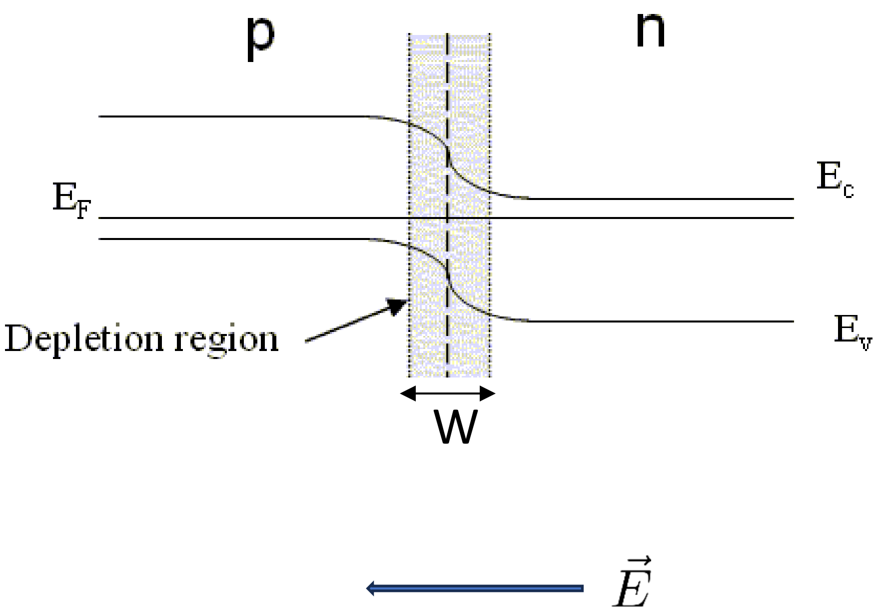
$$x_p = \frac{N_D W}{N_A + N_D} \qquad x_n = \frac{N_A W}{N_A + N_D}$$

$$V_{bi} = \frac{e}{2\varepsilon} \frac{N_D N_A}{N_D + N_A} W^2$$

$$W = \sqrt{\frac{2\varepsilon(N_D + N_A)V_{bi}}{e N_D N_A}}$$

light doping: wide depletion width

Depletion width



V_{bi} ~ 1V

W ~ 10 nm - 10 μm

E_{max} ~ 10^4 V/cm

The electric field pushes the electrons towards the n-region and the holes towards the p-region.

Diffusion sends electrons towards the p-region and holes towards the n-region.

Abrupt pn junctions in the depletion approximation

In an abrupt pn junction, the doping changes abruptly from p to n. It is common to solve for the band bending, the local electric field, the carrier concentration profiles, and the local conductivity in the depletion approximation. In this approximation it is assumed that there is a depletion width W around the transition from p to n where the charge carrier densities are negligible. Outside the depletion width the charge carrier densities are equal to the doping densities so that the semiconductor is electrically neutral outside the depletion width. Using this approximation it is possible to calculate the important properties of the pn junction.

$N_A =$ 1E15 1/cm³ $N_D =$ 1E15 1/cm³ $E_g =$ 1.166-4.73E-4*T*T/(T+636) eV

$N_v(300) =$ 9.84E18 1/cm³ $N_c(300) =$ 2.78E19 1/cm³ $\epsilon_r =$ 12 $T =$ 300 K

$\mu_p =$ 480 cm²/V s $\mu_n =$ 1350 cm²/V s $\tau_p =$ 1E-10 s $\tau_n =$ 1E-10 s

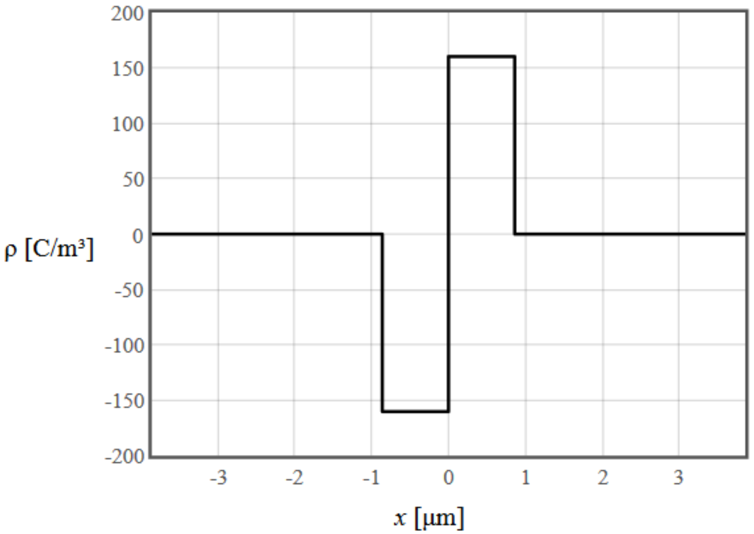
$V =$ -0.5 V Submit

V

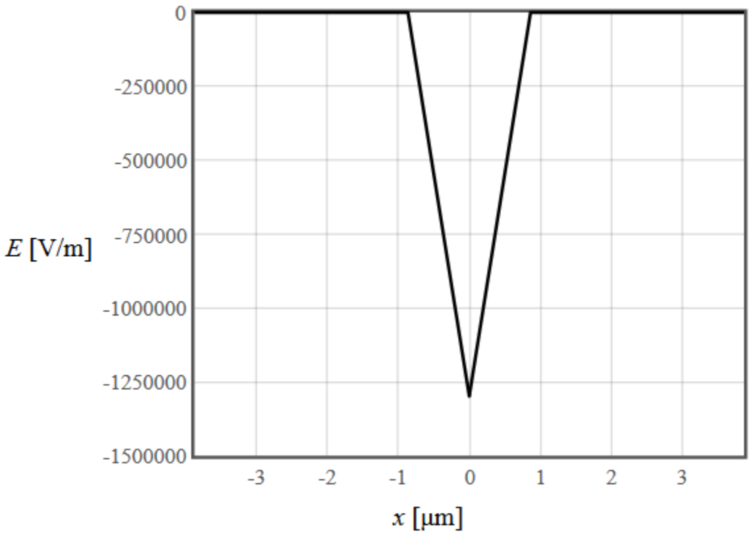
$E_g = 1.12$ eV $W = 1.72$ µm $x_p =$ -0.861 µm $x_n = 0.861$ µm $V_{bi} = 0.618$ V $C_j = 6.17$ nF/cm²

$D_p = 12.4$ cm²/s $D_n = 34.9$ cm²/s $L_p = 0.352$ µm $L_n = 0.591$ µm

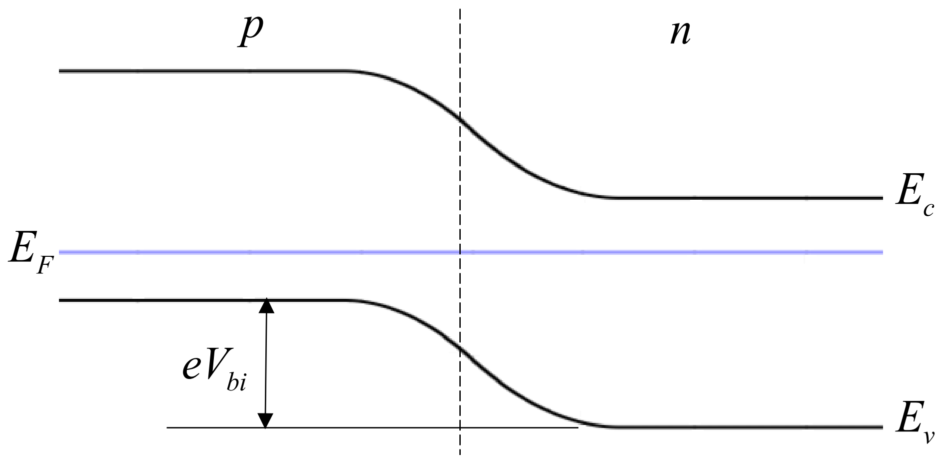
Charge density

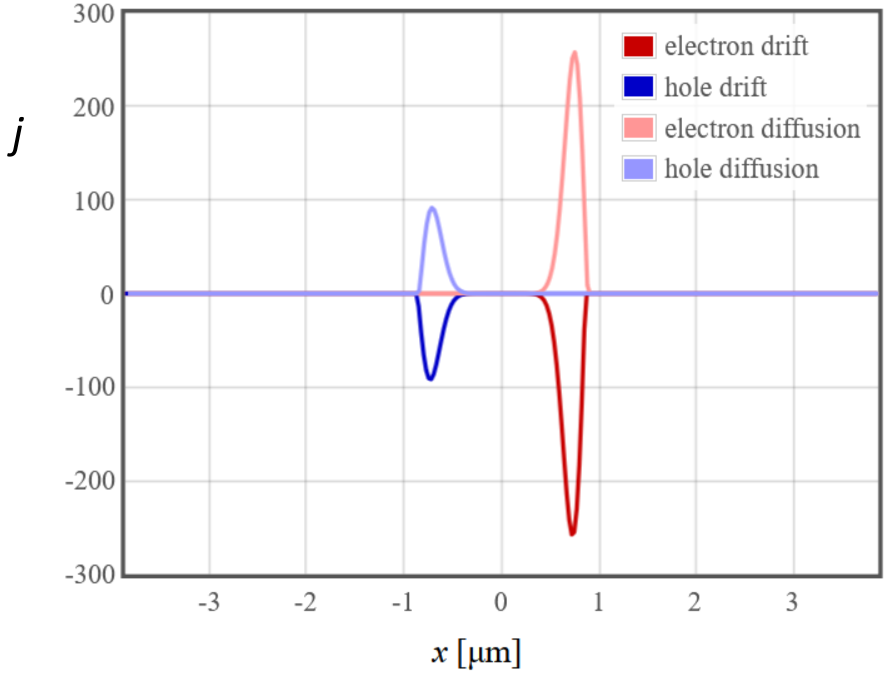


Electric field



Drift and diffusion





$$\vec{j}_n = en\mu_n\vec{E} + eD_n\nabla n$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$\nabla n = -\frac{\nabla E_c}{k_B T} N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = -\frac{\nabla E_c}{k_B T} n$$

