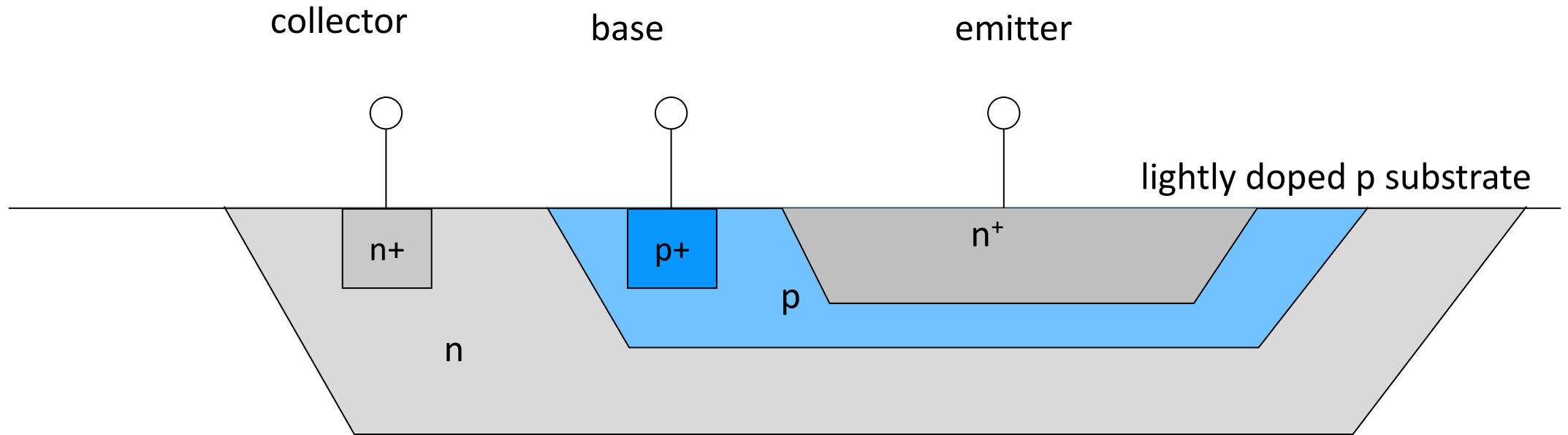


11

Bipolar junction transistors (BJT)

bipolar transistors

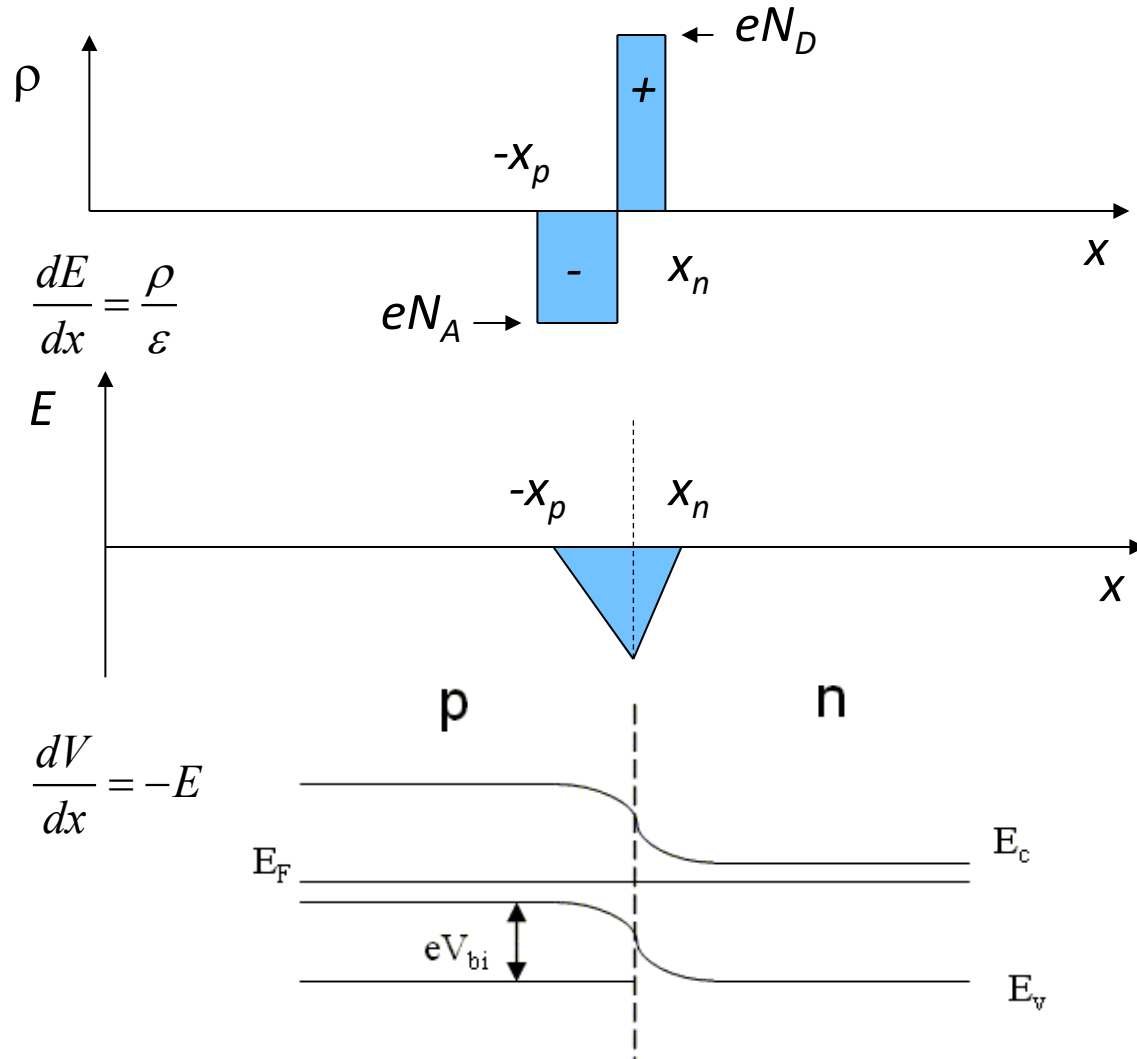
npn transistor



Used in

- front-end high-frequency receivers (mobile telephones)
- low input impedance amplifiers
- power devices

abrupt junction



$$eV_{bi} = k_B T \ln \left(\frac{N_D N_A}{n_i^2} \right)$$

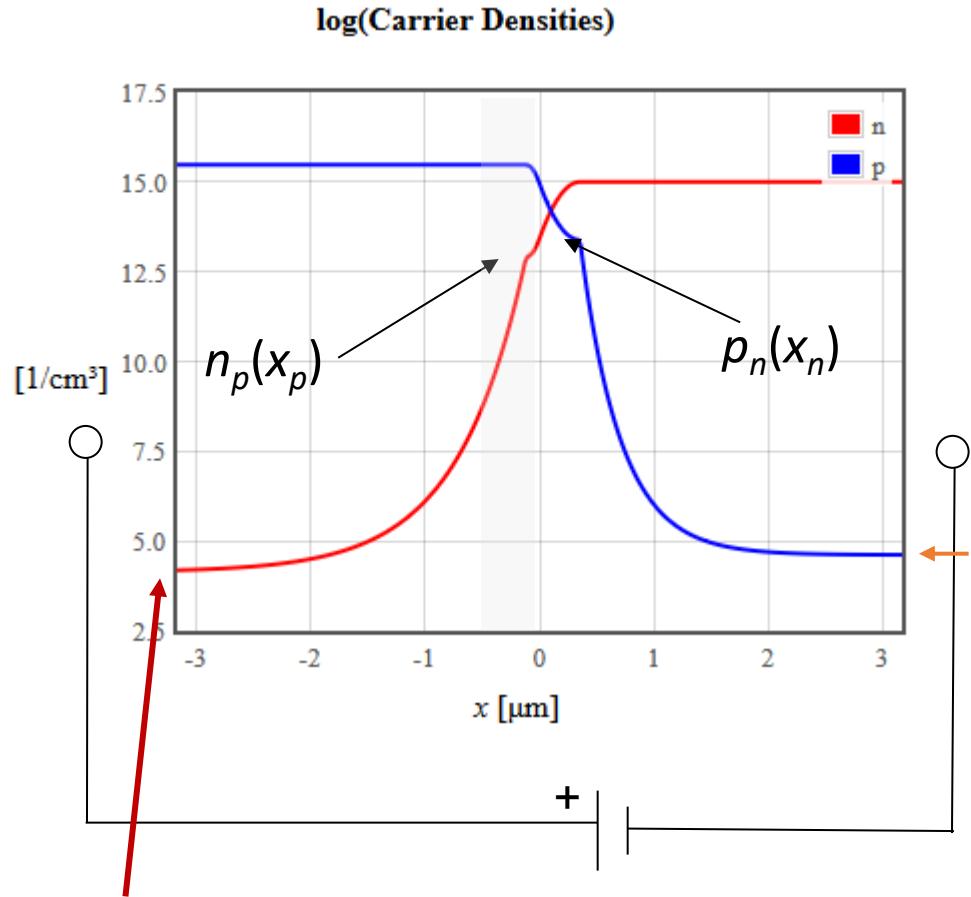
$$E = -\frac{eN_A}{\epsilon} (x + x_p) \quad -x_p > x > 0$$

$$E = \frac{eN_D}{\epsilon} (x - x_n) \quad 0 > x > x_n$$

$$V = \frac{eN_A}{\epsilon} \left(\frac{x^2}{2} + xx_p \right) \quad -x_p > x > 0$$

$$V = \frac{-eN_D}{\epsilon} \left(\frac{x^2}{2} - xx_n \right) \quad 0 > x > x_n$$

