

# Semiconductors

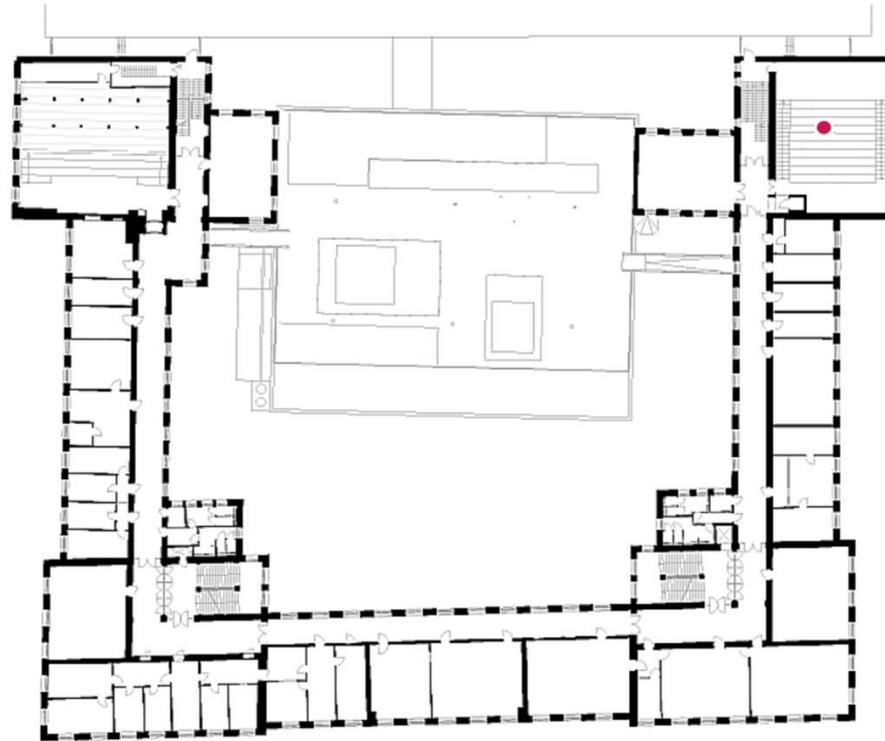
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# Exam

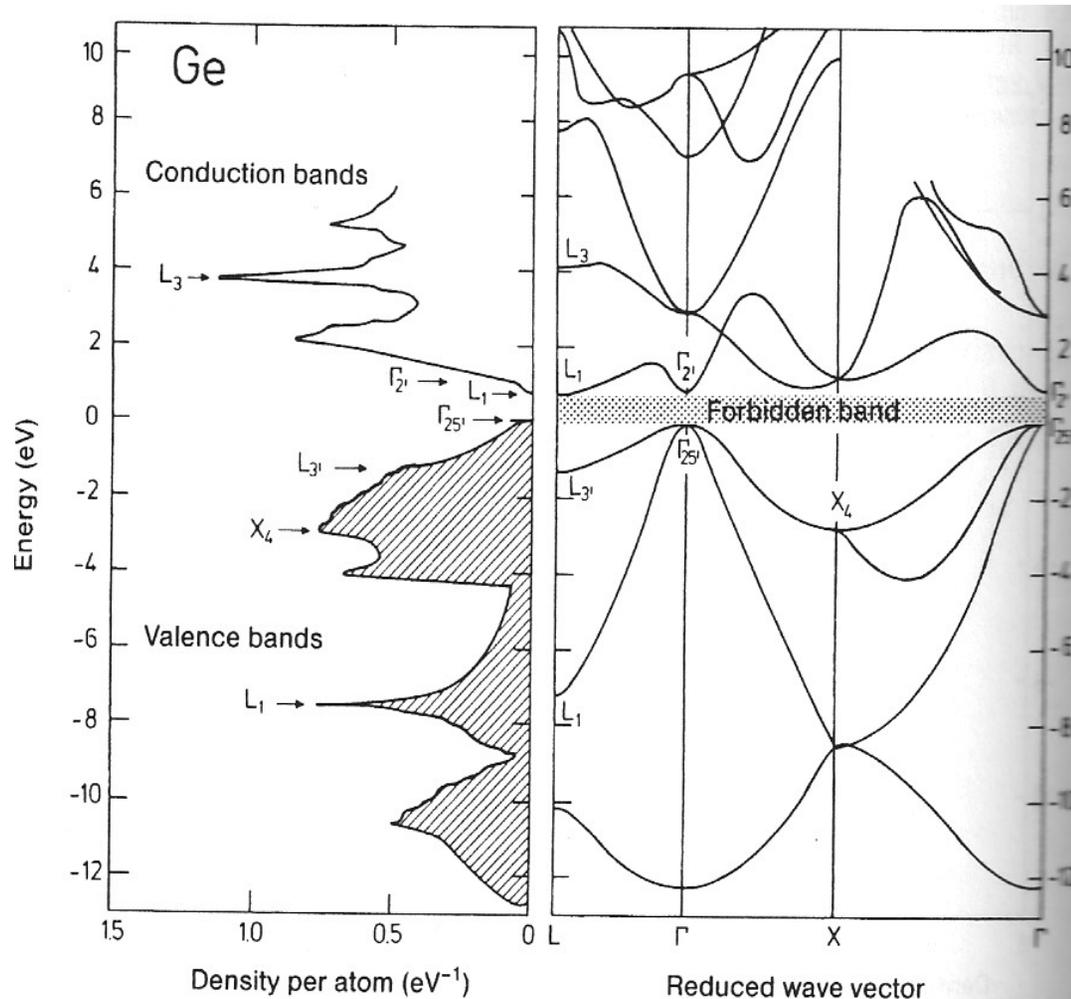
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HS B 10:15 – 11:15

1 A4 handwritten notes



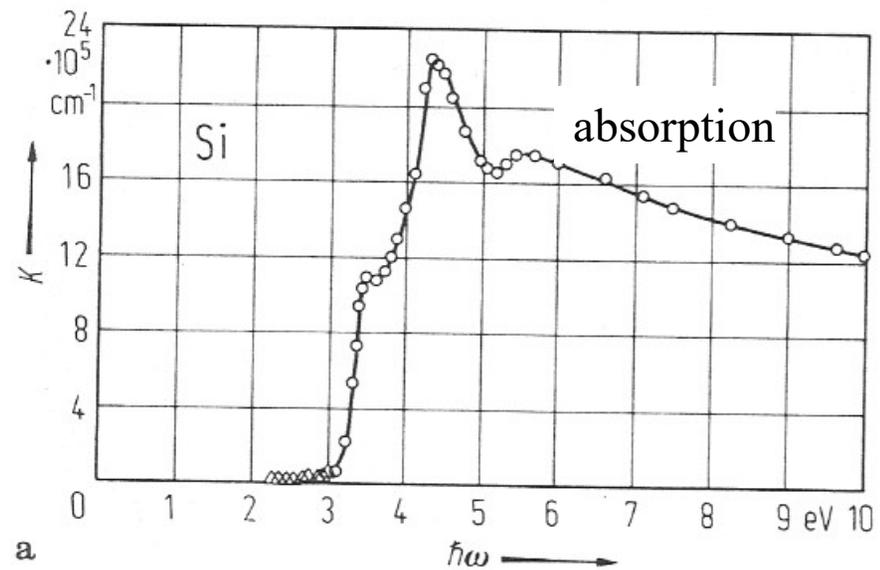
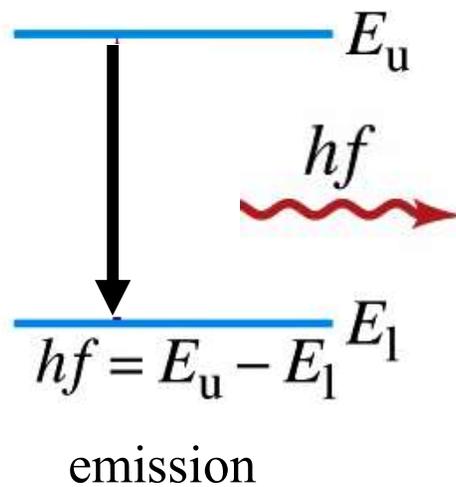
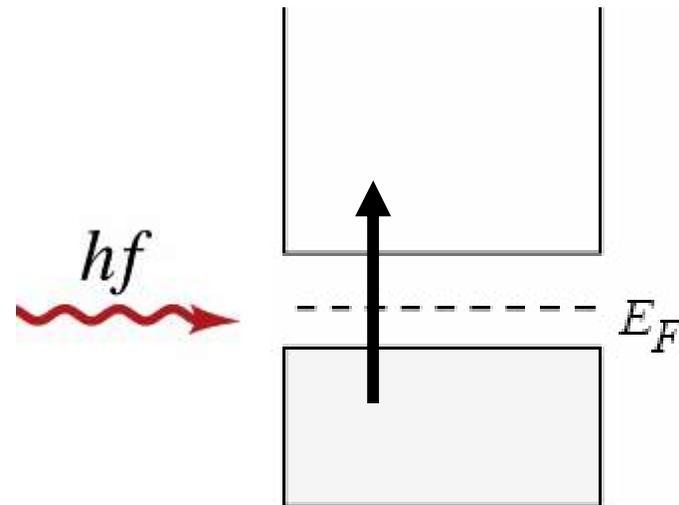
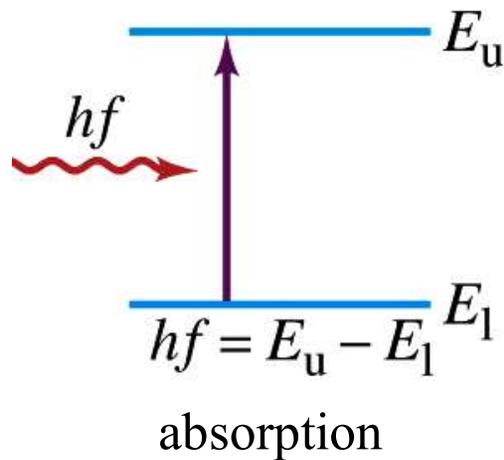
# Semiconductors



