

# Semiconductors

density of electrons  
in the conduction  
band

$$n = N_c \left( \frac{T}{300} \right)^{\frac{3}{2}} \exp \left( \frac{\mu - E_c}{k_B T} \right)$$

↑  
effective density of  
states in the  
conduction band at  
300 K

density of holes in  
the valence band

$$p = N_v \left( \frac{T}{300} \right)^{\frac{3}{2}} \exp \left( \frac{E_v - \mu}{k_B T} \right)$$

↑  
effective density of  
states in the valence  
band at 300 K

## Intrinsic semiconductors

In the Boltzmann approximation, the density of states of a semiconductor is,

$$D(E) = \begin{cases} \frac{(2m_h^*)^{3/2}}{2\pi^2\hbar^3} \sqrt{E_v - E}, & \text{if } E < E_v \\ 0, & \text{if } E_v < E < E_c \\ \frac{(2m_e^*)^{3/2}}{2\pi^2\hbar^3} \sqrt{E - E_c}, & \text{if } E_c < E \end{cases}$$

Here  $m_e^*$  and  $m_h^*$  are the 'density of states effective masses' for electrons and holes. Usually in the literature, effective density of states at 300 K is given instead of the 'density of states effective masses'. The relationship between the two is,

$$m_h^* = \frac{\pi\hbar^2}{300k_B} \left( \sqrt{2}N_v(300) \right)^{2/3}$$

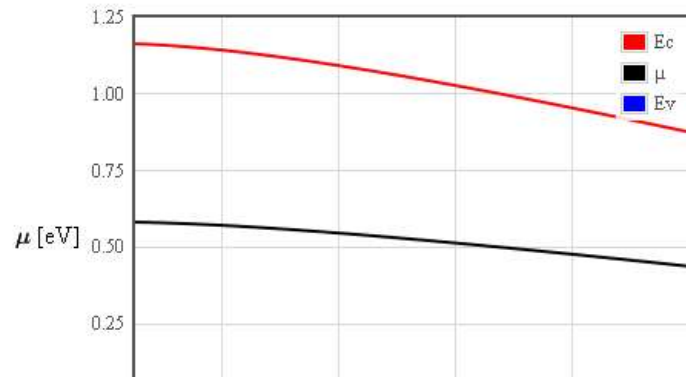
$$m_e^* = \frac{\pi\hbar^2}{300k_B} \left( \sqrt{2}N_c(300) \right)^{2/3}$$

In an intrinsic semiconductor, the density of electrons equals the density of holes,  $n = p = n_i = \sqrt{N_c \left(\frac{T}{300}\right)^{3/2} N_v \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right)}$ .

By setting the concentration of electrons equal to the concentration of holes, it is possible to solve for the chemical potential. The bandgap of most semiconductors is temperature dependent. The form below lets you input the temperature dependence of the bandgap. The bandgaps for some semiconductors can be loaded into the form with the buttons on the right.

$$n = N_c(300) \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{\mu - E_c}{k_B T}\right) = p = N_v(300) \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

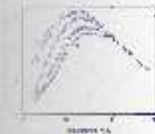
$$\mu = \frac{E_v + E_c}{2} + k_B T \ln\left(\frac{N_c(300)}{N_v(300)}\right)$$



$N_c(300 \text{ K}) =$	<input type="text" value="2.78E19"/>	1/cm <sup>3</sup>	Semiconductor <input type="button" value="Si"/> <input type="button" value="Ge"/> <input type="button" value="GaAs"/>
$N_v(300 \text{ K}) =$	<input type="text" value="9.84E18"/>	1/cm <sup>3</sup>	
$E_g =$	<input type="text" value="1.166-4.73E-4*T*(T+636)"/>	eV	
$T_1 =$	<input type="text" value="50"/>	K	
$T_2 =$	<input type="text" value="1000"/>	K	
<input type="button" value="Replot"/>			

# Semiconductors

## on NSM

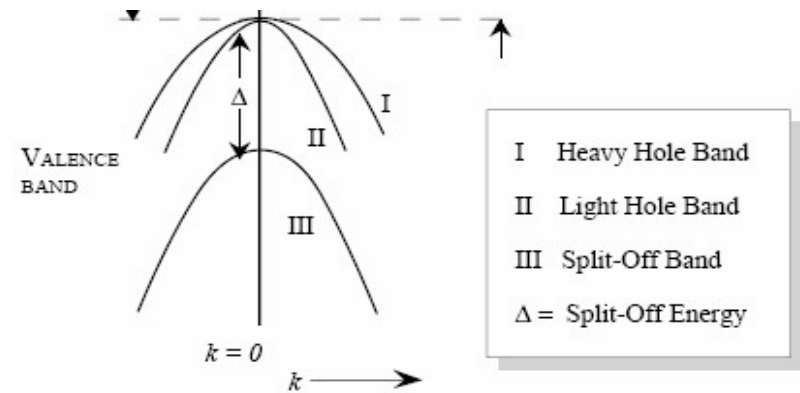
[Semiconductors](#)[n, k database](#)[InGaAsP](#)[Equivalents](#)

<a href="#">Si</a>	- Silicon	<a href="#">Ge</a>	- Germanium
<a href="#">GaP</a>	- Gallium Phosphide	<a href="#">GaAs</a>	- Gallium Arsenide
<a href="#">InAs</a>	- Indium Arsenide	<a href="#">C</a>	- Diamond
<a href="#">GaSb</a>	- Gallium Antimonide	<a href="#">InSb</a>	- Indium Antimonide
<a href="#">InP</a>	- Indium Phosphide	<a href="#">GaAs<sub>1-x</sub>Sb<sub>x</sub></a>	- Gallium Arsenide Antimonide
<a href="#">Al<sub>x</sub>Ga<sub>1-x</sub>As</a>	- Aluminium Gallium Arsenide		
<a href="#">AlN</a>	- Aluminium Nitride	<a href="#">InN</a>	- Indium Nitride
<a href="#">BN</a>	- Boron Nitride	<a href="#">GaN</a>	- Gallium Nitride

### We are going to add new data for:

<a href="#">Ga<sub>x</sub>In<sub>1-x</sub>As<sub>y</sub>Sb<sub>1-y</sub></a>	- Gallium Indium Arsenide Antimonide	<a href="#">Ga<sub>x</sub>In<sub>1-x</sub>P</a>	- Gallium Indium Phosphide
<a href="#">Ga<sub>x</sub>In<sub>1-x</sub>As</a>	- Gallium Indium Arsenide	<a href="#">Ga<sub>x</sub>In<sub>1-x</sub>Sb</a>	- Gallium Indium Antimonide
<a href="#">InAs<sub>1-x</sub>Sb<sub>x</sub></a>	- Indium Arsenide Antimonide	<a href="#">Ga<sub>x</sub>In<sub>1-x</sub>As<sub>y</sub>P<sub>1-y</sub></a>	- Gallium Indium Arsenide Phosphide
<a href="#">Si<sub>1-x</sub>Ge<sub>x</sub></a>	- Silicon Germanium	<a href="#">SiC</a>	- Silicon Carbide

# Effective Masses



## Electrons:

The surfaces of equal energy are ellipsoids.

$$m_l = 0.98m_0$$

$$m_t = 0.19m_0$$

Effective mass of density of states

$$m_c = 0.36m_0$$

There are 6 equivalent valleys in the conduction band.

$$m_{cc} = 0.26m_0$$

## Holes:

Heavy

$$m_h = 0.49m_0$$

Light

$$m_{lp} = 0.16m_0$$

Split-off band

$$m_{so} = 0.24m_0$$

Effective mass of density of states

$$m_v = 0.81m_0$$

## Intrinsic semiconductors with a split-off band

Many common semiconductors such as Si, Ge, and GaAs have a split-off band just below the valence band. The states in the split-off band change the temperature dependence of the concentration of holes. In the Boltzmann approximation, the density of states of a semiconductor with a split-off band just below the valence band is,

$$D(E) = \begin{cases} \frac{(2m_h^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E_v - E} + \frac{(2m_{so}^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E_{so} - E}, & \text{for } E < E_{so} \\ \frac{(2m_h^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E_v - E}, & \text{for } E_{so} < E < E_v \\ 0, & \text{for } E_v < E < E_c \\ \frac{(2m_c^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_c}, & \text{for } E_c < E \end{cases}$$

Here  $m_e^*$ ,  $m_h^*$ , and  $m_{so}^*$  are the 'density of states effective masses'. Often in the literature, effective density of states at 300 K is given instead of the 'density of states effective masses'. The relationship between the two is,

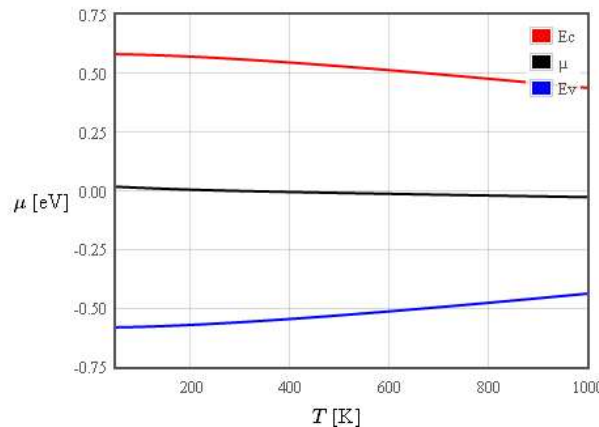
$$\begin{aligned} m_{so}^* &= \frac{\pi \hbar^3}{300k_B} (\sqrt{2} N_{so}(300))^{2/3} \\ m_h^* &= \frac{\pi \hbar^3}{300k_B} (\sqrt{2} N_v(300))^{2/3} \\ m_c^* &= \frac{\pi \hbar^3}{300k_B} (\sqrt{2} N_c(300))^{2/3} \end{aligned}$$

The density of states can therefore also be written as,

$$D(E) = \begin{cases} \frac{2N_c(300)}{\sqrt{\pi}} \left(\frac{1}{300k_B}\right)^{3/2} \sqrt{E_v - E} + \frac{2N_{so}(300)}{\sqrt{\pi}} \left(\frac{1}{300k_B}\right)^{3/2} \sqrt{E_{so} - E}, & \text{for } E < E_{so} \\ \frac{2N_v(300)}{\sqrt{\pi}} \left(\frac{1}{300k_B}\right)^{3/2} \sqrt{E_v - E}, & \text{for } E_{so} < E < E_v \end{cases}$$