forward bias $V > 0$



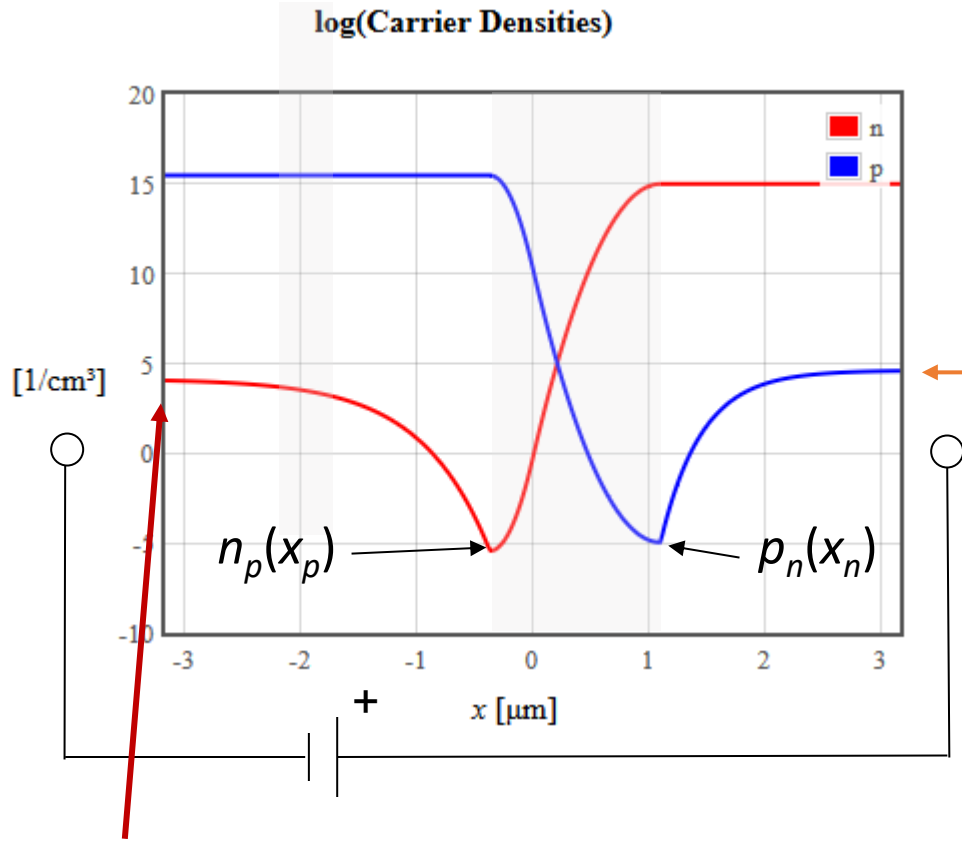
- electrons and holes are driven towards the junction
- depletion region becomes narrower

$$n_p(x_p) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

Minority electrons injected into the p-region
Minority holes injected into the n-region

reverse bias, $V < 0$



- electrons and holes are driven away from the junction
- depletion region becomes wider

$$p_{n0} = \frac{n_i^2}{N_D}$$

$$n_p(x_p) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

$$n_{p0} = \frac{n_i^2}{N_A}$$

Minority electrons are extracted from the p-region by the electric field
Minority holes are extracted from the n-region by the electric field

long / short diodes

$N_A =$ 1E15 1/cm ³	$N_D =$ 1E15 1/cm ³	$E_g =$ 1.166-4.73E-4*T*T/(T+636) eV	$d_p =$ -2 μm
$N_v(300) =$ 9.84E18 1/cm ³	$N_c(300) =$ 2.78E19 1/cm ³	$\epsilon_r =$ 12 $T =$ 300 K	$d_n =$ 2 μm
$\mu_p =$ 480 cm ² /V s	$\mu_n =$ 1350 cm ² /V s	$\tau_p =$ 1E-7 s	$\tau_n =$ 1E-7 s

$V =$ V Submit

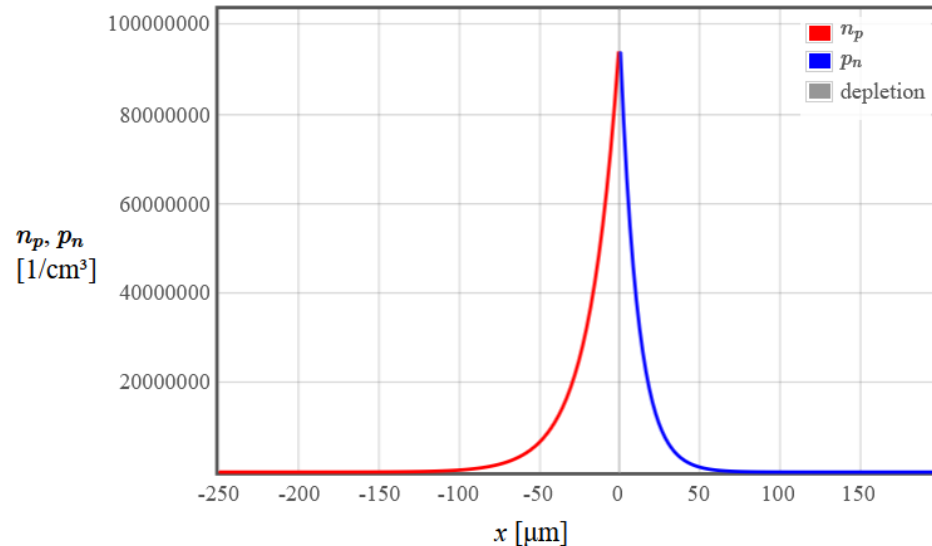
$$E_g = 1.12 \text{ eV} \quad W = |x_n| + |x_p| = 1.05 \text{ μm} \quad x_p = -0.527 \text{ μm} \quad x_n = 0.527 \text{ μm} \quad V_{bi} = 0.618 \text{ V} \quad n_i = 6.41\text{e}+9 \text{ cm}^{-3}$$

$$D_p = 12.4 \text{ cm}^2/\text{s} \quad D_n = 34.9 \text{ cm}^2/\text{s} \quad L_p = 11.1 \text{ μm} \quad L_n = 18.7 \text{ μm} \quad n_{p0} = 4.10\text{e}+4 \text{ cm}^{-3} \quad p_{n0} = 4.10\text{e}+4 \text{ cm}^{-3}$$

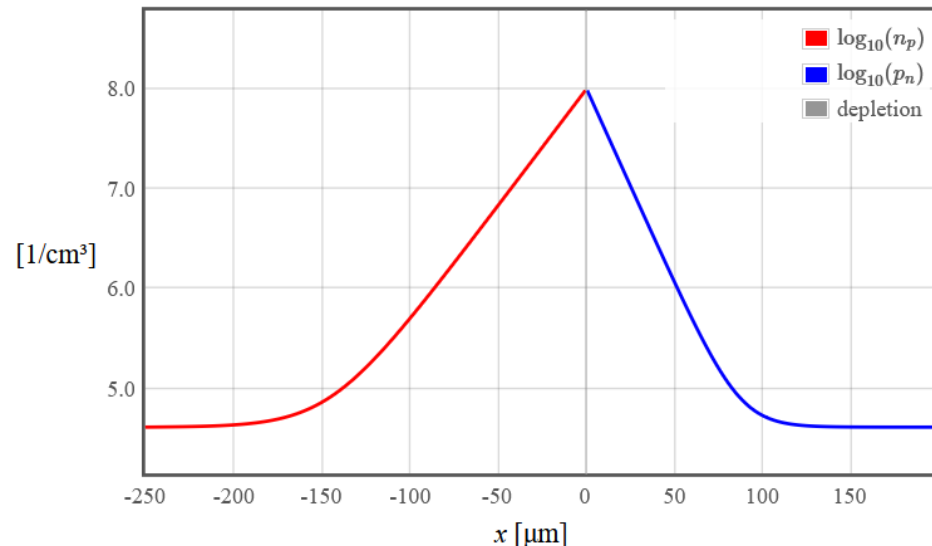
$$A_n = -5.35\text{e}+8 \text{ cm}^{-3} \quad B_n = 6.62\text{e}+8 \text{ cm}^{-3} \quad A_p = 4.24\text{e}+8 \text{ cm}^{-3} \quad B_p = -2.96\text{e}+8 \text{ cm}^{-3}$$

$$j_n = 0.00000357 \text{ A cm}^{-2} \quad j_p = 0.00000128 \text{ A cm}^{-2} \quad j_{\text{diff}} = 0.00000485 \text{ A cm}^{-2}$$