$$n = \int_{-\infty}^{\infty} D(E) f(E) dE$$

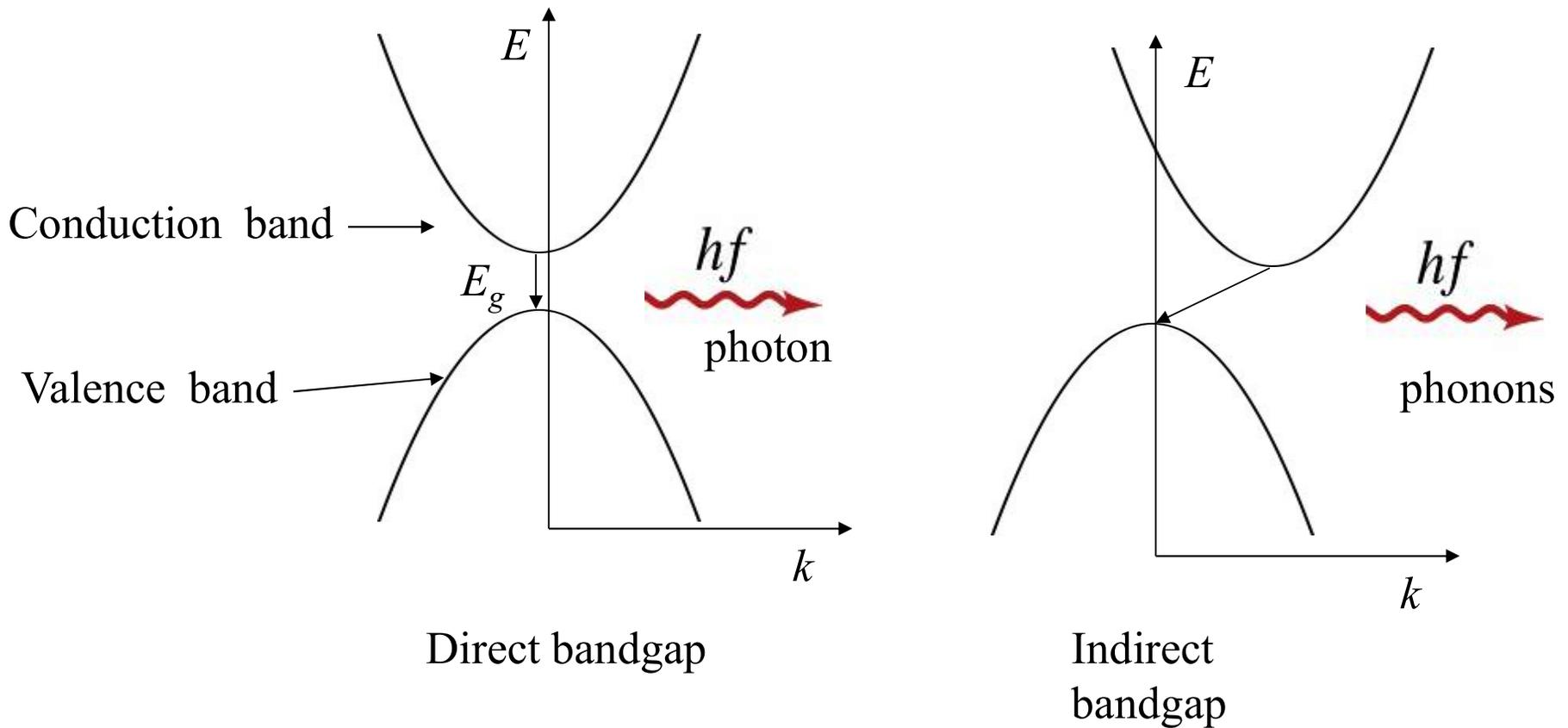
From Ibach & Lueth

# Absorption and emission of photons



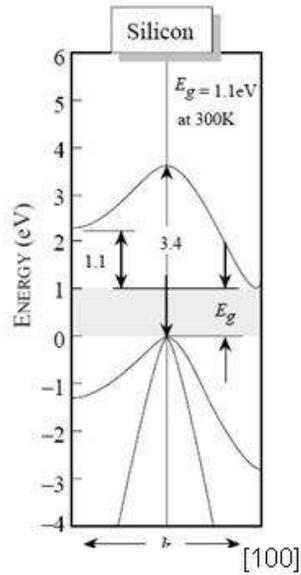
# Direct and indirect band gaps

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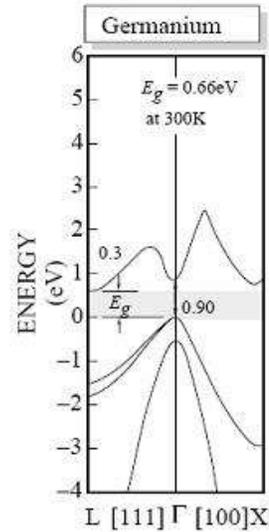


Direct bandgap semiconductors are used for optoelectronics

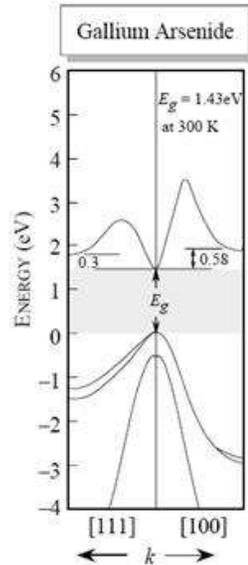
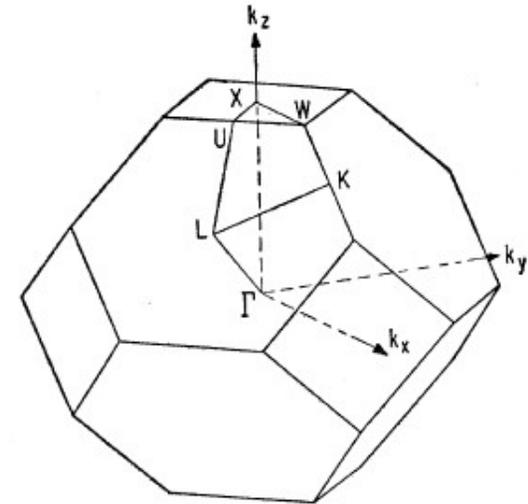
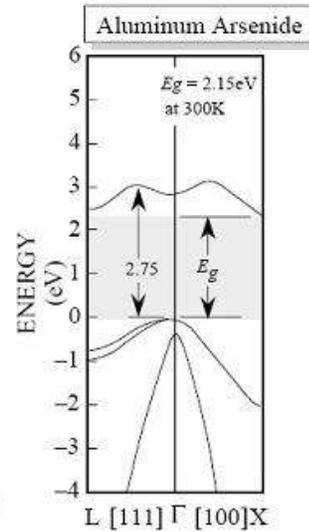
# Semiconductors



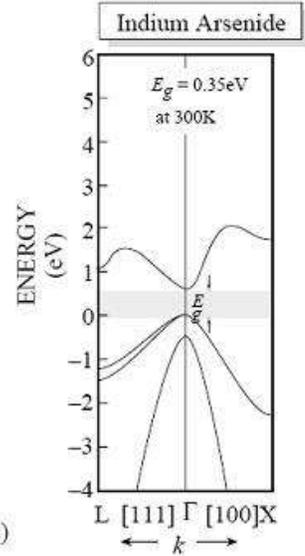
(a)



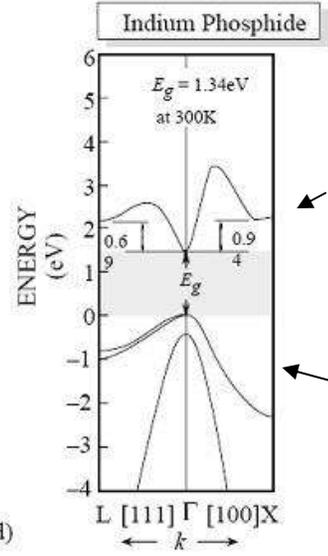
(b)



(c)



(d)



