

sp^2 hybrid orbitals 120°

The four orbitals are sp^2, sp^2, sp^2, p

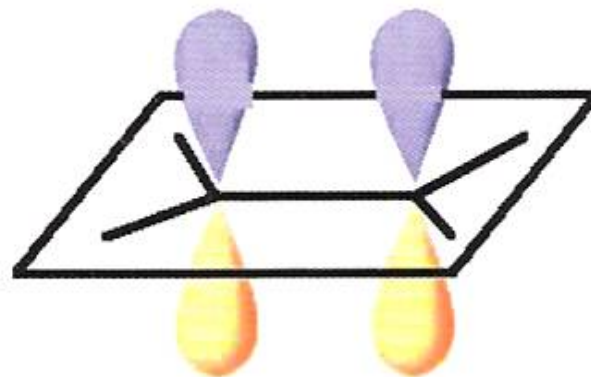
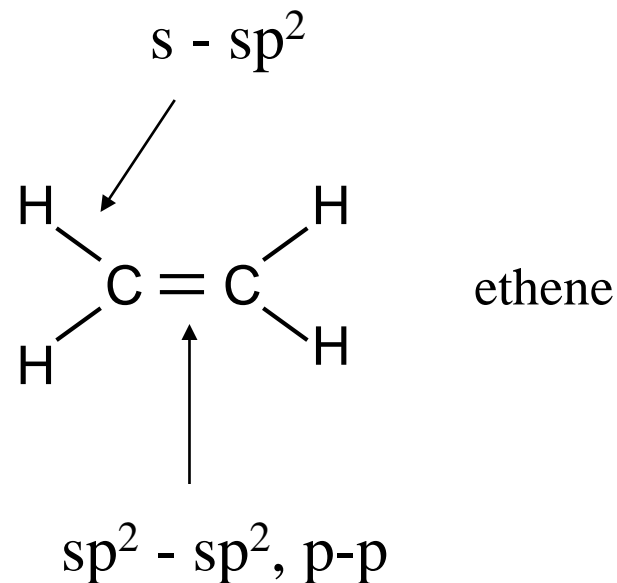
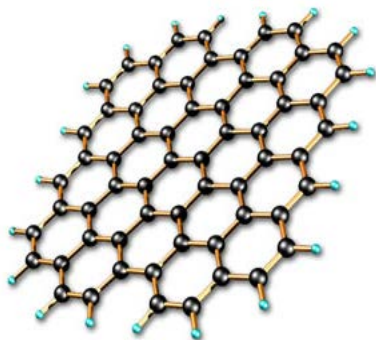
$$\psi_1 = \frac{1}{\sqrt{3}} (\phi_{2s} + \sqrt{2}\phi_{2p_x})$$

$$\psi_2 = \frac{1}{\sqrt{3}} \phi_{2s} - \frac{1}{\sqrt{6}} \phi_{2p_x} + \frac{1}{\sqrt{2}} \phi_{2p_y}$$

$$\psi_3 = \frac{1}{\sqrt{3}} \phi_{2s} - \frac{1}{\sqrt{6}} \phi_{2p_x} - \frac{1}{\sqrt{2}} \phi_{2p_y}$$