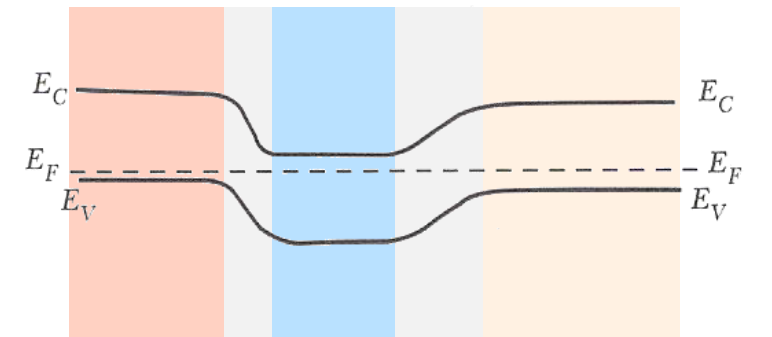
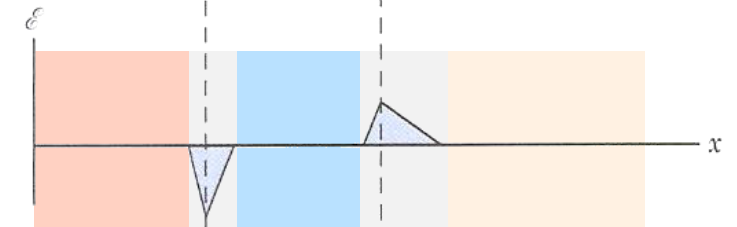
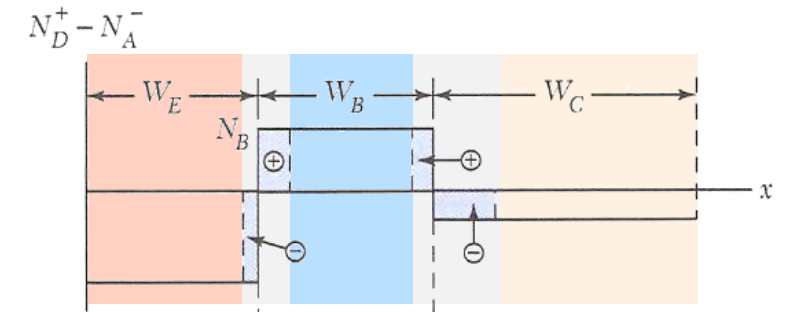
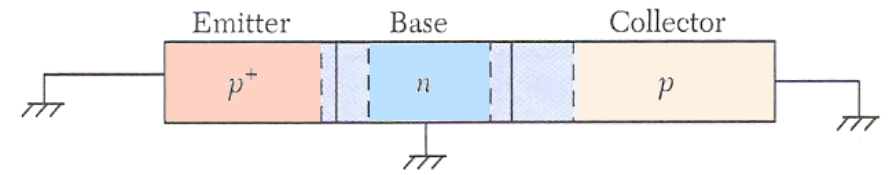
Minority Carrier Densities



$\log_{10}$ (Minority Carrier Densities)



pnp transistor, no bias



pn transistor, forward active bias

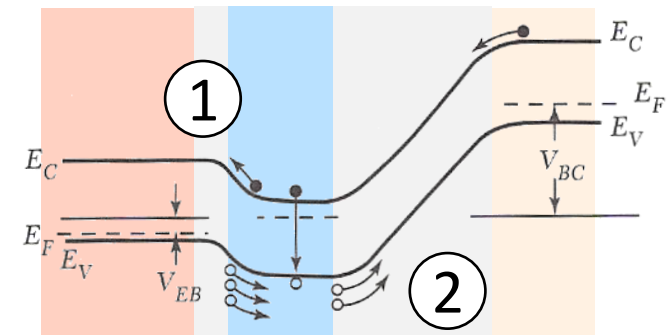
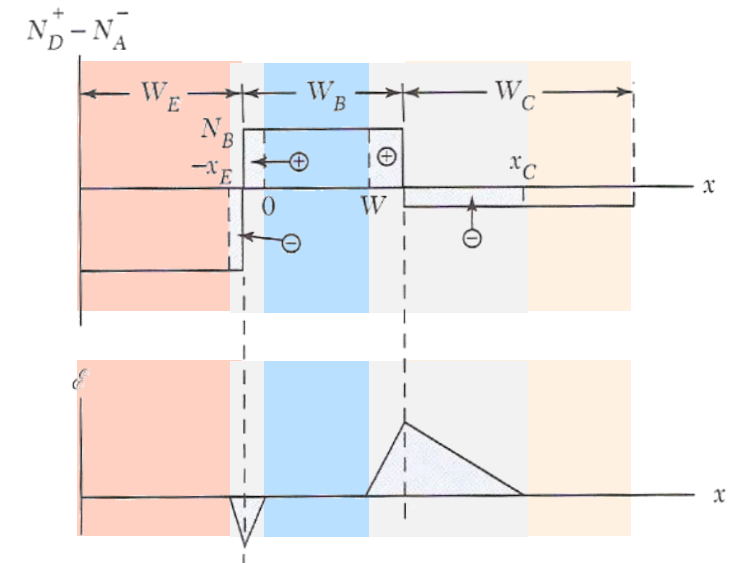
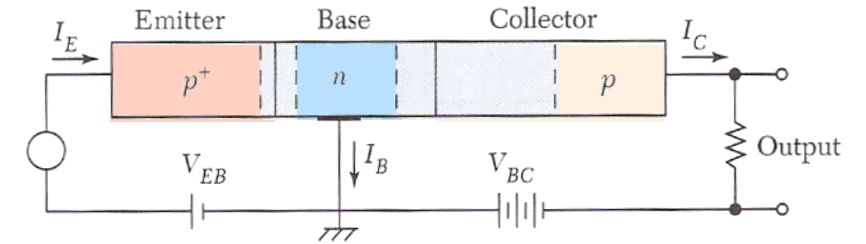
① base-emitter voltage:

controls the minority carriers injected from the emitter to the base

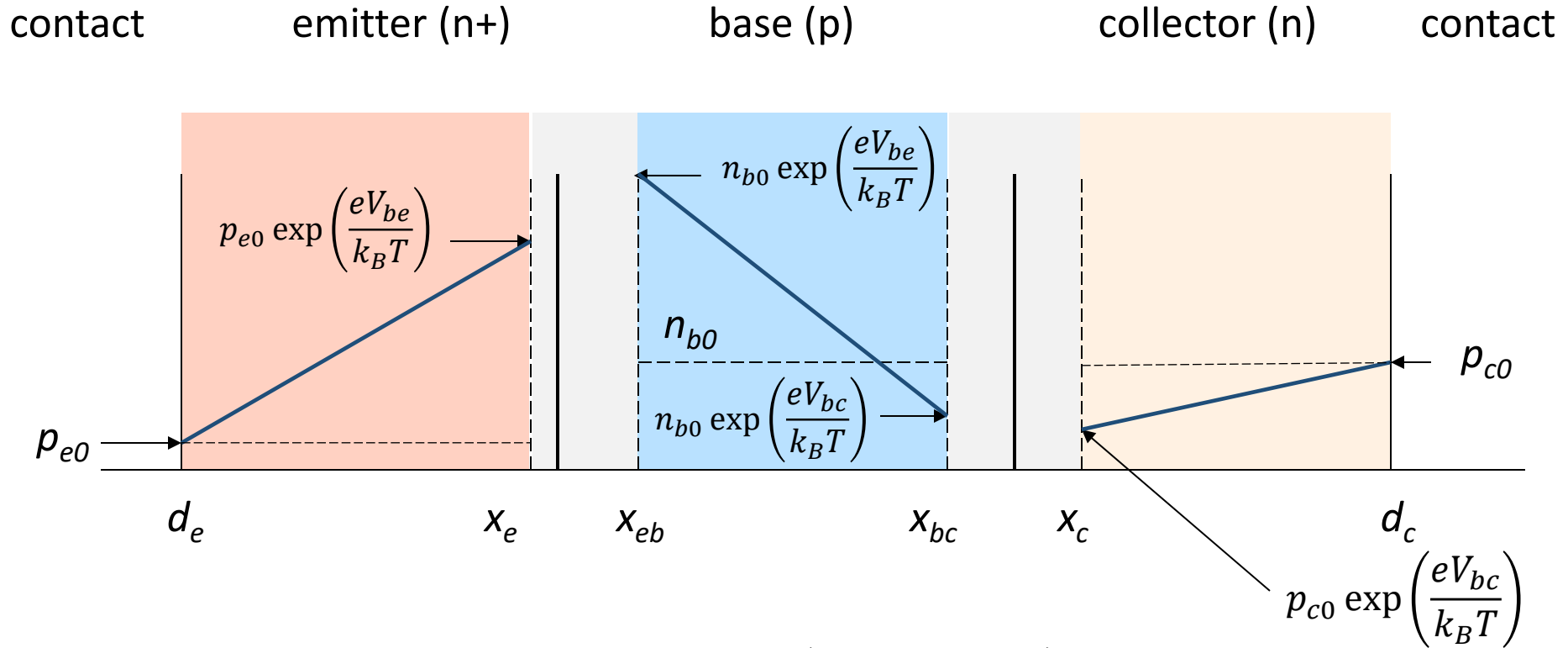
② minority carriers

diffuse to the base-collector junction and are swept into the collector

always dissipate power due to the forward bias



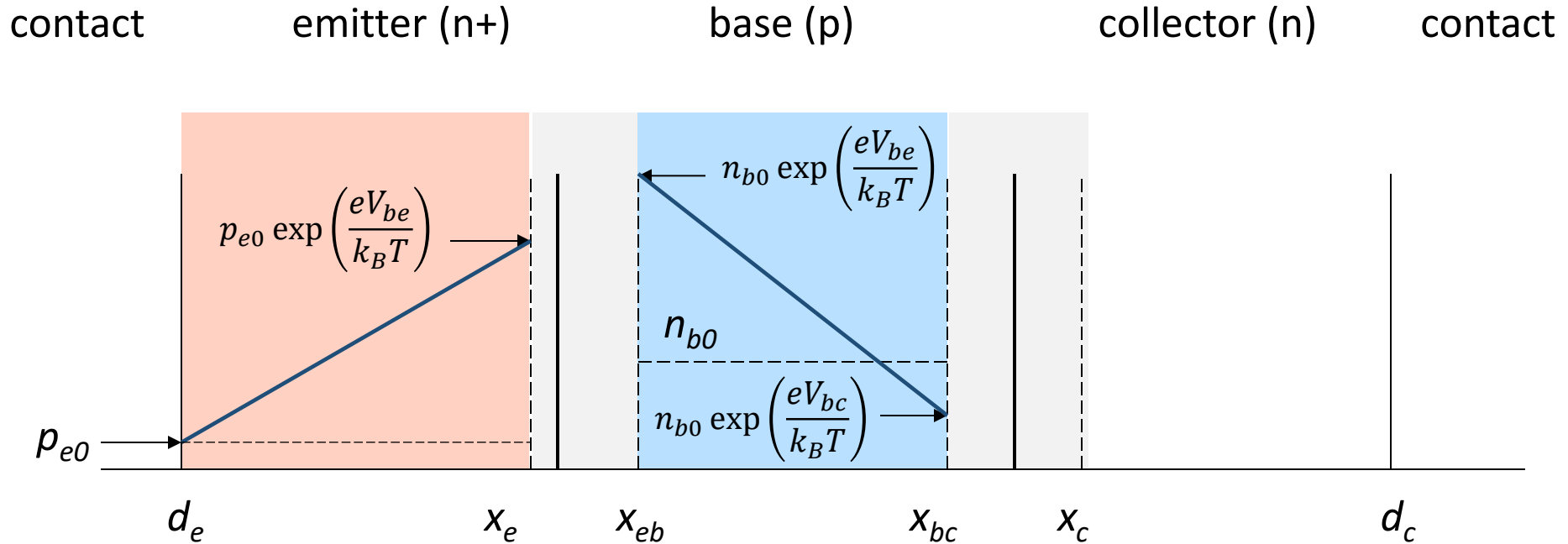
minority carrier concentration



$$I_{Ep} = -eA_{be}D_p \frac{p_{e0}(e^{eV_{be}/k_B T} - 1)}{x_e - d_e}$$

$$I_{En} = eA_{be}D_n \frac{n_{b0}(e^{eV_{bc}/k_B T} - e^{eV_{be}/k_B T})}{x_{bc} - x_{be}}$$

