3. Crystal Binding

What keeps crystals together ?

What causes bonding?



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Goal of this chapter:

What keeps crystals together ?

- •Types of attractive and repulsive forces
- Behavior as a function of distance
- Preferred direction, impact on geometry
- Impact on properties

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Crystals are classified with respect to their bonding type





What causes bonding?



Adapted from: http://www.smbc-comics.com/?id=3554

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For stability: relative energies are important

Cohesive energy:

· Energy needed to separate a crystal into neutral free atoms at rest

E(crystal) - E(atoms)

Lattice energy (ionic crystals):

· Energy needed to separate an ionic crystal into free ions (at infinite distance) at rest

E(crystal) - E(ions)

stable if energy of the combined system is lower than the separated fragments by more than $\sim k_B T$ (ca. 25meV at 300K)

immediate consequences for melting temperatures and bulk moduli

Cohesive energies:

- · Inert gas (and organic) crystals weakly bound
- Alkali metals have intermediate values
- Covalent crystals characterized by stronger bonding
- Transition metals quite strongly bound

		-			Ta	ble 1	1 Coh	esive	ener	gies					_							
Li 158. 1.63 37.7	Be 320. 3.32 76.5	Ener elect Prof.	Energy required to form separated neutral atoms in their ground electronic state from the solid at 0 K at 1 atm. The data were supplied by Prof. Leo Brewer.									nd B 55 51	61 .81 34	C 711. 7.37 170.	N 474 4.9 113	4. 2 3.4	O 251. 2.60. 60.03	F 81.0 0.84 19.37	Ne 1.92 0.020 0.46			
Na 107. 1.113 25.67	Mg 145. 1.51 34.7		kJ/mol							\rightarrow 3 \rightarrow 3 \rightarrow 7	I 27. .39 B.1	Si 446. 4.63 106.7	P 331 3.4 79.	1. 3 16	S 275. 2.85 65.75	CI 135. 1.40 32.2	Ar 7.74 0.080 1.85					
K 90.1 0.934 21.54	Ca 178. 1.84 42.5	Sc 376 3.90 89.9	Ti 468. 4.85 111.	V 51 5.3 12	2. 39 31 4. 2.4 94	5. 10. .5	Mn 282. 2.92 67.4	Fe 413 4.28 98.7	Cc 42 4.3 10	94. 39 1.3	Ni 428. 4.44 102.4	C 33 3. 80	u 36. 49 0.4	Zn 130 1.3 31.0	G 2 5 2 04 6	a 71. .81 4.8	Ge 372. 3.85 88.8	As 285 2.9 68.3	5.3 6 2	Se 237 2.46 56.7	Br 118. 1.22 28.18	Kr 11.2 0.116 2.68
Rb 82.2 0.852 19.64	Sr 166. 1.72 39.7	Y 422. 4.37 100.8	Zr 603. 6.25 144.	Nb 730 7.5 2 174	0. 65 67 6.1 4.5 15	8 32 7.2	Tc 661. 6.85 158.	Ru 650. 6.74 155.	RH 55 5.1 4 13	4. 75 2.5	Pd 376. 3.89 89.8	Ag 28 2.3 68	95 1.0	Cd 112 1.10 26.7	. 24 5 2. 73 58	43. .52 3.1	Sn 303. 3.14 72.4	Sb 265 2.7 63.4	5	Te 211 2.19 50.34	I 107. 1.11 25.62	Xe 15.9 0.16- 3.80
Cs 77.6 0.804 18.54	Ba 183. 1.90 43.7	La 431. 4.47 103.1	Hf 621. 6.44 148.4	Ta 782 8.1 186	2. 85 0 8.9 5.9 20	9. 90 5.2	Re 775. 8.03 185.2	Os 788 8.17 188	Ir 67 6.9 4 16	0. 94 0.1	Pt 564. 5.84 134.7	Au 36 3.4 87	a 8. 31 7.96	Hg 65. 0.67 15.5	TI 18 7 1. 5 43	32. 88 3.4	Pb 196. 2.03 46.78	Bi 210 2.11 50.2). B 2	Po 144. 1.50 34.5	At	Rn 19.5 0.202 4.66
Fr	Ra 160. 1.66 38.2	Ac 410. 4.25 98.	449	e 17. .32 9.7	Pr 357. 3.70 85.3	Nd 328 3.4 78.5	Pr 3. 0 5	n	Sm 206. 2.14 49.3	Eu 179 1.8 42.	9. 4 96 4 .8 9	ad 00. 1.14 5.5	Tb 39 4.0 93.	1.	Dy 294. 3.04 70.2	Ho 30 3.1 72	E 2. 3 4 3 .3 7	r 17. .29 5.8	Tm 233 2.4 55.4	YI 3. 15 2 1. 8 37	54. 42 60 4.4 7.1 10	8. 13 2.2
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Table from KI

3.1 Van der Waals crystals

typically noble gas crystals or molecular crystals

Crystals of noble (inert) gases:

Electron distribution in the crystalline state similar to that of the free atoms !

