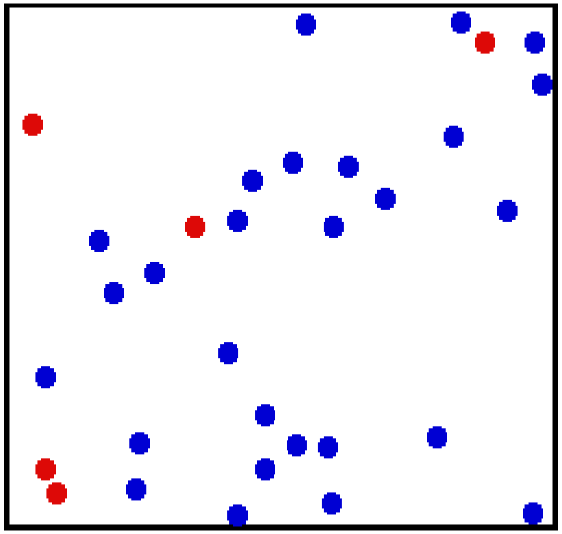
# 23. Kinetic theory

June 19, 2018

# kinetic theory

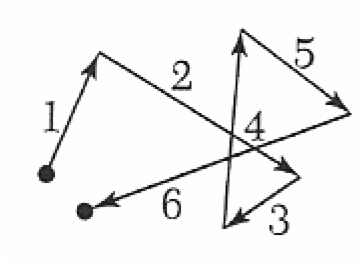


describe electrons as a gas of particles

$v_F = 10^8$ cm/s.

The average time between scattering events $\tau_{sc}$ can be calculated by Fermi's golden rule

mean free path: $\boldsymbol{l} = v_F \tau_{sc} \sim 1$ nm - 1 cm



# Electrons as waves or particles

Scattering of electrons can be thought of as transitions between *k* states or as collisions between particles.

Umklapp scattering of electrons by phonons makes large changes in the momentum of the electrons because of the reciprocal lattice vector **G**.

$$\vec{k}'_{el} = \vec{k}_{el} + \vec{k}_{ph} \longleftarrow \text{phonon emitted}$$

$$\vec{k}'_{el} = \vec{k}_{el} + \vec{k}_{ph} + \vec{G} \longleftarrow \text{Umklapp scattering}$$