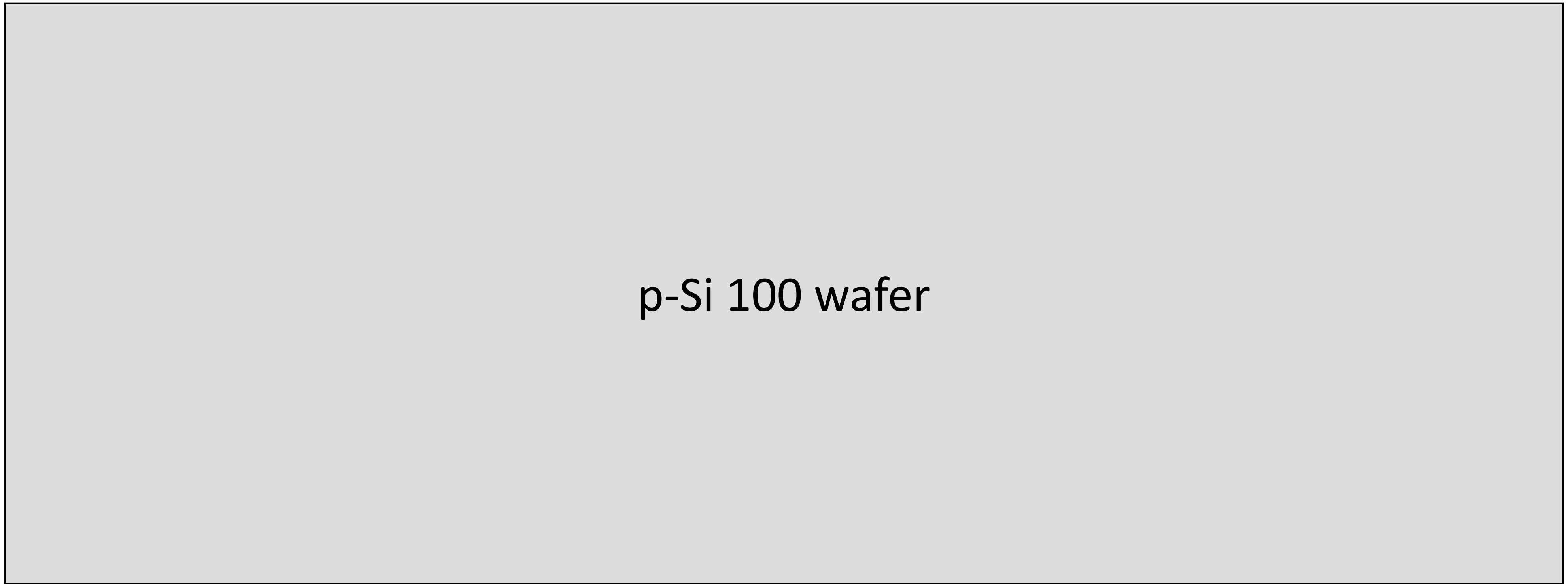
$$e\vec{E} = \nabla E_c$$

$$\vec{j}_n = n\nabla E_c\left(\mu_n - \frac{eD_n}{k_B T}\right)$$

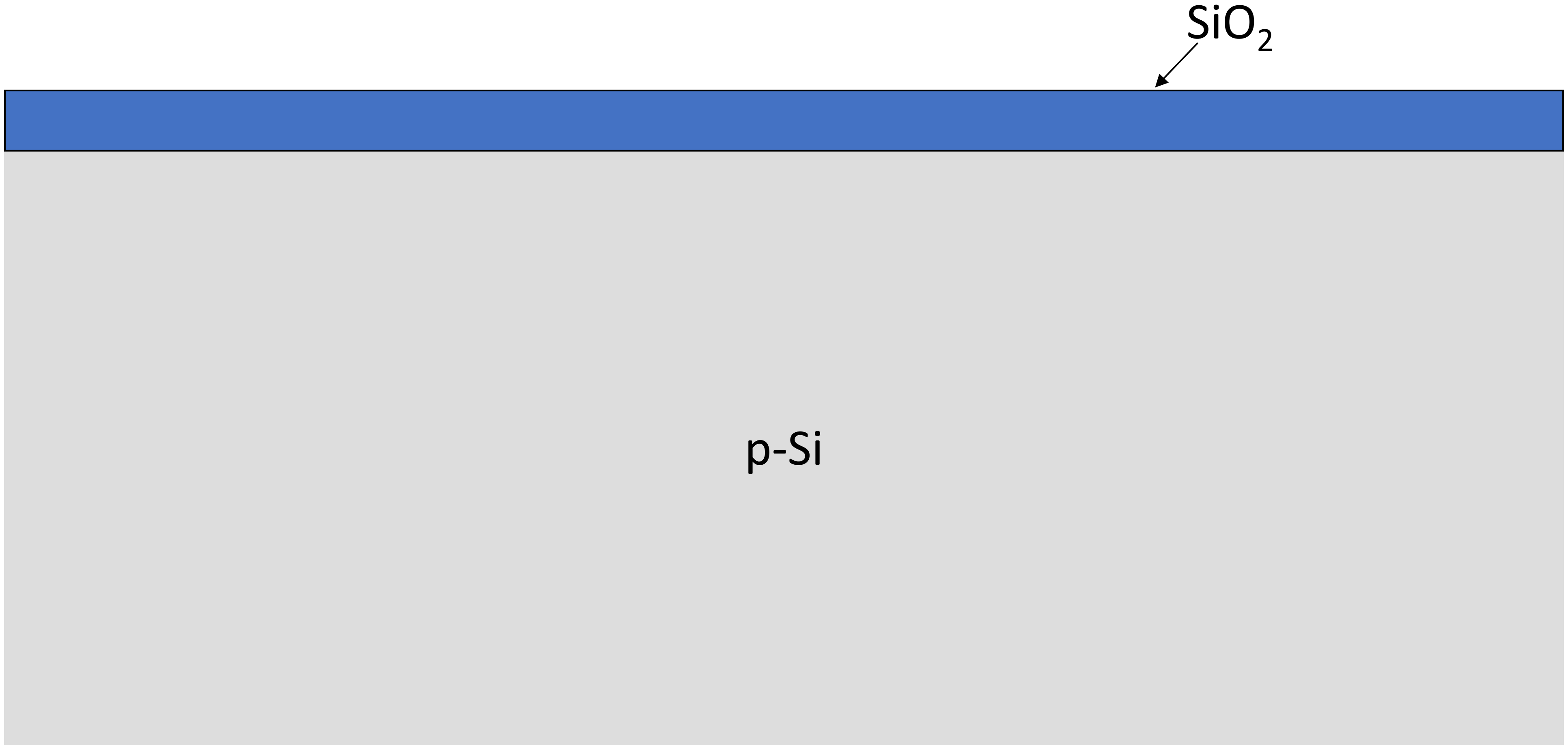
Einstein relation

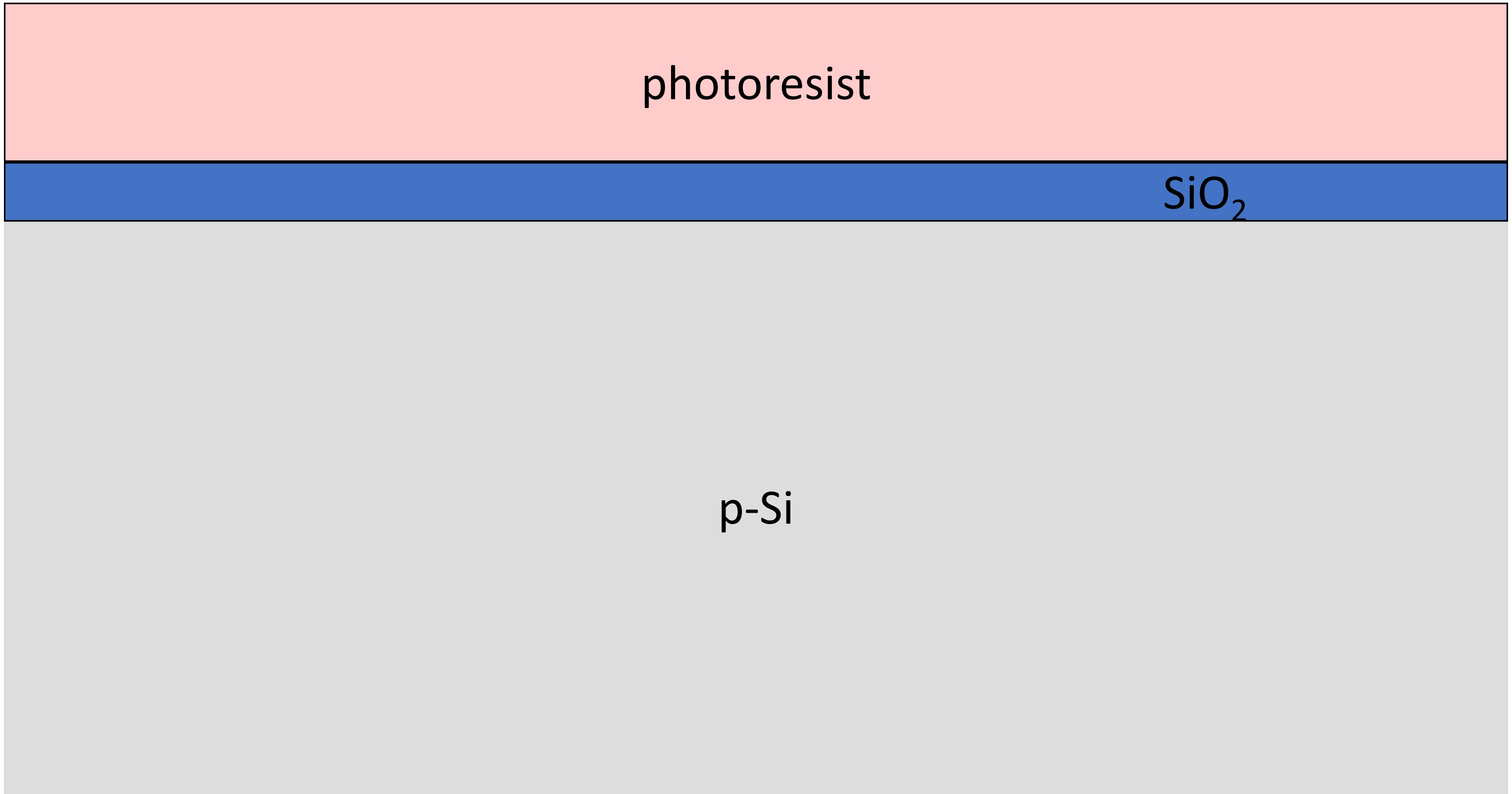
If the E_F is constant, j = 0.