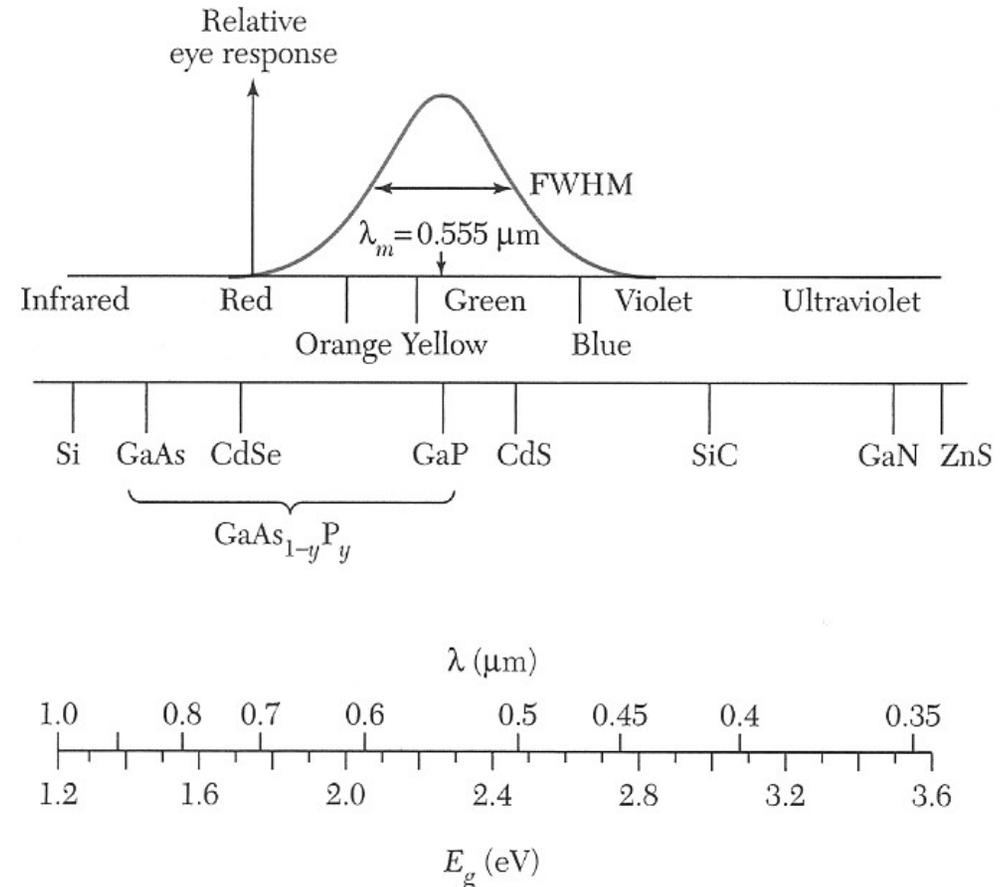
← Conduction band

← Valence band

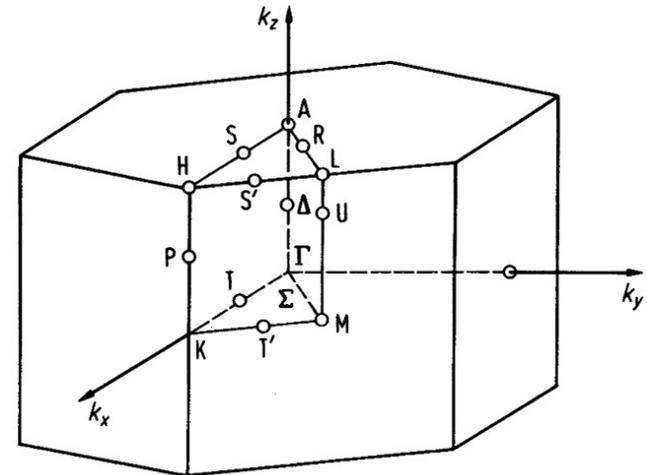
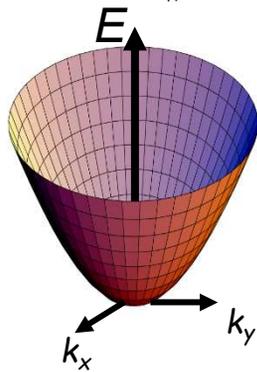
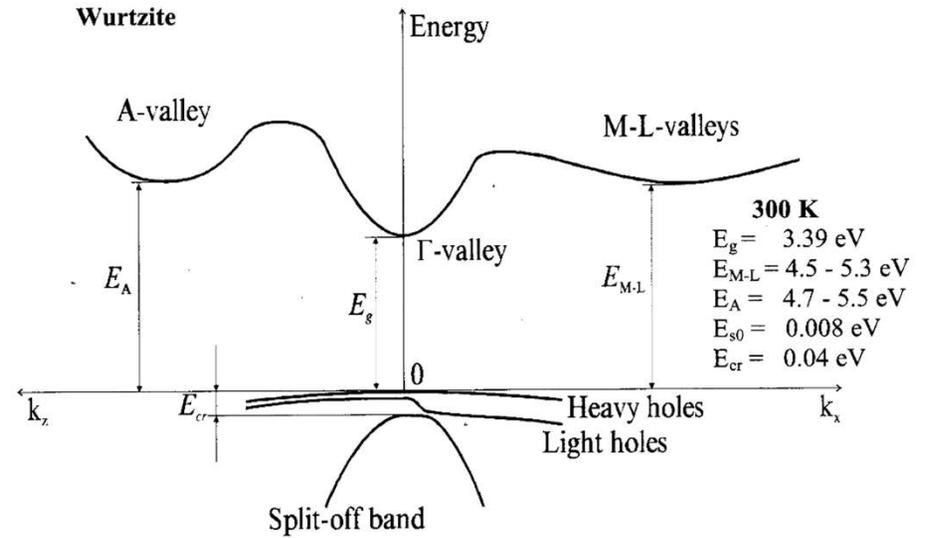
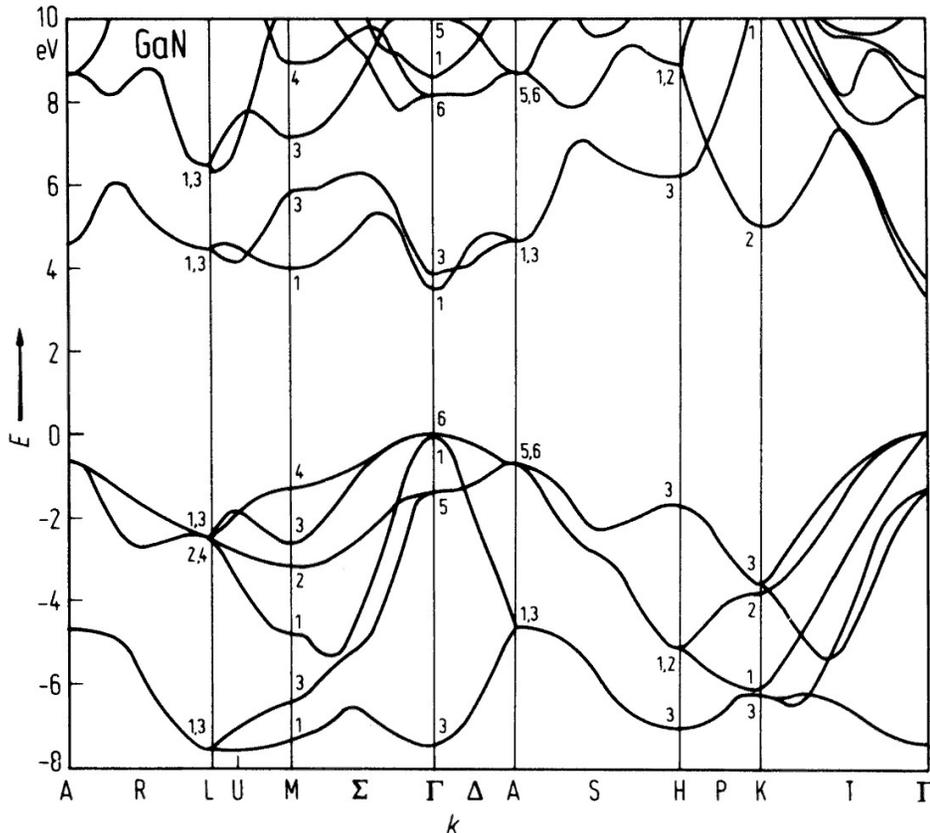
**TABLE 1 Common III-V materials used to produce LEDs and their emission wavelengths.**

Material	Wavelength (nm)
InAsSbP/InAs	4200
InAs	3800
GaInAsP/GaSb	2000
GaSb	1800
$Ga_xIn_{1-x}As_{1-y}P_y$	1100-1600
$Ga_{0.47}In_{0.53}As$	1550
$Ga_{0.27}In_{0.73}As_{0.63}P_{0.37}$	1300
GaAs:Er, InP:Er	1540
Si:C	1300
GaAs:Yb, InP:Yb	1000
$Al_xGa_{1-x}As:Si$	650-940
GaAs:Si	940
$Al_{0.11}Ga_{0.89}As:Si$	830
$Al_{0.4}Ga_{0.6}As:Si$	650
$GaAs_{0.6}P_{0.4}$	660
$GaAs_{0.4}P_{0.6}$	620
$GaAs_{0.15}P_{0.85}$	590
$(Al_xGa_{1-x})_{0.5}In_{0.5}P$	655
GaP	690
GaP:N	550-570
$Ga_xIn_{1-x}N$	340,430,590
SiC	400-460
BN	260,310,490