# Ballistic transport

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{d\vec{v}}{dt}$$

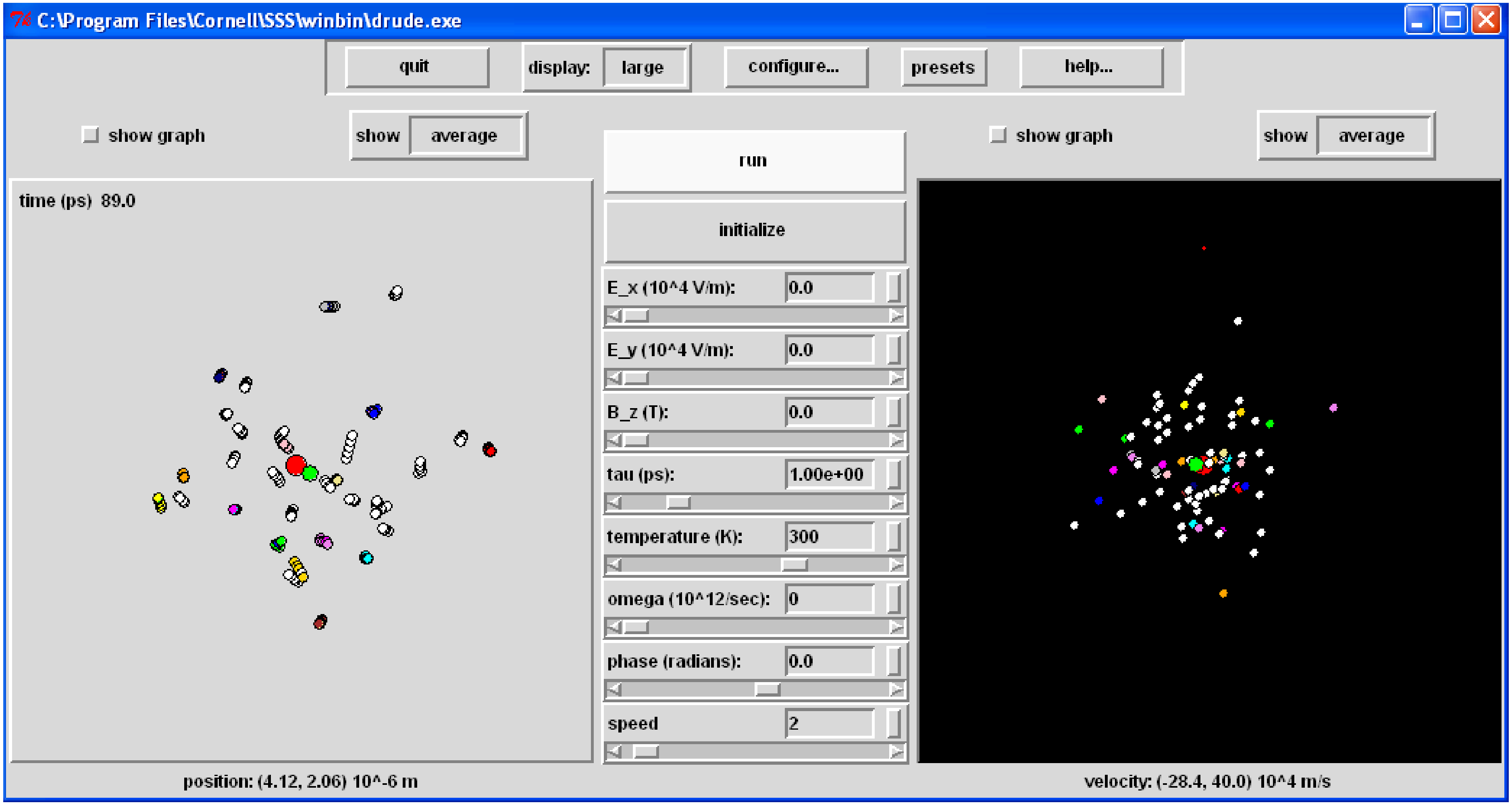
$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0 t + \vec{x}_0$$

electrons in an electric field follow a parabola.

electrons in a magnetic field move in a spiral

electrons crossed electric and magnetic fields spiral along the direction perpendicular to the electric and magnetic fields