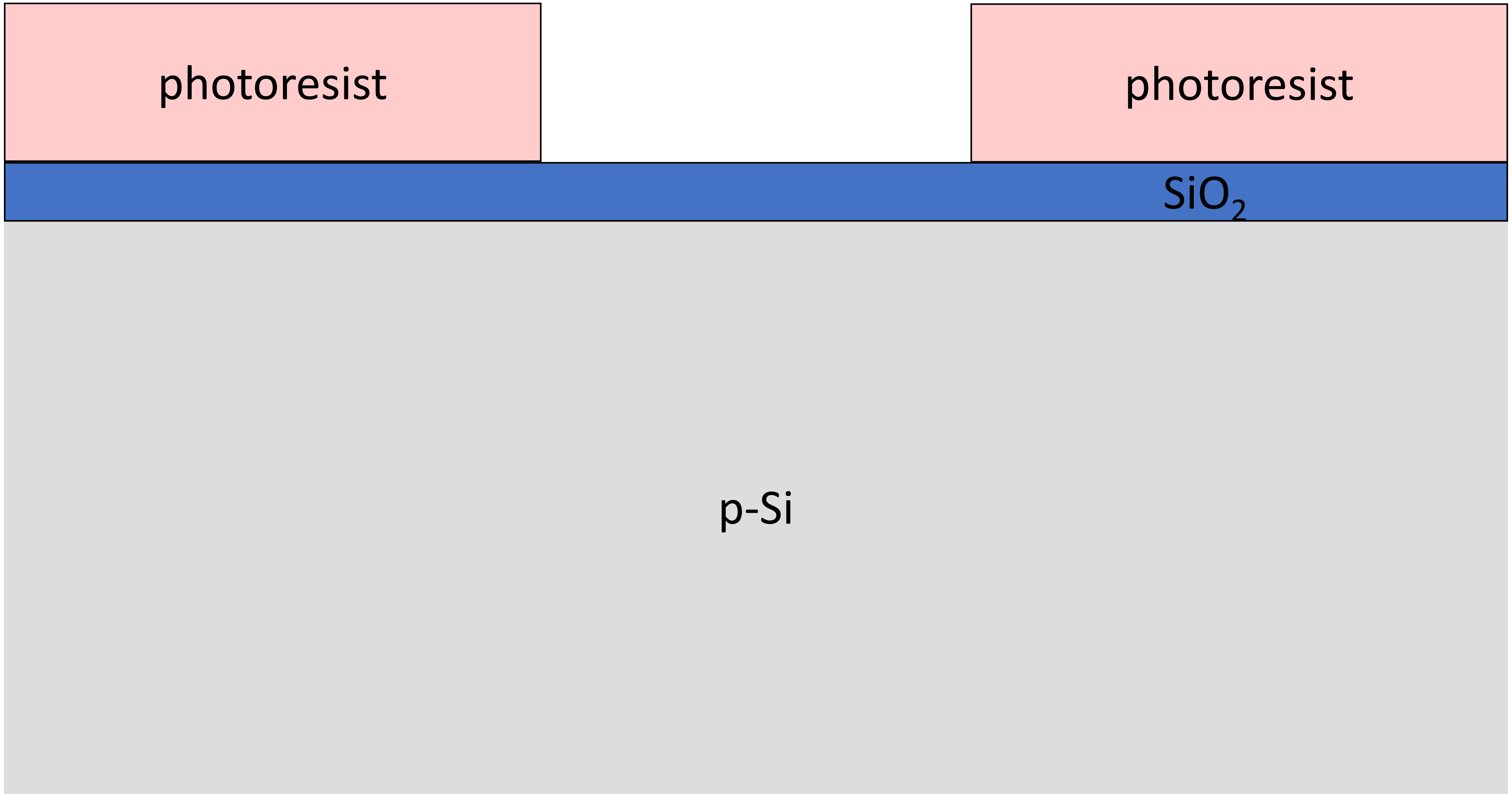
diode fabrication

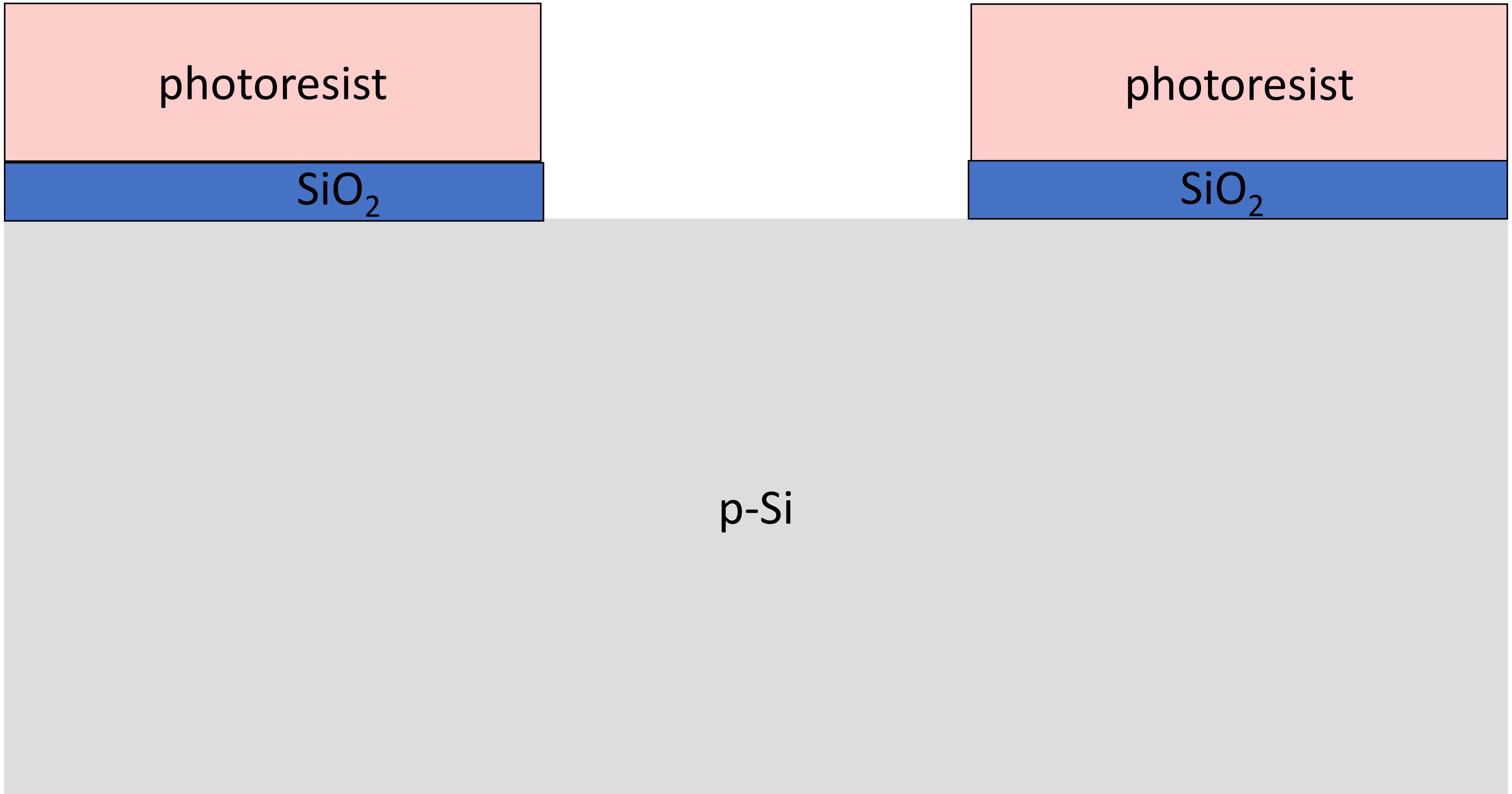


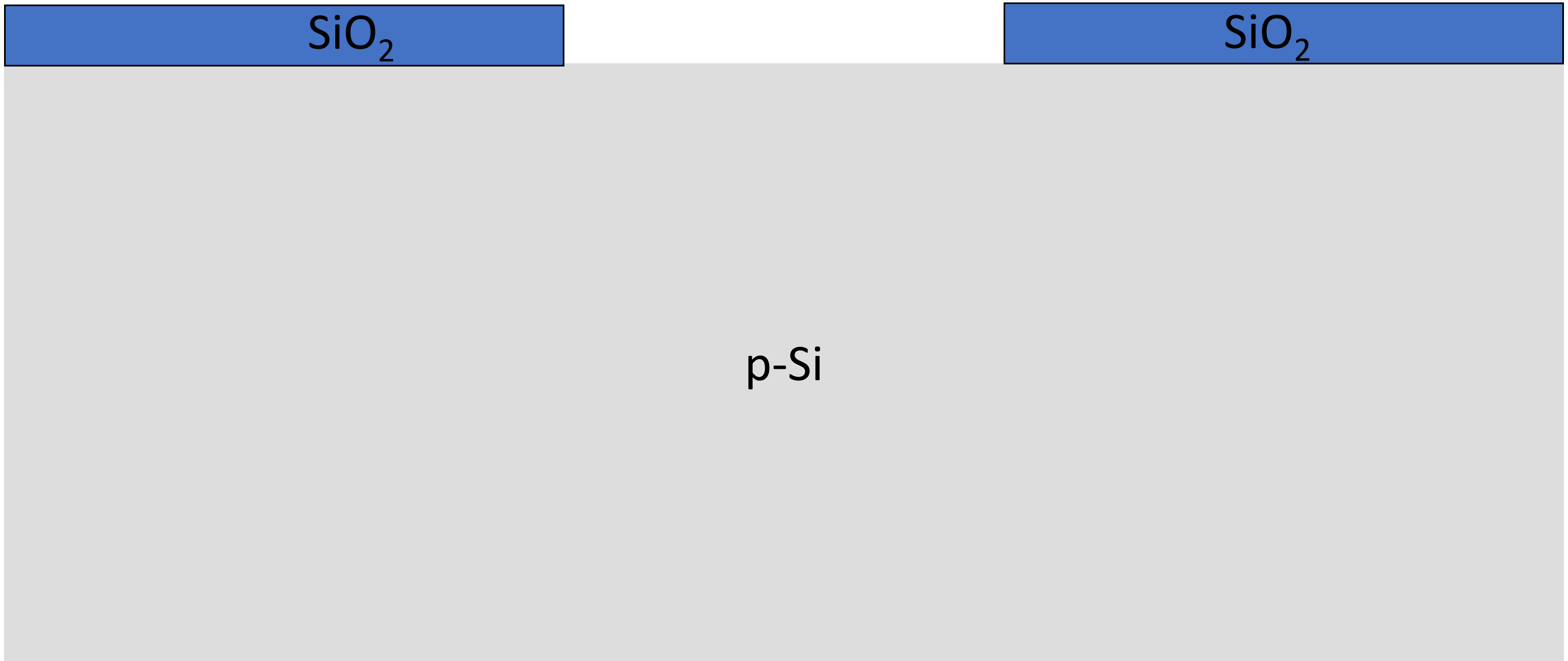
CVD oxide



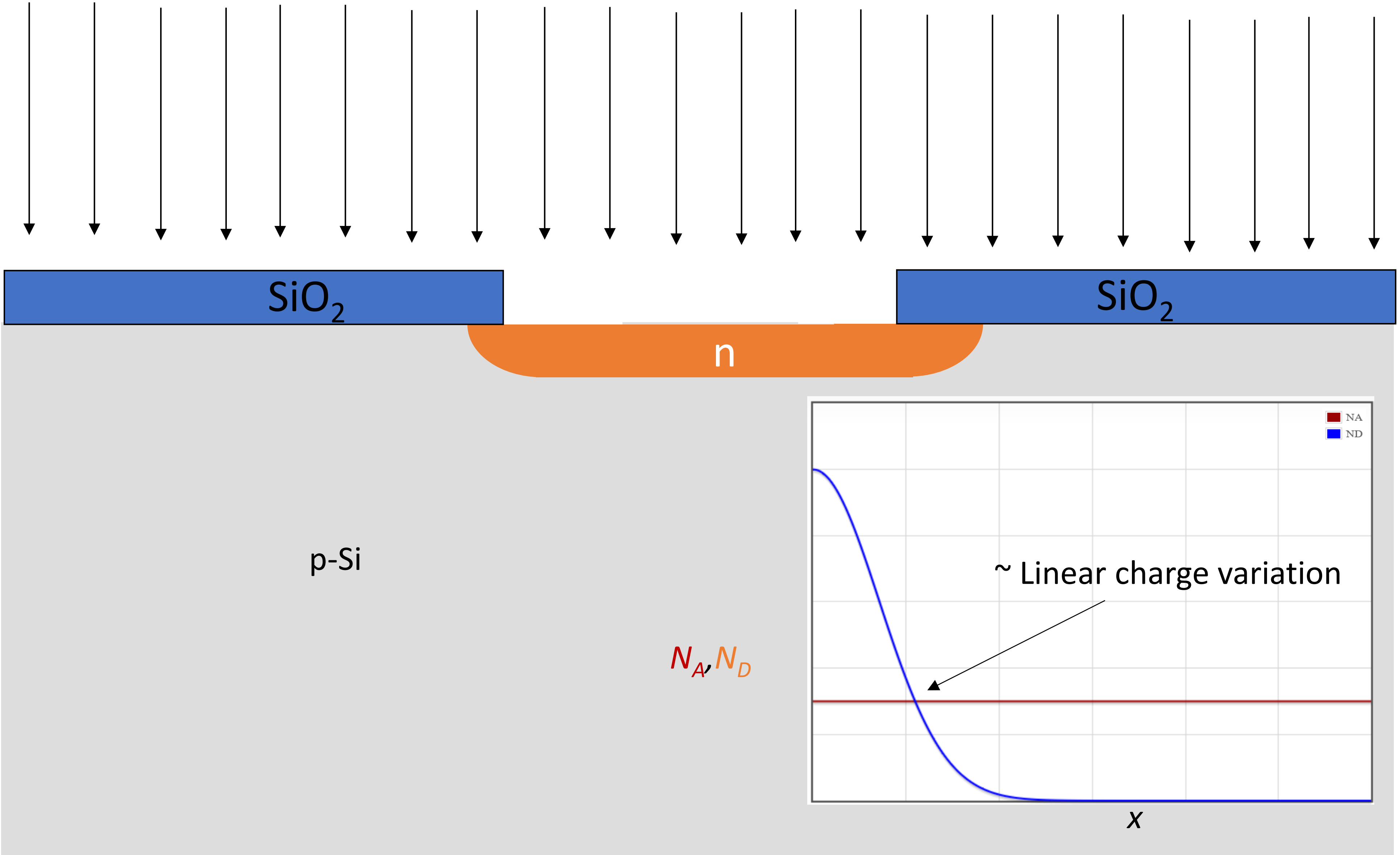

photoresist
SiO_2
p-Si


photoresist
photoresist
SiO_2
p-Si


photoresist
photoresist
SiO_2
SiO_2
p-Si

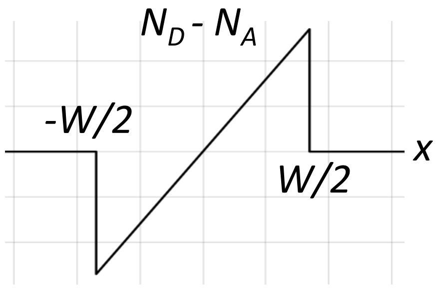

SiO_2
SiO_2
p-Si

Diffuse or Implant

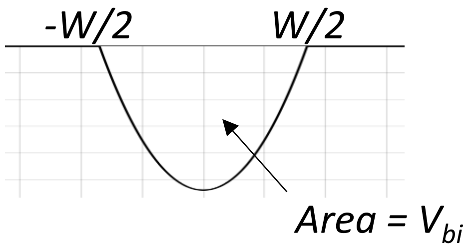


~ Linear charge variation

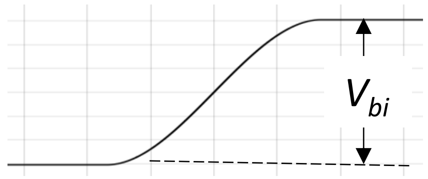
linearly graded junction



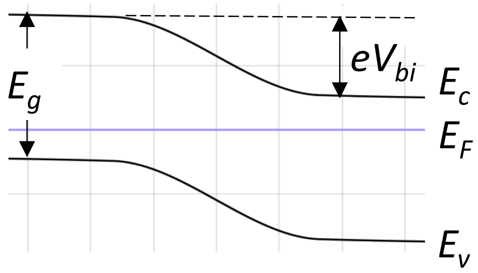
$$\rho = e(N_D(x) - N_A(x)) = eax$$



$$E = \int \frac{\rho}{\varepsilon} dx = \frac{-ea}{2\varepsilon}\left(\left(\frac{W}{2}\right)^2 - x^2\right) \qquad E_{max} = \frac{-eaW^2}{8\varepsilon}$$



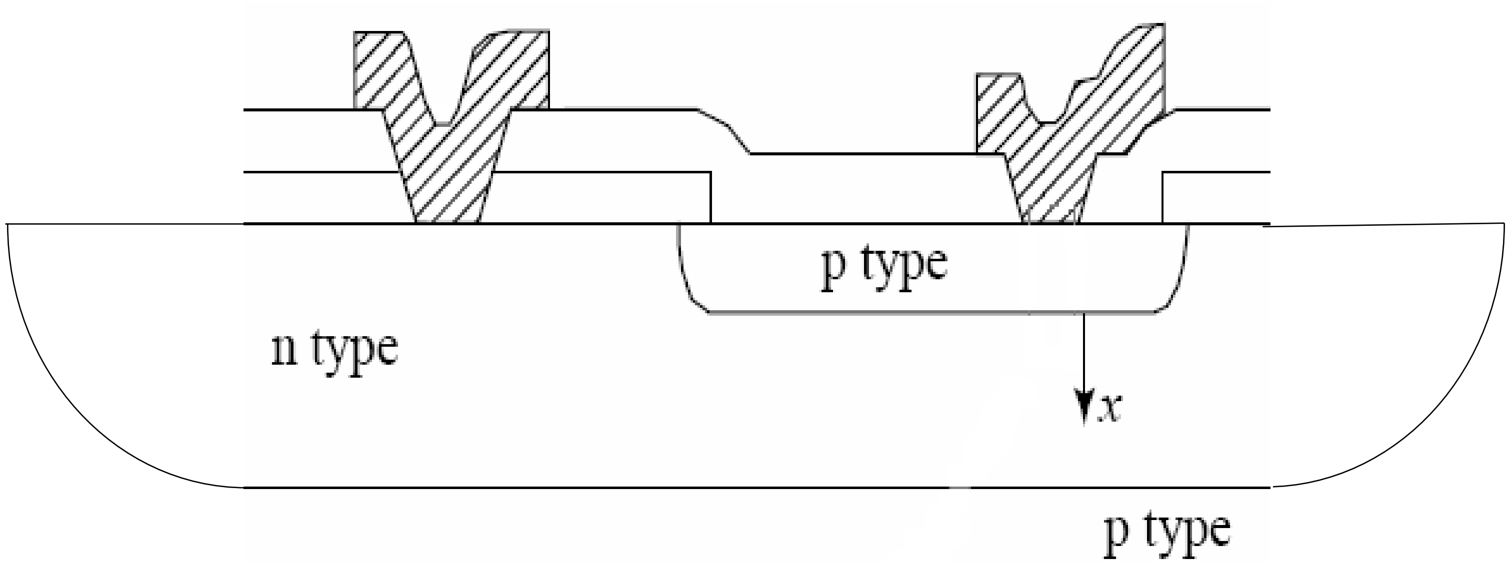