emitter current



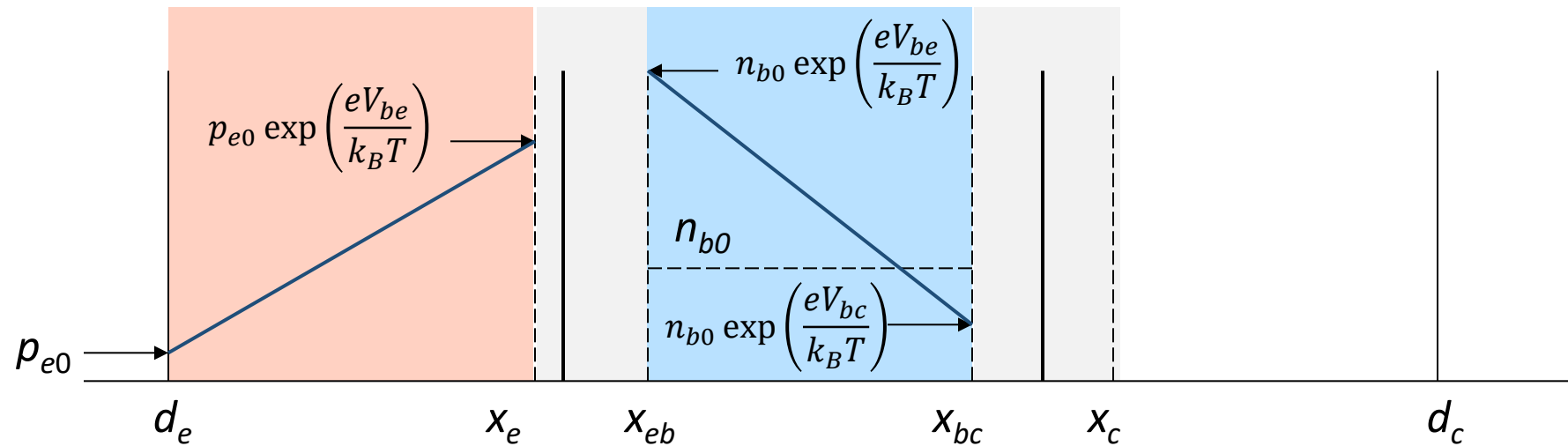
$$I_{Ep} = -eA_{be}D_p \frac{p_{e0}(e^{eV_{be}/k_B T} - 1)}{x_e - d_e}$$

$$I_{En} = eA_{be}D_n \frac{n_{b0}(e^{eV_{bc}/k_B T} - e^{eV_{be}/k_B T})}{x_{bc} - x_{be}}$$

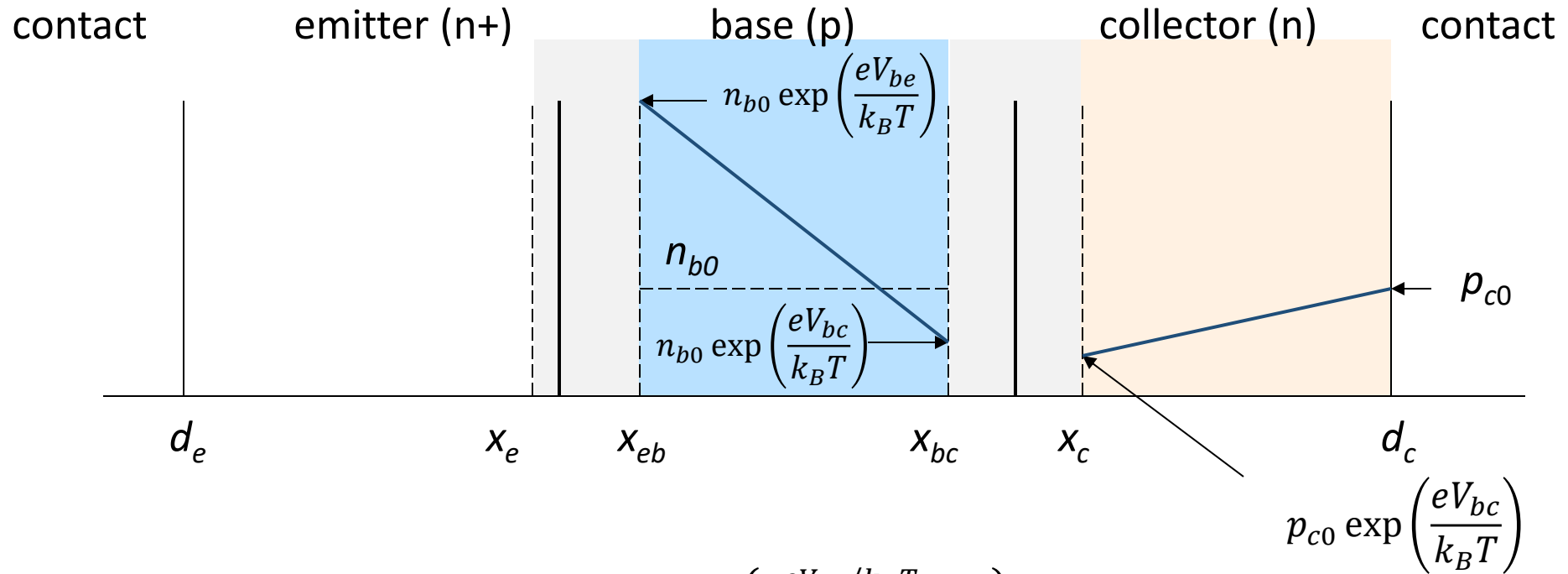
Emitter current

$$I_E = I_{En} + I_{Ep} = \left[\frac{eA_{be}D_p p_{e0}}{x_{be} - d_e} + \frac{eA_{be}D_n n_{b0}}{x_{bc} - x_{be}} \right] (e^{eV_{be}/k_B T} - 1) - \frac{eA_{be}D_n n_{b0}}{x_{bc} - x_{be}} (e^{eV_{bc}/k_B T} - 1)$$

$$I_E = I_{ES} (e^{eV_{be}/k_B T} - 1) - \alpha_R I_{CS} (e^{eV_{bc}/k_B T} - 1)$$



collector current



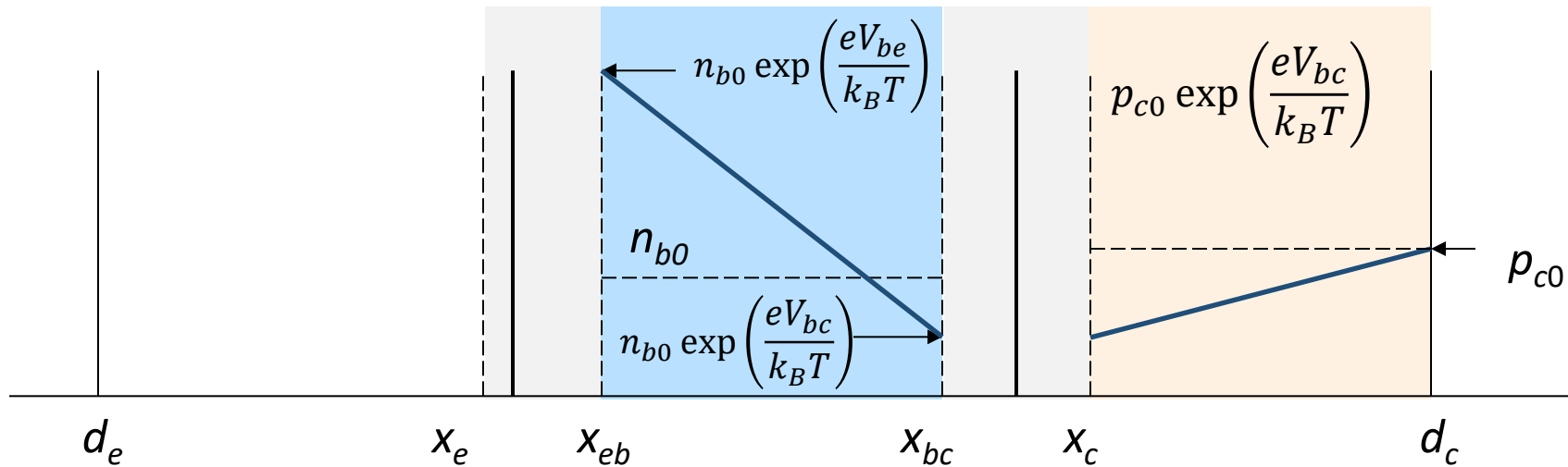
$$I_{cp} = -eA_{bc}D_p \frac{p_{c0}(e^{eV_{bc}/k_B T} - 1)}{d_c - x_c}$$

$$I_{cn} = -eA_{bc}D_n \frac{n_{b0}(e^{eV_{be}/k_B T} - e^{eV_{bc}/k_B T})}{x_{bc} - x_{eb}}$$