# Light emitting diodes



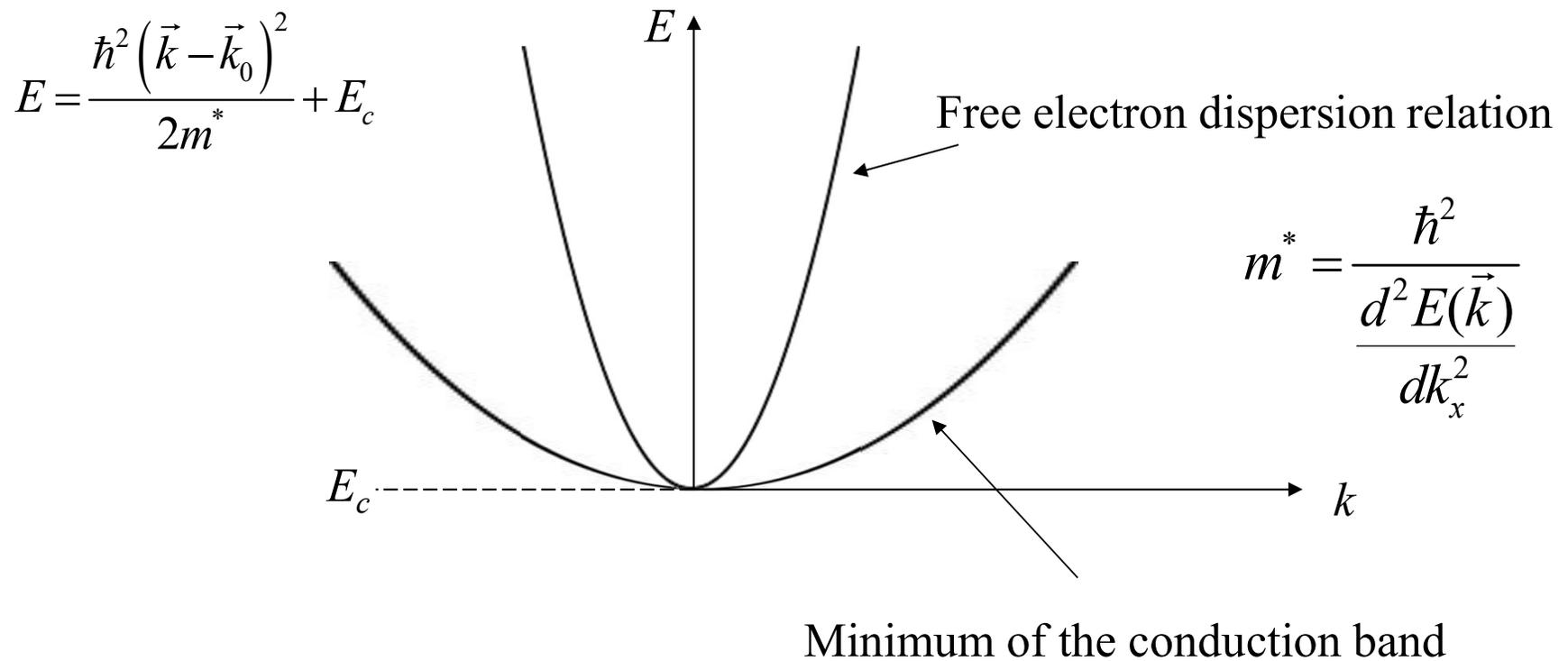
# GaN



1st Brillouin zone of hcp

# Conduction band minimum

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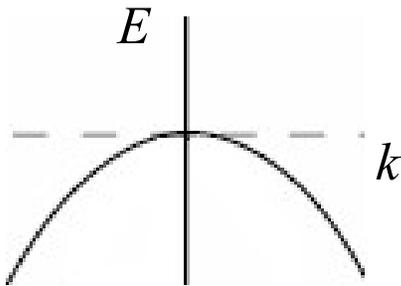


Near the conduction band minimum, the bands are approximately parabolic.

# Top of the valence band

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In the valence band, the effective mass is negative.



$$m^* = \frac{\hbar^2}{\frac{d^2 E(\vec{k})}{dk_x^2}} < 0$$

Charge carriers in the valence band are positively charged holes.

$m_h^*$  = effective mass of holes

$$m_h^* = \frac{-\hbar^2}{\frac{d^2 E(\vec{k})}{dk_x^2}}$$

# Holes

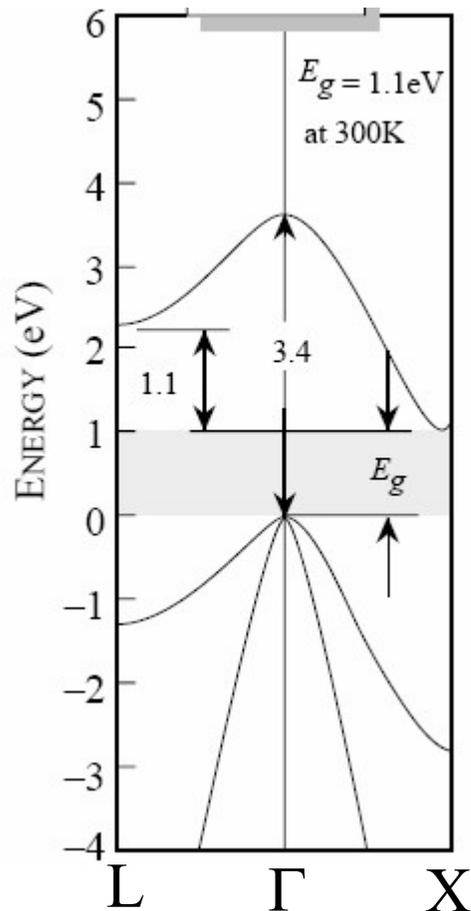
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A completely filled band does not contribute to the current.

$$\begin{aligned}\vec{j} &= \int_{\text{filled states}} -e\vec{v}(\vec{k})D(\vec{k})f(\vec{k})d\vec{k} \\ &= \int_{\text{band}} -e\vec{v}(\vec{k})D(\vec{k})f(\vec{k})d\vec{k} - \int_{\text{empty states}} -e\vec{v}(\vec{k})D(\vec{k})f(\vec{k})d\vec{k} \\ &= \int_{\text{empty states}} e\vec{v}(\vec{k})D(\vec{k})f(\vec{k})d\vec{k}\end{aligned}$$

Holes have a positive charge and a positive mass.

# Light holes and heavy holes



$$E = \frac{\hbar^2 (\vec{k} - \vec{k}_0)^2}{2m^*} + E_c$$

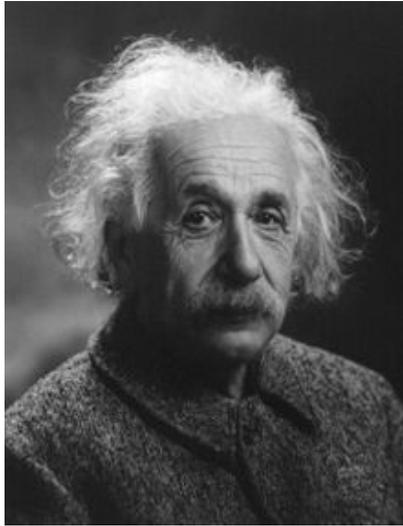
$$m_e^* = \frac{\hbar^2}{\frac{d^2 E}{dk_x^2}}$$

$$E = \frac{-\hbar^2 (\vec{k} - \vec{k}_0)^2}{2m^*} + E_v$$

$$m_h^* = \frac{-\hbar^2}{\frac{d^2 E}{dk_x^2}}$$

# Holes

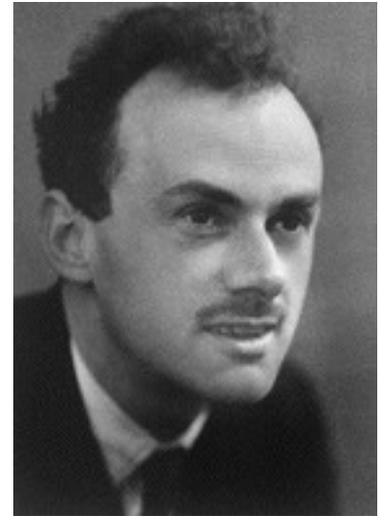
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Albert Einstein