$$V = \int E\,dx = \frac{ea}{2\varepsilon}\left(\left(\frac{W}{2}\right)^2 x - \frac{x^3}{3}\right)$$



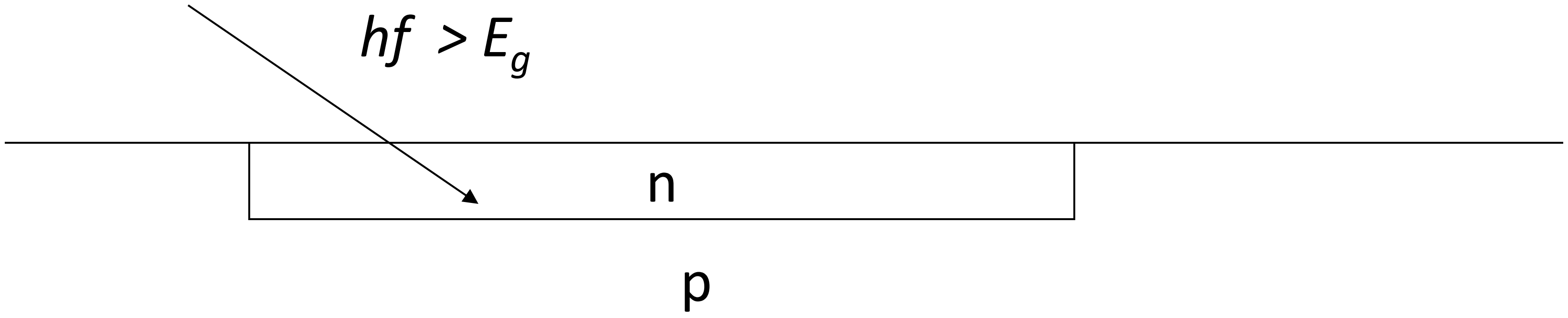
$$V_{bi} = \frac{eaW^3}{12\varepsilon}$$

http://lampz.tugraz.at/~hadley/psd/L6/linearly_graded.html

Isolation

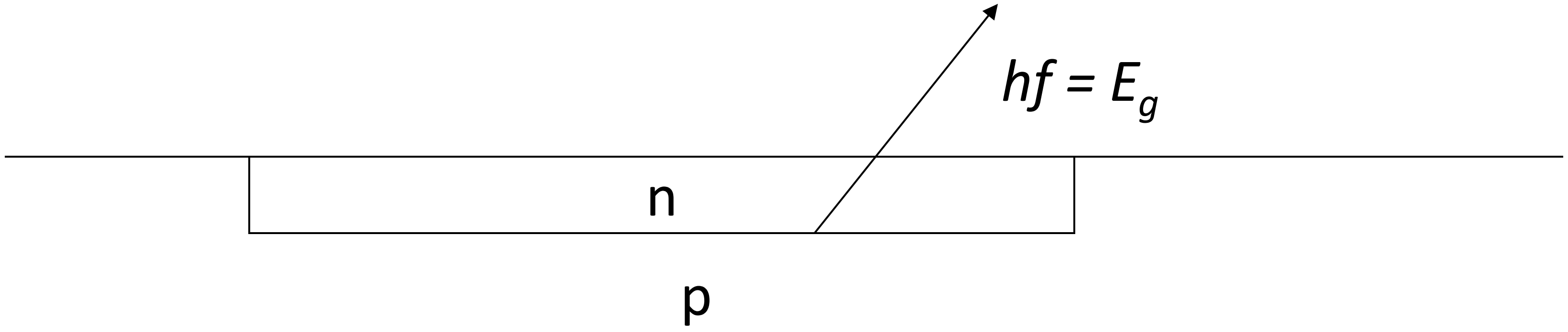


solar cell



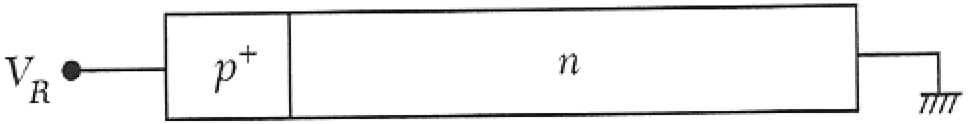
Light creates an electron-hole pair in the depletion region. The electric field sweeps the electrons towards the n-region and the holes towards the p-region.

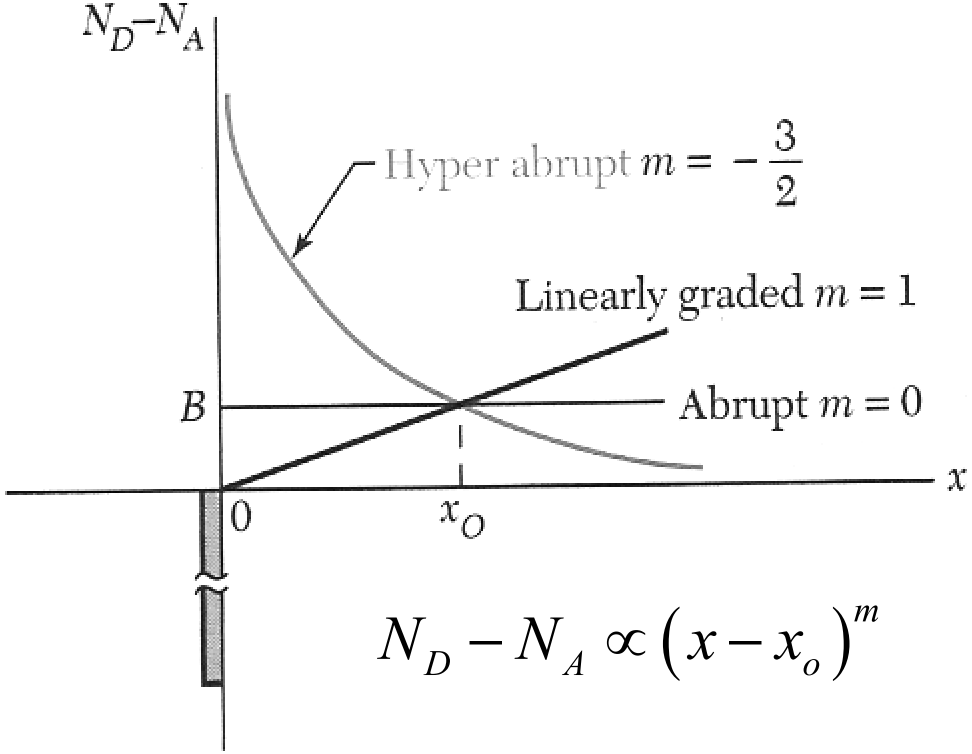
Light emitting diode



Electrons and holes are injected into the depletion region by forward biasing the junction. The electrons fall in the holes. For direct bandgap semiconductors, photons are emitted. For indirect bandgap semiconductors, phonons are emitted.