In an intrinsic semiconductor, the concentration of electrons and holes is equal,  $n = p =$

$$n = p =$$



$N_c(300 \text{ K}) = 2.78\text{E}19$	$1/\text{cm}^3$
$N_v(300 \text{ K}) = 9.84\text{E}18$	$1/\text{cm}^3$
$N_{so}(300 \text{ K}) = 2.98\text{E}18$	$1/\text{cm}^3$
$E_g = 1.166 - 4.73\text{E}-4 * T * T / (T + 636)$	eV
$E_v - E_{so} = 0.044$	eV
$T_1 = 50$	K
$T_2 = 1000$	K
<input type="button" value="Replot"/>	

Semiconductor

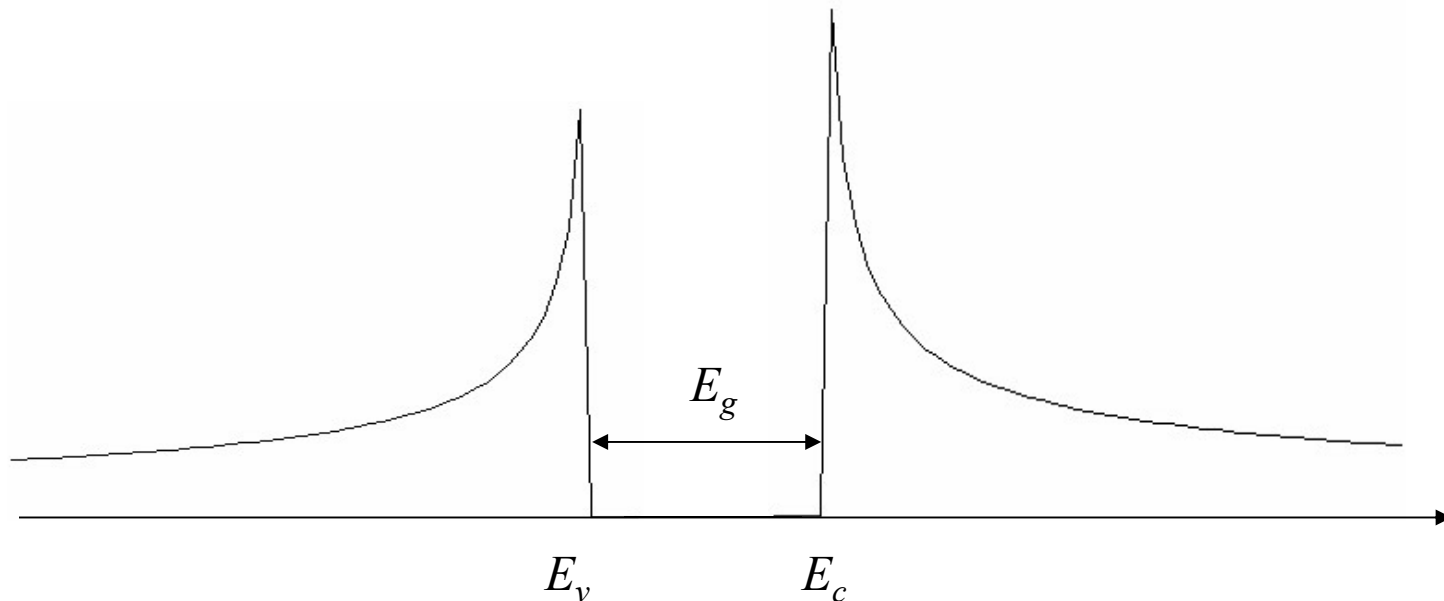
ndgap,  $E_g$ . For most

$\left(\frac{1}{\sqrt{\pi}}\right)$

# Semiconductors and insulators - 1d

---

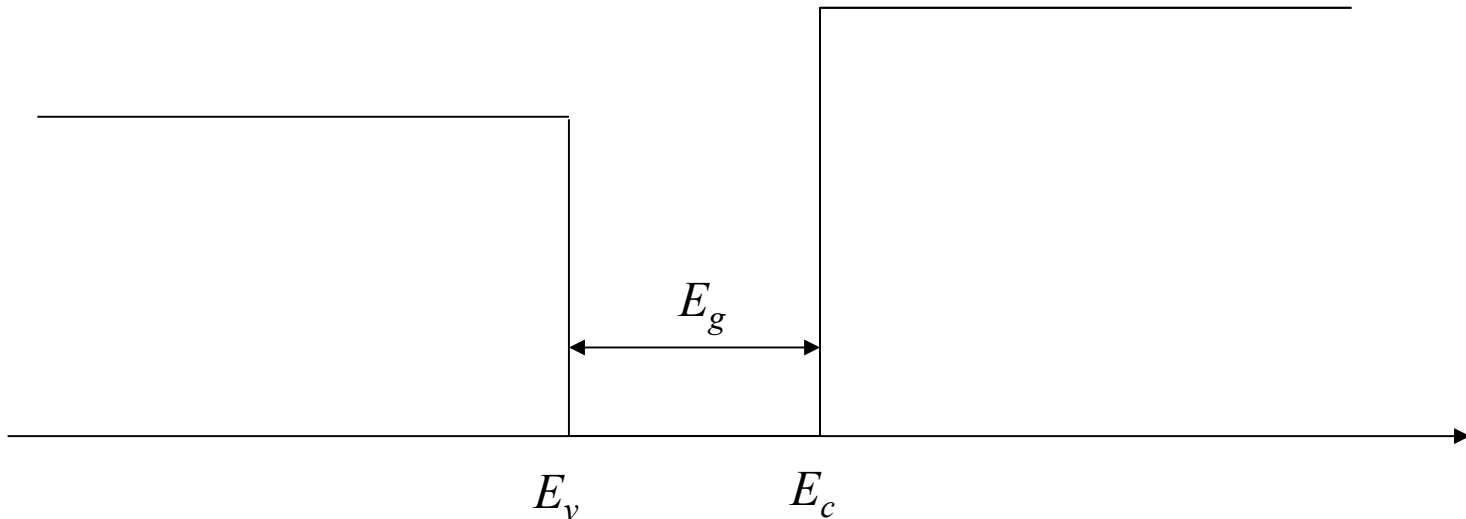
$$E = \frac{\hbar^2 (\vec{k} - \vec{k}_0)^2}{2m^*}$$
$$D(E) = \begin{cases} \sqrt{\frac{2m_h^*}{\hbar^2 \pi^2 (E_v - E)}} & E < E_v \\ 0 & E_v < E < E_c \\ \sqrt{\frac{2m_e^*}{\hbar^2 \pi^2 (E - E_c)}} & E_c < E \end{cases} \quad \text{J}^{-1} \text{m}^{-3}$$



# Semiconductors and insulators - 2d

---

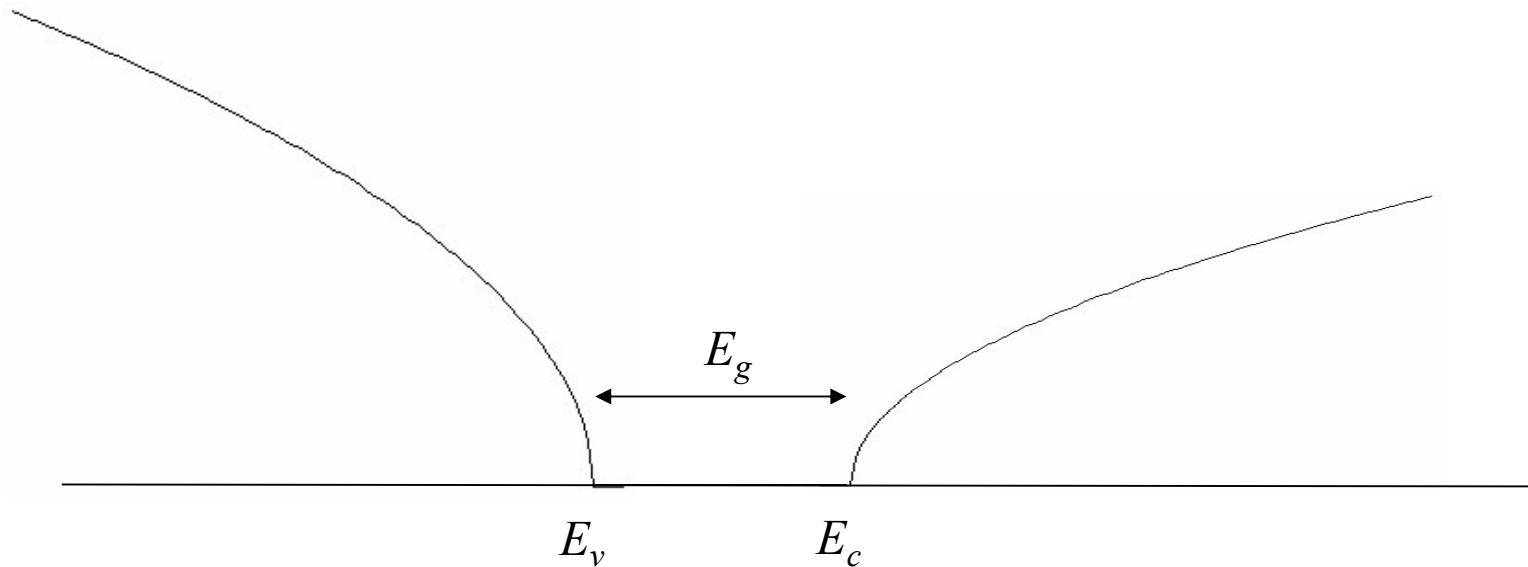
$$D(E) = \begin{cases} \frac{m_h^*}{\hbar^2 \pi} & E < E_v \\ 0 & E_v < E < E_c \\ \frac{m_e^*}{\hbar^2 \pi} & E_c < E \end{cases} \quad \text{J}^{-1}\text{m}^{-3}$$



# Semiconductors and insulators - 3d

---

$$D(E) = \begin{cases} \frac{(2m_h^*)^{\frac{3}{2}}}{2\pi^2\hbar^3} \sqrt{E_v - E} & E < E_v \\ 0 & E_v < E < E_c \\ \frac{(2m_e^*)^{\frac{3}{2}}}{2\pi^2\hbar^3} \sqrt{E - E_c} & E_c < E \end{cases} \quad \text{J}^{-1}\text{m}^{-3}$$