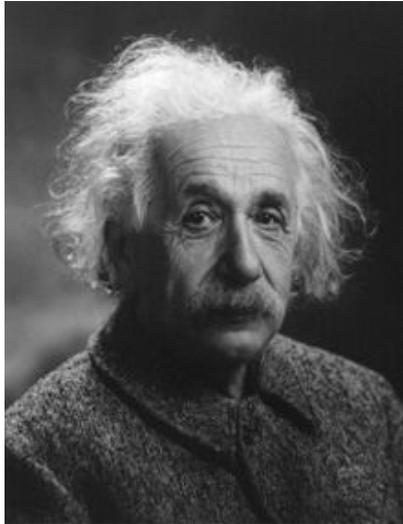
Erwin Schrödinger



Paul Adrien Maurice Dirac

# Holes

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Albert Einstein



Erwin Schrödinger



Paul Adrien Maurice Dirac

$$\frac{d^2u}{dt^2} = c^2 \frac{d^2u}{dx^2}$$

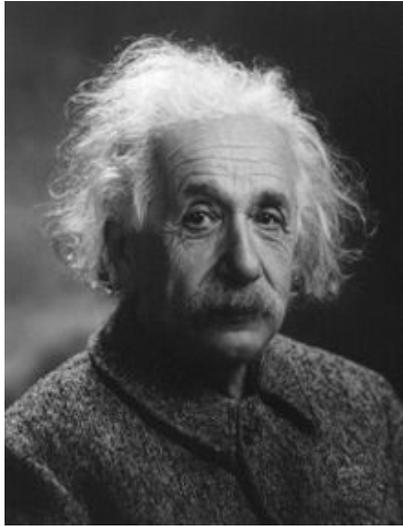
Wave equation

$$\frac{du}{dt} = k \frac{d^2u}{dx^2}$$

Heat equation

# Holes

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Albert Einstein



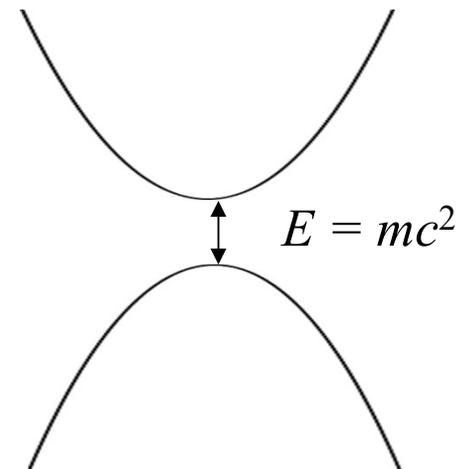
Erwin Schrödinger



Paul Adrien Maurice Dirac

$$\left( \beta mc^2 + \sum_{j=1}^3 \alpha_j p_j c \right) \psi = i\hbar \frac{\partial \psi}{\partial t}$$

Dirac equation

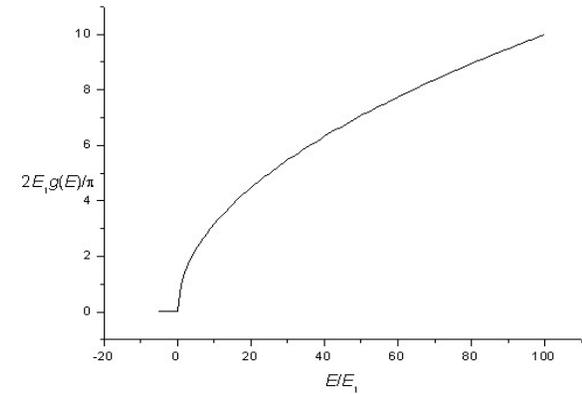
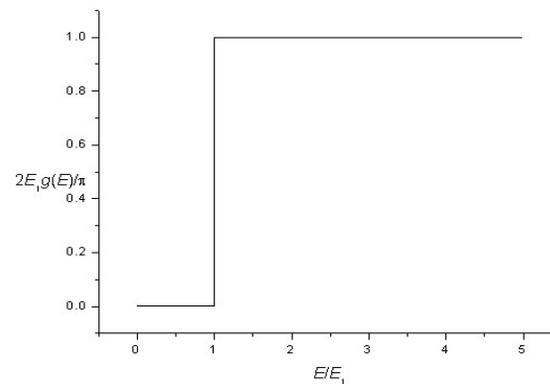
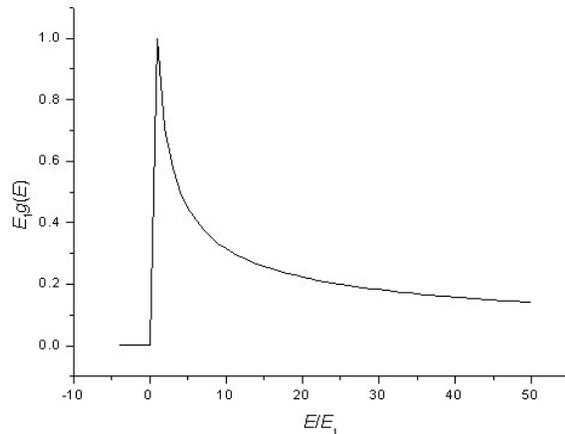


# Free electron Fermi gas

1 - d  $D(E) = \sqrt{\frac{2m}{\hbar^2 \pi^2 E}} = \frac{n}{2\sqrt{E_F E}} \quad \text{J}^{-1} \text{m}^{-1}$

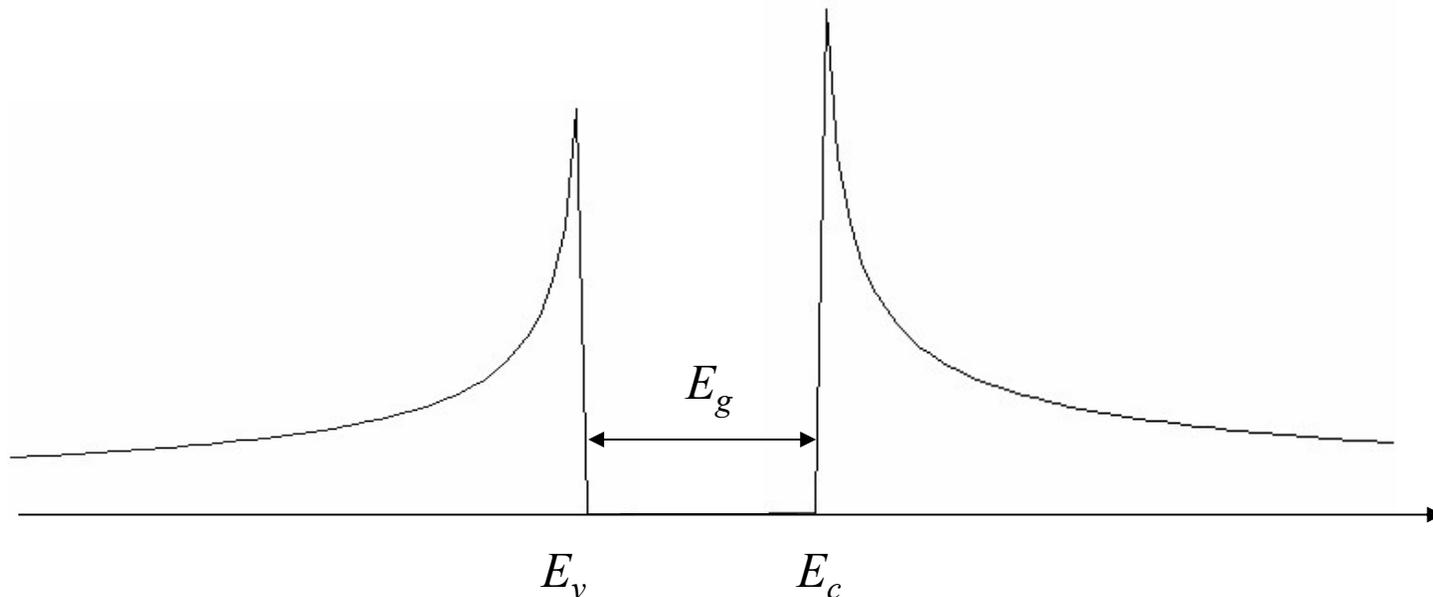
2 - d  $D(E) = \frac{m}{\hbar^2 \pi} = \frac{n}{E_F} \quad \text{J}^{-1} \text{m}^{-2}$

3 - d  $D(E) = \frac{\pi}{2} \left( \frac{2m}{\hbar^2 \pi^2} \right)^{3/2} \sqrt{E} = \frac{3n}{2E_F^{3/2}} \sqrt{E} \quad \text{J}^{-1} \text{m}^{-3}$