Varactor





$$N_D - N_A \propto (x - x_o)^m$$

$$C_j \propto (V_{bi} + V_R)^{-n}$$

abrupt: *n* = 1/2
linearly graded: *n* = 1/3

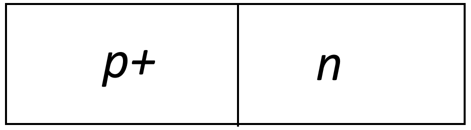
n = 1/(*m*+2)

Capacitance-voltage characteristics

specific capacitance $C_j = \dfrac{\varepsilon}{W} \quad \mathrm{F\,m^{-2}}$

abrupt junction: $W = \dfrac{\varepsilon}{C_j} = \sqrt{\dfrac{2\varepsilon(N_D + N_A)(V_{bi} - V)}{eN_D N_A}}$

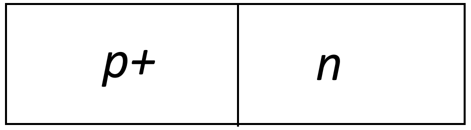
a one-sided abrupt junction in reverse bias:



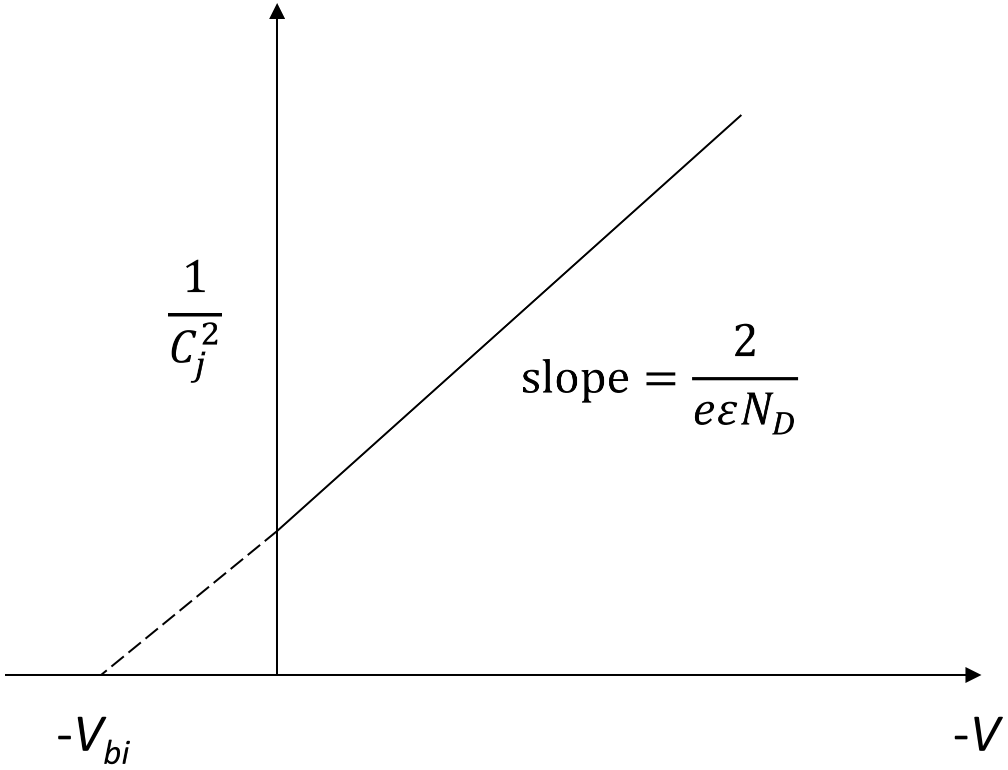
$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{e\varepsilon N_D}$$

Capacitance-voltage characteristics

a one-sided
abrupt junction in
reverse bias:

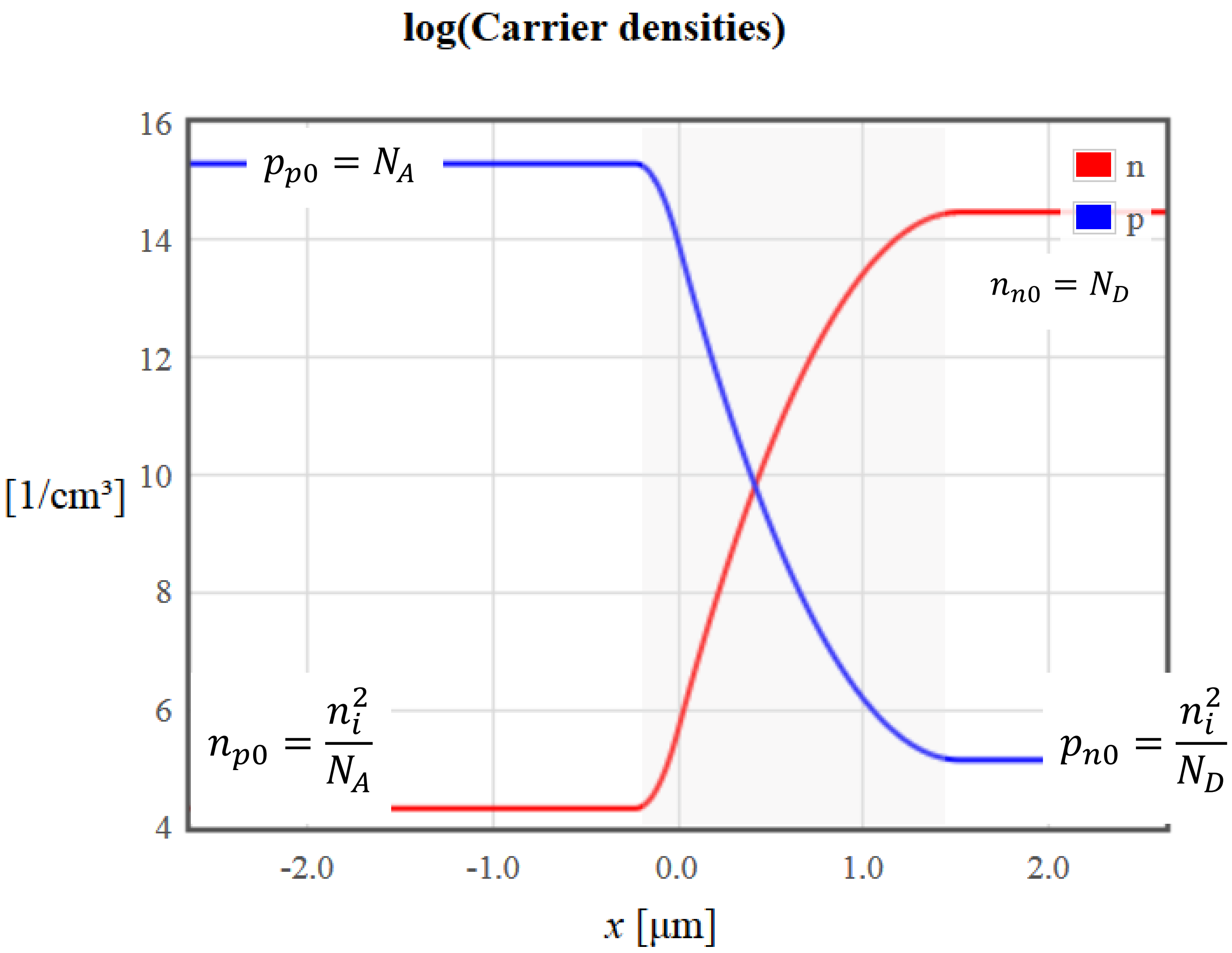


$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{e\varepsilon N_D}$$



slope gives impurity concentration and the intercept gives V_{bi}

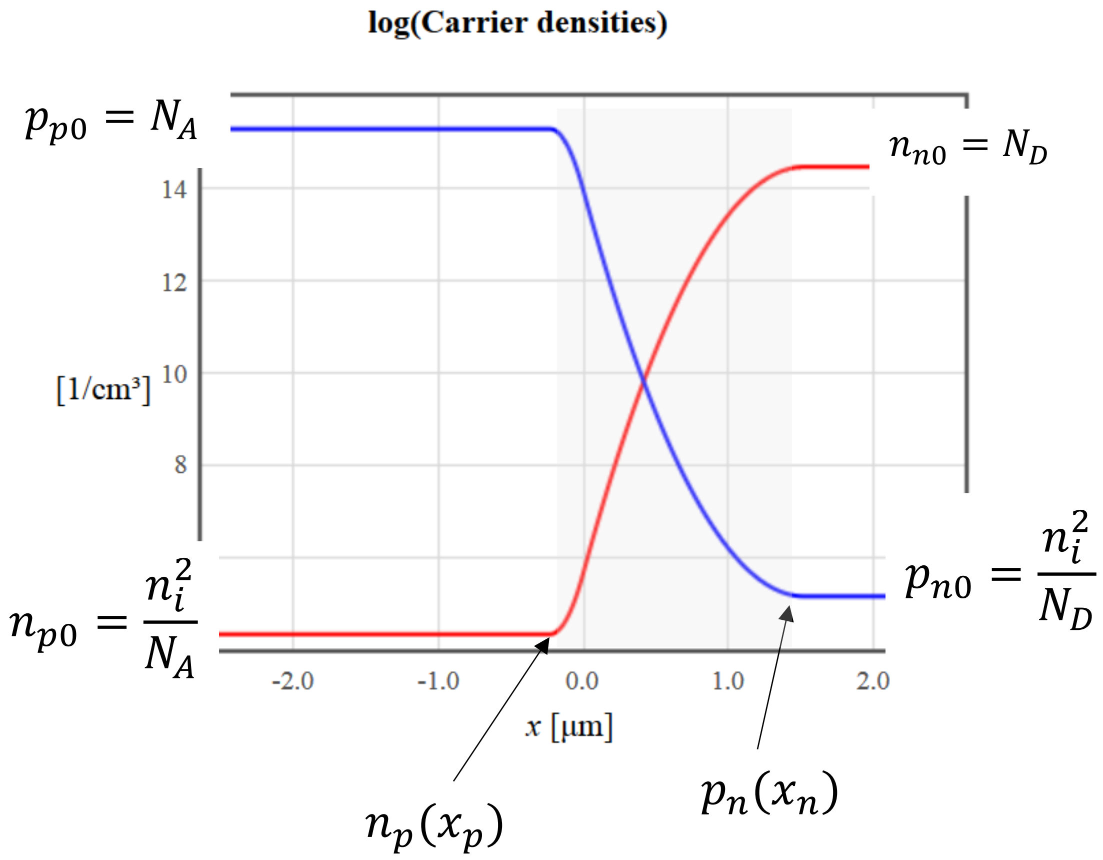
equilibrium concentrations, $V = 0$



$$n_{p0}p_{p0} = n_{n0}p_{n0} = n_i^2$$

bias voltage, $V = 0$

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right) = k_B T \ln\left(\frac{N_D}{n_{p0}}\right) = k_B T \ln\left(\frac{N_A}{p_{n0}}\right)$$