C:\Program Files\Cornell\SSS\winbin\drude.exe

time (ps) 89.0

E_x (10^4 V/m): 0.0

E_y (10^4 V/m): 0.0

B_z (T): 0.0

tau (ps): 1.00e+00

temperature (K): 300

omega (10^12/sec): 0

phase (radians): 0.0

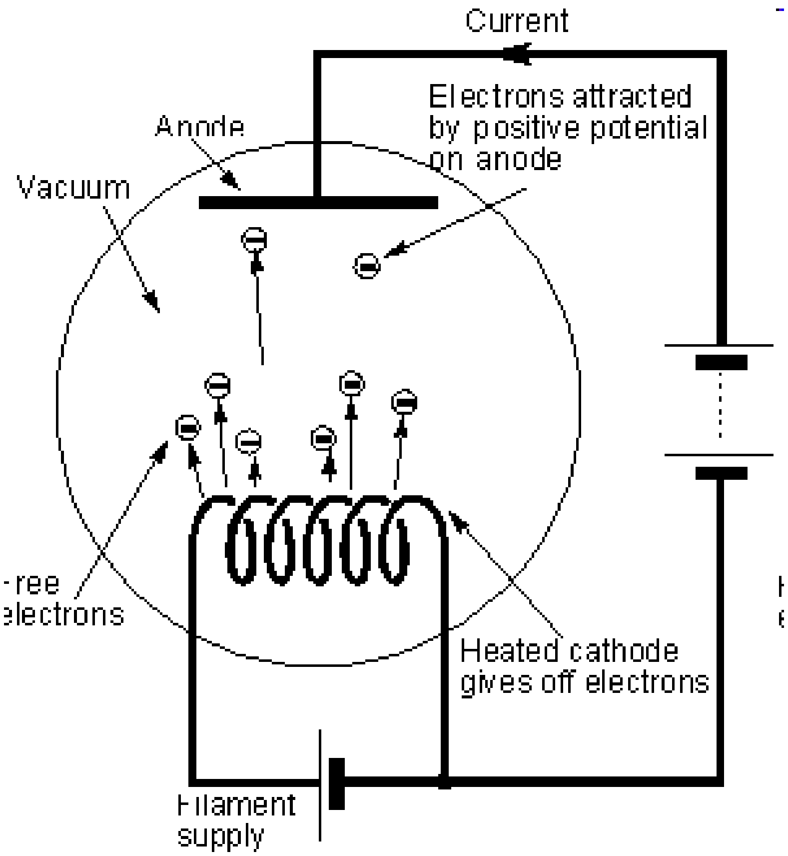
speed 2

position: (4.12, 2.06) 10^-6 m

velocity: (-28.4, 40.0) 10^4 m/s

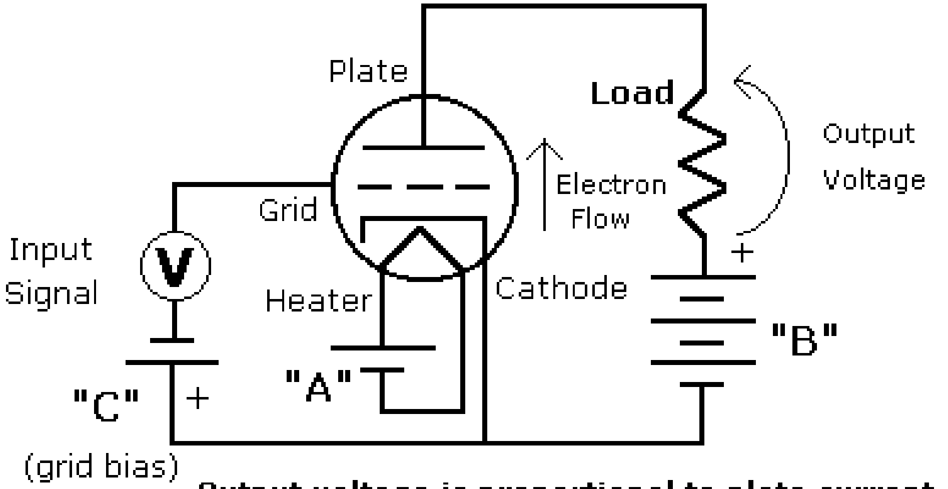
If no forces are applied, the electrons diffuse.
The average velocity moves against an electric field.
In just a magnetic field, the average velocity is zero.
In an electric and magnetic field, the electrons move in a straight line at the Hall angle.

# Vacuum diodes



diode

The Common-cathode Triode Amplifier



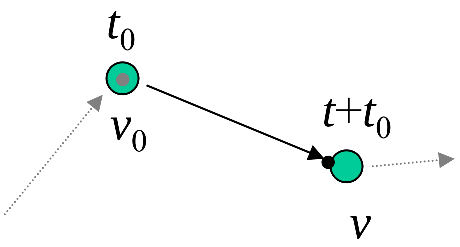
**Output voltage is proportional to plate current, which is controlled by grid voltage.**

# Diffusive transport

$$\vec{F} = -e\vec{E} = m^*\vec{a} = m^*\frac{d\vec{v}}{dt}$$

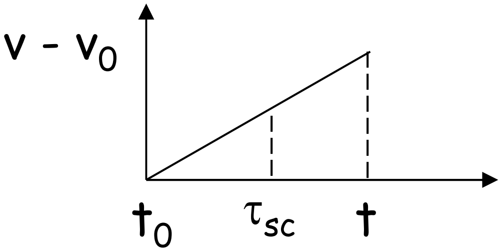
$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*}(t - t_0)$$

*time between two collisions*



$\langle v_0 \rangle = 0$

$\langle t - t_0 \rangle = \tau_{sc}$ < average time between scattering events



$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v_F}$$

drift velocity: $\vec{v}_d = -\mu\vec{E}$

Ohm's law: $\vec{j} = -ne\vec{v}_d = ne\mu\vec{E} = \sigma\vec{E}$

# Matthiessen's rule

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

mostly temperature independent

phonons, temperature dependent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