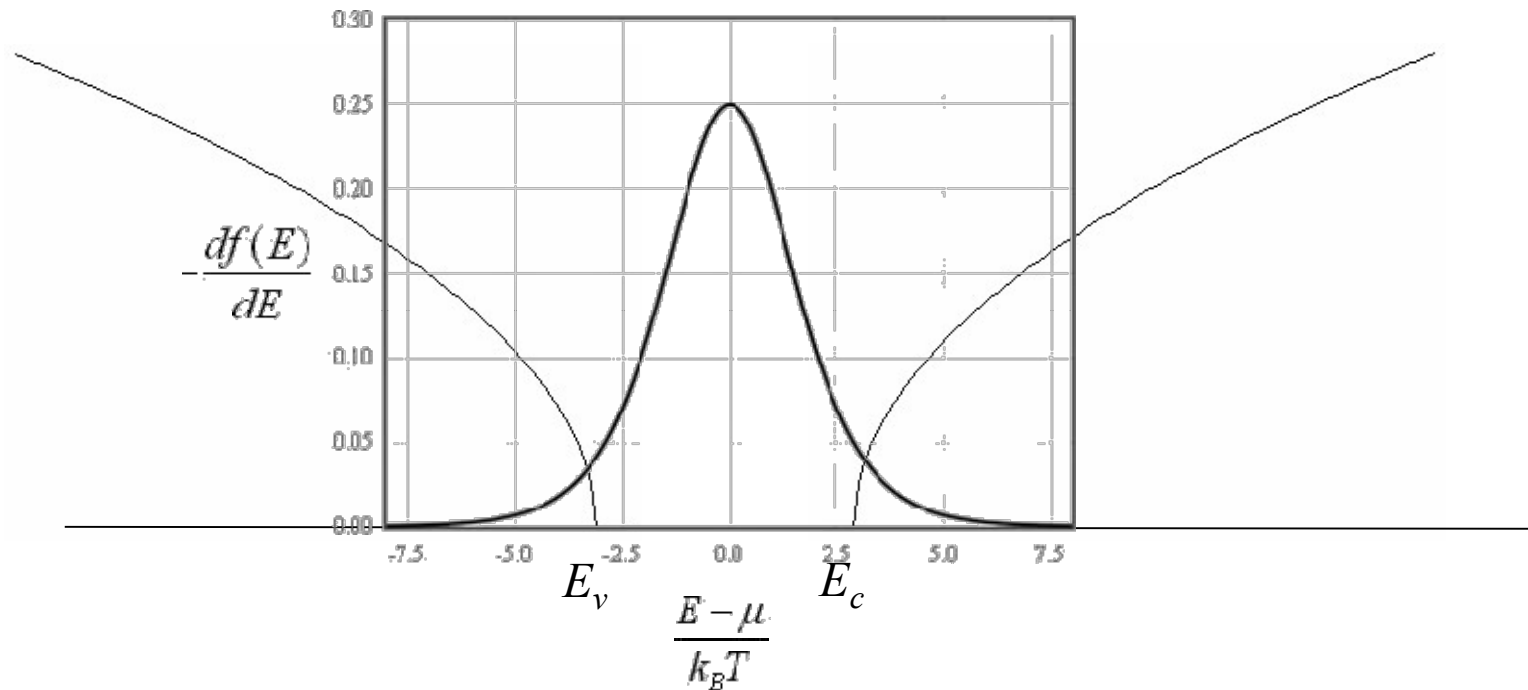
## Boltzmann approximation

The table below gives the contribution of electrons in intrinsic semiconductors and insulators to some thermodynamic quantities. These results were calculated in the Boltzmann approximation where it is assumed that the chemical potential lies in the band gap more than  $3k_B T$  from the band edge. The electronic contribution to the thermodynamic quantities are usually much smaller than the contribution of the phonons and thus the electronic components are often simply ignored.

	1-d	2-d	3-d
Density of states	$D(E) = \begin{cases} \frac{1}{\hbar\pi} \sqrt{\frac{2m_h^*}{(E_v - E)}} & E < E_v \\ 0 & E_v < E < E_c \\ \frac{1}{\hbar\pi} \sqrt{\frac{2m_e^*}{(E - E_c)}} & E > E_c \end{cases} \quad \text{J}^{-1} \text{m}^{-1}$	$D(E) = \begin{cases} \frac{m_h^*}{\hbar^2\pi} H(E_v - E) & E < E_v \\ 0 & E_v < E < E_c \\ \frac{m_e^*}{\hbar^2\pi} H(E - E_c) & E > E_c \end{cases} \quad \text{J}^{-1} \text{m}^{-2}$ $H(x) = 0 \text{ for } x < 0 \text{ and } H(x) = 1 \text{ for } x > 0$	$D(E) = \begin{cases} \frac{(2m_h^*)^{3/2}}{2\pi^2\hbar^3} \sqrt{E_v - E} & E < E_v \\ 0 & E_v < E < E_c \\ \frac{(2m_e^*)^{3/2}}{2\pi^2\hbar^3} \sqrt{E - E_c} & E > E_c \end{cases}$
Density of electrons in the conduction band $n = \int_{E_c}^{\infty} D(E) f(E) dE$	$n = \sqrt{\frac{m_e^* k_B T}{\hbar^2 \pi}} \exp\left(\frac{\mu - E_c}{k_B T}\right) \text{ m}^{-1}$	$n = \frac{m_e^* k_B T}{\hbar^2 \pi} \exp\left(\frac{\mu - E_c}{k_B T}\right) \text{ m}^{-2}$	$n = \frac{1}{\sqrt{2}} \left(\frac{m_e^* k_B T}{\pi \hbar^2}\right)^{3/2} \exp\left(\frac{\mu - E_c}{k_B T}\right)$
Density of holes in the valence band $p = \int_{-\infty}^{E_v} D(E) (1 - f(E)) dE$	$p = \sqrt{\frac{m_h^* k_B T}{\hbar^2 \pi}} \exp\left(\frac{E_v - \mu}{k_B T}\right) \text{ m}^{-1}$	$p = \frac{m_h^* k_B T}{\hbar^2 \pi} \exp\left(\frac{E_v - \mu}{k_B T}\right) \text{ m}^{-2}$	$p = \frac{1}{\sqrt{2}} \left(\frac{m_h^* k_B T}{\pi \hbar^2}\right)^{3/2} \exp\left(\frac{E_v - \mu}{k_B T}\right)$
Law of mass action	$np = \frac{k_B T}{\hbar^2 \pi} \sqrt{m_e^* m_h^*} \exp\left(\frac{-E_g}{k_B T}\right) \text{ m}^{-2}$	$np = \left(\frac{k_B T}{\pi \hbar^2}\right)^2 m_e^* m_h^* \exp\left(\frac{-E_g}{k_B T}\right) \text{ m}^{-4}$	$np = \frac{1}{2} \left(\frac{k_B T}{\pi \hbar^2}\right)^3 (m_e^* m_h^*)^{3/2} \exp\left(\frac{-E_g}{k_B T}\right)$
Intrinsic carrier density $n_i = \sqrt{np}$	$n_i = \sqrt{\frac{k_B T}{\hbar^2 \pi}} (m_e^* m_h^*)^{1/4} \exp\left(\frac{-E_g}{2k_B T}\right) \text{ m}^{-1}$	$n_i = \frac{k_B T}{\pi \hbar^2} \sqrt{m_e^* m_h^*} \exp\left(\frac{-E_g}{2k_B T}\right) \text{ m}^{-2}$	$n_i = \frac{1}{\sqrt{2}} \left(\frac{k_B T}{\pi \hbar^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right)$
Chemical potential Set $n = p$ , solve for $\mu$	$\mu = \frac{E_v + E_c}{2} - \frac{k_B T}{4} \ln\left(\frac{m_e^*}{m_h^*}\right) \text{ J}$	$\mu = \frac{E_v + E_c}{2} - \frac{k_B T}{2} \ln\left(\frac{m_e^*}{m_h^*}\right) \text{ J}$	$\mu = \frac{E_v + E_c}{2} - \frac{3}{4} k_B T \ln\left(\frac{m_e^*}{m_h^*}\right)$
Internal energy density $u = \int_{-\infty}^{\infty} E D(E) f(E) dE$	$u = u(T=0) + \sqrt{\frac{2k_B T}{\hbar^2 \pi}} (m_e^* m_h^*)^{1/4} \exp\left(\frac{-E_g}{2k_B T}\right) (E_g + k_B T) \text{ J m}^{-1}$	$u = u(T=0) + \frac{k_B T}{\hbar^2 \pi} \sqrt{m_e^* m_h^*} \exp\left(\frac{-E_g}{2k_B T}\right) (E_g + 2k_B T) \text{ J m}^{-2}$	$u = u(T=0) + \frac{\sqrt{2\pi}}{2\pi^2 \hbar^3} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right) (k_B T)^{3/2}$
Helmholtz free energy $f = u - Ts$	$f = u(T=0) - 2\sqrt{\frac{2k_B T}{\hbar^2 \pi}} (m_e^* m_h^*)^{1/4} \exp\left(\frac{-E_g}{2k_B T}\right) k_B T^{3/2} \text{ J m}^{-1}$	$f = u(T=0) - \frac{2\sqrt{m_e^* m_h^*}}{\hbar^2 \pi} k_B^2 T^2 \exp\left(\frac{-E_g}{2k_B T}\right) \text{ J m}^{-2}$	$f = u(T=0) - \frac{\sqrt{2\pi}}{\pi^2 \hbar^3} (m_e^* m_h^*)^{3/4} (k_B T)^{5/2} \text{ e:}$
Specific heat $c_v = \left(\frac{\partial u}{\partial T}\right)_{V=const}$	$c_v = \sqrt{\frac{2k_B T}{\hbar^2 \pi}} (m_e^* m_h^*)^{1/4} \exp\left(\frac{-E_g}{2k_B T}\right) \left(\frac{E_g^2 \sqrt{T}}{2k_B T^2} + \frac{E_g}{\sqrt{T}} + \frac{3k_B \sqrt{T}}{2}\right) \text{ J K}^{-1} \text{ m}^{-1}$	$c_v = \frac{\sqrt{m_e^* m_h^*}}{\hbar^2 \pi} \exp\left(\frac{-E_g}{2k_B T}\right) \left(\frac{E_g^2}{2T} + 2k_B E_g + 4k_B^2 T\right) \text{ J K}^{-1} \text{ m}^{-2}$	$c_v = \frac{\sqrt{2\pi}}{2\pi^2 \hbar^3} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right) (k_B T)^{3/2} \left(\frac{15}{2} k_B T\right)$
Entropy $s = \int \frac{c_v}{T} dT$	$s = \sqrt{\frac{2k_B T}{\hbar^2 \pi}} (m_e^* m_h^*)^{1/4} \exp\left(\frac{-E_g}{2k_B T}\right) \left(\frac{E_g}{\sqrt{T}} + 3k_B \sqrt{T}\right) \text{ J K}^{-1} \text{ m}^{-1}$	$s = \frac{\sqrt{m_e^* m_h^*}}{\hbar^2 \pi} \exp\left(\frac{-E_g}{2k_B T}\right) (k_B E_g + 4k_B^2 T) \text{ J K}^{-1} \text{ m}^{-2}$	$s = \frac{\sqrt{2\pi}}{2\pi^2 \hbar^3} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right) (k_B T)^{3/2} \left(\frac{15}{2} k_B T\right)$