$p_{p0} = N_A$

$n_{n0} = N_D$

$n_{p0} = \dfrac{n_i^2}{N_A}$

$p_{n0} = \dfrac{n_i^2}{N_D}$

$n_p(x_p)$

$p_n(x_n)$

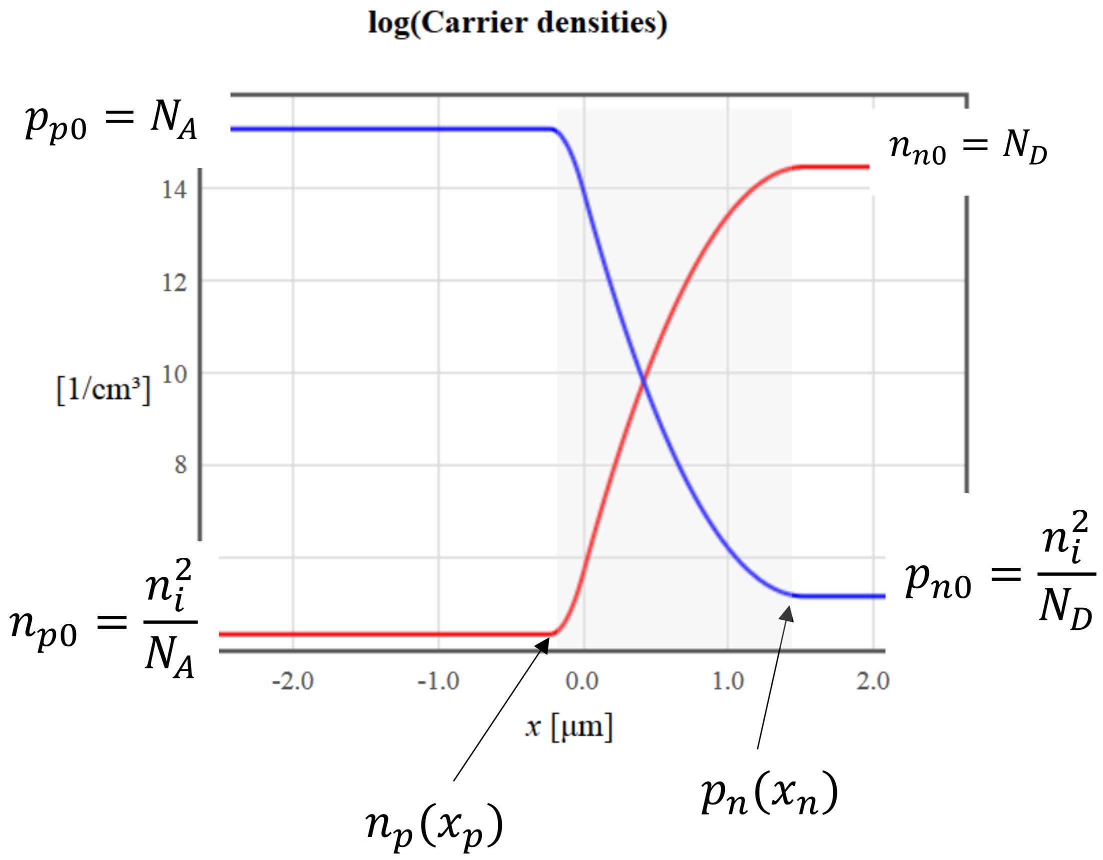
$$n_{p0} p_{p0} = n_{n0} p_{n0} = n_i^2$$

$$\boxed{V = 0}$$

$$\boxed{\begin{aligned} n_{p0} &= N_D \exp\left(\frac{-eV_{bi}}{k_B T}\right) \\ p_{n0} &= N_A \exp\left(\frac{-eV_{bi}}{k_B T}\right) \end{aligned}}$$

bias voltage, $V \neq 0$

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right) = k_B T \ln\left(\frac{N_D}{n_{p0}}\right) = k_B T \ln\left(\frac{N_A}{p_{n0}}\right)$$



$$n_{p0} p_{p0} = n_{n0} p_{n0} = n_i^2$$

$V = 0$

$$n_{p0} = N_D \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

$$p_{n0} = N_A \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

$V \neq 0$

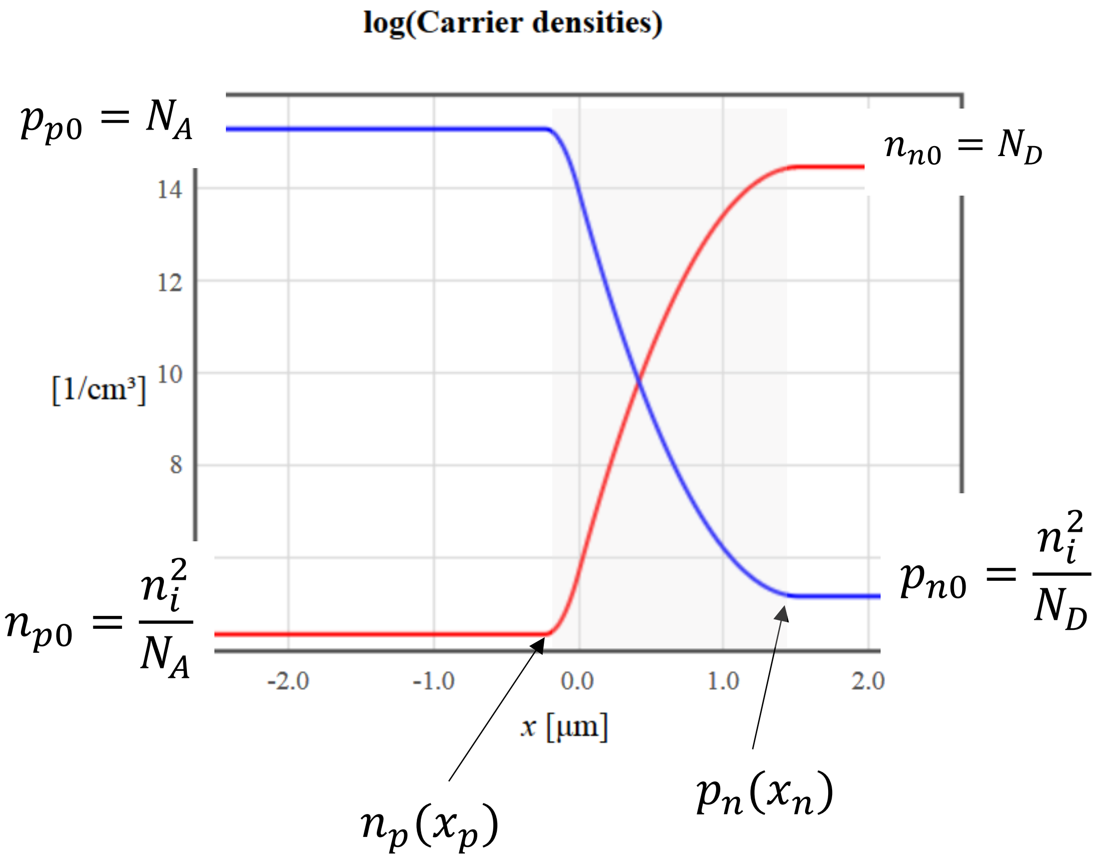
$$n_p(x_p) = N_D \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$

$$p_n(x_n) = N_A \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$

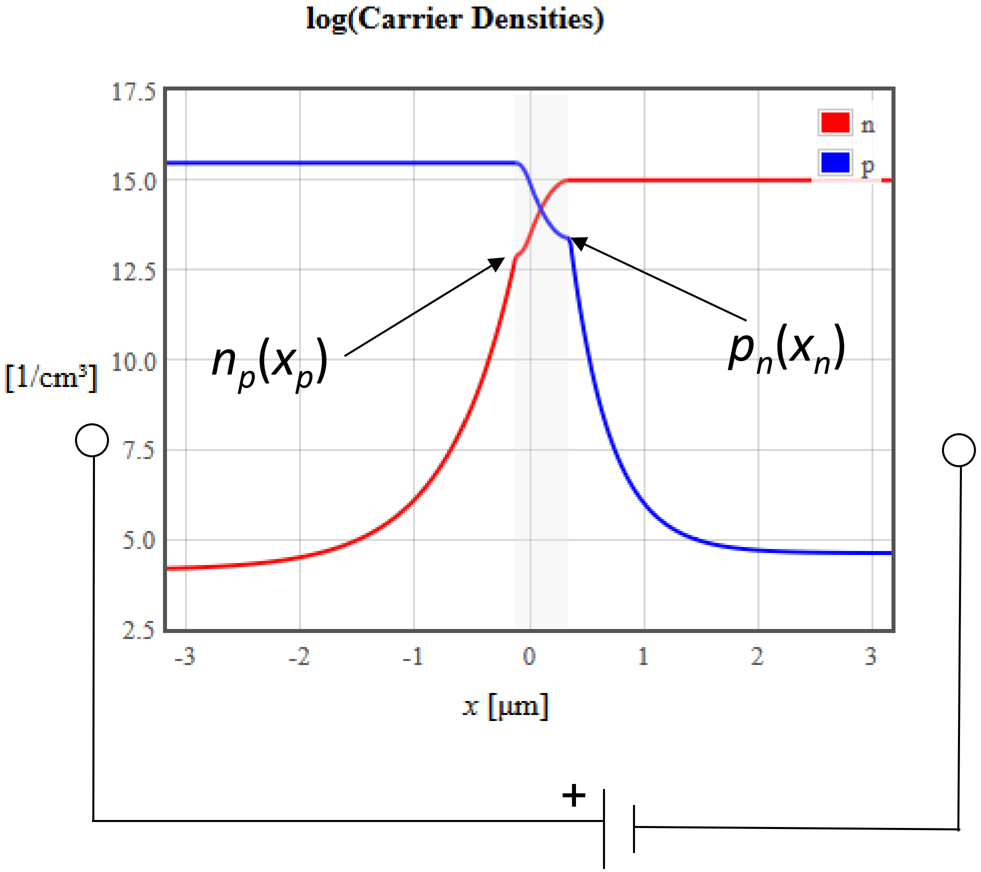
bias voltage, $V \neq 0$

$$n_p(x_p) = N_D \exp\left(\frac{-e(V_{bi}-V)}{k_BT}\right) = n_{p0}\exp\left(\frac{eV}{k_BT}\right)$$

$$p_n(x_n) = N_A \exp\left(\frac{-e(V_{bi}-V)}{k_BT}\right) = p_{n0}\exp\left(\frac{eV}{k_BT}\right)$$



forward bias, *V* > 0

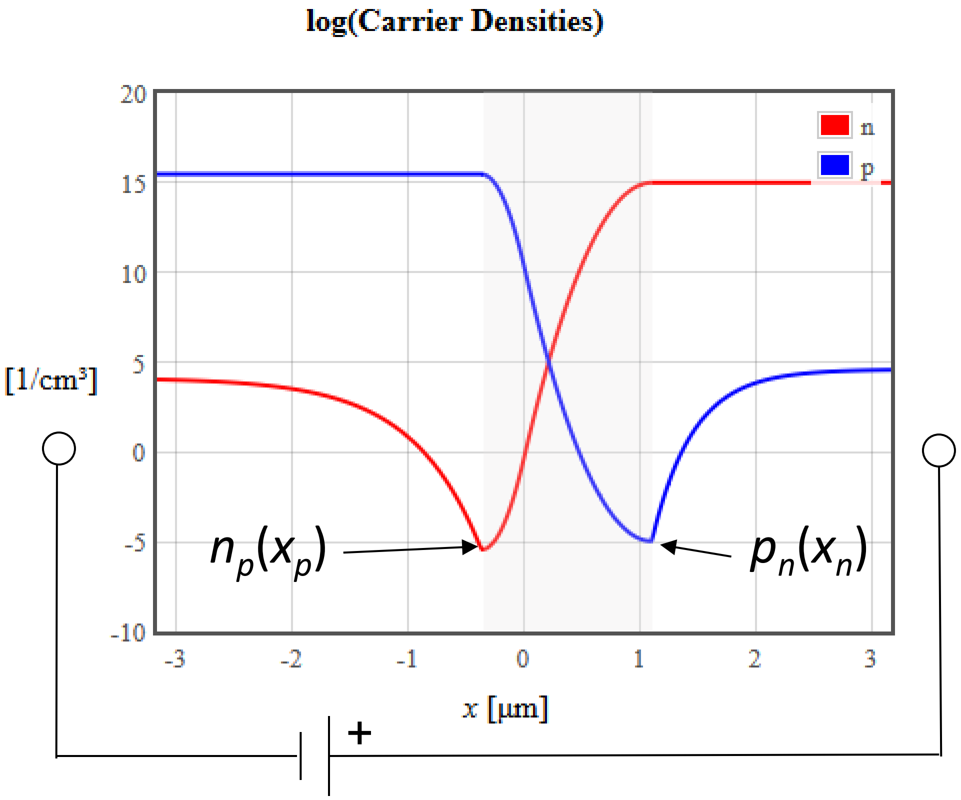


Electrons and holes are driven towards the junction.
The depletion region becomes narrower

$$n_p(x_p) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$
$$p_n(x_n) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