Collector current

$$I_C = I_{Cp} + I_{Cn} = \frac{eA_{bc}D_n n_{b0}}{x_{bc} - x_{be}} (e^{eV_{be}/k_B T} - 1) - \left[\frac{eA_{bc}D_p p_{c0}}{d_c - x_c} + \frac{eA_{bc}D_n n_{b0}}{x_{bc} - x_{be}} \right] (e^{eV_{bc}/k_B T} - 1)$$

$$\alpha_F I_{ES} (e^{eV_{be}/k_B T} - 1) - I_{CS} (e^{eV_{bc}/k_B T} - 1)$$

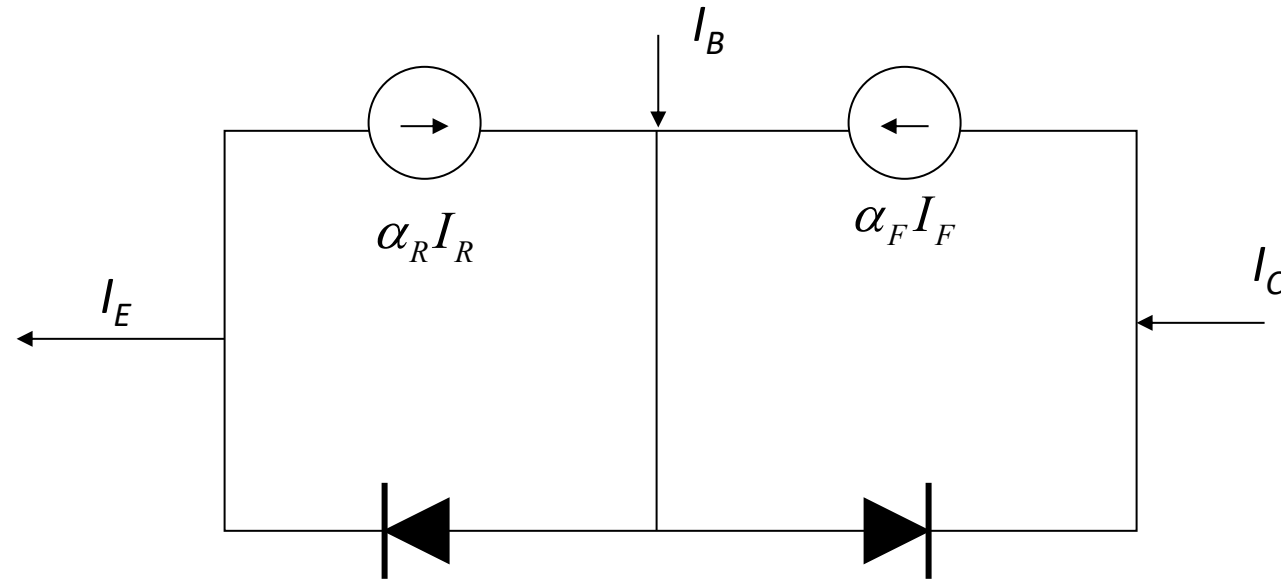


Ebers-Moll model

$$I_E = I_{ES}(e^{eV_{be}/k_B T} - 1) - \alpha_R I_{CS}(e^{eV_{bc}/k_B T} - 1)$$

$$I_C = \alpha_F I_{ES}(e^{eV_{be}/k_B T} - 1) - I_{CS}(e^{eV_{bc}/k_B T} - 1)$$

$$I_B = I_E - I_C$$

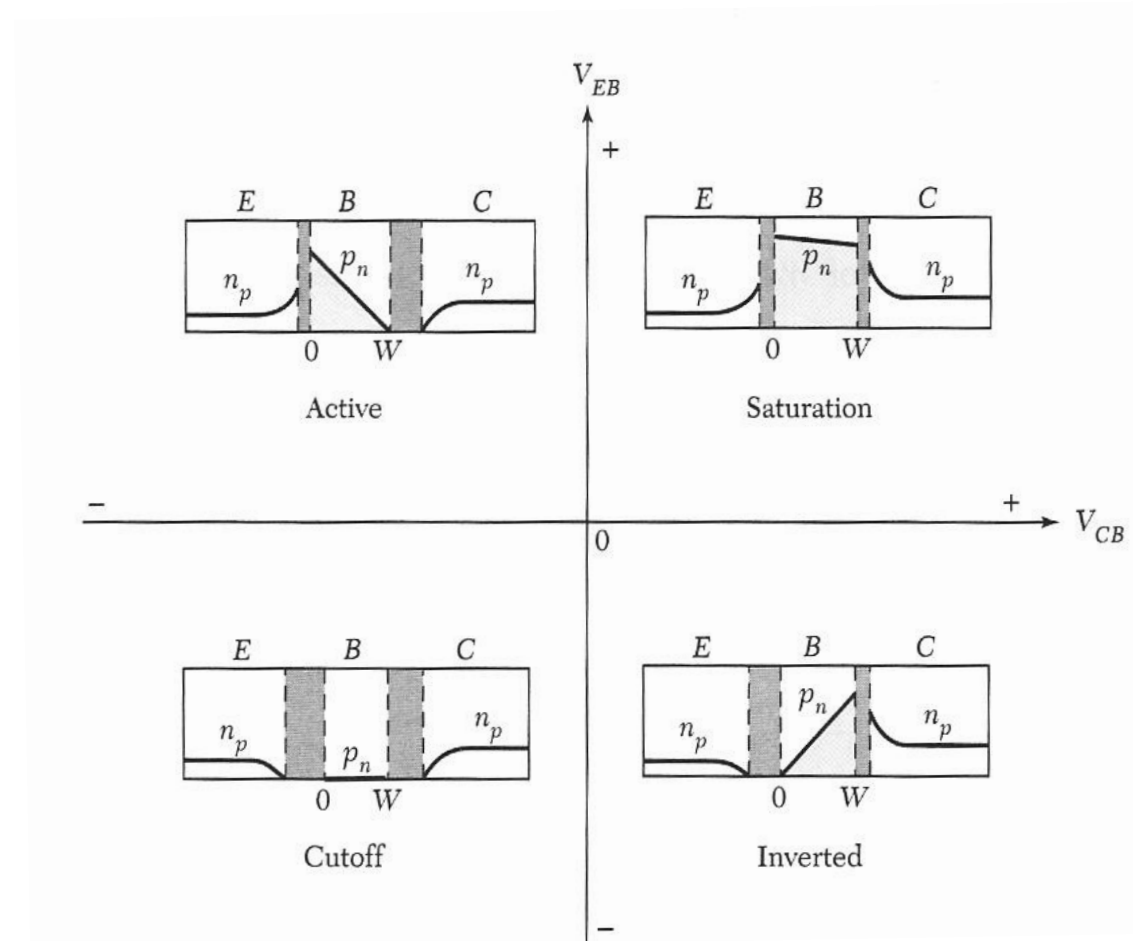


$$I_F = I_{ES}(e^{eV_{be}/k_B T} - 1) \quad I_R = I_{CS}(e^{eV_{bc}/k_B T} - 1)$$

transistor modes

1. forward active:
emitter-base **forward**,
base-collector **reverse**

4. cut-off:
emitter-base **reverse**,
base-collector **reverse**



2. saturation:
emitter-base **forward**,
base-collector **forward**

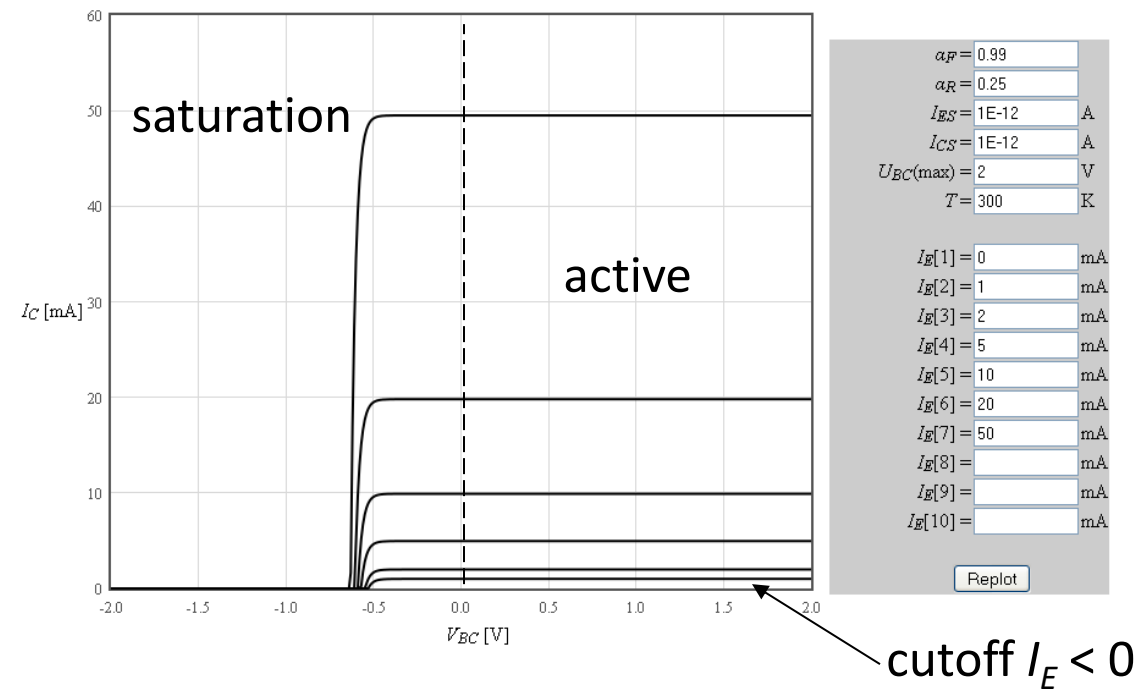
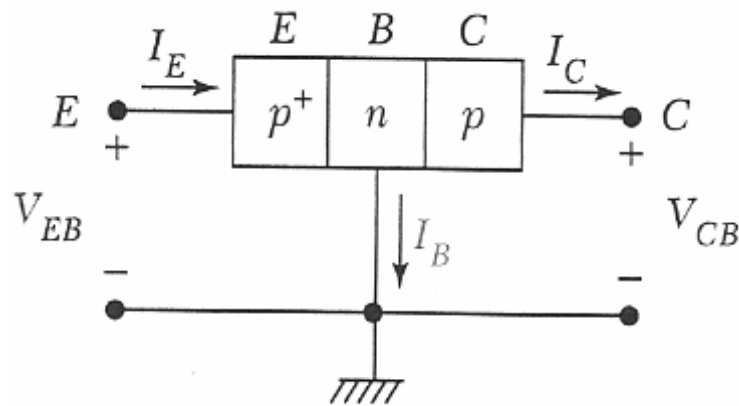
3. reverse active:
emitter-base **reverse**,
base-collector **forward**