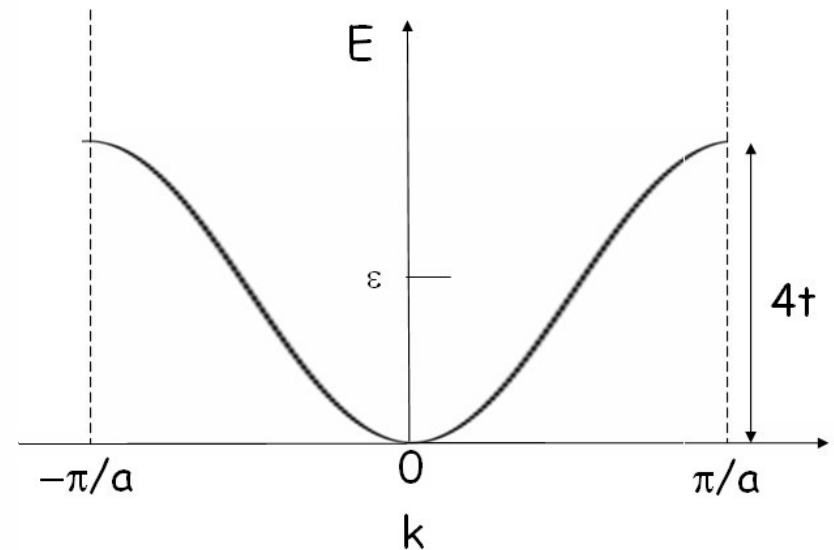
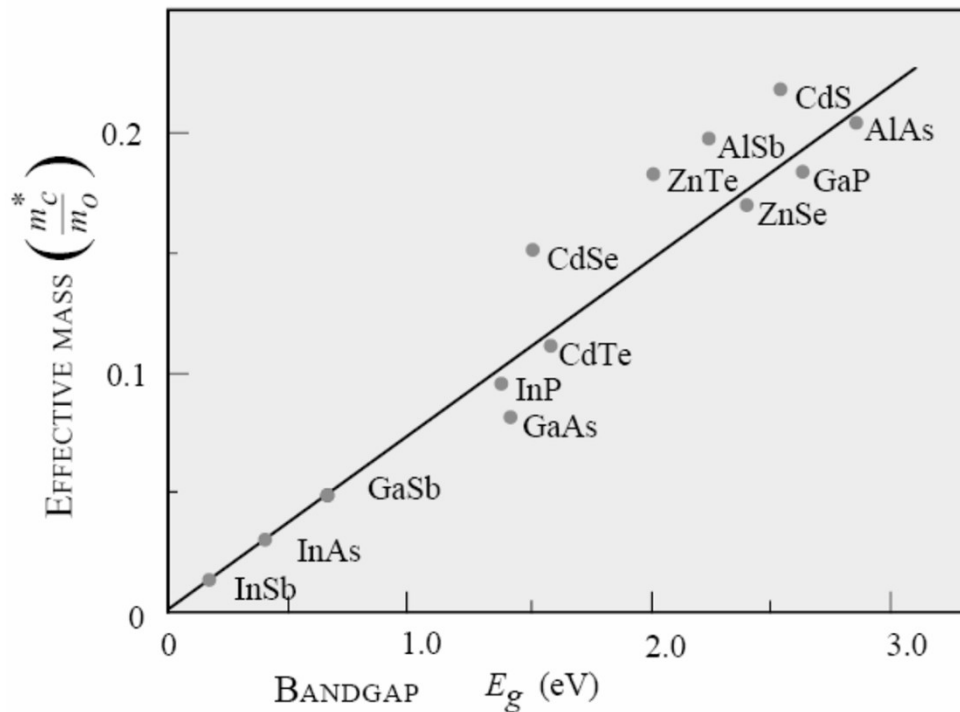
# Narrow bandgap semiconductors

---



Use the programs for metals for small bandgap semiconductors.

# Large gap -> large effective mass



$$E_k = \epsilon - 2t \cos(ka)$$

$$m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}}$$

$$\frac{d^2 E}{dk^2} = 2ta^2$$

$$m^* \propto \frac{1}{t}$$

narrow bands -> large effective mass

# Measuring the effective mass

---

Cyclotron resonance  $\omega_c = \frac{eB}{m^*}$

Resonant absorption occurs when rf waves with the cyclotron resonance frequency are applied. This can be used to experimentally determine the effective mass.

Knowing the effective mass, the scattering time can be calculated from the measured conductivity.

$$\sigma = \frac{ne^2\tau_{sc}}{m^*}$$

# Extrinsic semiconductors

---

The introduction of impurity atoms that can add electrons or holes is called doping.

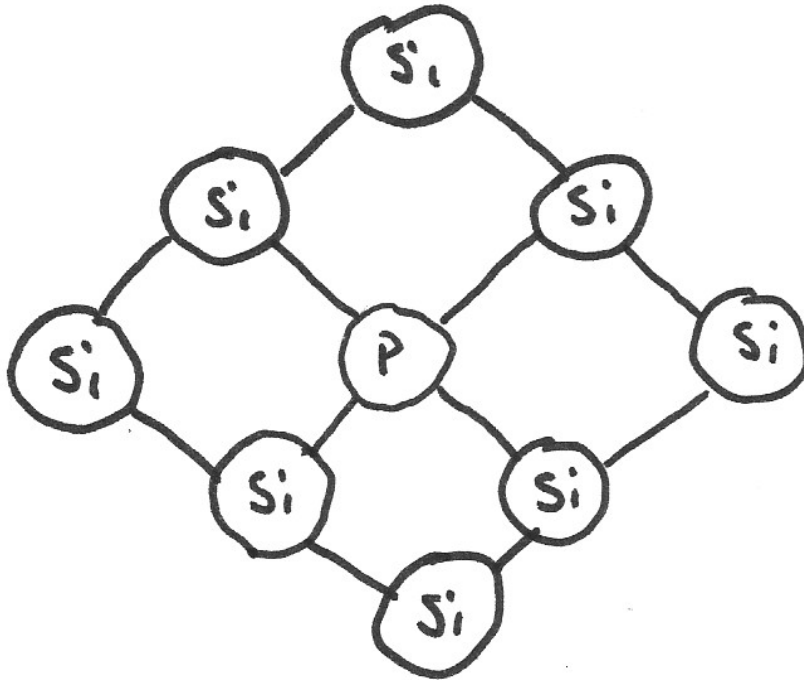
n-type : donor atoms contribute electrons to the conduction band.  
Examples: P, As in Si.

p-type : acceptor atoms contribute holes to the valence band.  
Examples: B, Ga, Al in Si.

	IIIA	IVA	VA	VIA	
	<sup>5</sup> B	<sup>6</sup> C	<sup>7</sup> N	<sup>8</sup> O	
	<sup>13</sup> Al	<sup>14</sup> Si	<sup>15</sup> P	<sup>16</sup> S	
IIB	<sup>30</sup> Zn	<sup>31</sup> Ga	<sup>32</sup> Ge	<sup>33</sup> As	<sup>34</sup> Se
	<sup>48</sup> Cd	<sup>49</sup> In	<sup>50</sup> Sn	<sup>51</sup> Sb	<sup>52</sup> Te

# Ionization of dopants

---



Easier to ionize a P atom in Si than a free P atom

$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2 n^2}$$

Ionization energy is smaller by a factor:  $\frac{m^*}{m} \left( \frac{\epsilon_0}{\epsilon_r \epsilon_0} \right)^2$

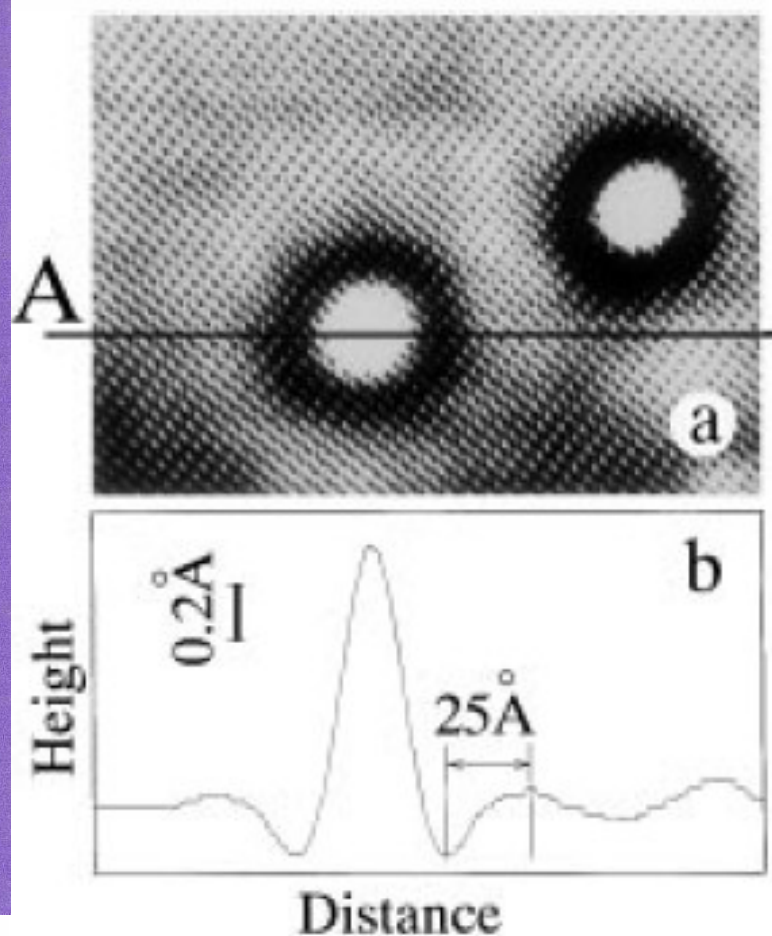
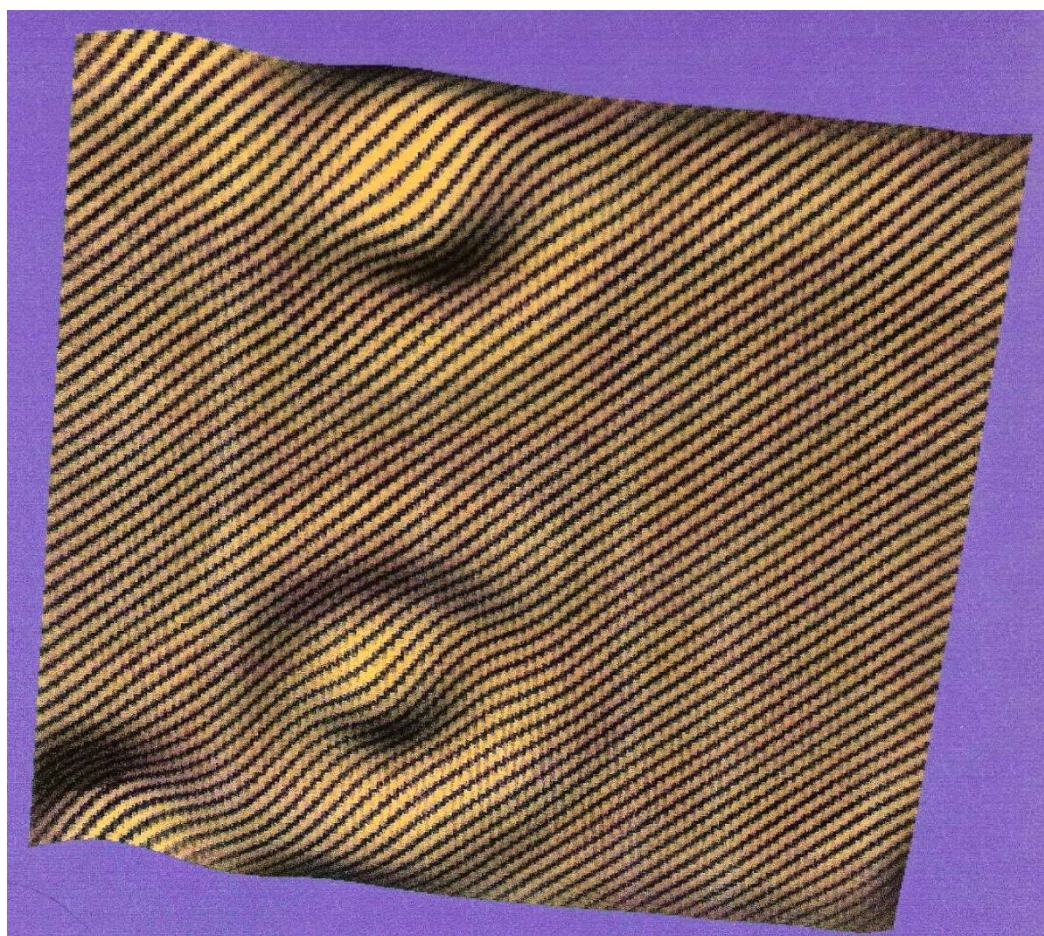
Ionization energy  $\sim 25$  meV