Minority electrons are injected into the p-region
Minority holes are injected into the n-region

reverse bias, $V < 0$



Electrons and holes are driven away from the junction.
The depletion region becomes wider

$$n_p(x_p) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

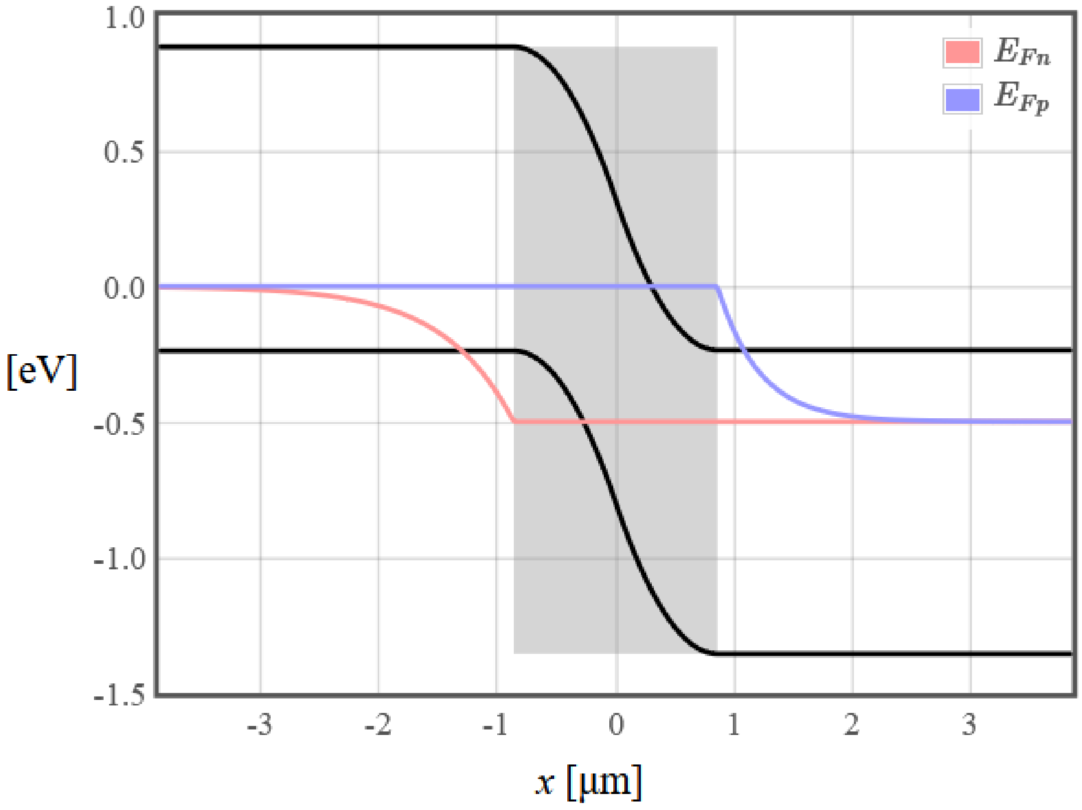
Minority electrons are extracted from the p-region by the electric field
Minority holes are extracted from the n-region by the electric field

Quasi Fermi level

When the charge carriers are not in equilibrium the Fermi energy can be different for electrons and holes.

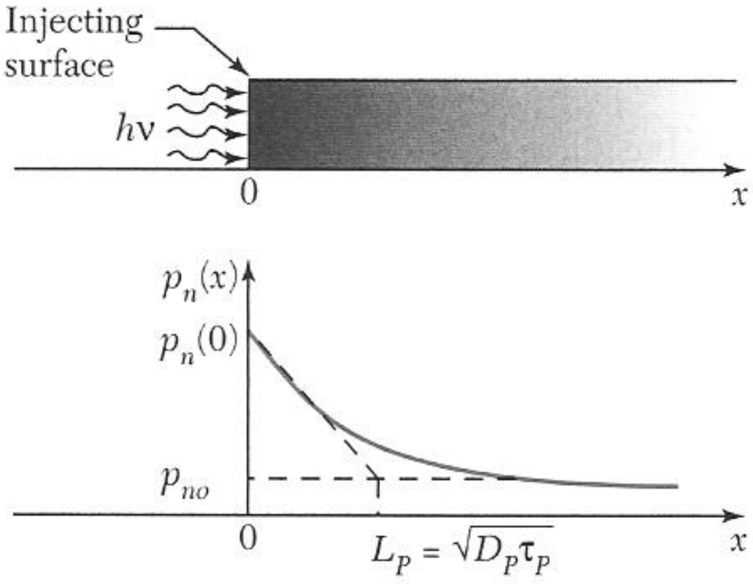
$$n = N_c \exp\left(\frac{E_{Fn} - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_{Fp}}{k_B T}\right)$$



Review of diffusion

n-type



$$D_p \frac{\partial^2 p_n}{\partial x^2} = \frac{p_n - p_{n0}}{\tau_p}$$

recombination time

$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

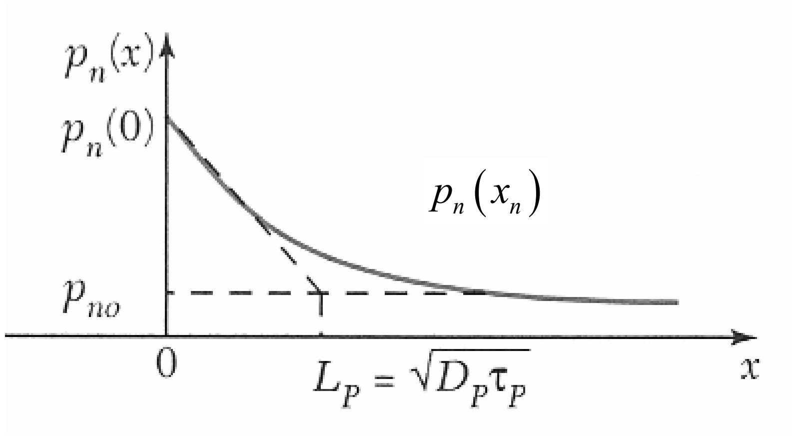
$$L_p = \sqrt{D_p \tau_p}$$

diffusion length

Injection only occurs at the surface.
There the minority carrier density is $p_n(0)$.

Diffusion current

n-type



$$p_n(x) = p_{n0} + (p_n(x_n) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

$$J_{diff,p} = -eD_p \frac{dp}{dx}$$

$$J_{diff,p} = -eD_p \frac{dp}{dx} = (p_n(x_n) - p_{n0}) \frac{eD_p}{L_p} \exp\left(\frac{-x}{L_p}\right)$$

At the edge of the depletion region:

$$J_{diff,p} = -eD_p \frac{dp}{dx} = (p_n(x_n) - p_{n0}) \frac{eD_p}{L_p}$$

Diffusion current

$$J_{diff,p} = (p_n(x_n) - p_{n0})\frac{eD_p}{L_p}$$

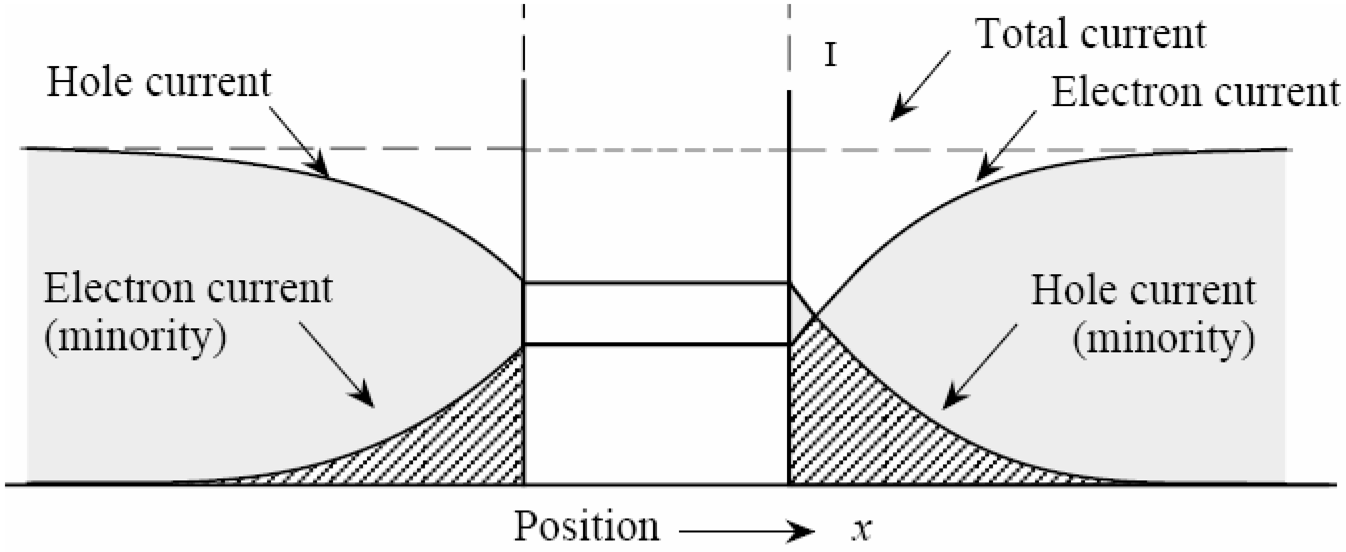
$$p_n(x_n) = p_{n0}\exp\left(\frac{eV}{k_B T}\right)$$

$$J_{diff,p} = p_{n0}\frac{eD_p}{L_p}\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

Diffusion current

$$J_{diff,p} = \frac{p_{n0} e D_p}{L_p}\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

$$J_{diff,n} = \frac{n_{p0} e D_n}{L_n}\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