$$\psi_4 = \phi_{2p_z}$$

Graphene



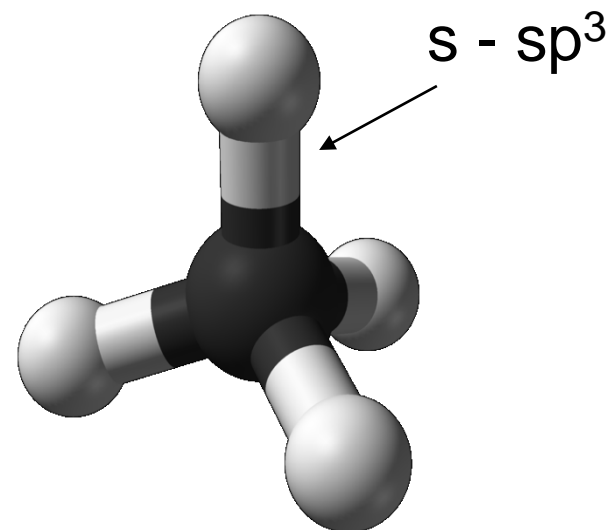
sp^3 hybrid orbitals 109°

$$\psi_1 = \frac{1}{2}(\phi_{2s} + \phi_{2p_x} + \phi_{2p_y} + \phi_{2p_z})$$

$$\psi_2 = \frac{1}{2}(\phi_{2s} + \phi_{2p_x} - \phi_{2p_y} - \phi_{2p_z})$$

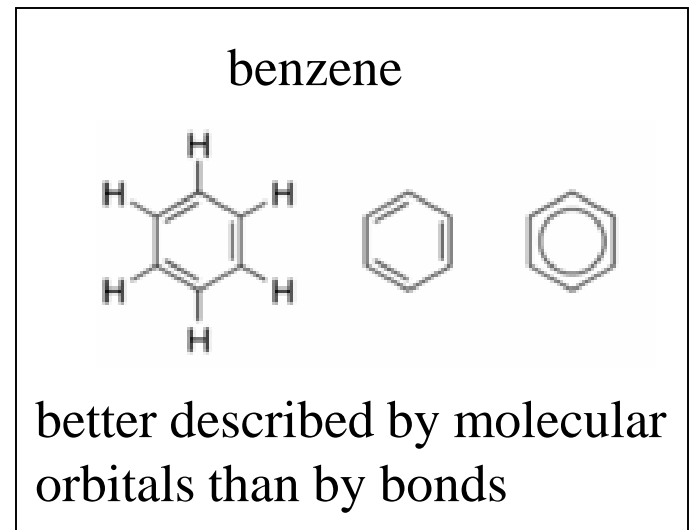
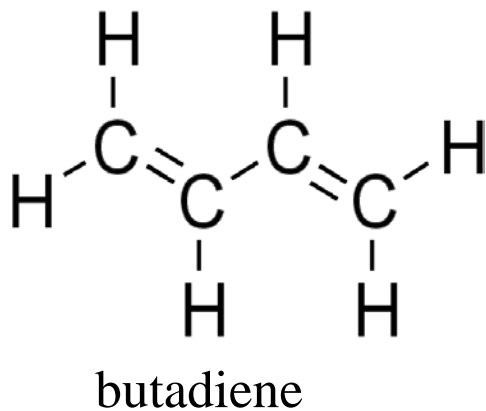
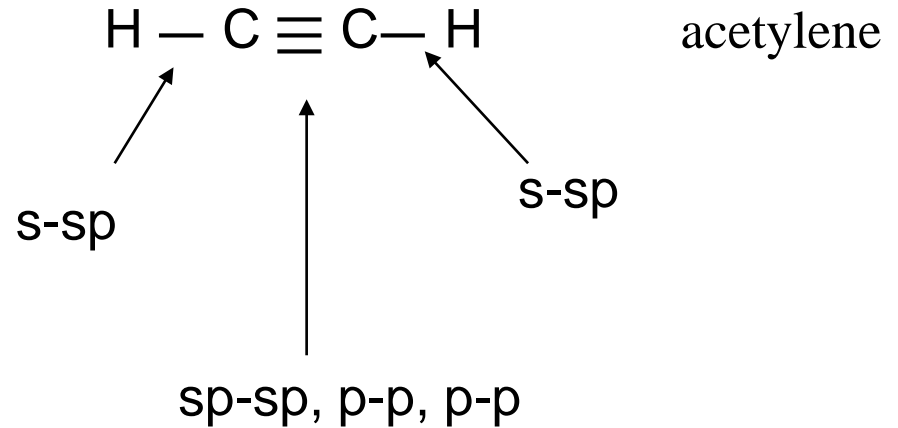
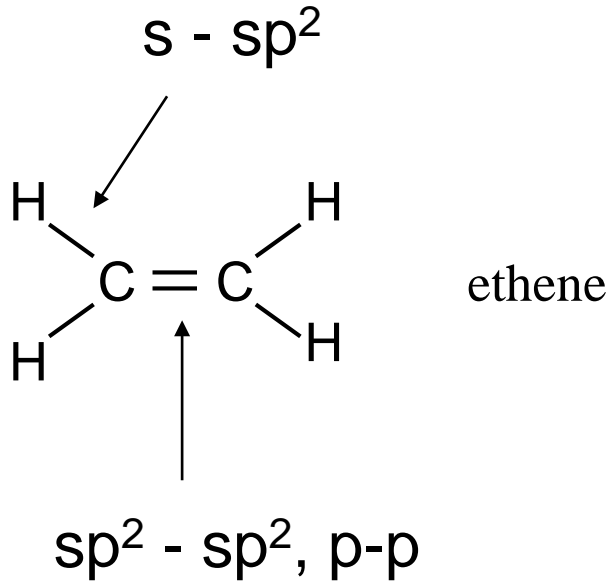
$$\psi_3 = \frac{1}{2}(\phi_{2s} - \phi_{2p_x} + \phi_{2p_y} - \phi_{2p_z})$$

$$\psi_4 = \frac{1}{2}(\phi_{2s} - \phi_{2p_x} - \phi_{2p_y} + \phi_{2p_z})$$



In this molecular orbital, the coefficients of these 4 atomic orbitals are about $c_{2s} = 1$, $c_{2p_x} = -1$, $c_{2p_y} = -1$, $c_{2p_z} = 1$.

Examples of bonds



Symmetries

Molecules can be classified by their symmetries. The eigenfunctions of the Hamiltonian will also be eigenfunctions of the symmetry operators.

Symmetries belong to a group. for $A, B \in G, AB \in G$

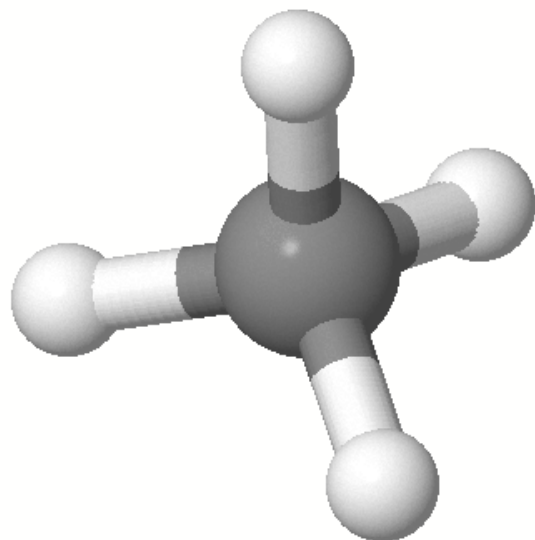
Point symmetries

If one point remains fixed during transformation, symmetries can be represented by 3×3 matrices.

$AB \in G$ for $A, B \in G$

Rotation about the x axis by angle α :

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



Point Group = T_d

Jmol

Element Operation

Show All Proper

C_3 axis

C_3 axis

C_3 axis

C_3 axis

C_2 axis

C_2 axis

C_2 axis

S_4 axis

S_4 axis

S_4 axis

Element Operation

Show All Planes

plane (σ_d)

plane (σ_d)

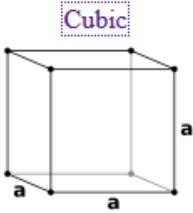
plane (σ_d)

plane (σ_d)

plane (σ_d)

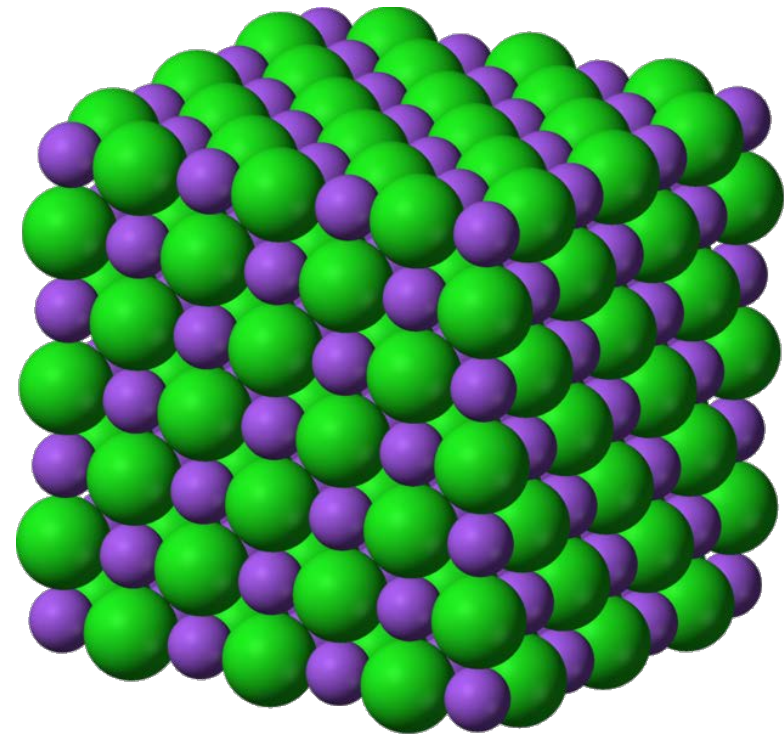
plane (σ_d)