All electron shells completely filled (stable !!!)

\rightarrow spherically symmetric charge distributions

 $\rightarrow\,$ close packed structures (fcc) except He: zero point motion !

Table 4 Properties of inert gas crystals (Extrapolated to 0 K and zero pressure)

Nearest-		Experimental cohesive			Ionization	Paramete Lennard-J potential, H	rs in ones Eq. 10	•Wea
	distance, in Å	ene kJ/mol	eV/atom	Melting point, K	of free atom, eV	$\epsilon_{,}$ in 10^{-16} erg	σ , in Å	•Lov
He	(liquid	l at zero pr	essure)		24.58	14	2.56	
Ne	3.13	1.88	0.02	24.56	21.56	50	2.74	
Ar	3.76	7.74	0.080	83.81	15.76	167	3.40	
Kr	4.01	11.2	0.116	115.8	14.00	225	3.65	
Xe	4.35	16.0	0.17	161.4	12.13	320	3.98	

Transparent insulators

Weakly bound

Low melting temperatures

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3.98	
3.65	
3.40	Table from KI
2.74	

What keeps those crystals together ?

3.1.1 Van der Waals-London interactions (dispersion forces)

- Manifestation of electron correlation (correlated motion of electrons)
- "time averaged charge distribution" insufficient description of actual situation !

dipole-dipole interaction (as a 1st approximation)









following: Schwoerer and Wolf, "Organische Molekulare Festkörper" (Wiley-VCH)

b.) interaction between two dipoles



Interaction potentials and forces

$$V = \frac{1}{4\pi\varepsilon_0} \frac{\vec{\mathbf{p}}_1 \vec{\mathbf{p}}_2 - 3(\vec{\mathbf{p}}_1 \vec{\mathbf{e}}_r)(\vec{\mathbf{p}}_2 \vec{\mathbf{e}}_r)}{r^3}$$

Parallel dipoles:

general case:

$$V = -\frac{1}{4\pi\varepsilon_0} \frac{2p_1p_2}{r^3} F = -\frac{dV}{dr} = -\frac{1}{4\pi\varepsilon_0} \frac{6p_1p_2}{r^4}$$

Figure from Schwoerer and Wolf







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Side remark- molecular crystals:

$$V_{\rm disp} = -\int_{\infty}^{r} F dr = -\frac{1}{(4\pi\varepsilon_0)^2} \frac{A''\alpha^2}{\underline{r^6}}$$

A' (and A"=A'/6) molecule specific factors	melting points (°C) of various organic crystals			
Consider:	o-Xylol	-25		
deviations from speherical symmetry	Cyclooctatetraen	-4		
E anisotropic: in fact higher moments that	<i>n</i> -Hexan	-95		
dipole moments need to be considered	Benzol	5.5		
	Naphthalin	80		
$F \sim 1/r' \rightarrow \text{short range }!$	Anthracen	216		
$F \sim \alpha^2 \rightarrow$ large for highly polarizable	Tetracen	357		
systems with large number of π -electrons	Hexacen	380		
	Schwoerer and Wol	f		
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- van-der-Waals bonding is weak, but
- every atom counts (~0.1eV/atom)





Bonding ? – e.g., in the case of covalent bonds (depends on "shape" of overlap distribution and occupation of hybrid states)

Repulsion ? Certainly at very short distances – Why ?

To a large part because of the Pauli exclusion principle

- Two electrons cannot have their quantum numbers equal !
- I.e., each state specified by its quantum numbers can be occupied only by a single electron

Qualitative picture:

- Electron distributions overlap → electrons from atom A tend in part to "occupy states" of atom B, which are already occupied by electrons from B.
- → atoms with closed shells can overlap only if this is accompanied by the "promotion" of electrons to some sort of unoccupied high energy states of the atoms
- QM more appropriate: Occupation of "antibonding"states; consequence of requirement of orthogonality of wavefunctions Fundamentals of Solid State Physics

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- · Shape of the potential hard to determine from first principles
- · Empirical determination by fitting to experimental data on noble gases

either: $V(r) = \frac{B}{r^{12}}$ or: $V(r) = \lambda \exp\left(-\frac{r}{\rho}\right)$