## Direct Observation of Friedel Oscillations around Incorporated $\text{Si}_{\text{Ga}}$ Dopants in GaAs by Low-Temperature Scanning Tunneling Microscopy

M. C. M. M. van der Wielen, A. J. A. van Roij, and H. van Kempen

*Research Institute for Materials, University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands*

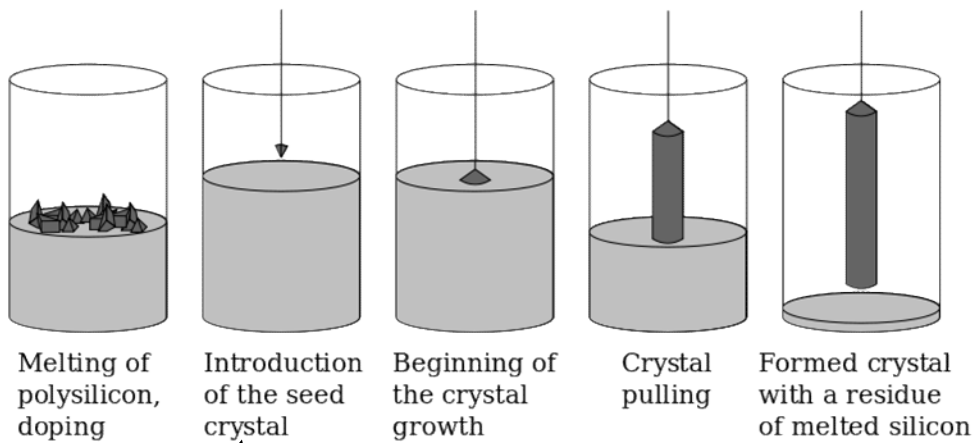
(Received 25 July 1995)



# Crystal growth

---

## Czochralski Process



add dopants to the melt



images from wikipedia



# Crystal growth

---

## Float zone Process

Neutron transmutation

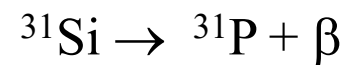
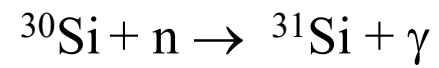
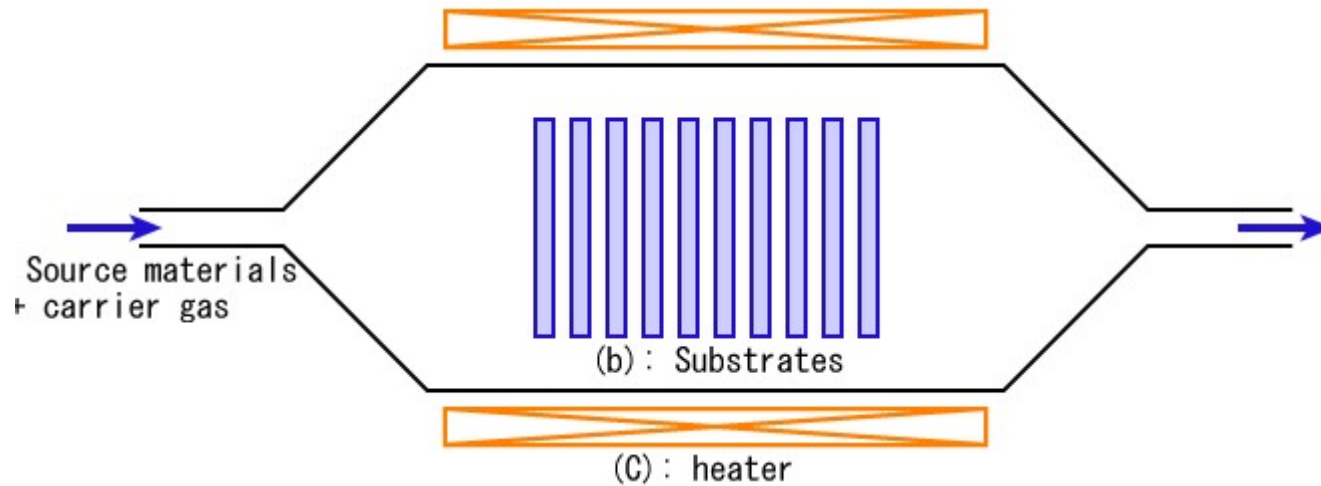


image from wikipedia

# Chemical vapor deposition

---



Epitaxial silicon CVD  $\text{SiH}_4$  (silane) or  $\text{SiH}_2\text{Cl}_2$  (dichlorosilane)  
 $\text{PH}_3$  (phosphine) for n-doping or  $\text{B}_2\text{H}_6$  (diborane) for p-doping.

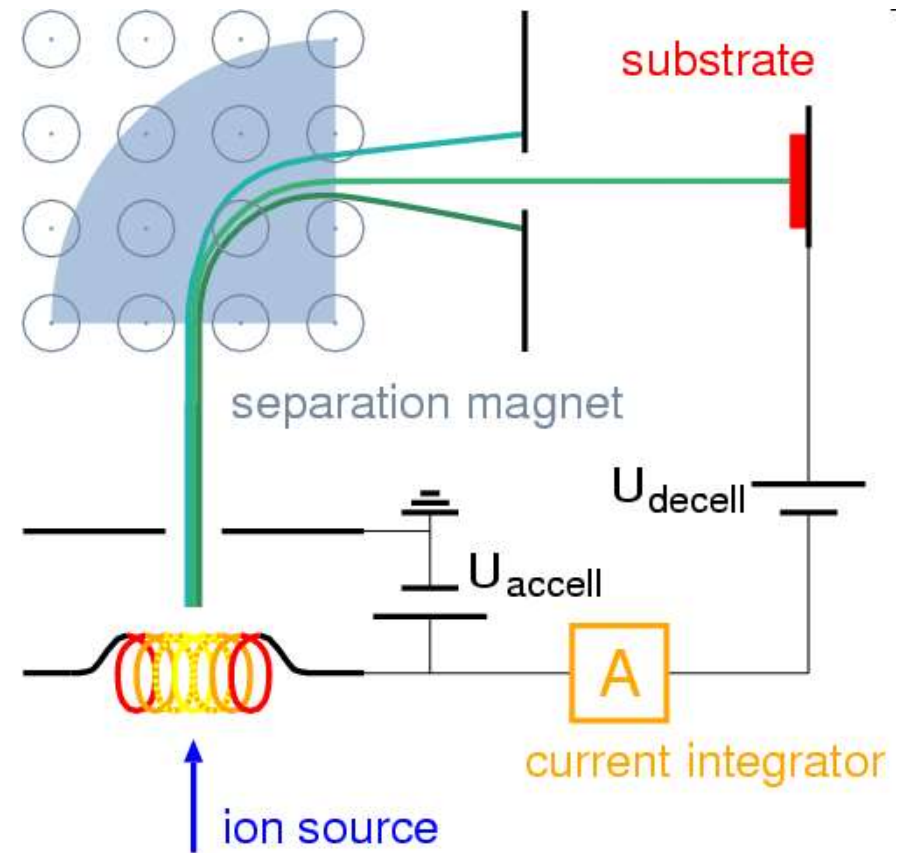
# Gas phase diffusion

---



$\text{AsH}_3$  (Arsine) or  $\text{PH}_3$  (phosphine) for n-doping  
 $\text{B}_2\text{H}_6$  (diborane) for p-doping.