The 32 Crystal Classes

Crystal system	Crystal Class	International symbol	Schoenflies symbol	Space groups	2-fold axes	3-fold axes	4-fold axes	6-fold axes	mirror planes	inversion	Examples	Number of symmetry elements
	tetrahedral	23	T	195-199	3	4	-	-	-	n		12
	diploidal	$m\bar{3}$	T_h	200-206	3	4	-	-	3	y		24
	gyroidal	432	O	207-214	6	4	3	-	-	n		24
	hextetrahedral	$\bar{4}3m$	T_d	215-220	3	4	-	-	6	n	216: Zincblende, ZnS, GaAs, GaP, InAs, SiC	24
	hexoctahedral	$m\bar{3}m$	O_h	221-230	6	4	3	-	9	y	221: CsCl, cubic perovskite 225: fcc, Al, Cu, Ni, Ag, Pt, Au, Pb, γ -Fe, NaCl 227: diamond, C, Si, Ge, α -Sn, spinel 229: bcc, Na, K, Cr, α -Fe, β -Ti, Nb, Mo, Ta	48

Crystal structure

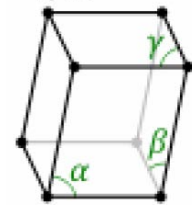
A crystal is a three dimensional periodic arrangement of atoms.



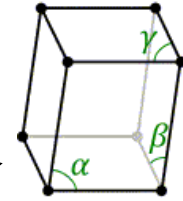
7 Crystal Systems

triclinic: $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90^\circ$

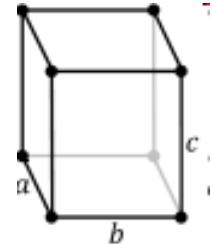
$\alpha, \beta, \gamma \neq 90^\circ$



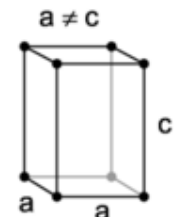
monoclinic: $a \neq b \neq c$ and $\alpha \neq 90^\circ$ $\beta = \gamma = 90^\circ$



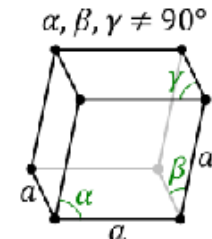
orthorhombic: $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$



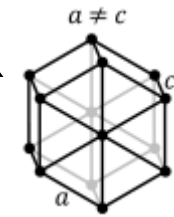
tetragonal: $a = b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$



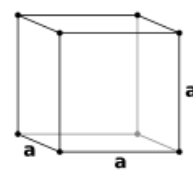
rhombohedral: $a = b = c$ and $\alpha \neq \beta \neq \gamma \neq 90^\circ$



hexagonal: $a = b \neq c$ and $\alpha = \beta = 90^\circ$, $\gamma = 120^\circ$

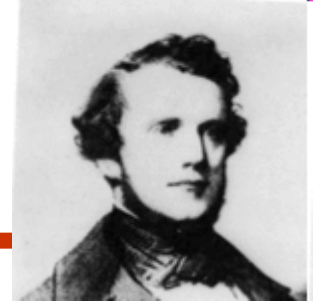


cubic $a = b = c$ and $\alpha = \beta = \gamma = 90^\circ$

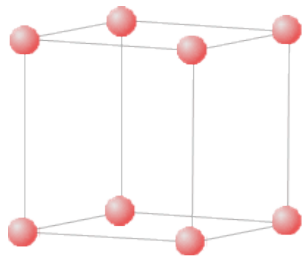
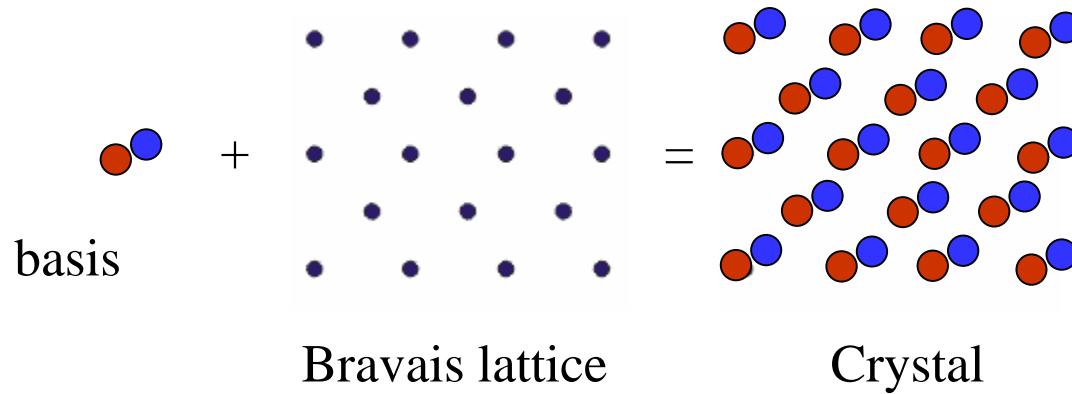