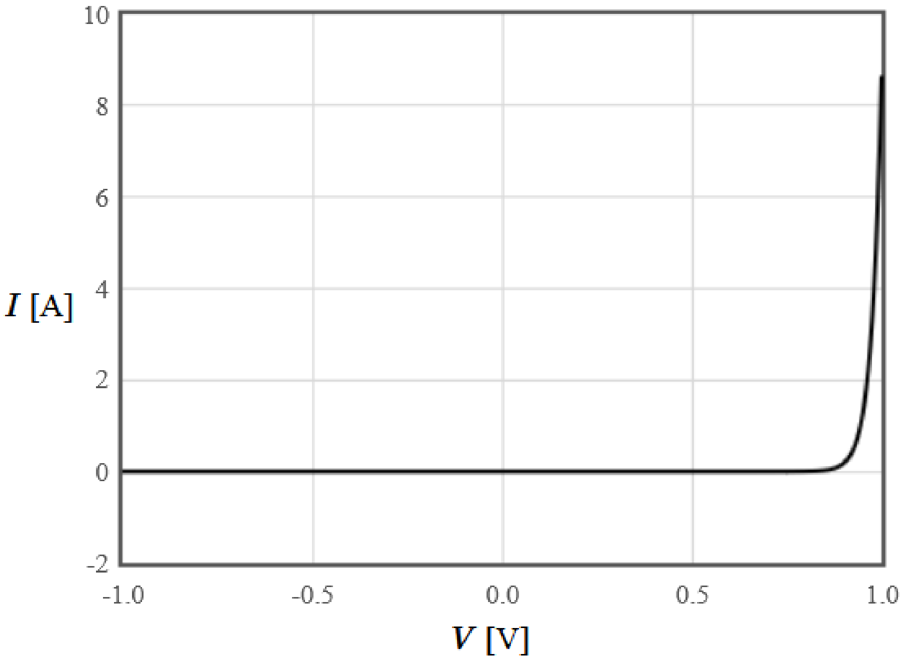


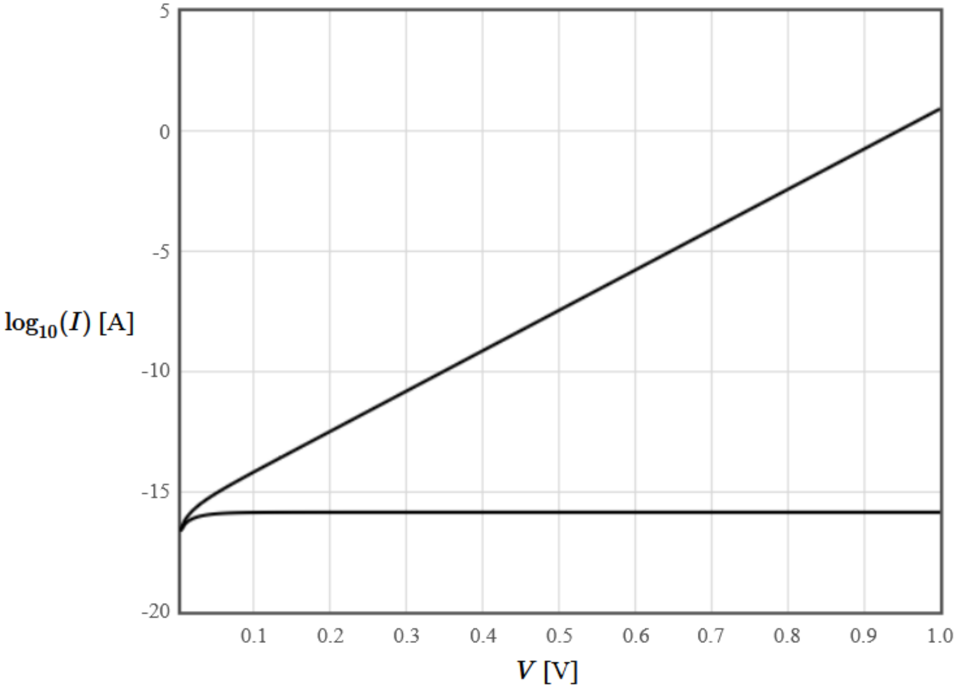
Diode current

$$I = eA\left(\frac{p_{n0}D_p}{L_p} + \frac{n_{p0}D_n}{L_n}\right)\left(\exp\left(\frac{eV}{k_BT}\right) - 1\right) = I_s\left(\exp\left(\frac{eV}{k_BT}\right) - 1\right)$$

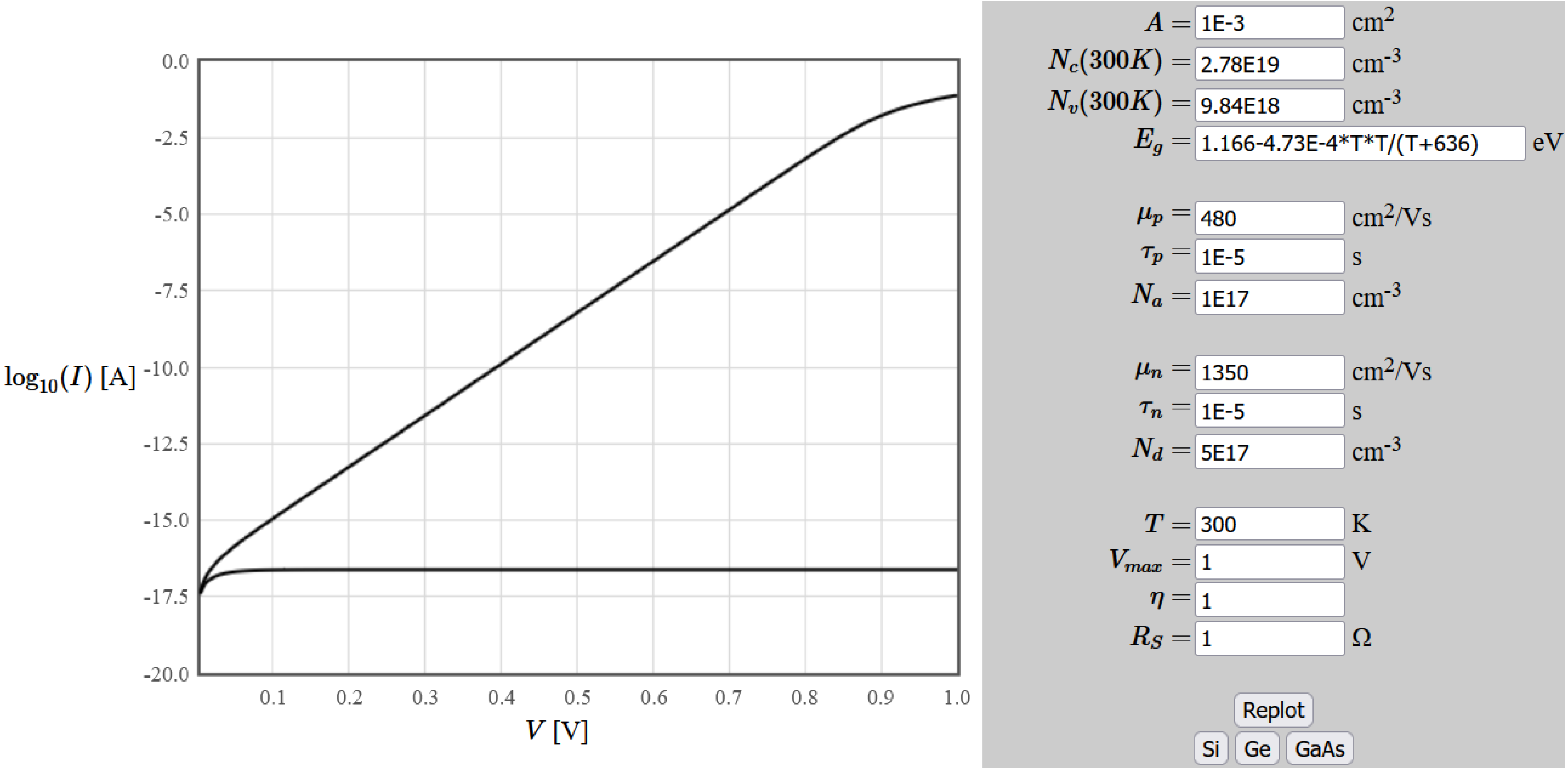
Area

Saturation current





Diode I-V characteristics



$A =$ 1E-3 cm^2
$N_c(300K) =$ 2.78E19 cm^{-3}
$N_v(300K) =$ 9.84E18 cm^{-3}
$E_g =$ 1.166-4.73E-4*T*T/(T+636) eV

$\mu_p =$ 480 cm^2/Vs
$\tau_p =$ 1E-5 s
$N_a =$ 1E17 cm^{-3}

$\mu_n =$ 1350 cm^2/Vs
$\tau_n =$ 1E-5 s
$N_d =$ 5E17 cm^{-3}

$T =$ 300 K
$V_{max} =$ 1 V
$\eta =$ 1
$R_S =$ 1 Ω

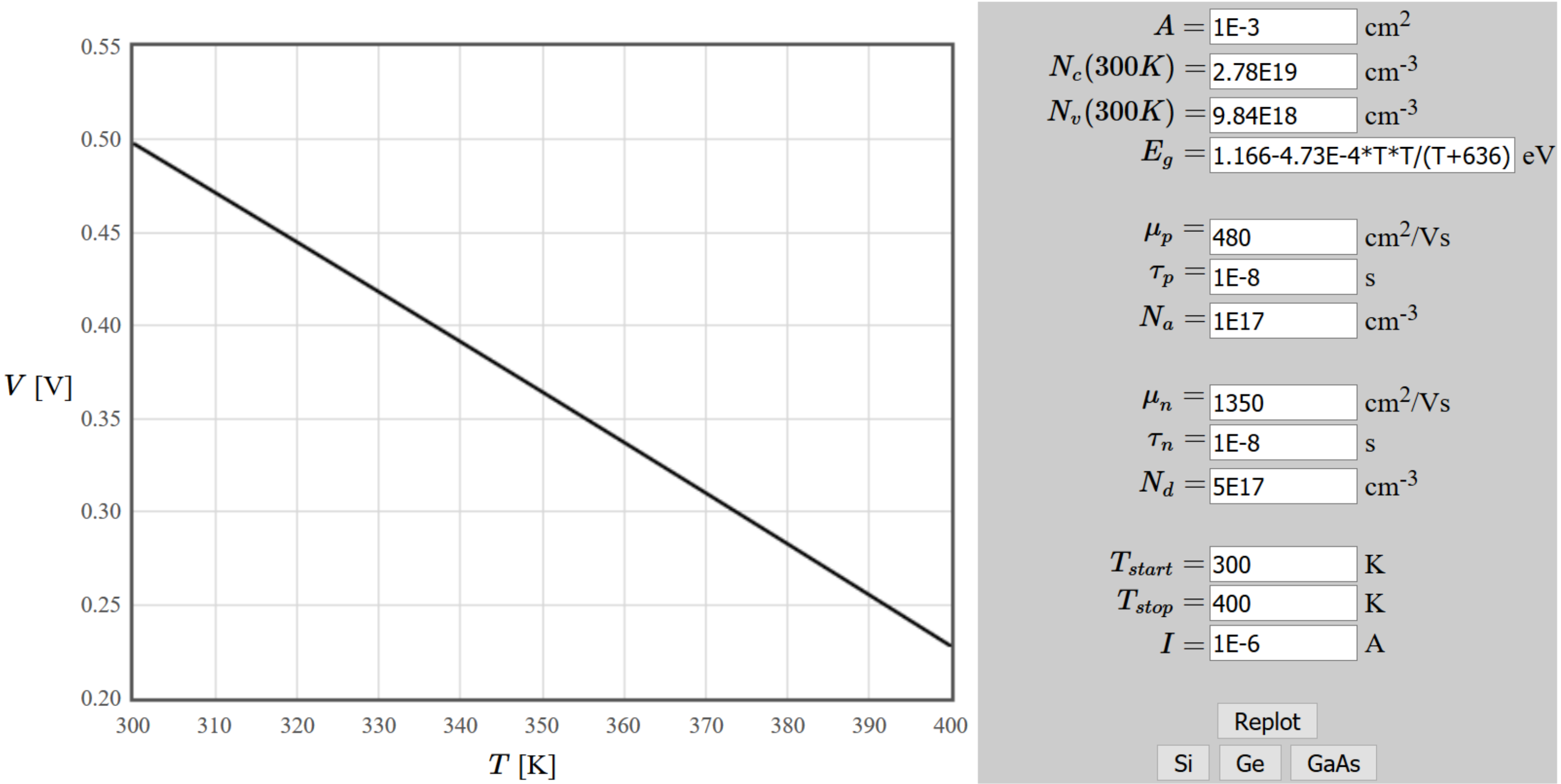
http://lampz.tugraz.at/~hadley/psd/L6/pnIV.php

Thermometer

$$I_S = Aen_i^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)$$

$$n_i = \sqrt{N_c\left(\frac{T}{300}\right)^{3/2} N_v\left(\frac{T}{300}\right)^{3/2}} \exp\left(\frac{-E_g}{2k_BT}\right)$$

$$D_n = \frac{\mu_n k_B T}{e}$$



$A =$ 1E-3 cm^2

$N_c(300K) =$ 2.78E19 cm^{-3}

$N_v(300K) =$ 9.84E18 cm^{-3}

$E_g =$ 1.166-4.73E-4*T*T/(T+636) eV

$\mu_p =$ 480 cm^2/Vs

$\tau_p =$ 1E-8 s

$N_a =$ 1E17 cm^{-3}

$\mu_n =$ 1350 cm^2/Vs

$\tau_n =$ 1E-8 s

$N_d =$ 5E17 cm^{-3}

$T_{start} =$ 300 K

$T_{stop} =$ 400 K

$I =$ 1E-6 A