common base configuration

$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

solve for V_{be}

$$I_C = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

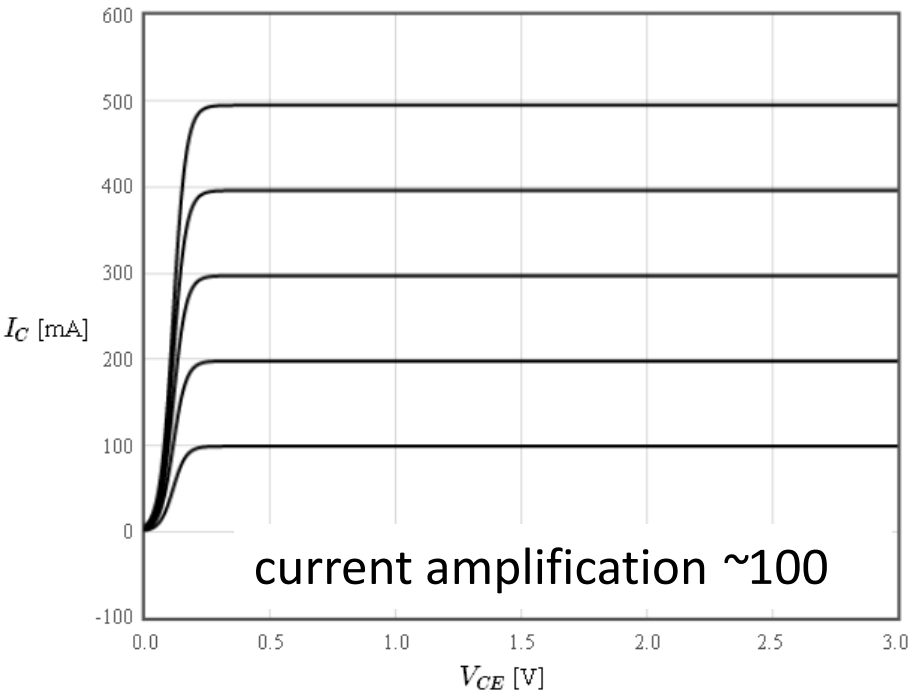
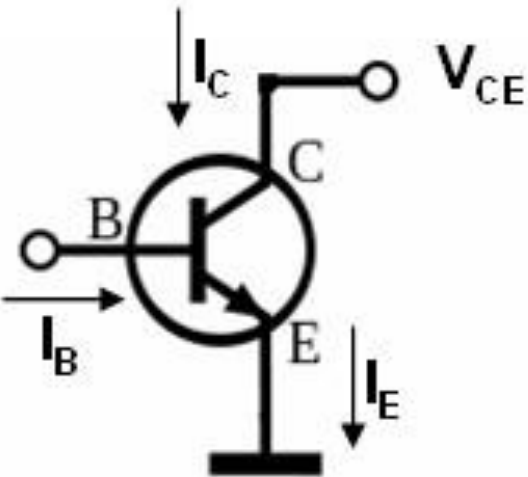


common emitter configuration

$$I_E = I_{ES}(e^{eV_{be}/k_BT} - 1) - \alpha_R I_{CS}(e^{eV_{bc}/k_BT} - 1)$$

$$I_B = I_E - I_C$$

$$I_C = \alpha_F I_{ES}(e^{eV_{be}/k_BT} - 1) - I_{CS}(e^{eV_{bc}/k_BT} - 1)$$



α_F

α_R

I_{ES}

A

I_{CS}

A

$V_{CE(max)}$

V

T

K

$I_B[1]$

mA

$I_B[2]$

mA

$I_B[3]$

mA

$I_B[4]$

mA

$I_B[5]$

mA

$I_B[6]$

$I_B[7]$

$I_B[8]$

$I_B[9]$

$I_B[10]$

Replot

Emitter efficiency

for npn

$$\gamma_e = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{1}{1 + I_{Ep}/I_{En}} \quad \leftarrow$$

$$I_{Ep} = eA_{be}D_p \frac{p_{e0}(e^{eV_{be}/k_B T} - 1)}{x_{eb} - d_e}$$

$$I_{En} = -eA_{be}D_n \frac{n_{b0}(e^{eV_{be}/k_B T} - e^{eV_{bc}/k_B T})}{x_{bc} - x_{be}}$$

For $\gamma_e \sim 1$, $x_{bc} - x_{be} \ll L_b$, $x_{eb} - d_e$ and $n_{b0} \gg p_{e0}$

neutral base width $\nearrow$

$\frac{n_i^2}{N_{Ab}} \nearrow$ $\nwarrow \frac{n_i^2}{N_{De}}$

Small base width and heavy emitter doping

Zegbroek gets a simpler formula by setting $V_{bc} = 0$ so that the exponential terms are the same.

Base transport factor

$$B = \frac{I_C}{I_{En}}$$

ratio of the injected current to the collected current

- recombination in the base would reduce the base transport factor
- a thin base with low doping results in a base transport factor ~ 1

Current transfer ratio

$$\alpha = \frac{I_C}{I_E} = B\gamma_e$$

$\alpha \sim 1$ for a good BJT

current amplification factor

$$\beta = h_{fe} = \frac{I_C}{I_B}$$

$$I_B = I_E - I_C$$

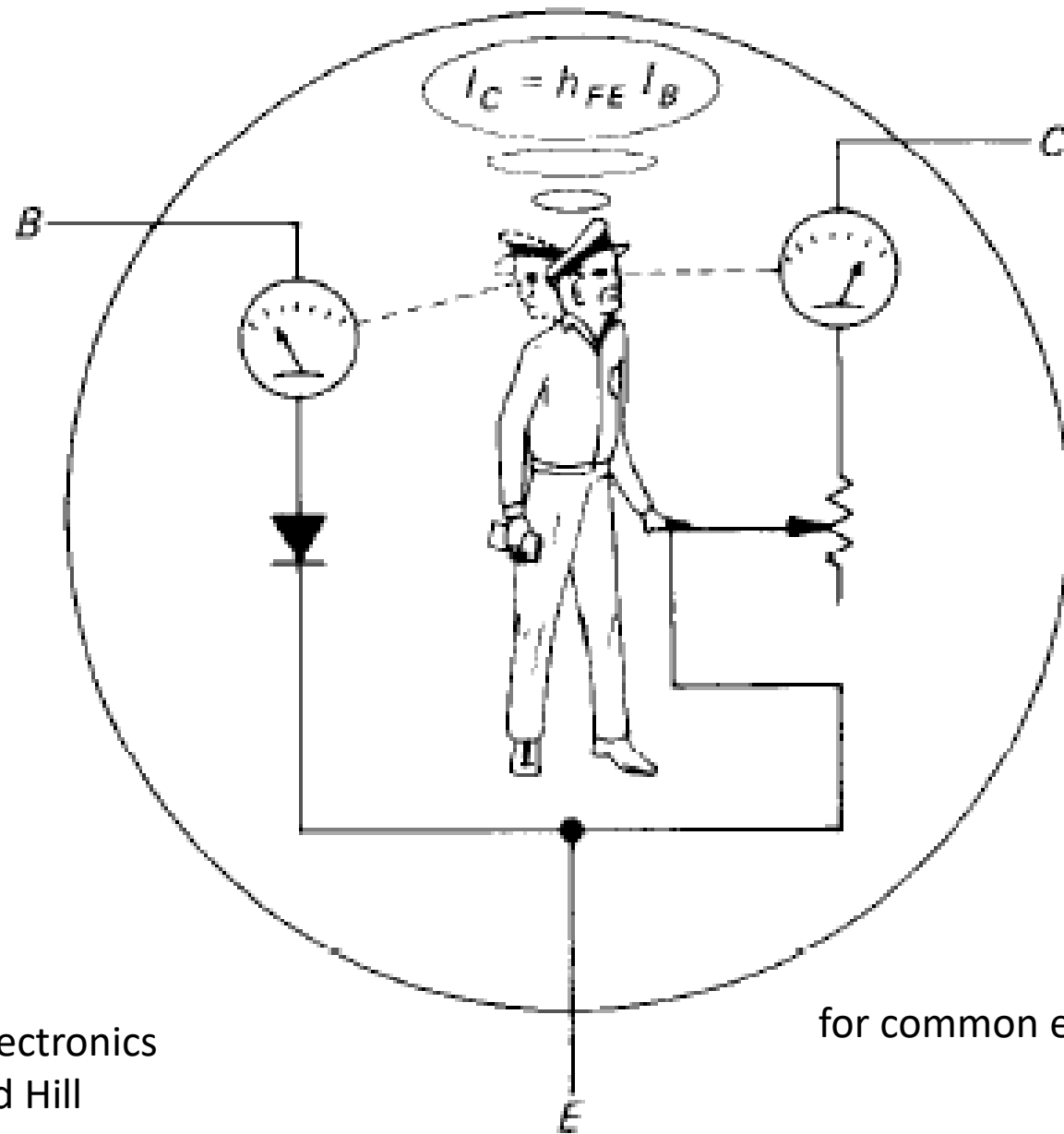
$$I_C = \alpha I_E$$

$$I_B = \left(\frac{1}{\alpha} - 1 \right) I_C$$

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1 - \alpha} = \frac{B\gamma_e}{1 - B\gamma_e}$$

$$\beta \sim 50 - 500$$

no causality: $I_B \rightarrow I_C$



The Art of Electronics
Horowitz and Hill

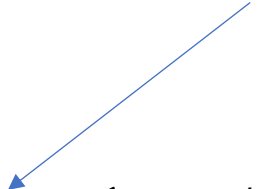
for common emitter configuration

"Transistor man"

Transconductance

$$g_m = \frac{\partial I_C}{\partial V_{be}}$$

operated in forward active mode


$$I_C = \alpha_F I_{ES} (e^{eV_{be}/k_B T} - 1) - I_{CS} (e^{eV_{bc}/k_B T} - 1)$$

first term depends on V_{be}

$$g_m = \frac{e\alpha_F I_{ES}}{k_B T} e^{eV_{be}/k_B T} \approx \frac{eI_C}{k_B T} = \frac{e\beta I_B}{k_B T}$$

The transconductance can be very high.