α is the angle between b and c

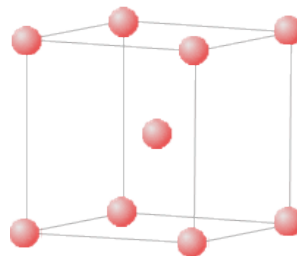
Bravais lattice



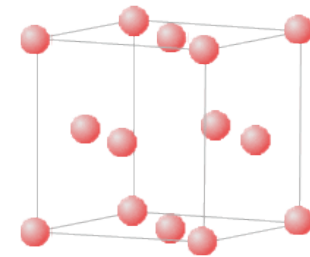
Auguste Bravais



simple cubic



body centered
cubic, bcc



face centered
cubic, fcc

14 Bravais lattices

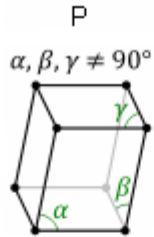
Crystal system

Bravais lattices

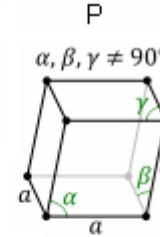
Crystal system

Bravais lattices

triclinic

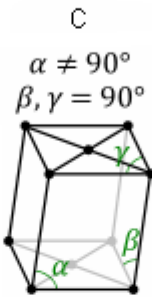
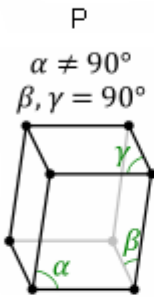


rhombohedral
 (trigonal)

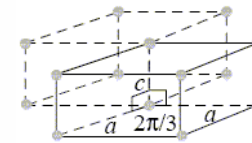


Points of a Bravais lattice do not necessarily represent atoms.

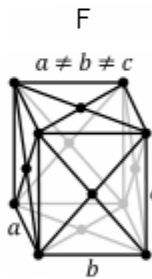
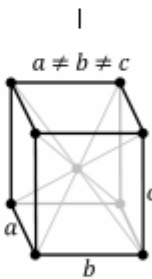
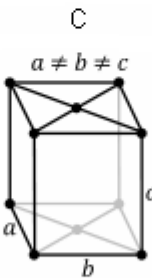
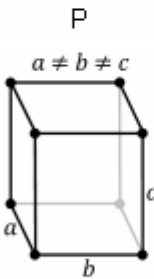
monoclinic



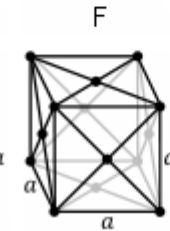
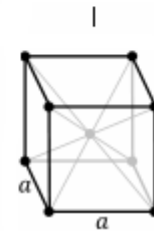
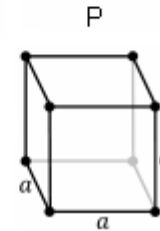
hexagonal



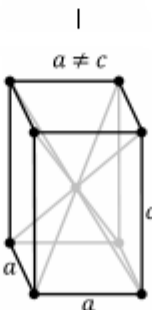
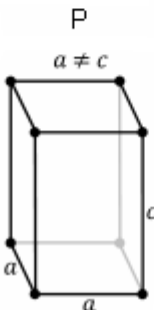
orthorhombic



cubic



tetragonal



P ... primitive

I ... body centered

F ... face centered

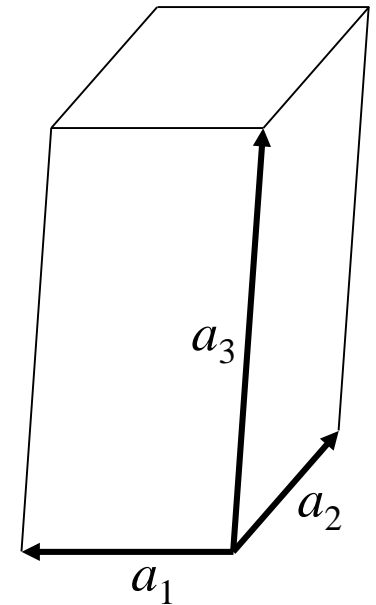
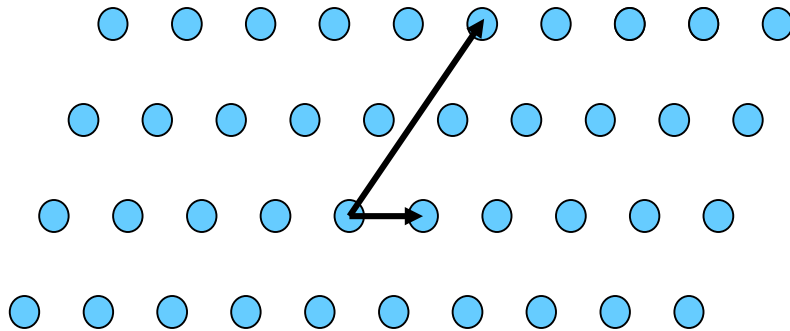
C ... centered

Primitive lattice vectors

Every point of a Bravais lattice can be reached from another point on the lattice by a translation vector

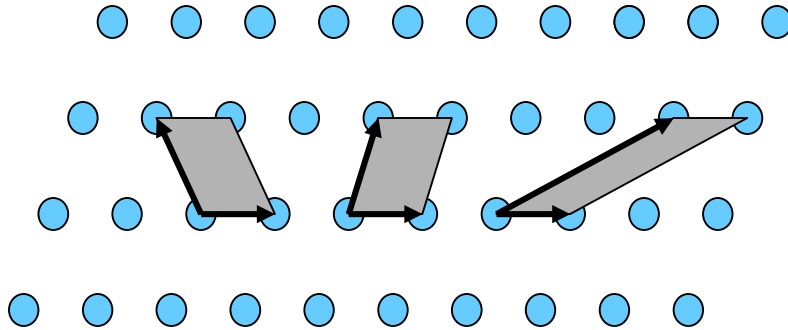
Translation vector

$$\vec{T} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3 \quad n_1, n_2, n_3 = \dots -2, -1, 0, 1, 2, \dots$$