# Ballistic transport in transistors

The mean free path ~100 nm > gate length ~ 20 nm

*v* not proportional to *E*

$$\xcancel{\vec{v} = \mu\vec{E}}$$

*j* not proportional to *E*

$$\xcancel{\vec{j} = \sigma\vec{E}}$$

nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

# Magnetic field (diffusive regime)

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{\text{sc}}}$$

$$-\frac{e\tau_{\text{sc}}}{m}\vec{E} = \vec{v}_d$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right) = m\frac{\vec{v}_d}{\tau_{\text{sc}}}$$

If $B$ is in the $z$-direction, the three components of the force are

$$-e\left(E_x + v_{dy}B_z\right) = m\frac{v_{dx}}{\tau_{\text{sc}}}$$

$$-e\left(E_y - v_{dx}B_z\right) = m\frac{v_{dy}}{\tau_{\text{sc}}}$$

$$-e\left(E_z\right) = m\frac{v_{dz}}{\tau_{\text{sc}}}$$

# Magnetic field (diffusive regime)

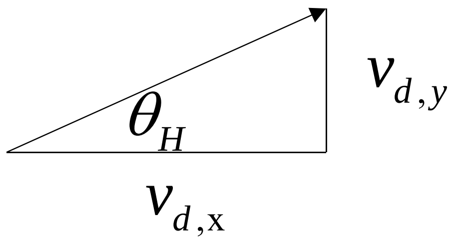
$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