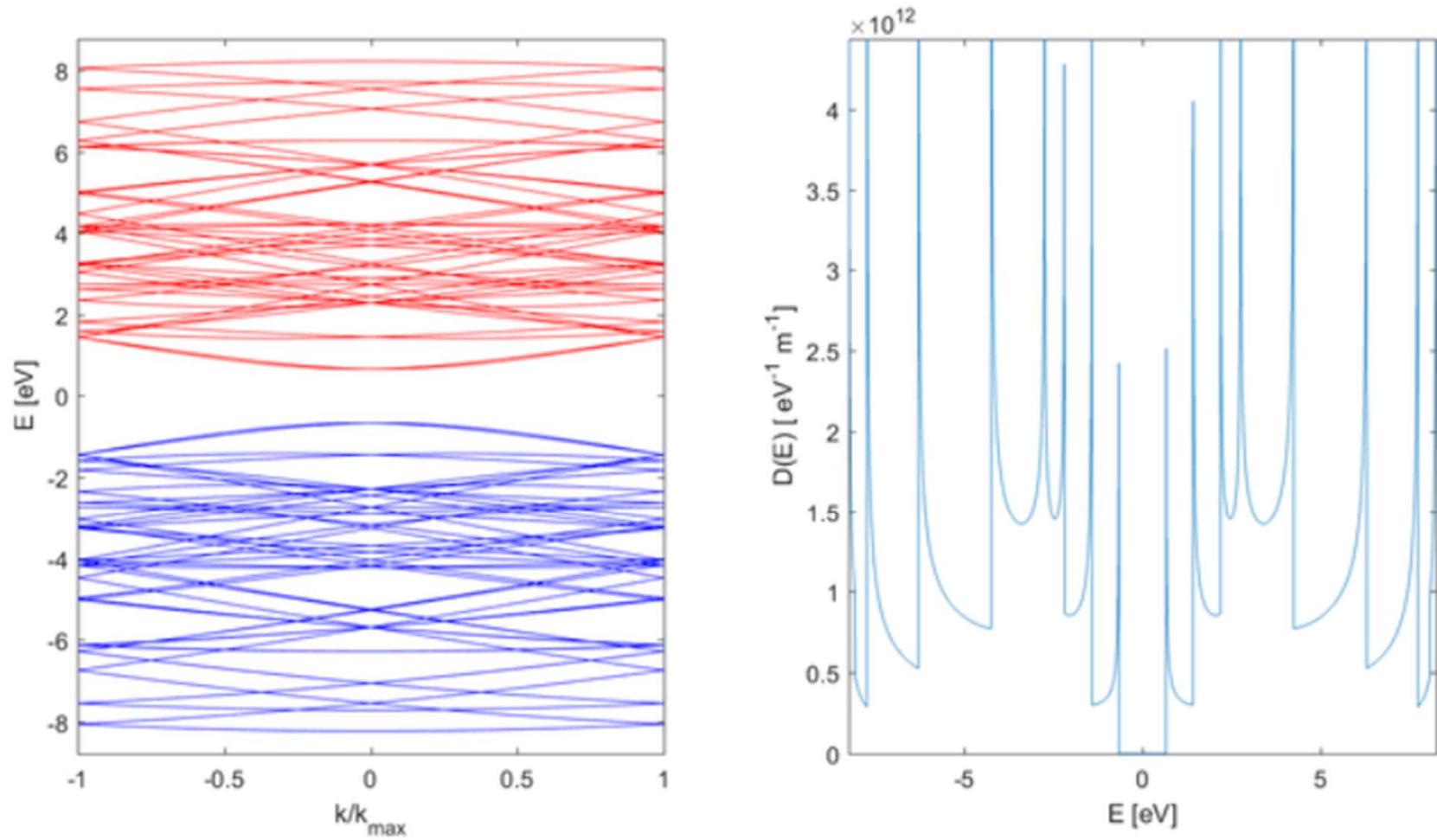


# Semiconductors and insulators - 1d

$$E = \frac{\hbar^2(\vec{k} - \vec{k}_0)^2}{2m^*}$$
$$D(E) = \begin{cases} \frac{D_c}{\sqrt{(E_v - E)}} & E < E_v \\ 0 & E_v < E < E_c \\ \frac{D_v}{\sqrt{(E - E_c)}} & E_c < E \end{cases} \quad \text{J}^{-1}\text{m}^{-3}$$



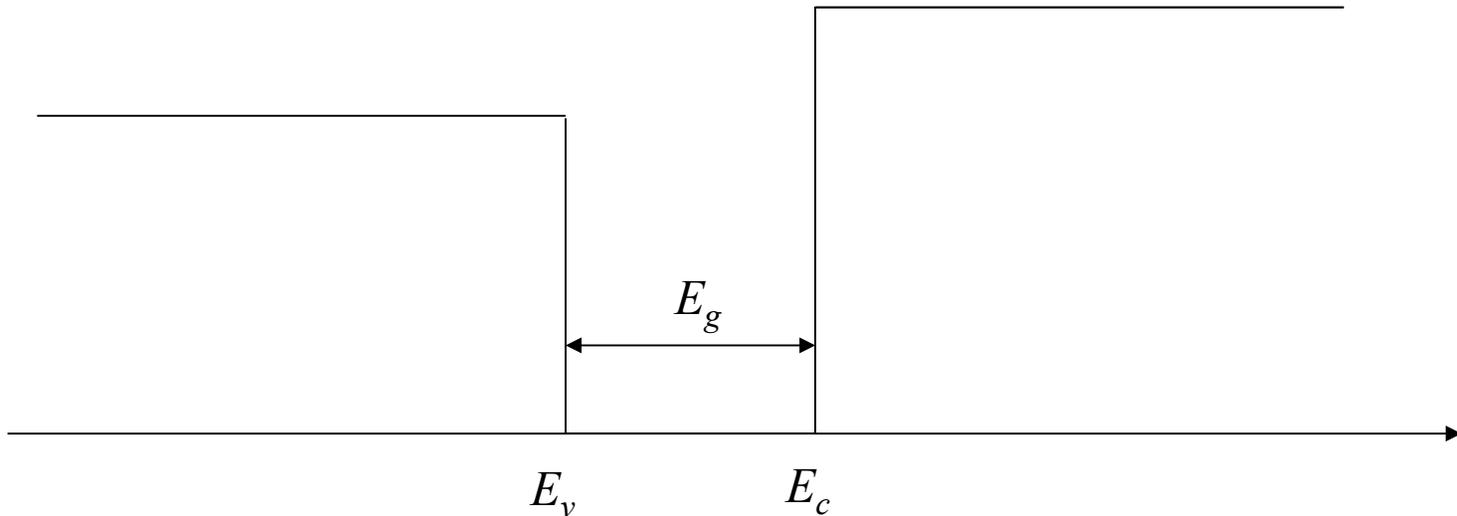
## Dispersion relation and density of states of a 6,2 carbon nanotube



# Semiconductors and insulators - 2d

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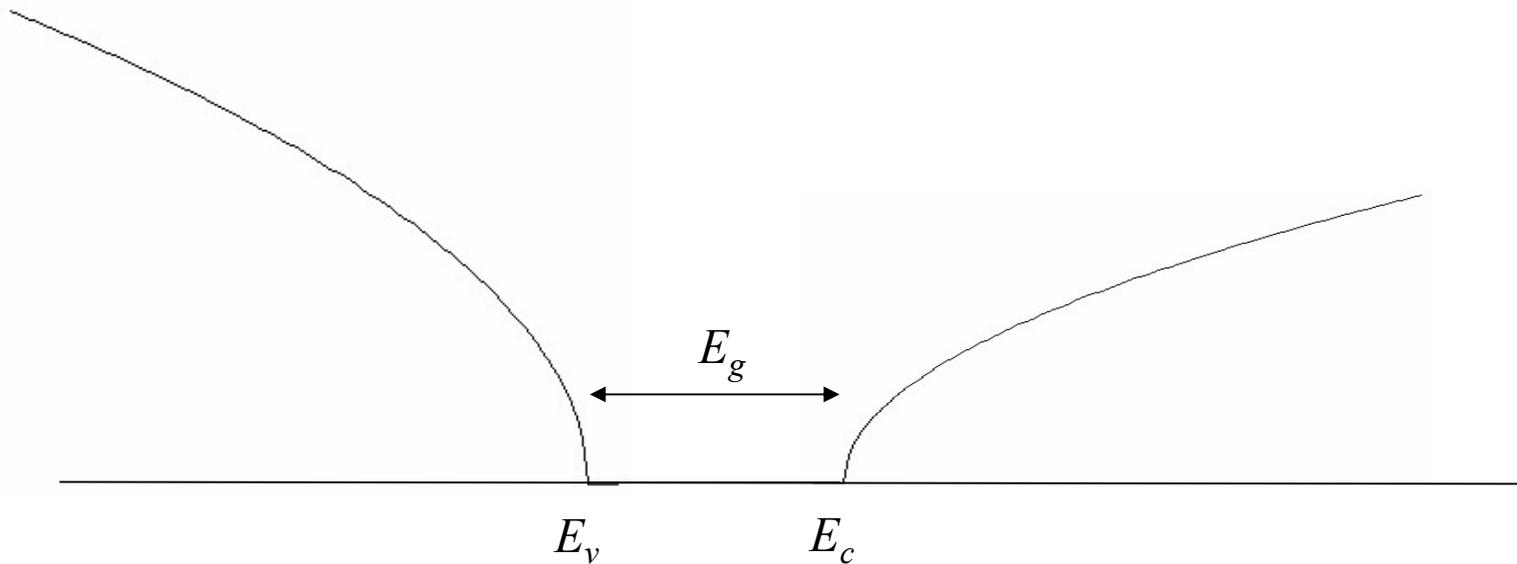
$$D(E) = \begin{cases} D_c & E < E_v \\ 0 & E_v < E < E_c \\ D_v & E_c < E \end{cases} \quad \text{J}^{-1}\text{m}^{-3}$$