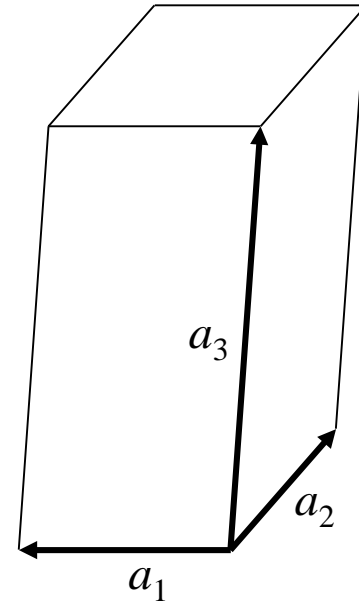


Primitive lattice vectors

Primitive Unit Cell



There is more than one choice for a primitive unit cell



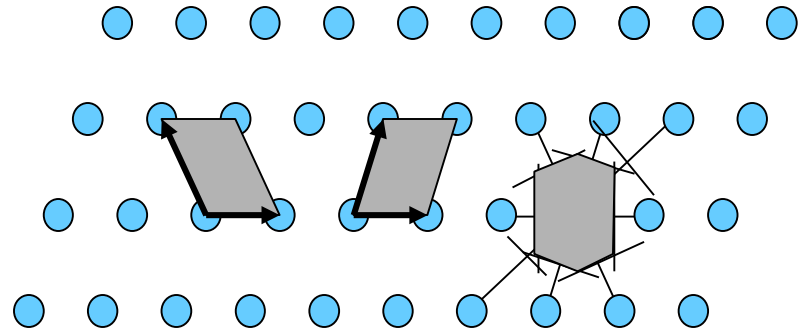
volume of a unit cell =

$$\vec{T} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3 \quad n_1, n_2, n_3 = \dots -2, -1, 0, 1, 2, \dots$$

$$\left| \vec{a}_1 \cdot \vec{a}_2 \times \vec{a}_3 \right|$$

Unit Cells

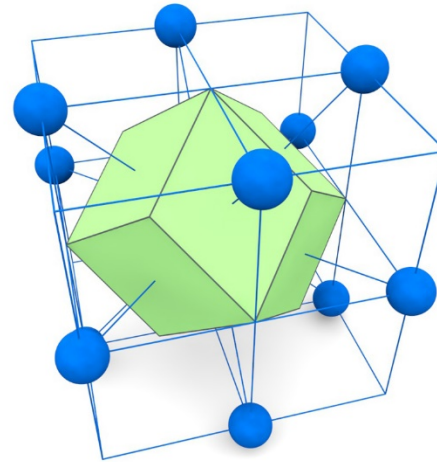
There is more than one choice for a primitive unit cell



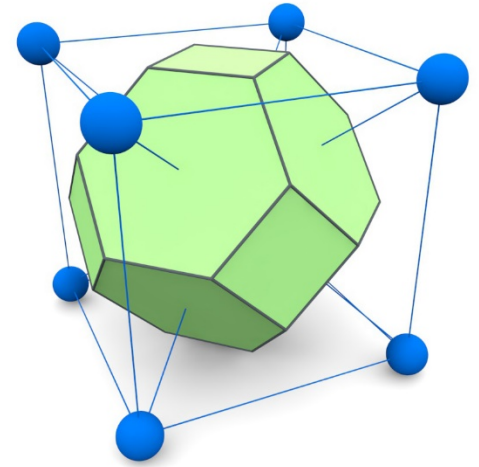
Eugene
Wigner



Frederick
Seitz



fcc

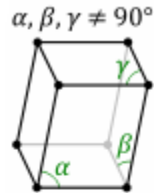


bcc

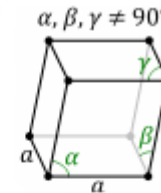
Wigner-Seitz primitive unit cell

Conventional (crystallographic) unit cell

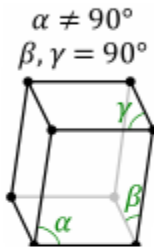
triclinic



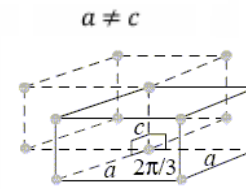
rhombohedral
(trigonal)



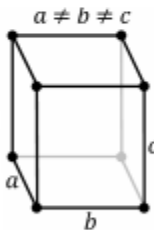
monoclinic



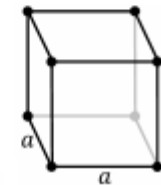
hexagonal



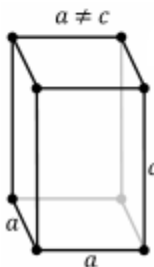
orthorhombic



cubic



tetragonal

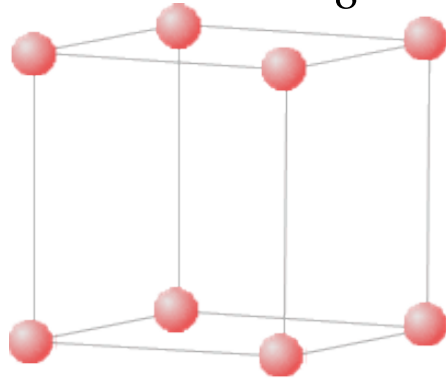


6 faces, 8 corners

http://en.wikipedia.org/wiki/Bravais_lattice

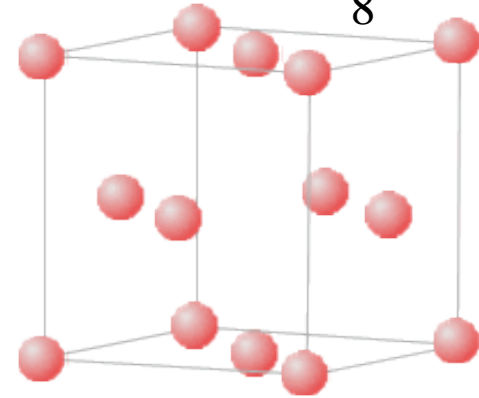
Conventional (crystallographic) unit cell