If $E_y = 0, E_z = 0$

$$v_{d,y} = -\frac{eB_z}{m}\tau_{sc}v_{d,x}$$

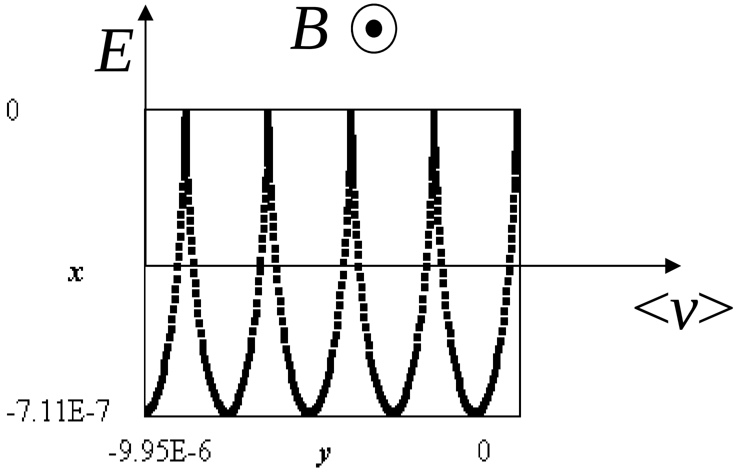


$$\tan\theta_H = -\frac{eB_z}{m}\tau_{sc}$$

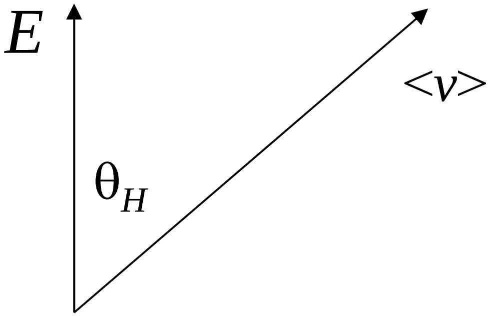
# Crossed *E* and *B* fields

Ballistic transport

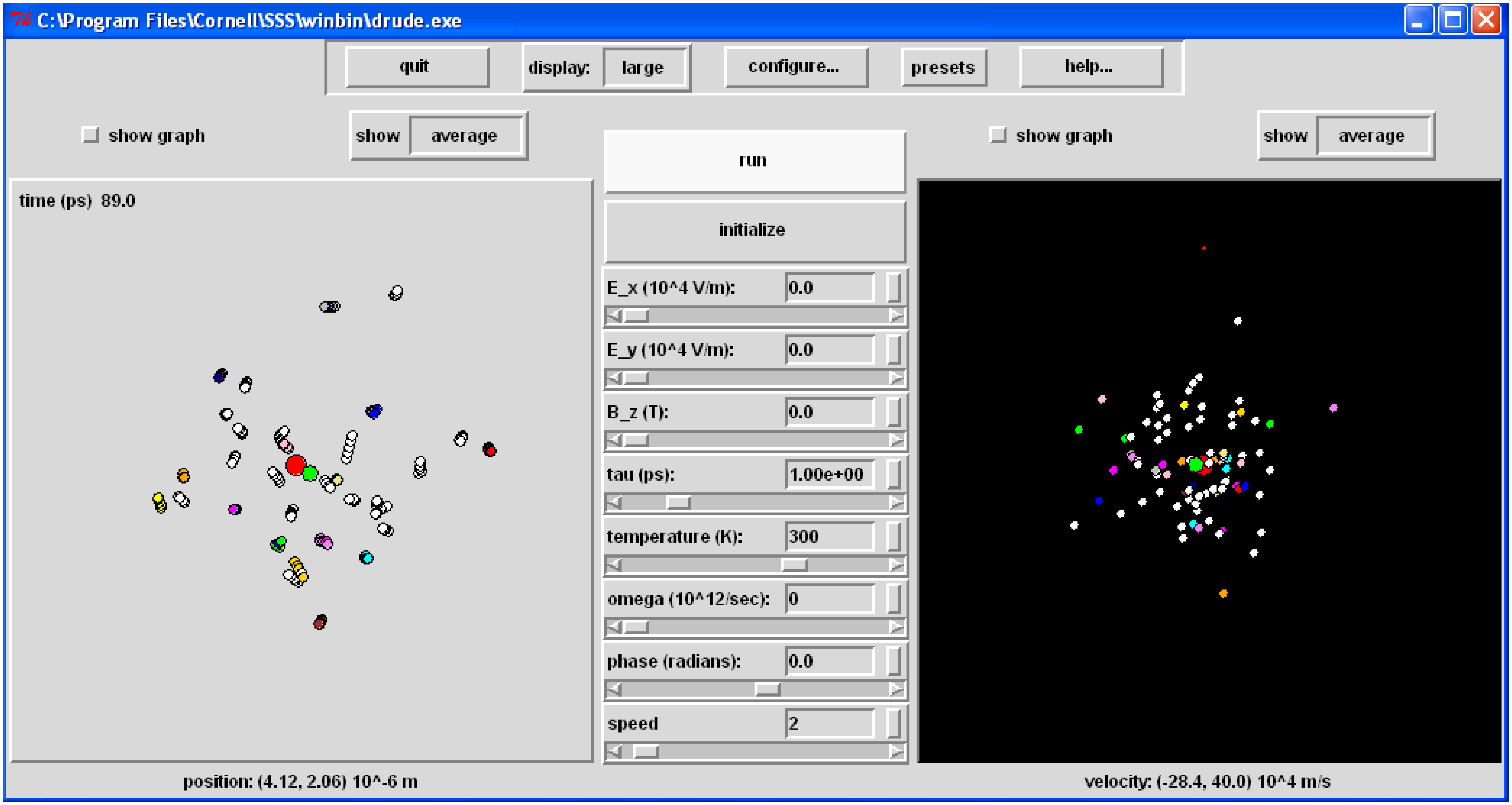
$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right)$$