# Ion implantation

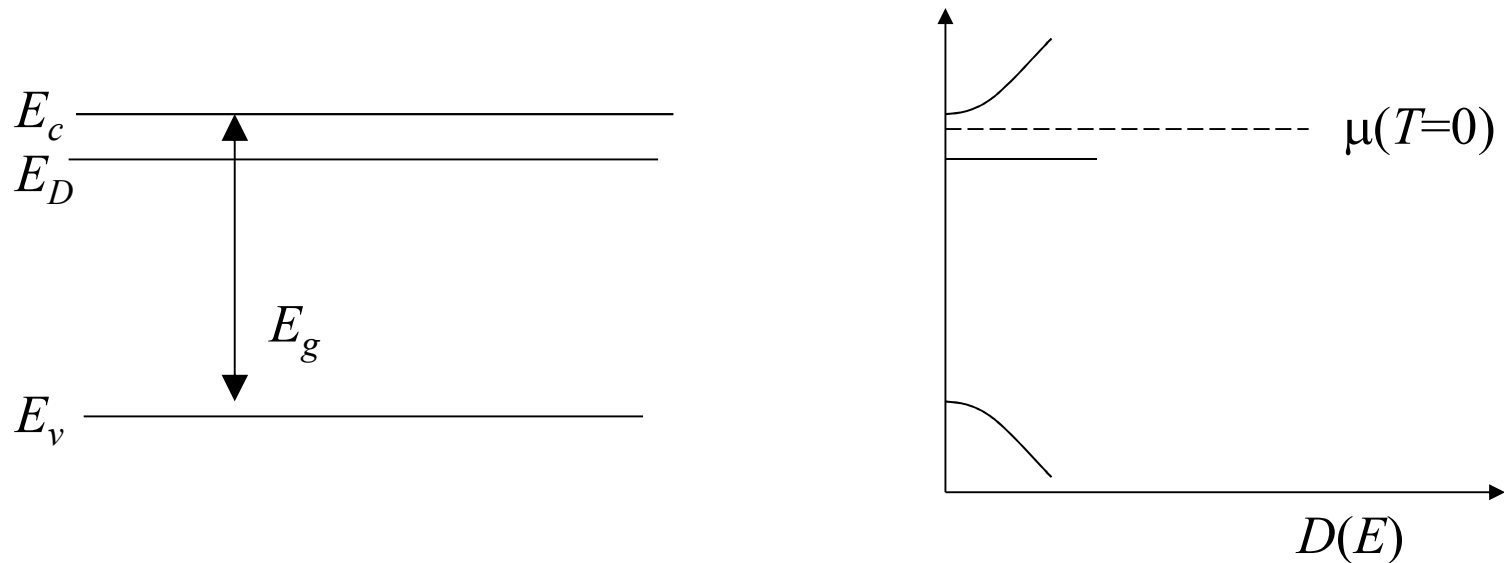


Implant at  $7^\circ$  to avoid channeling

# Donors

Five valence electrons: P, As

States are added in the band gap just below the conduction band



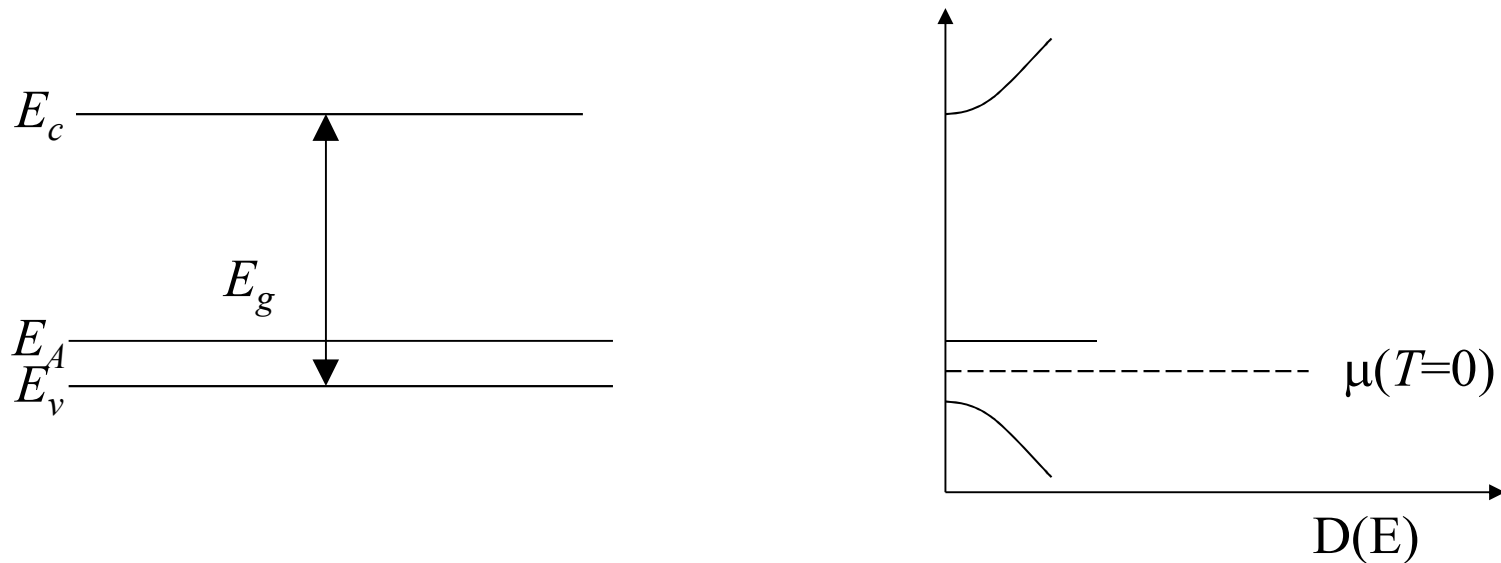
n-type:  $n \sim N_D$  Many more electrons in the conduction band than holes in the valence band.

majority carriers: electrons; minority carriers: holes

# Acceptors

Three valence electrons: B, Al, Ga

States are added in the band gap just above the valence band



p-type:  $p \sim N_A$  Many more holes in the valence band than electrons in the conduction band.

majority carriers: holes; minority carriers: electrons

# Donor and Acceptor Energies

---


Semiconductor	Donor	Energy (meV)
Si	Li	33
	Sb	39
	P	45
	As	54
Ge	Li	9.3
	Sb	9.6
	P	12
	As	13
GaAs	Si	5.8
	Ge	6.0
	S	6.0
	Sn	6.0

Energy below the conduction band



Semiconductor	Acceptor	Energy (meV)
Si	B	45
	Al	67
	Ga	72
	In	160
Ge	B	10
	Al	10
	Ga	11
	In	11
GaAs	C	26
	Be	28
	Mg	28
	Si	35

Energy above the valence band



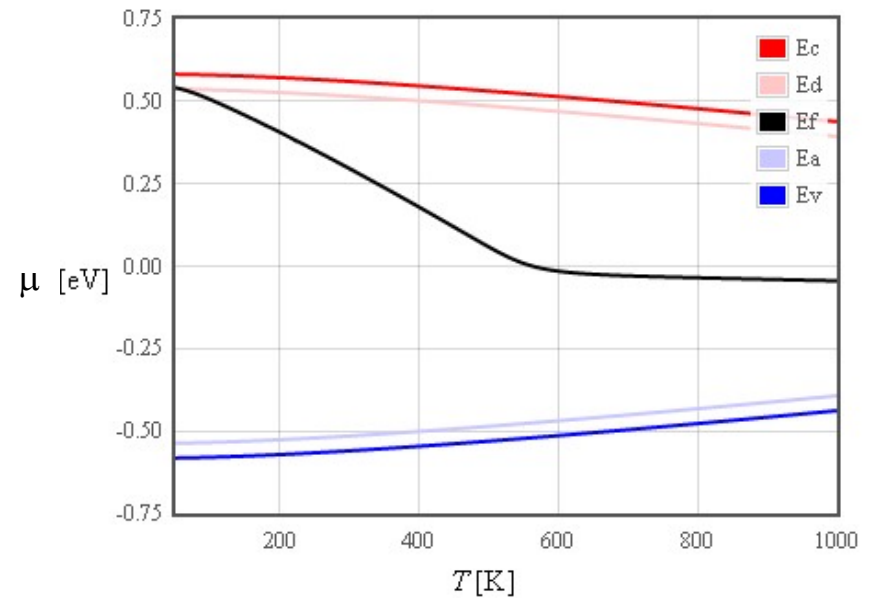
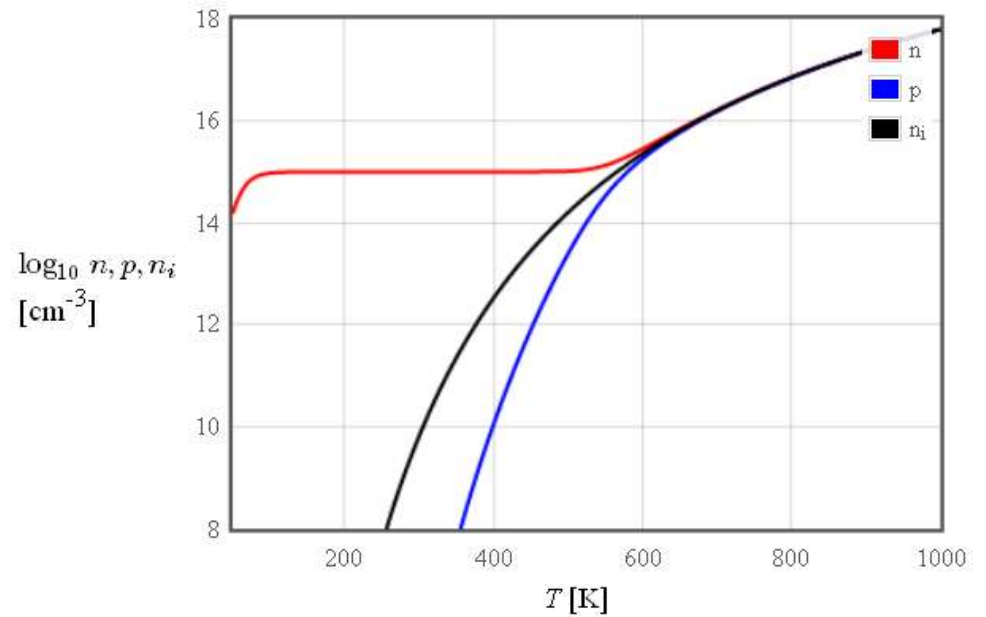
# n-type