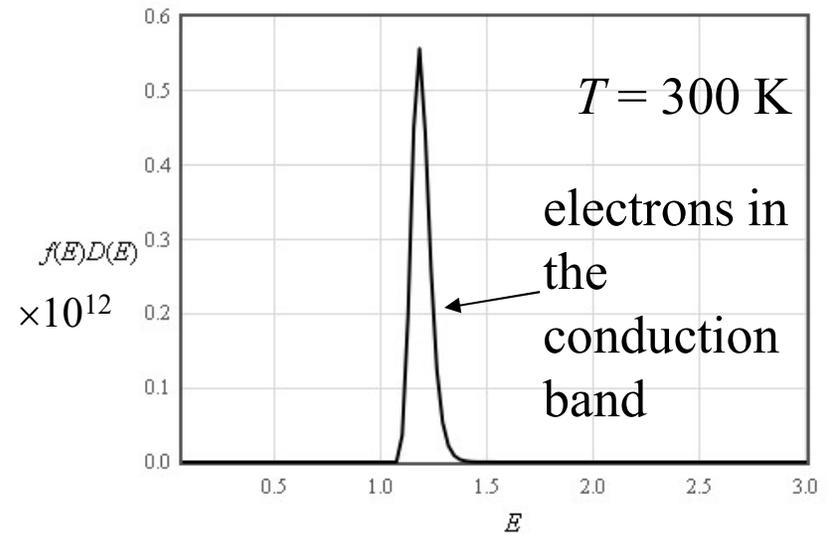
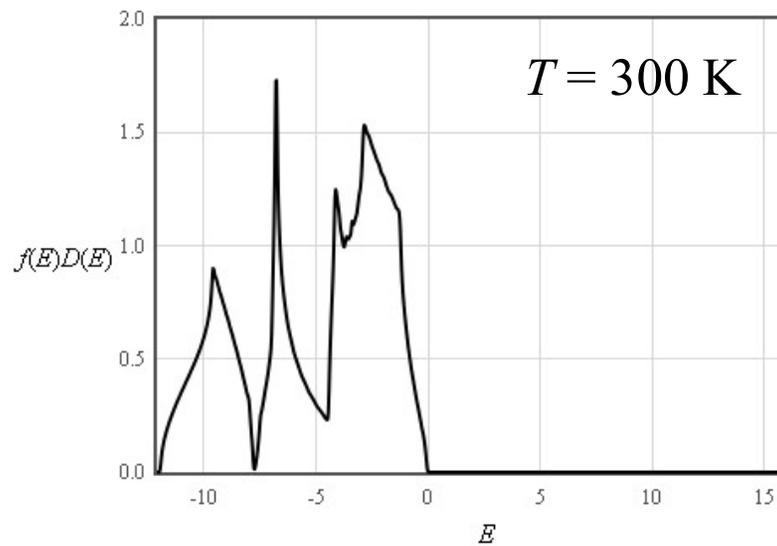
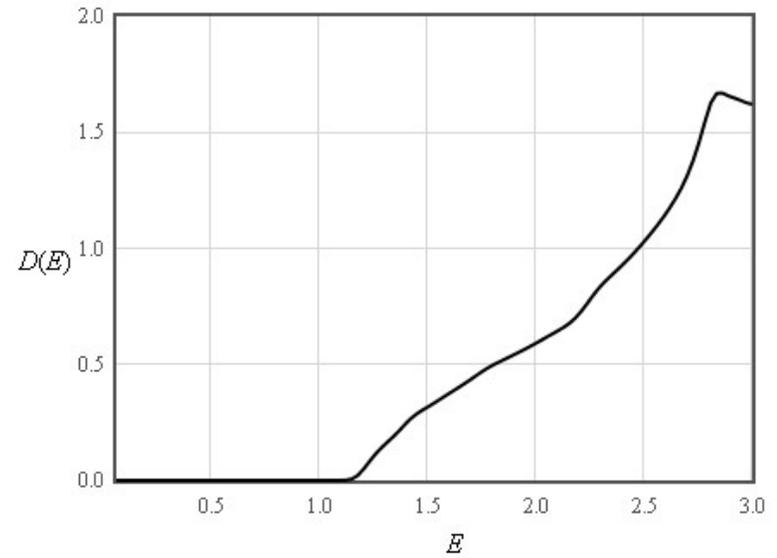
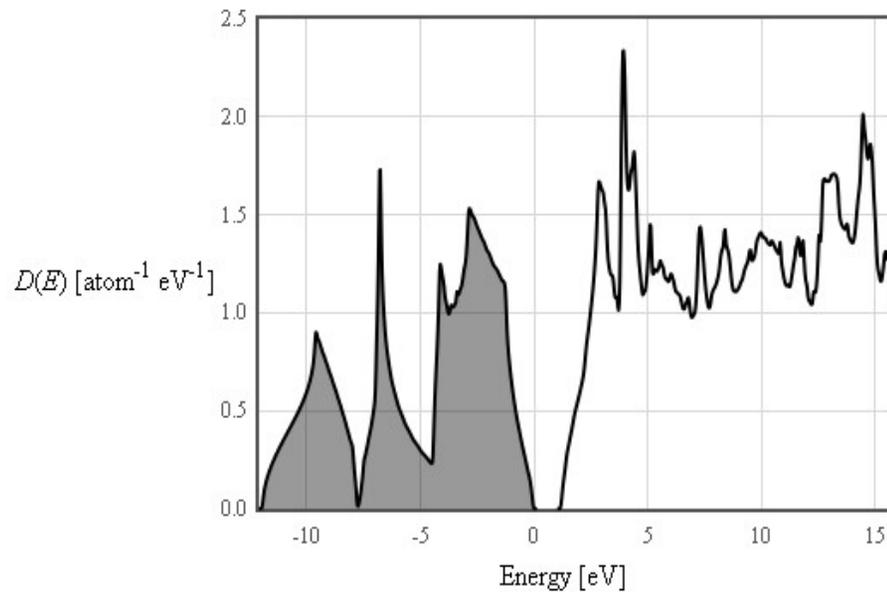
# Semiconductors and insulators - 3d

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$$D(E) = \begin{cases} D_c \sqrt{E_v - E} & E < E_v \\ 0 & E_v < E < E_c \\ D_v \sqrt{E - E_c} & E_c < E \end{cases} \quad \text{J}^{-1}\text{m}^{-3}$$



# Silicon density of states

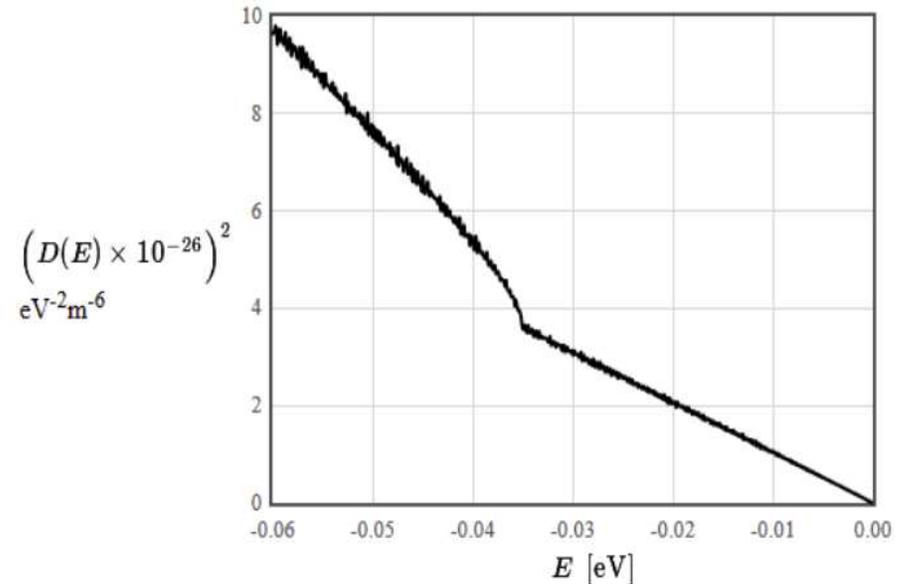
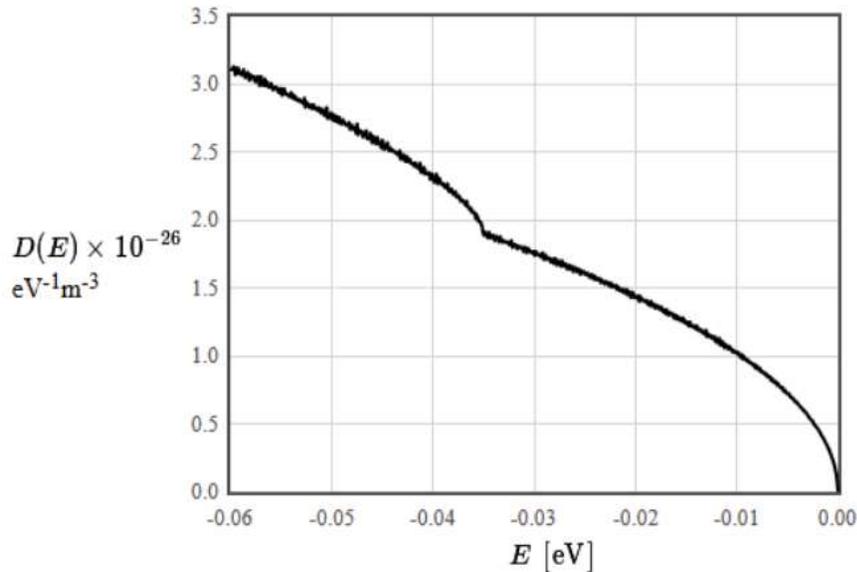