$$8 \times \frac{1}{8} = 1$$



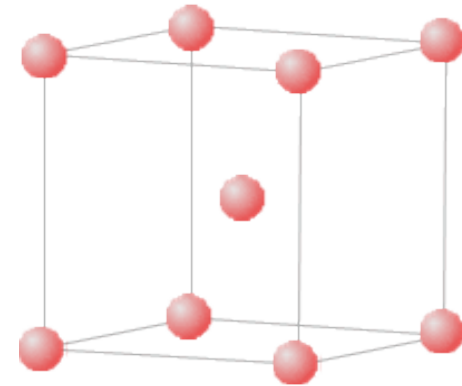
simple cubic

$$8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$

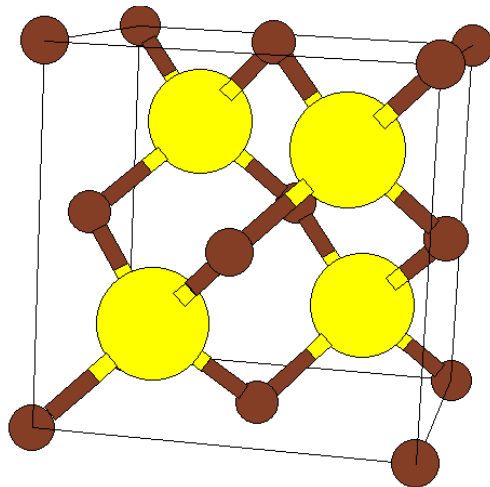


fcc

$$8 \times \frac{1}{8} + 1 = 2$$

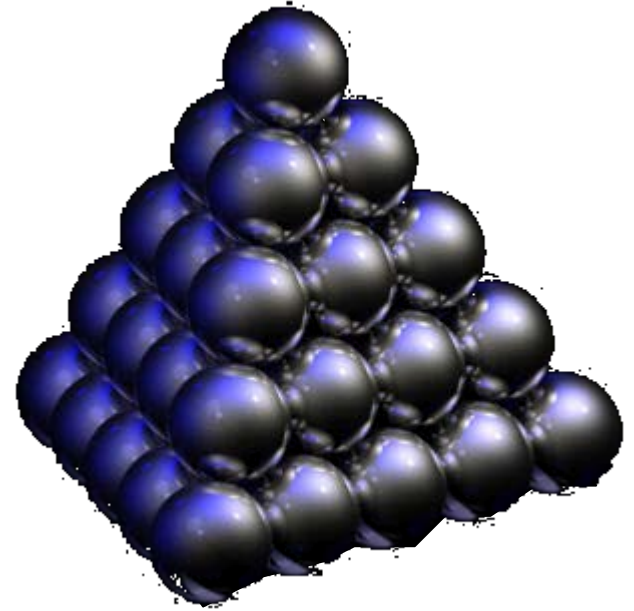
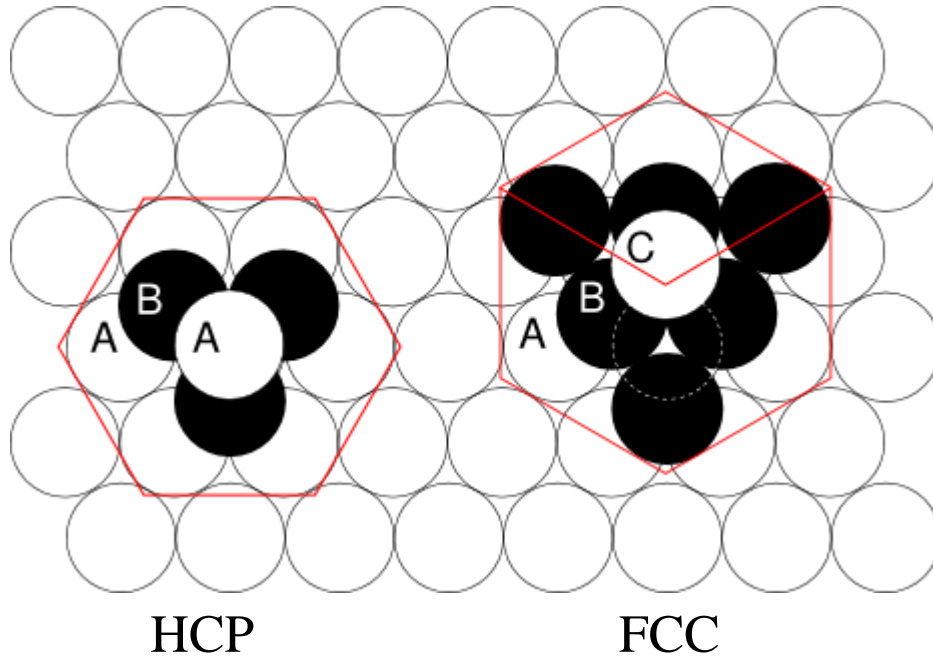


bcc



zincblende

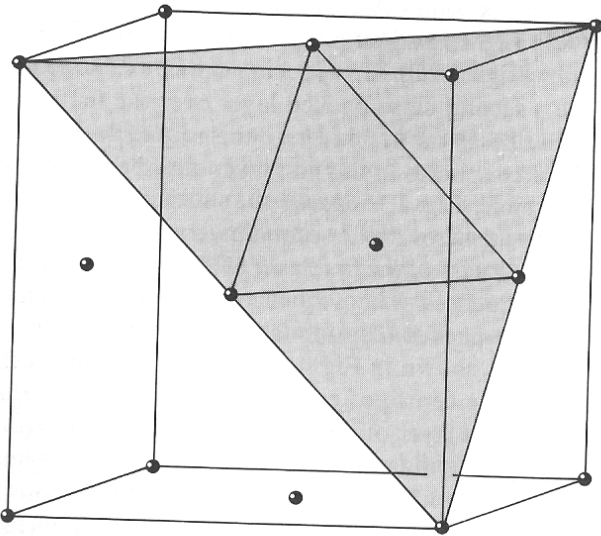
Close packing



HCP = Hexagonal close pack

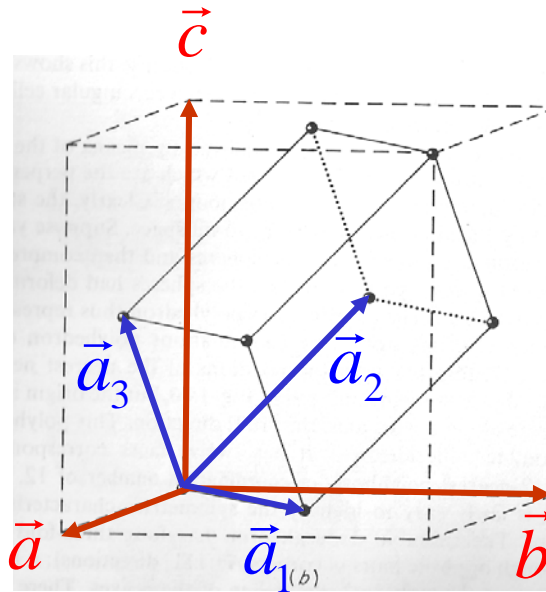
Hexagonal Bravais lattice with two atoms in the basis.