Early effect

Ebers - Moll:

$$I_E = I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - \alpha_R I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

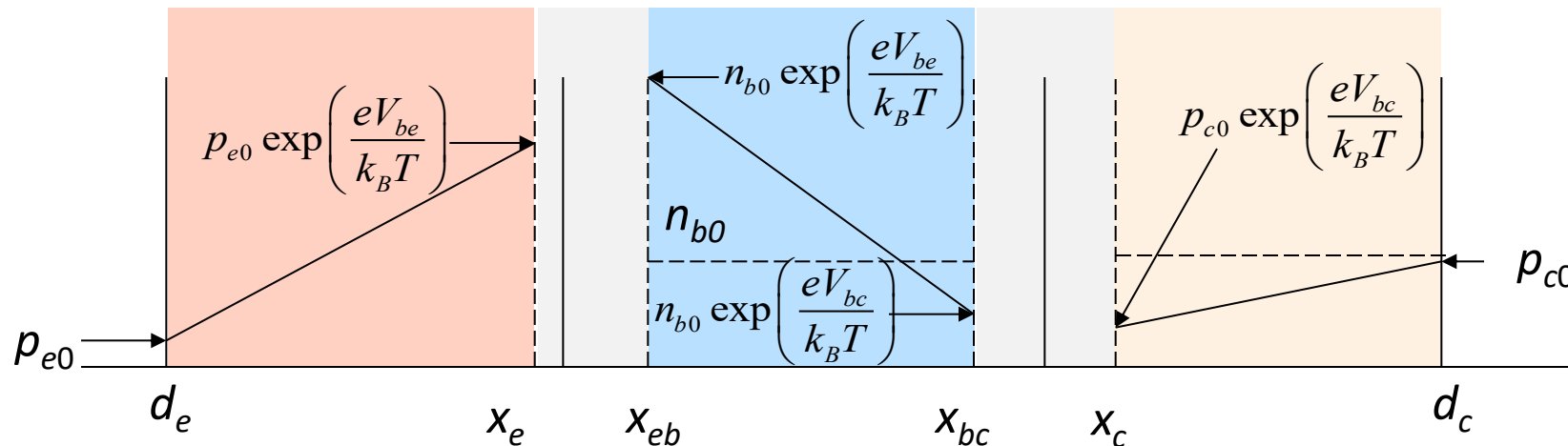
$$I_C = \alpha_F I_{ES} \left(e^{eV_{be}/k_B T} - 1 \right) - I_{CS} \left(e^{eV_{bc}/k_B T} - 1 \right)$$

$$I_B = I_E - I_C$$

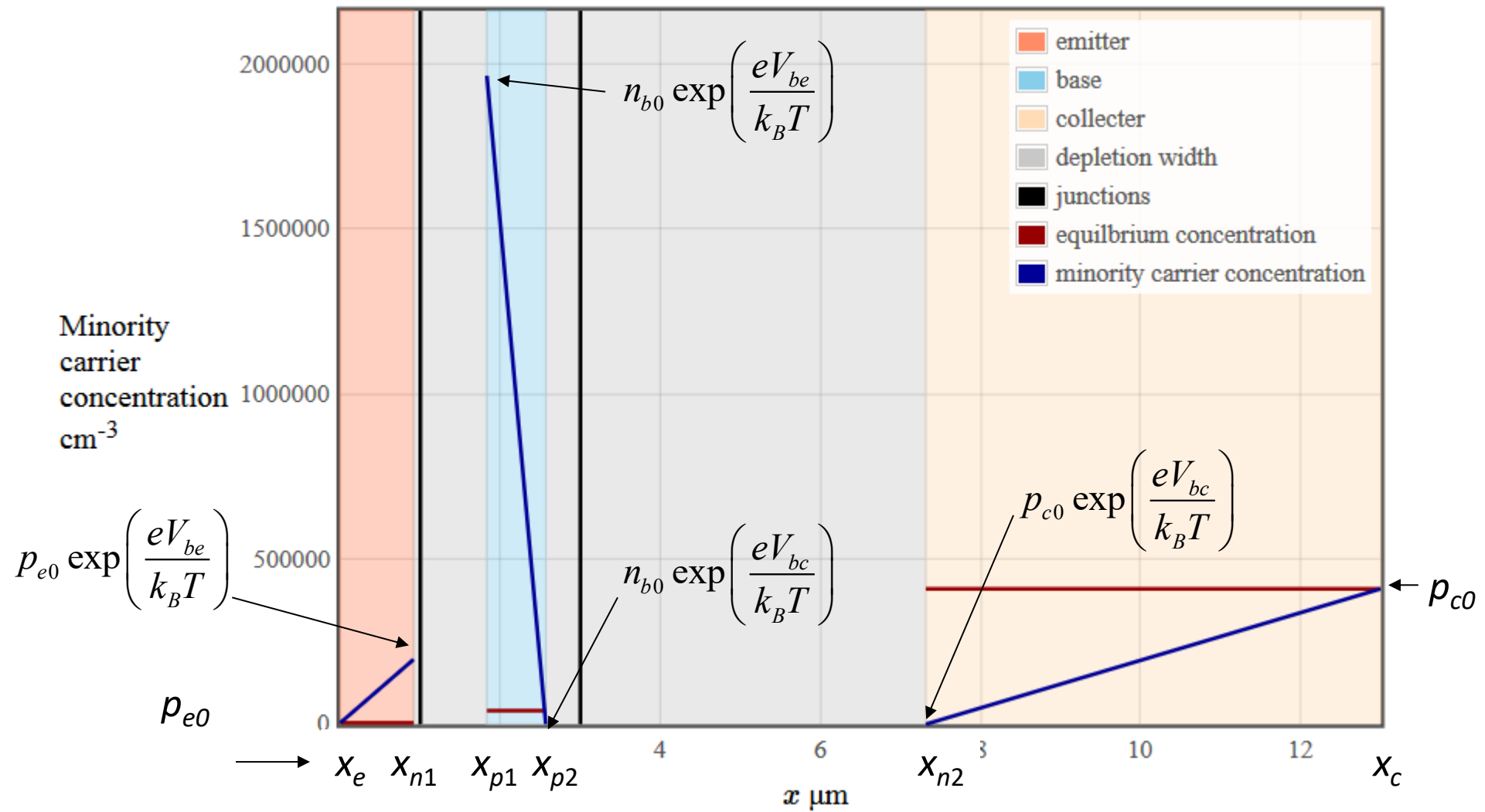
$$I_{ES} = \left[\frac{eA_{be}D_p p_{e0}}{x_e - d_e} + \frac{eA_{be}D_n n_{b0}}{x_{bc} - d_{be}} \right]$$

$$I_{CS} = \left[\frac{eA_{bc}D_p p_{c0}}{d_c - x_c} + \frac{eA_{bc}D_n n_{b0}}{x_{bc} - x_{be}} \right]$$

I_{ES} and I_{CS} are treated as constants but the depletion widths W_{bc} , W_{be} , W_c and W_e depend on the voltages.