Diffusive transport



Hall angle: $\theta_H = \tan^{-1}\left(-\frac{eB_z\tau_{sc}}{m}\right)$



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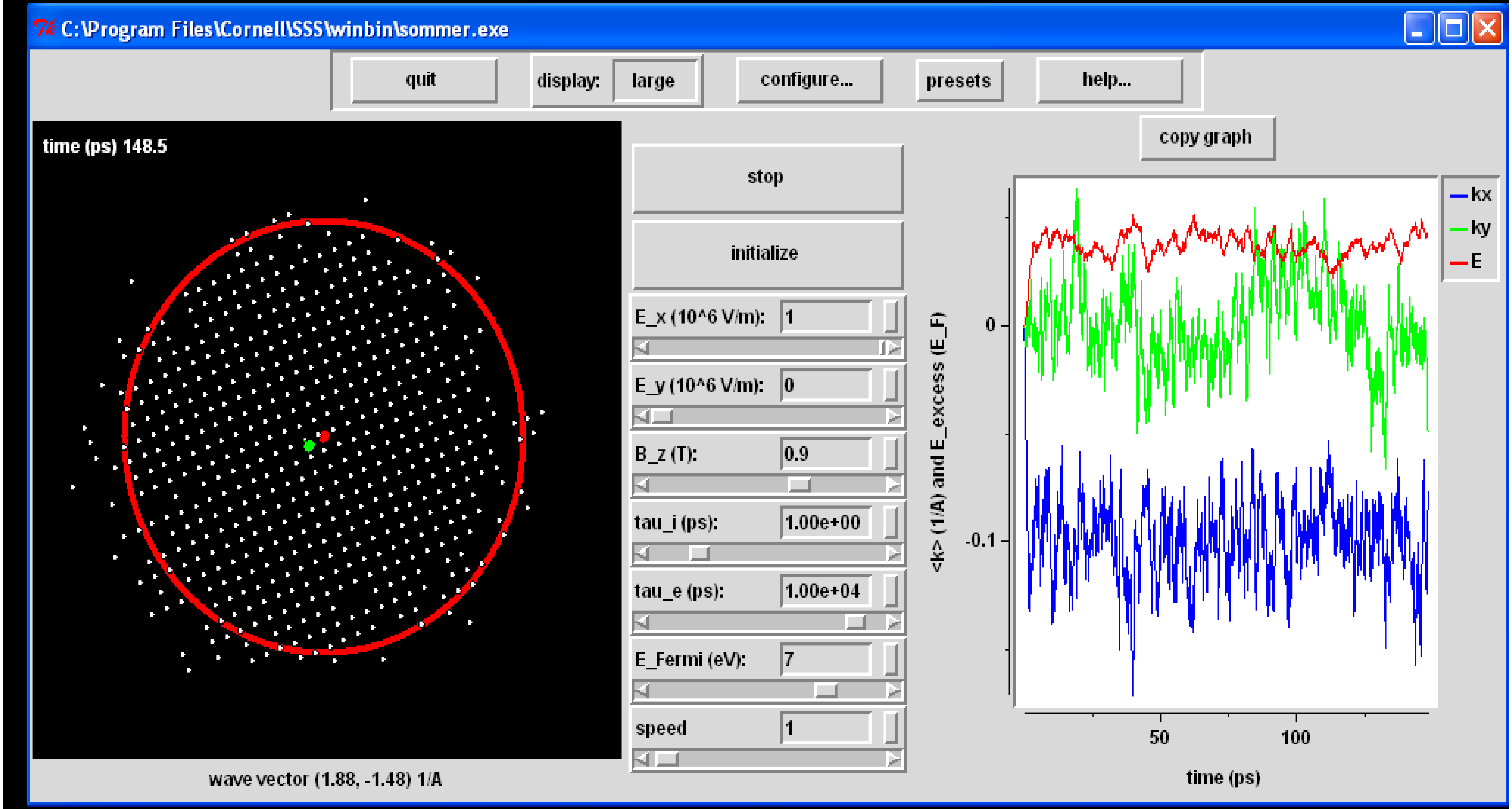