Fcc

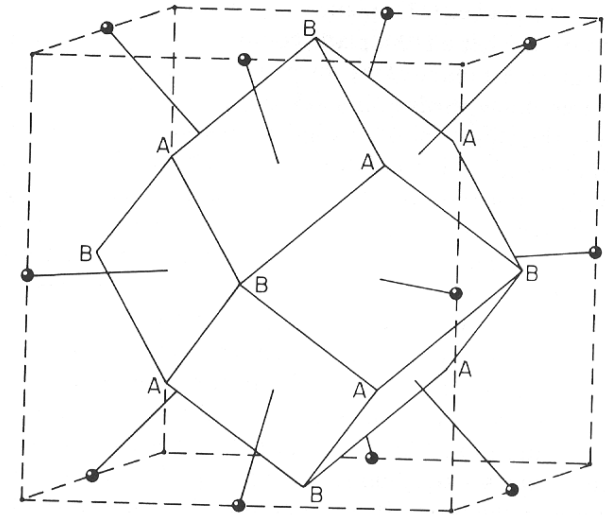


(a)

Crystallographic unit cell
showing close packed
plane



Crystallographic lattice
vectors
Primitive lattice vectors



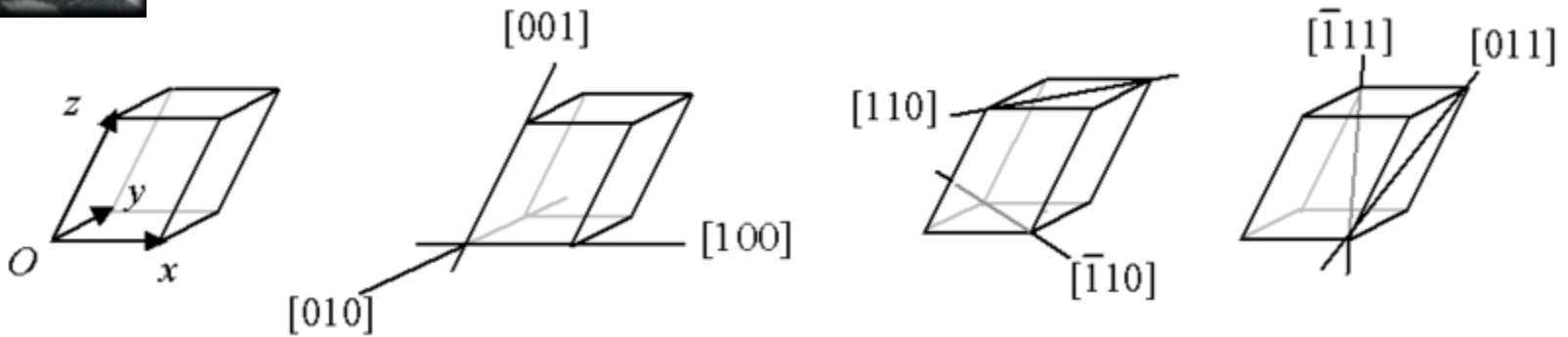
Wigner-Seitz cell

Miller indices: Crystal direction $[uvw]$



$[uvw]$ = vector in direction $u \mathbf{a} + v \mathbf{b} + w \mathbf{c}$

lattice vectors of the crystallographic unit cell

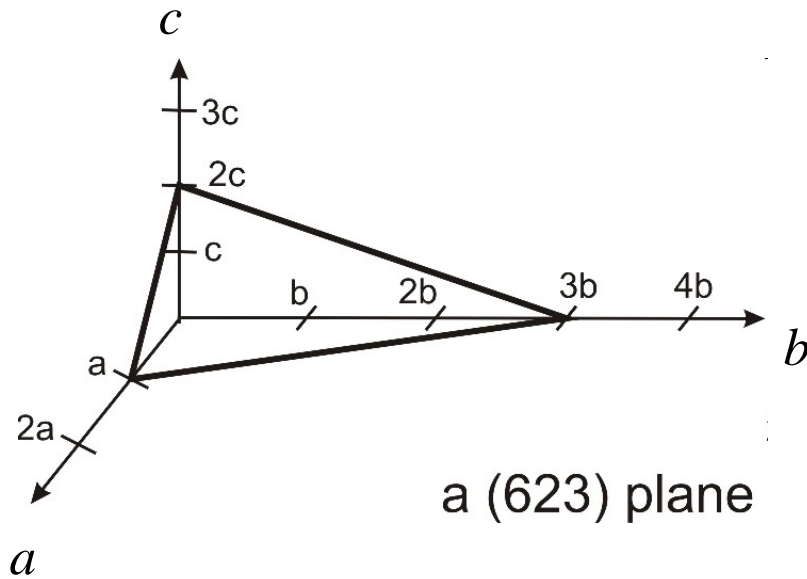


notation: $-1 = \bar{1}$

$[]$ specific direction