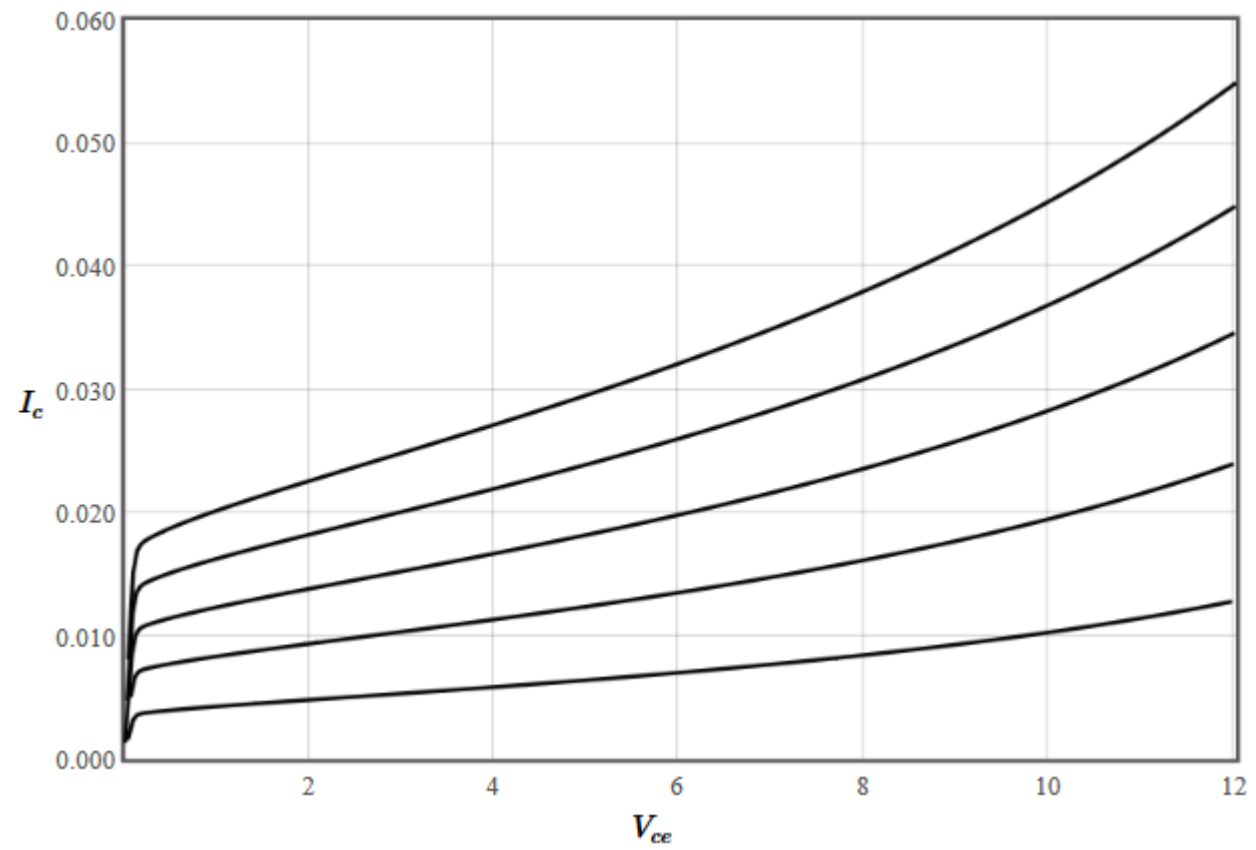
minority carrier concentration



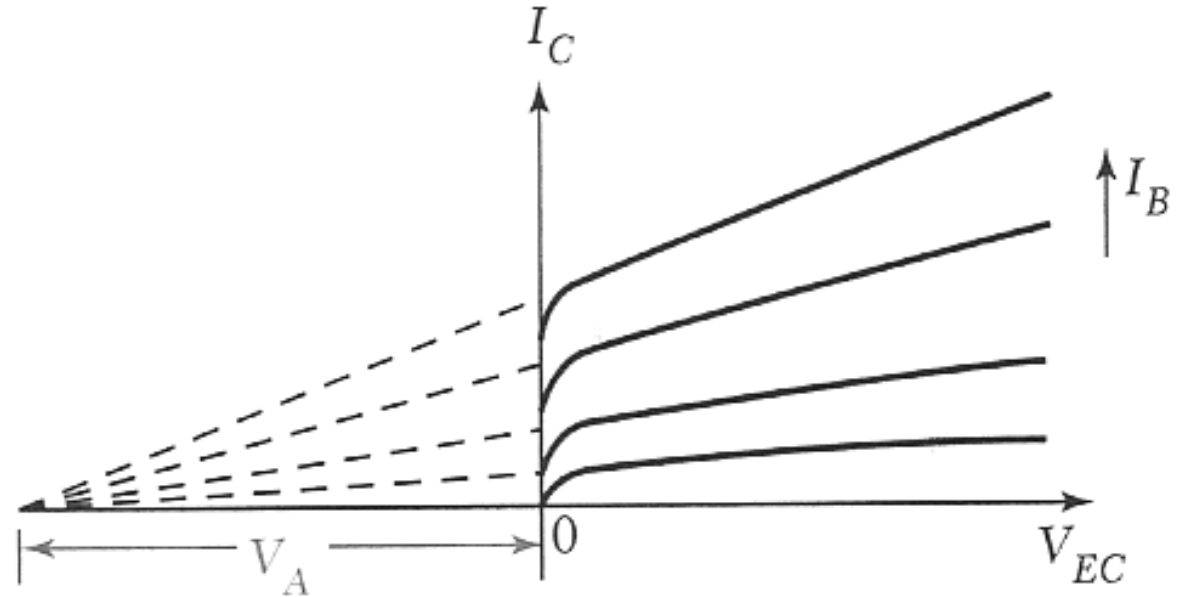
NPN common emitter configuration

n-Emitter		$A_{eb} = 1E-3$ cm ²	
Minority $\mu_{pe} = 480$ cm ² /Vs		$N_c(300K) = 2.78E19$ cm ⁻³	
$N_{de} = 1E16$ cm ⁻³		$N_v(300K) = 9.84E18$ cm ⁻³	
$\tau_{pe} = 1E-5$ s		$E_g = 1.166-4.73E-4*T*(T/(T+636))$ eV	
p-Base		$\epsilon_r = 11.9$	
Minority $\mu_{nb} = 1350$ cm ² /Vs		$I_{b\max} = 0.001$ eV	
$N_{ab} = 1E15$ cm ⁻³		$V_{ce\max} = 12$ eV	
$\tau_{nb} = 1E-5$ s		$x_1 - x_e = 1$ μm	
n-Collector		$x_2 - x_1 = 2$ μm	
Minority $\mu_{pc} = 480$ cm ² /Vs		$x_c - x_2 = 10$ μm	
$N_{dc} = 1E14$ cm ⁻³		$T = 300$ K	
$\tau_{pc} = 1E-5$ s			
Calculate			

$I_C \sim \beta I_B$



Early effect



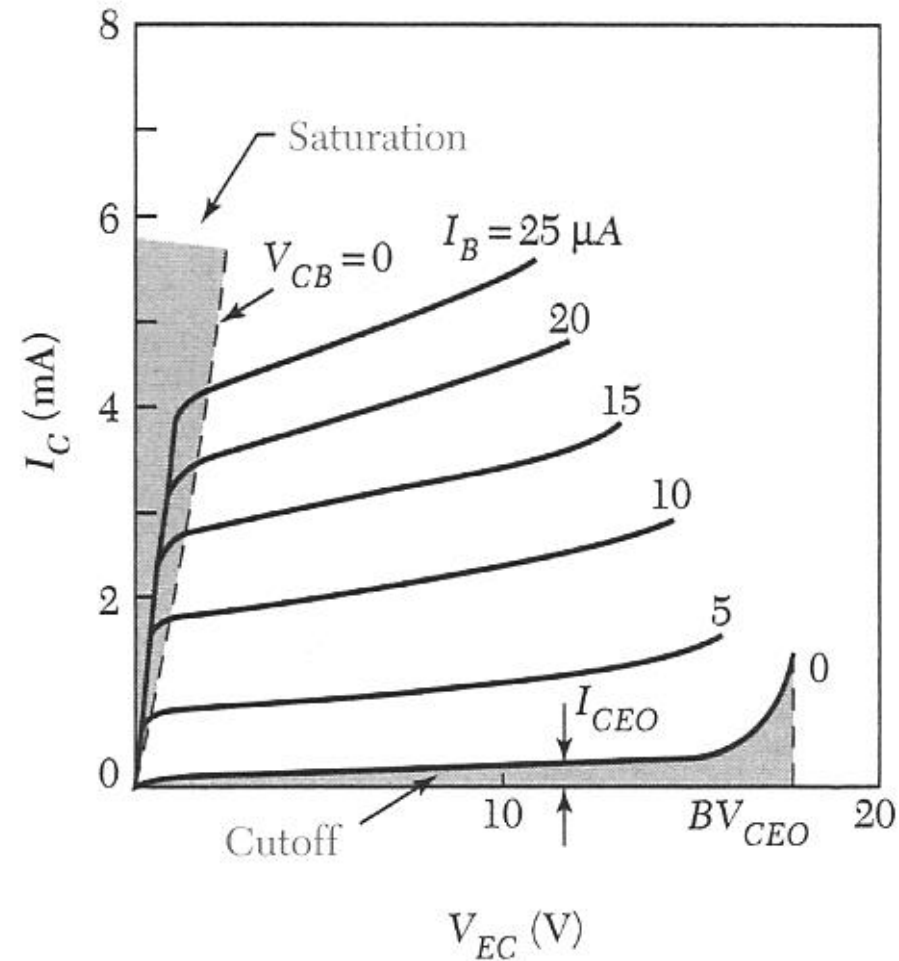
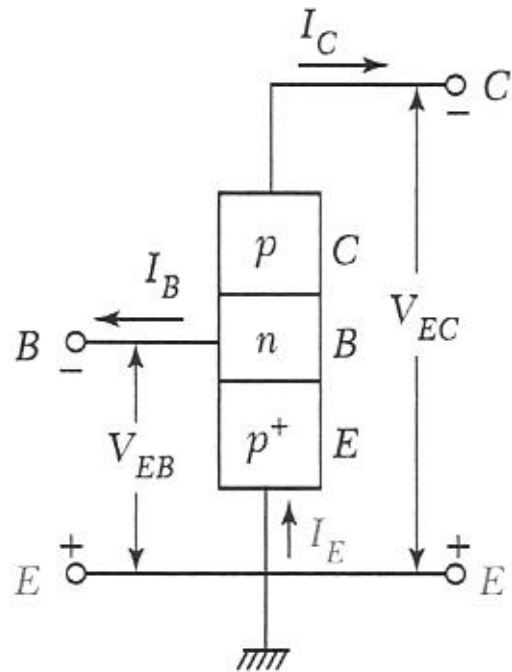
Common emitter configuration

Base width modulation: smaller width increases the diffusion current and increases the gain.

Punchthrough: The neutral base width goes to zero and all gain is lost.

Lightly dope the collector -> voltage drops in collector. Makes circuit slower.

Common emitter configuration



$$I_C \sim \beta I_B \text{ amplifier}$$