n-type  $N_D > N_A$ ,  $p \sim 0$

$$n = N_D = N_c \exp\left(\frac{\mu - E_c}{k_B T}\right)$$

$$\mu = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)$$

For n-type,  $n \sim$  density of donors,  
 $p = n_i^2 / n$





# p-type

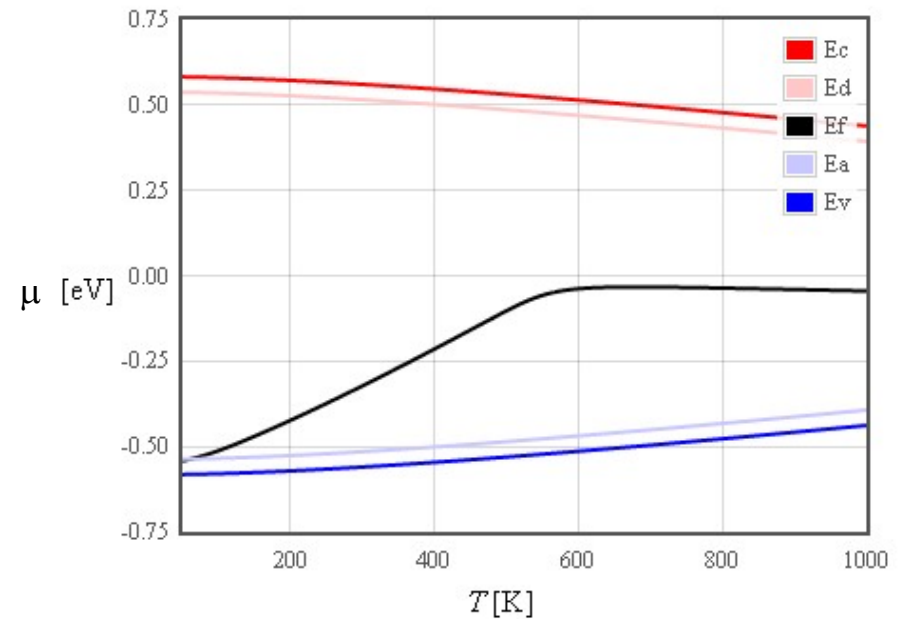
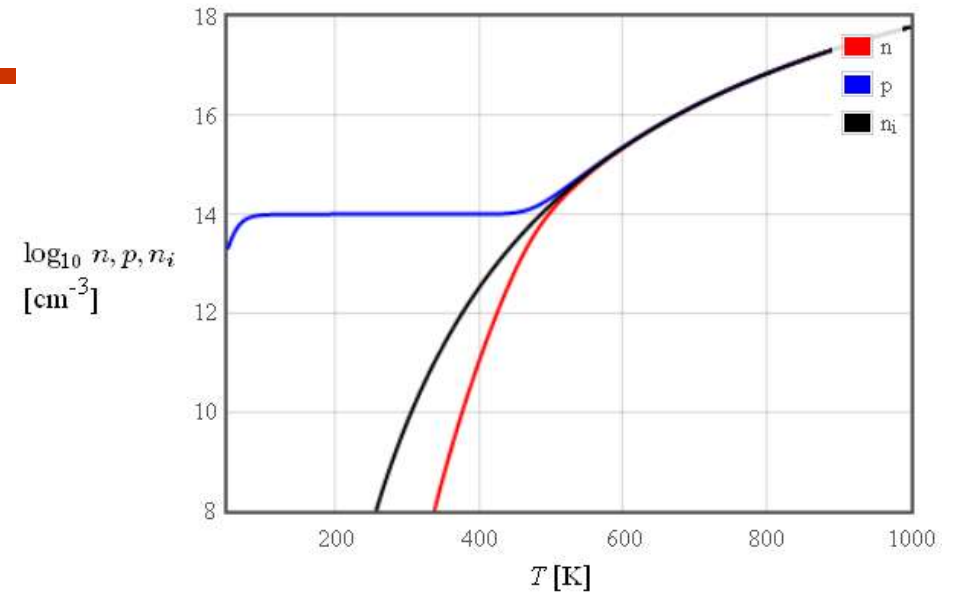
---

p-type  $N_A > N_D$ ,  $n \sim 0$

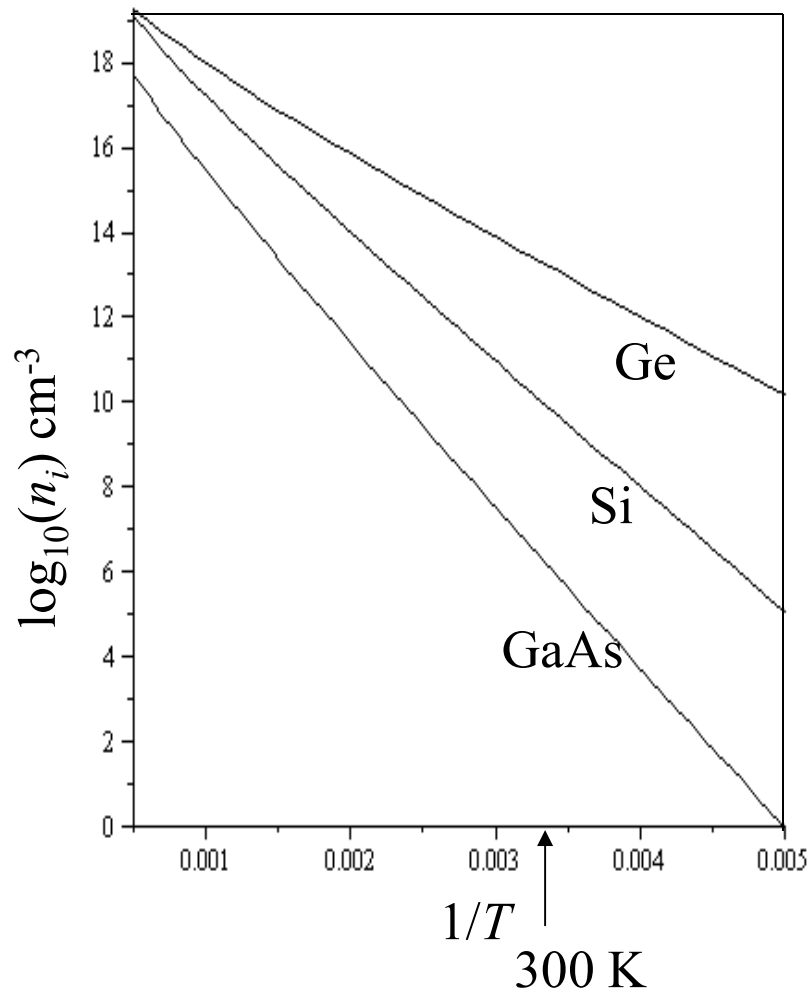
$$p = N_A = N_v \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

$$\mu = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)$$

For p-type,  $p \sim$  density of acceptors,  
 $n = n_i^2/p$

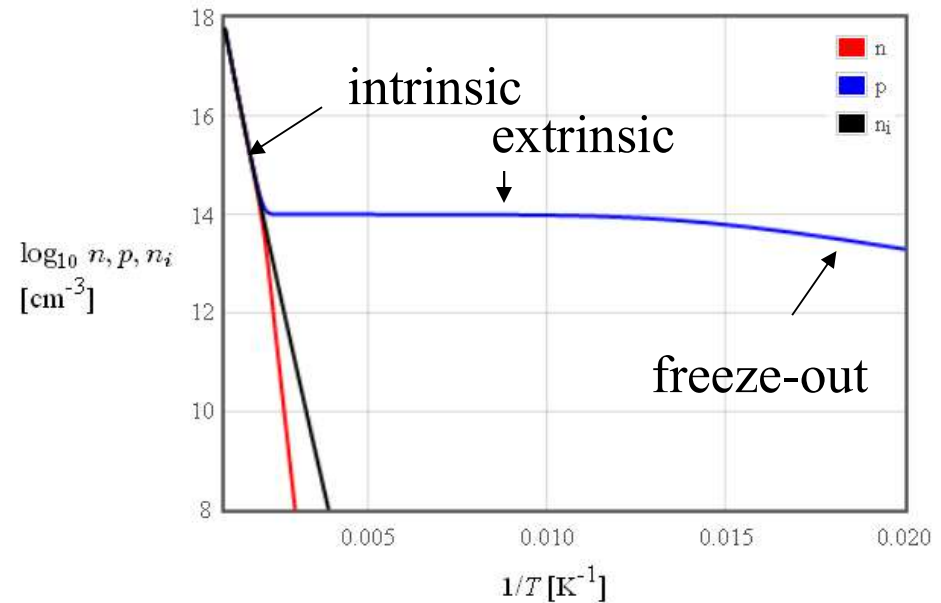


# Intrinsic semiconductors



$$n_i = \sqrt{N_v N_c} \exp\left(-\frac{E_g}{2k_B T}\right)$$

# Extrinsic semiconductors



At high temperatures, extrinsic semiconductors have the same temperature dependence as intrinsic semiconductors.

# Ionized donors and acceptors

---

For  $E_v + 3k_B T < \mu < E_c - 3k_B T$  Boltzmann approximation

$$N_D^+ = \frac{N_D}{1 + 2 \exp\left(\frac{\mu - E_D}{k_B T}\right)}$$

$$N_A^- = \frac{N_A}{1 + 4 \exp\left(\frac{E_A - \mu}{k_B T}\right)}$$

4 for materials with light  
holes and heavy holes (Si)  
2 otherwise

$N_D$  = donor density  $\text{cm}^{-3}$

$N_D^+$  = ionized donor density  $\text{cm}^{-3}$

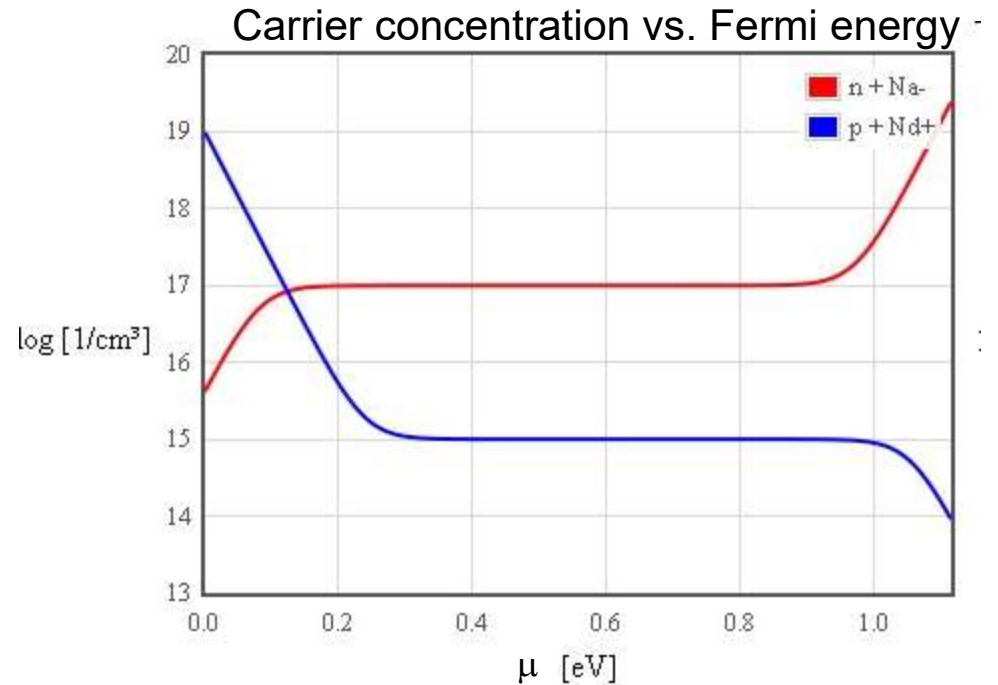
$N_A$  = acceptor density  $\text{cm}^{-3}$