C:\Program Files\Cornell\SSS\winbin\sommer.exe

time (ps) 148.5

E_x (10^6 V/m): 1

E_y (10^6 V/m): 0

B_z (T): 0.9

tau_i (ps): 1.00e+00

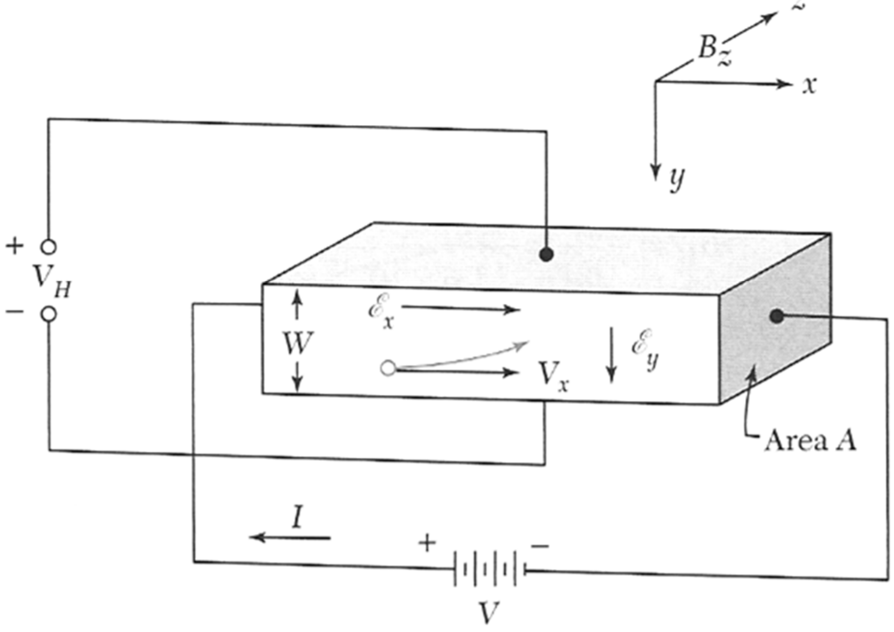
tau_e (ps): 1.00e+04

E_Fermi (eV): 7

speed 1

wave vector (1.88, -1.48) 1/A

# The Hall Effect (diffusive regime)



$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$

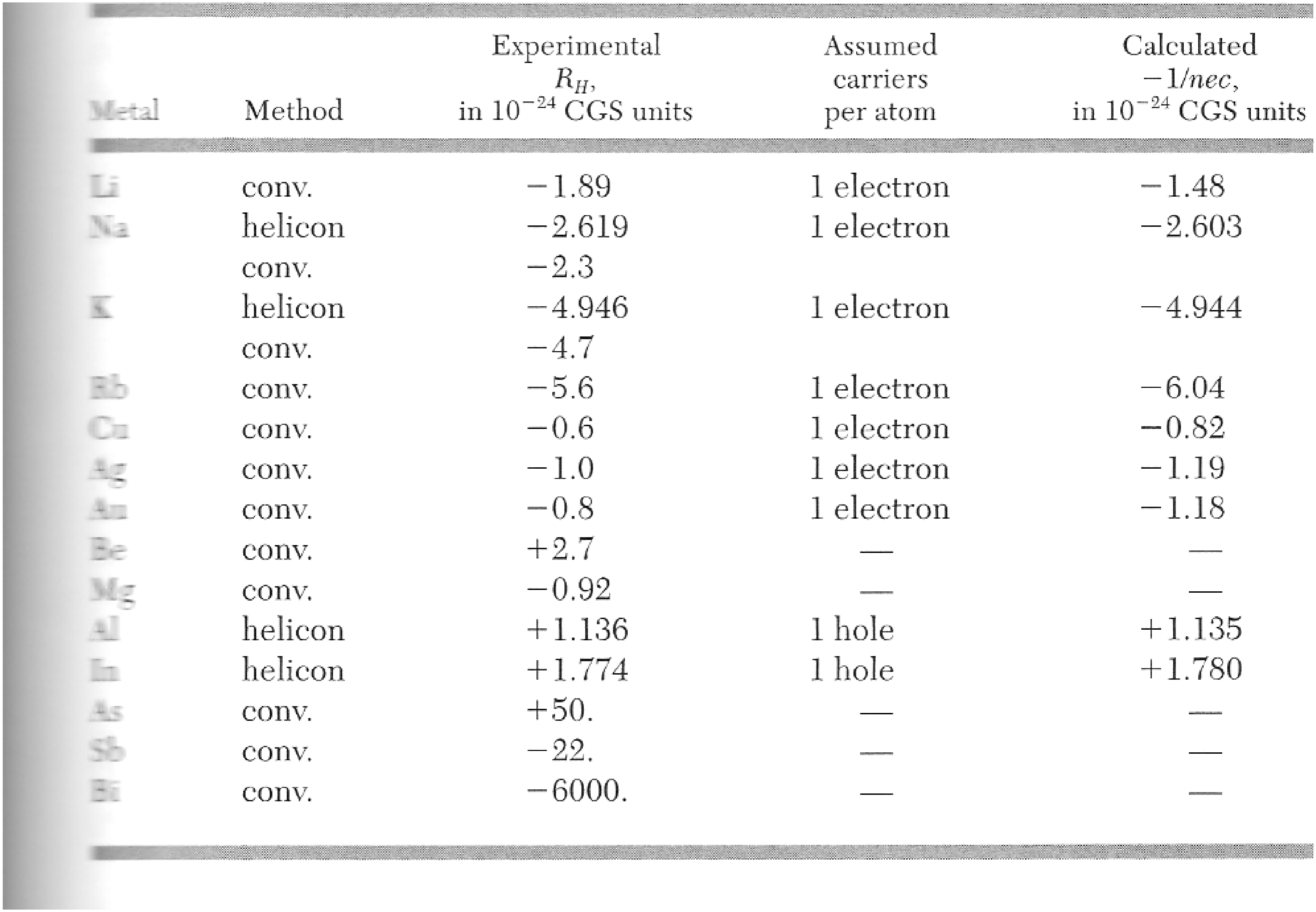
$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