$\langle \rangle$ family of equivalent directions

Miller indices: Crystal planes



() specific plane

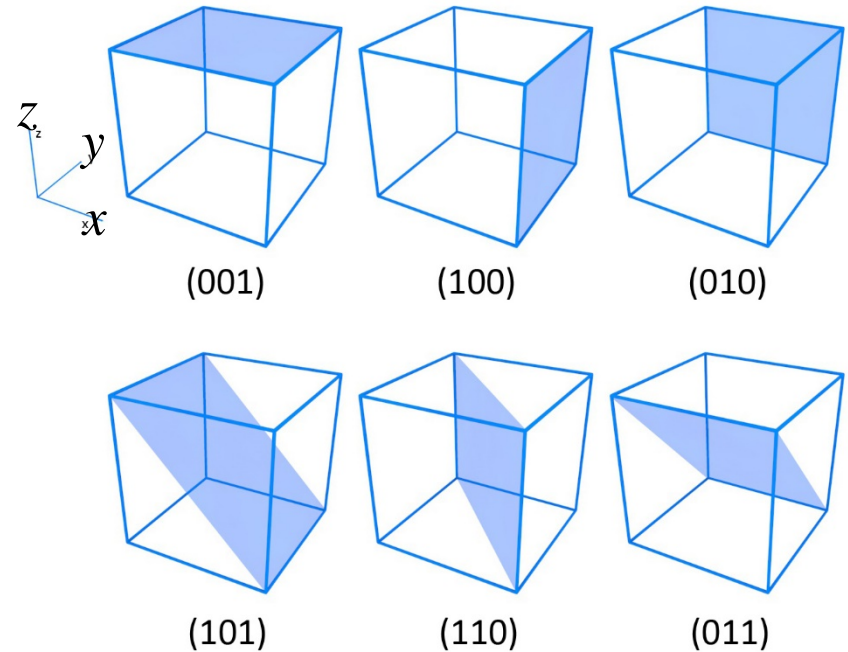
{ } family of equivalent planes



MOSFETs are made on $\langle 100 \rangle$ wafers

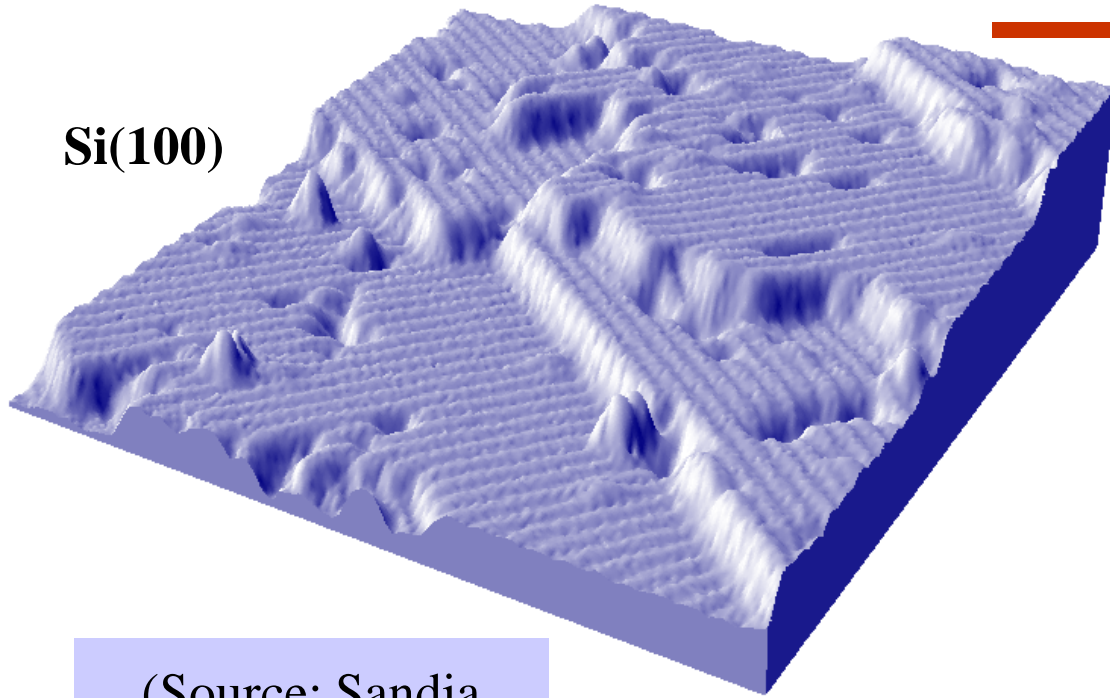
A plane with the intercepts $1/h, 1/k, 1/l$ is the (h,k,l) plane.

always use integers for h,k,l

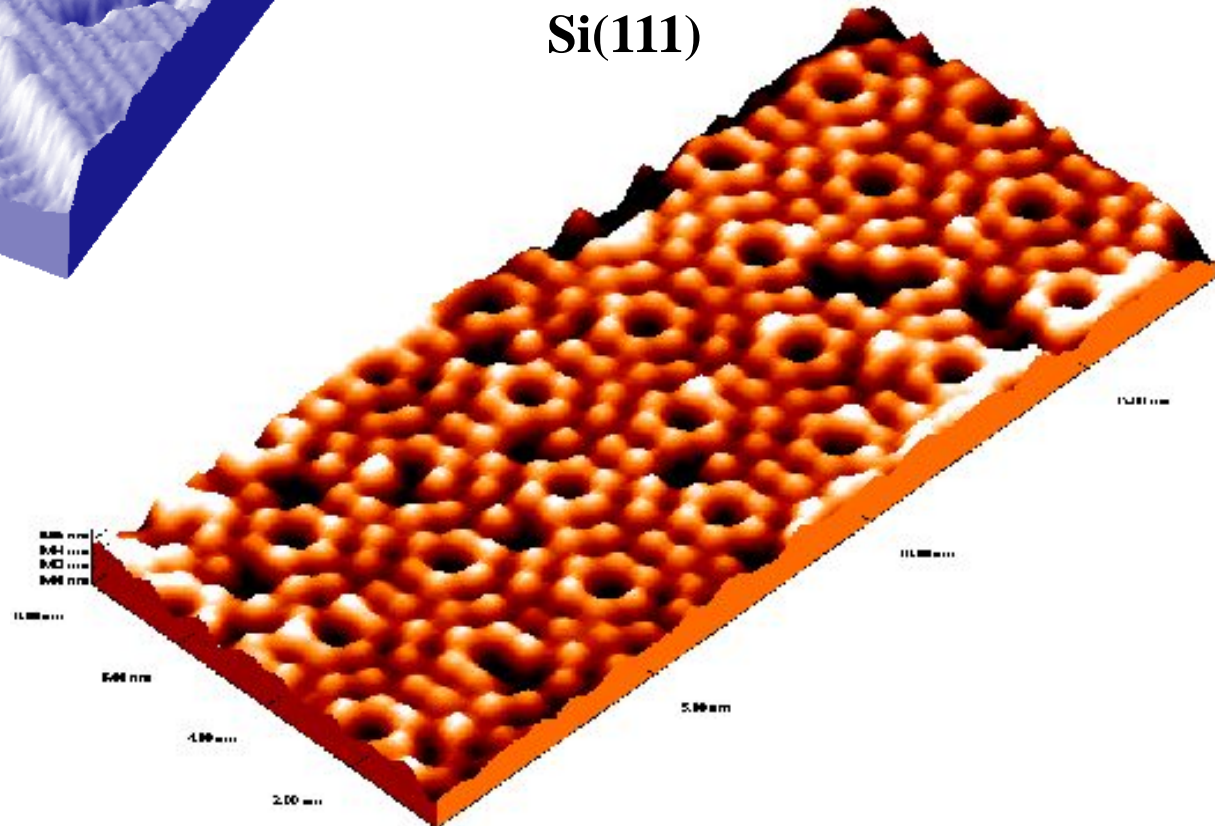


Silicon surfaces

Si(100)



Si(111)



(Source: Sandia
Nat.Labs.)