$N_A^-$  = ionized acceptor density  $\text{cm}^{-3}$

Mostly,  $N_D^+ = N_D$  and  $N_A^- = N_A$

# Charge neutrality

$$n + N_A^- = p + N_D^+$$

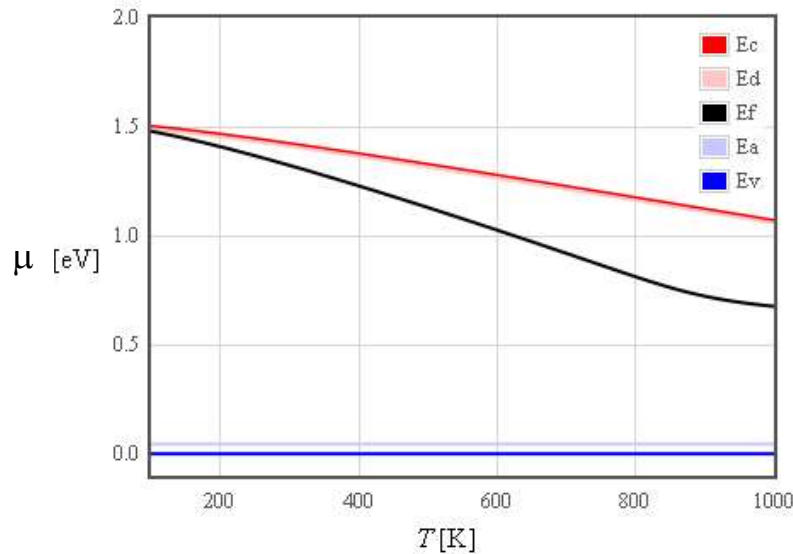


```
for ($i=0; $i<500; $i++) {
    $Ef = $i*$Eg/500;
    $n=$Nc*pow($T/300, 1.5) *exp(1.6022E-19*($Ef-$Eg) / (1.38E-23*$T));
    $p=$Nv*pow($T/300, 1.5) *exp(1.6022E-19*(-$Ef) / (1.38E-23*$T));
    $Namin = $Na/(1+4*exp(1.6022E-19*($Ea-$Ef) / (1.38E-23*$T)));
    $Ndplus = $Nd/(1+2*exp(1.6022E-19*($Ef-$Ed) / (1.38E-23*$T)));
}
```

$E_f$	$n$	$p$	$N_d^+$	$N_a^-$	$\log(n+N_a^-)$	$\log(p+N_d^+)$
0	4.16629283405	9.84E+18	1E+15	4.19743393218E+15	15.622983869	18.9930392318
0.00224	4.54358211887	9.0229075682E+18	1E+15	4.56020949614E+15	15.6589847946	18.9553946382
0.00448	4.95503779816	8.27366473417E+18	1E+15	4.95271809535E+15	15.694843609	18.9177504064
0.00672	5.40375389699	7.58663741327E+18	1E+15	5.37710747619E+15	15.7305487171	18.8801065693
0.00896	5.89210460791	6.95665026215E+18	1E+15	5.8256000025E+15	15.7660076057	18.8404621605

## Fermi energy vs. temperature

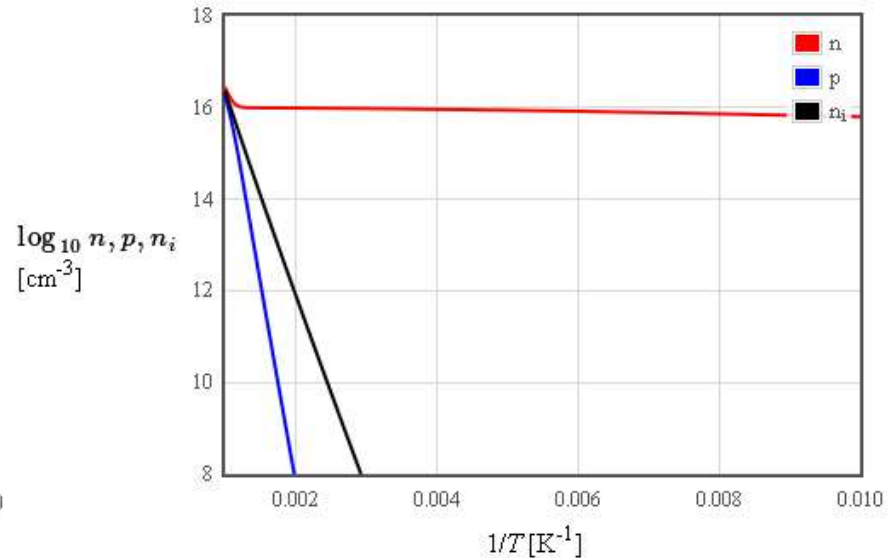
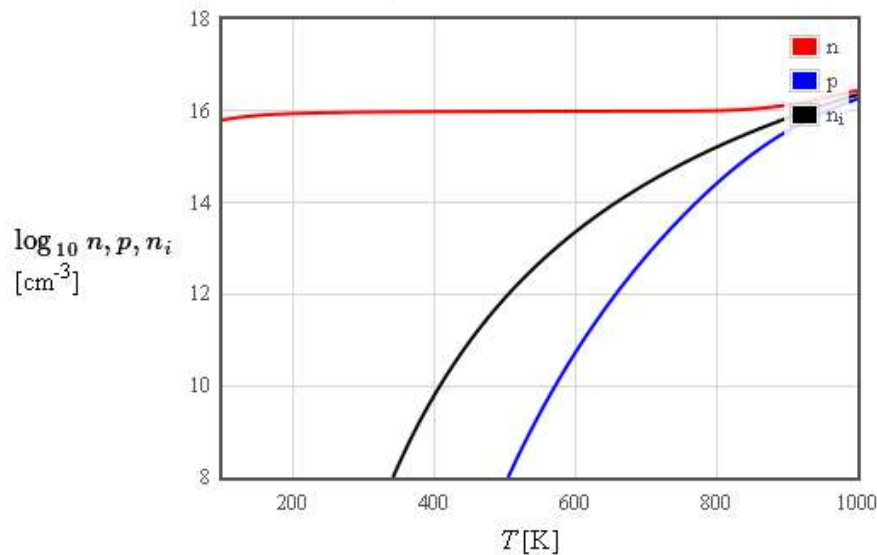
Fermi energy of an extrinsic semiconductor is plotted as a function of temperature. At each temperature the Fermi energy was calculated by requiring that charge neutrality be satisfied.



$N_c(300\text{ K}) = 4.45\text{E}17$	1/cm <sup>3</sup>	
$N_v(300\text{ K}) = 7.72\text{E}18$	1/cm <sup>3</sup>	
$E_g = 1.519 - 5.41\text{E-}4 * T * T / (T + 204)$		eV
Semiconductor		
		<input type="button" value="Si"/> <input type="button" value="Ge"/> <input type="button" value="GaAs"/>
Donor		
$N_d = 1\text{E}16$		1/cm <sup>3</sup>
$E_c - E_d = 0.012$		eV
		<input type="button" value="P in Si"/> <input type="button" value="P in Ge"/> <input type="button" value="Si in GaAs"/>
Acceptor		
$N_a = 1\text{E}12$		1/cm <sup>3</sup>
$E_a - E_v = 0.045$		eV
		<input type="button" value="B in Si"/> <input type="button" value="B in Ge"/> <input type="button" value="Si in GaAs"/>
$T_1 = 100$		K
$T_2 = 1000$		K
<input type="button" value="Replot"/>		

Once the Fermi energy is known, the carrier densities  $n$  and  $p$  can be calculated from the formulas,  $n = N_c \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{E_F - E_c}{k_B T}\right)$  and  $p = N_v \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{E_v - E_F}{k_B T}\right)$ .

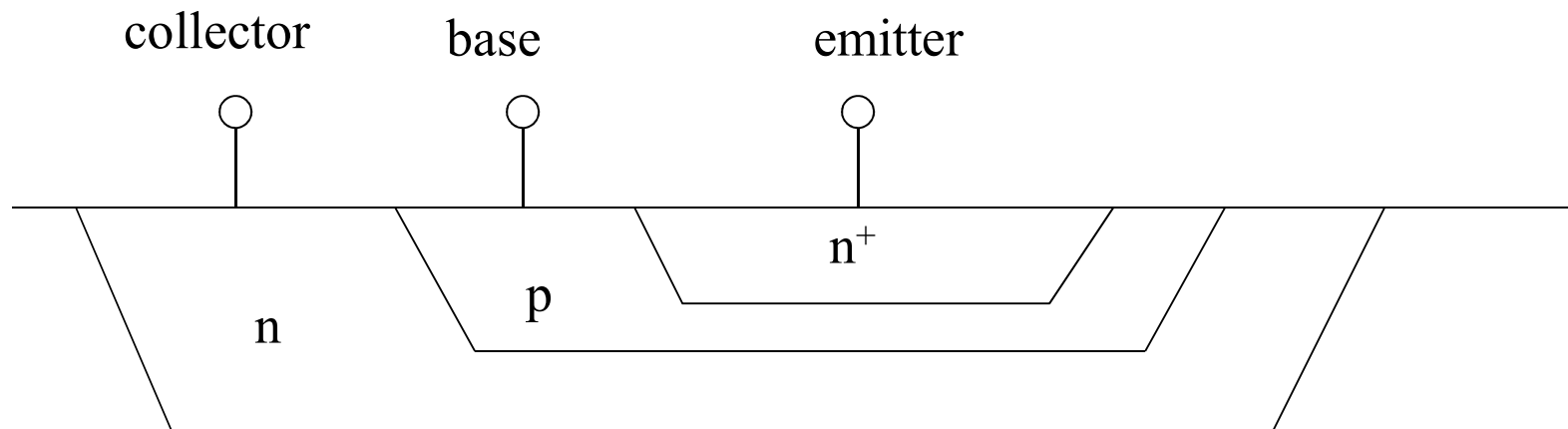
The intrinsic carrier density is  $n_i = \sqrt{N_c \left(\frac{T}{300}\right)^{3/2} N_v \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right)}$ .



# Why dope with donors AND acceptors?

---

## Bipolar transistor



lightly doped p substrate