If $v_{d,y} = 0$,

$E_y = v_{d,x}B_z = V_H/W = R_H j_x B_z$ $V_H$ = Hall voltage, $R_H$ = Hall Constant

$j_x$=- $nev_{d,x}$

$$\boxed{R_H = E_y/j_xB_z = - 1/ne}$$



| Metal | Method | Experimental $R_H$, in $10^{-24}$ CGS units | Assumed carriers per atom | Calculated $-1/nec$, in $10^{-24}$ CGS units |
|---|---|---|---|---|
| Li | conv. | −1.89 | 1 electron | −1.48 |
| Na | helicon | −2.619 | 1 electron | −2.603 |
|  | conv. | −2.3 |  |  |
| K | helicon | −4.946 | 1 electron | −4.944 |
|  | conv. | −4.7 |  |  |
| Rb | conv. | −5.6 | 1 electron | −6.04 |
| Cu | conv. | −0.6 | 1 electron | −0.82 |
| Ag | conv. | −1.0 | 1 electron | −1.19 |
| Au | conv. | −0.8 | 1 electron | −1.18 |
| Be | conv. | +2.7 | — | — |
| Mg | conv. | −0.92 | — | — |
| Al | helicon | +1.136 | 1 hole | +1.135 |
| In | helicon | +1.774 | 1 hole | +1.780 |
| As | conv. | +50. | — | — |
| Sb | conv. | −22. | — | — |
| Bi | conv. | −6000. | — | — |

Kittel

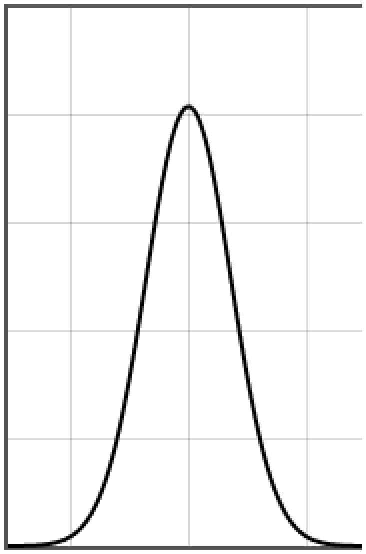
# Diffusion equation/ heat equation

$$\frac{dn}{dt} = -D\nabla^2 n$$

Diffusion constant

Fick's law $\vec{j} = -D\nabla n$

Continuity equation $\frac{dn}{dt} = \nabla \cdot \vec{j}$



$$n = \frac{1}{\sqrt{4\pi Dt}} \exp\left( \frac{-r^2}{4Dt} \right)$$

# Einstein relation

$$\vec{F} = -\nabla U_{pot} = -e\vec{E}$$

$$n(x) = A\exp\left(\frac{-U_{pot}(x)}{k_B T}\right)$$ Boltzmann factor

In equilibrium, drift = diffusion

$$en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{1}{k_B T} A\exp\left(\frac{-U_{pot}}{k_B T}\right)\nabla U_{pot} = -\frac{n}{k_B T}\nabla U_{pot} = \frac{-en\vec{E}}{k_B T}$$

$$en\mu\vec{E} - e^2 D\frac{n\vec{E}}{k_B T} = 0$$

$$\boxed{D = \frac{\mu k_B T}{e}}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen, A. Einstein (1905).