B, λ , ρ empirical parameters

 Parameters can be determined from independent measurements in the gas phase

Together with the van der Waals term:

$$V = -\frac{A}{r^6}$$





3.1.3 From the interatomic potential (pair of atoms) to the cohesive energy (the crystal)

Cohesive energy (neglecting kinetic energy of the noble gas atoms):

$$U_{tot} = \frac{1}{2}N4\varepsilon \left\{ \sum_{j} \left(\frac{\sigma}{p_{ij}R} \right)^{12} - \sum_{j} \left(\frac{\sigma}{p_{ij}R} \right)^{6} \right\} \quad \begin{array}{l} \text{Independent of the chosen "j"!} \\ \text{with:} \\ r_{ij} = p_{ij}R \end{array}$$

add energetic contributions of all atoms, but avoid double counting

summation over all "other" atoms distance between reference atom i and all other atoms (R ... nearest-neighbor distance)

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Evaluate for fcc structure:
$$\sum_{j} \frac{1}{p_{ij}^{12}} = 12.1318$$
 $\sum_{j} \frac{1}{p_{ij}^{6}} = 14.4539$

Close to number of nearest neighbors in fcc structure (i.e., 12)

→ the nearest neighbours count primarily ! Fundamentals of Solid State Physics

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Equilibrium distance:

$$\frac{dU_{tot}}{dR} \equiv 0 \qquad \Longrightarrow \qquad \frac{R_0}{\sigma} = 1.09$$

Experimentally determined values:

$$\frac{R_0}{\sigma}(He) = 1.14; \frac{R_0}{\sigma}(Ar) = 1.11; \frac{R_0}{\sigma}(Kr) = 1.10; \frac{R_0}{\sigma}(Ne) = 1.09$$

Light atoms: deviation due to zero point energy !

Directional	Weak
Bond Energies	5 – 50 meV / atom
Occurs when	Always
Examples:	Rare gas clusters, organic crystals
Model Potential:	Lennard-Jones
Packing density	Dense
Decays	r-6

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3.2 Ionic crystals

- · Consist of positively and negatively charged ions
- Primarily consequence of electrostatic interaction between oppositely charged ions
- Typical structures: NaCl and CsCl

lons of a simple ionic crystal:

- Closed shell structure
- Low ionisation energies
- Thus, spherical charge distribution

Na: 1s² 2s² 2p⁶ 3s Na⁺: 1s² 2s² 2p⁶ Cl: 1s² 2s² 2p⁶ 3s² 3p⁵ Cl⁻: 1s² 2s² 2p⁶ 3s² 3p⁶

Figure 7 Electron density distribution in the base plane of NaCl, after x-ray studies by G. Schoknecht. The numbers on the contours give the relative electron concentration.



Figure from KI

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with $r_{ii} = p_{ii}R$

and considering the repulsive interaction only for nearest neighbors

$$U_{ij} \begin{cases} \lambda \exp\left(-\frac{R}{\rho}\right) - \frac{1}{4\pi\varepsilon_0} \frac{q^2}{R} & (nearest neighbors) \\ \pm \frac{1}{4\pi\varepsilon_0} \frac{q^2}{p_{ii}R} & (other atoms) \end{cases}$$

one obtains

With N being the number of K⁺A⁻ pairs, z being the number of nearest neighbors and α, the Madelung constant given by

 $U_{tot} = NU_i = N\left(z\lambda \exp\left(-\frac{R}{\rho}\right) - \frac{1}{4\pi\varepsilon_0}\frac{\alpha q^2}{R}\right)$

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Calculating the equilibrium distance from $dU_{tot}/dR=0$, and inserting the obtained expression into the equation for U_{tot}, one obtains:

$$U_{tot} = \frac{N\alpha q^2}{4\pi\varepsilon_0 R_0} \left(1 - \frac{\rho}{R_0}\right)$$

α typically calculated numerically using a computer

Example: Madelung constant of a linear chain:



 $\alpha \coloneqq \sum_{j}^{'} \frac{(\pm)}{p_{ij}}$ Fundamentals of Solid State Physics

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Directional	None
Bond Energies	1 – 9 eV / 100 – 900 kJ/mol
Occurs when	 Atoms have strongly different ionization energies, electron affinities, electronegativities
Examples:	NaCl, CaF _{2,} CsCl,
Potential:	Coulomb + Pauli repulsion
Packing density	Dense
Decays	r -1

3.3 Covalently bonded crystals

- · classical electron pair or homopolar bond
- bonding involves "pairing of electrons" between atoms
- typically two electrons involved
- preferentially located between bonded atoms
- antiparallel spin
- covalent bond highly directional





Some orientations favor the overlap others are unfavorable



Fig. 1.3 a, b. Pictorial representation of the spatial overlap between an *s*- and a *p*-wavefunction of hydrogen. The "size" of the wavefunction is illustrated by a surface of constant wavefunction amplitude. (a) A situation in which the overlap cancels due to sign reversal of the p_z -wavefunction. (b) Nonvanishing overlap between *s*- and p_x -wavefunctions

Figure from IL

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Covalent bond highly directional !

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What about C, Si, Ge?

Lack 4 electrons for a closed shell (noble gas) configuration

C-atom: 1s²2s²2p²

 To be able to make 4 bonds needs to have 4 singly occupied orbitals

1s²2s2p³ (costs a a significant amount of energy)

 \rightarrow formation of hybrid orbitals





Figure 11 Calculated <u>valence electron</u> concentration in germanium. The numbers on the contours give the electron concentration per primitive cell, with four valence electrons per atom (eight electrons per primitive cell). Note the high concentration midway along the Ge-Ge bond, as we expect for covalent bonding. (After J. R. Chelikowsky and M. L. Cohen.)



Directional	Strong
Bond Energies	1.4 – 9 eV / 140 – 900 kJ/mol
Occurs when	- Unpaired electrons
Examples:	Si, Ge, organic molecules
Potential:	wave-function overlap
Packing density	Not necessarily dense
Decays	e-r

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Covalent and ionic bonding only two limiting cases

Continuous transition between these "extremes"

Table from KI Table 8 Fractional ionic character of bonds in binary crystals Fractional ionic character Fractional ionic character Crystal Crystal Si SiC 0.00 GaAs 0.31 0.180.26 Ge 0.00GaSb ZnO 0.62AgCl 0.86 ZnS0.62AgBr 0.85ZnSe 0.63 AgI 0.77ZnTe 0.61 MgO 0.84 CdO 0.79 0.79MgS CdS CdSe 0.69 0.79 MgSe 0.70 0.92 CdTe 0.67LiF 0.92 0.94 0.96 NaCl RbF $0.42 \\ 0.36$ InP InAs InSb -0.32After J. C. Phillips, Bonds and bands in semiconductors. olid State Physics

3.4 Bonding in metals

- Valence electrons (1 or 2) in metals become conducting electrons
- highly mobile ("smeared out", "delocalized")
- · "collective" effect



Cohesive energy:

Picture from: http://en.wikipedia.org/wiki/Metallic_bond

- · Energy of valence electrons reduced compared to atoms
- · Interaction of nuclei with conducting electrons









Transistion metals:

Higher cohesive energies because of additional bonding !

d-electrons:

- more localized with weaker overlap
- produce kind of covalent framework
- main contribution to the binding energy



3.5 Hydrogen bonds

Partly following: http://en.wikipedia.org/wiki/Hydrogen_bond

H attached to highly electronegative atom like N, O, or F

+ another electronegative atom is in its vicinity

· H to a large extent "looses" electron and becomes positively charged attracted by two atoms with strong forces

strongly ionic character

Figure 13 The hydrogen difluoride ion $HF_2^$ is stabilized by a hydrogen bond. The sketch is of an extreme model of the bond, extreme in the sense that the proton is shown bare of electrons.



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http://en.wikipedia.org/wiki/Hydrogen_bond

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Hydrogen bonds can vary in strength from (1-2 kJ mol⁻¹ to extremely strong >155 kJ mol⁻¹:

- F—H...:F (155 kJ/mol or 40 kcal/mol)
- O—H...:N (29 kJ/mol or 6.9 kcal/mol)
- O-H...:O (21 kJ/mol or 5.0 kcal/mol)
- N—H...:N (13 kJ/mol or 3.1 kcal/mol)
- N—H...:O (8 kJ/mol or 1.9 kcal/mol)





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a summary about forces

interaction type	range	directional	energy [kJ / mol]	energy [k _B T]
charge – charge	~ r ⁻¹	no	100 - 900	40 - 360
metal bond		no	100 - 900	40 – 360
covalent	very short	yes	100 - 900	40 - 360
hydrogen bond	very short	yes	10 - 155	4 - 62
dipole – dipole	~ r ⁻³	yes	20	8
van der Waals	~ r ⁻⁶	weakly	0,5 - 5	0,2 - 2

but: in organic systems are many van der Waals bonds parallel to each other and the bond energies sum up, for large organic objects the van der Waals forces can even become long range forces !!