# Silicon valence bands

$$E_{v,th} = -\frac{\hbar^2}{2m_e} \left( 4.1k^2 - \sqrt{1.21k^4 + 4.1(k_x^2k_y^2 + k_x^2k_z^2 + k_y^2k_z^2)} \right),$$

$$E_{v,hh} = -\frac{\hbar^2}{2m_e} \left( 4.1k^2 + \sqrt{1.21k^4 + 4.1(k_x^2k_y^2 + k_x^2k_z^2 + k_y^2k_z^2)} \right),$$

$$E_{v,so} = -E_{so} - \frac{\hbar^2k^2}{2m_{so}}.$$



# Boltzmann Approximation

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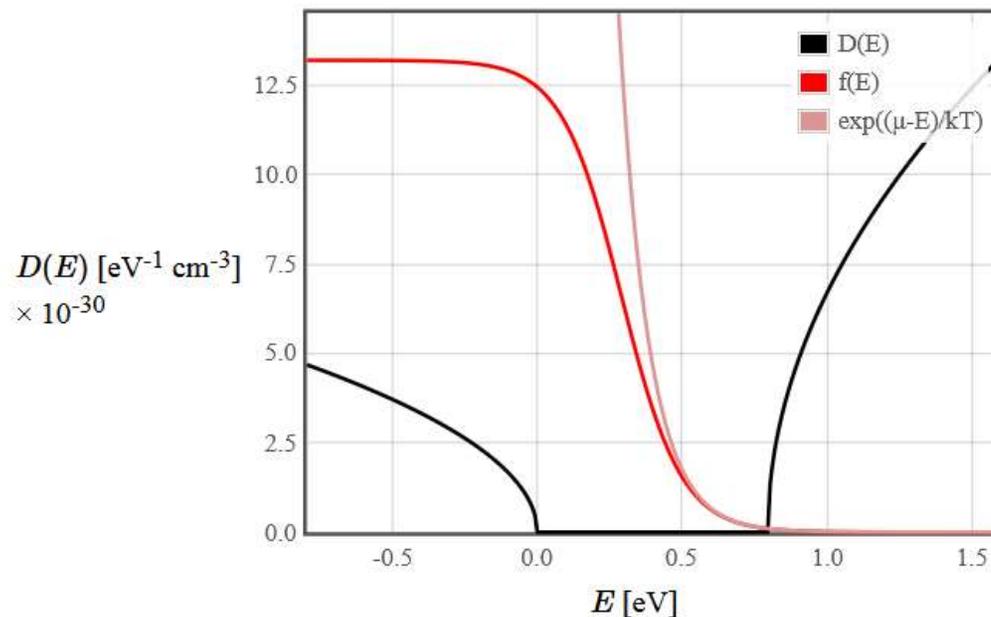
# Density of electrons in the conduction band

The free electron density of states is modified by the effective mass.

$$D(E) = D_c \sqrt{E - E_c}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - \mu}{k_B T}\right)} \approx \exp\left(\frac{\mu - E}{k_B T}\right)$$

Boltzmann approximation  
 $E_c - \mu > 3k_B T$



$$n = \int_{E_c}^{\infty} D(E) f(E) dE \approx D_c \int_{E_c}^{\infty} \exp\left(\frac{\mu - E}{k_B T}\right) \sqrt{E - E_c} dE$$

# Density of electrons in the conduction band

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$$n = \int_{E_c}^{\infty} D(E) f(E) dE \approx D_c \int_{E_c}^{\infty} \exp\left(\frac{\mu - E}{k_B T}\right) \sqrt{E - E_c} dE$$
$$= D_c \exp\left(\frac{\mu - E_c}{k_B T}\right) \int_{E_c}^{\infty} \exp\left(-\frac{E - E_c}{k_B T}\right) \sqrt{E - E_c} dE$$

$$x = E - E_c \quad \int_0^{\infty} \sqrt{x} \exp\left(\frac{-x}{k_B T}\right) dx = \frac{2}{\sqrt{\pi}} (k_B T)^{3/2}$$

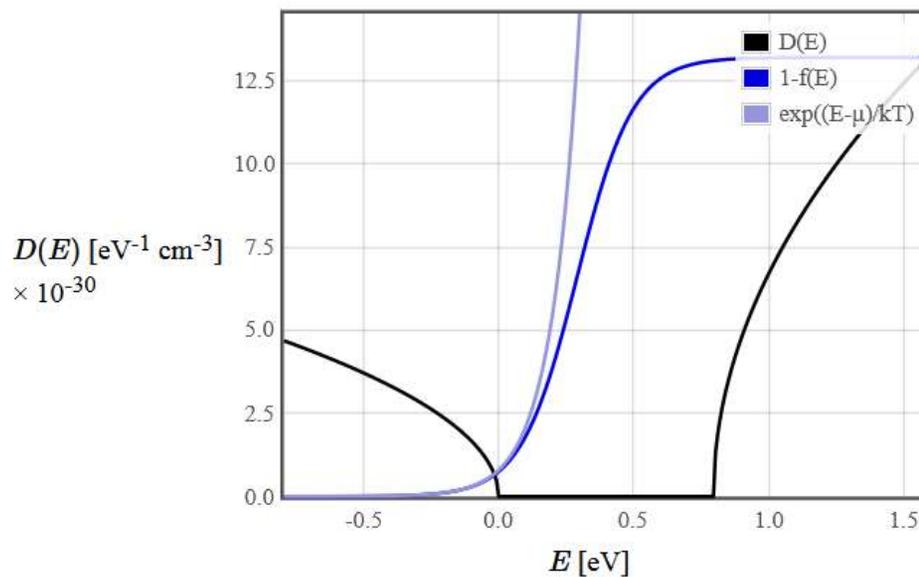
$$n = N_c(T) \exp\left(\frac{\mu - E_c}{k_B T}\right) = \frac{2D_c}{\sqrt{\pi}} (k_B T)^{3/2} \exp\left(\frac{\mu - E_c}{k_B T}\right)$$

$$N_c = \frac{2D_c}{\sqrt{\pi}} (k_B T)^{3/2} = 2 \left( \frac{m^* k_B T}{2\pi \hbar^2} \right)^{3/2} = \text{effective density of states}$$

# Density of holes in the valence band

$$D(E) = D_v \sqrt{E_v - E}$$

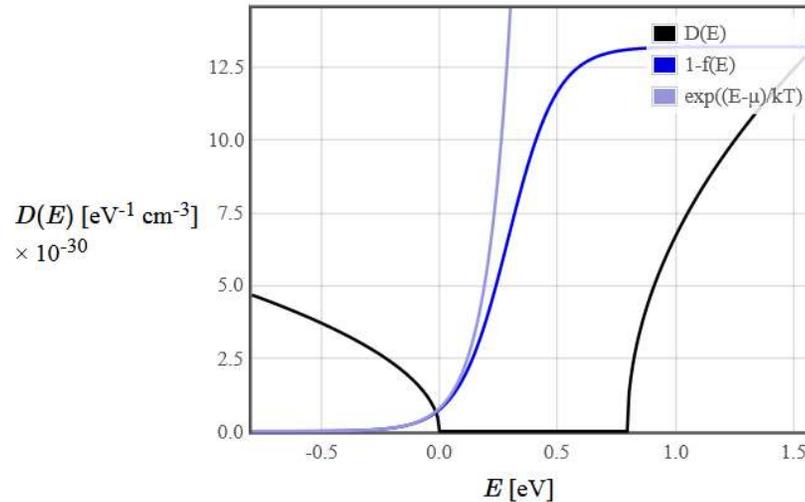
$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - \mu}{k_B T}\right)} \approx \exp\left(\frac{E - \mu}{k_B T}\right)$$



Boltzmann  
approximation  
 $\mu - E_v > 3k_B T$

$$p = \int_{-\infty}^{E_v} D(E) (1 - f(E)) dE \approx D_v \int_{-\infty}^{E_v} \exp\left(\frac{E - \mu}{k_B T}\right) \sqrt{E_v - E} dE$$

# Density of holes in the valence band



$$p = \int_{-\infty}^{E_v} D(E)(1-f(E))dE \approx D_v \int_{-\infty}^{E_v} \exp\left(\frac{E-\mu}{k_B T}\right) \sqrt{E_v-E} dE$$

$$p = N_v \exp\left(\frac{E_v - \mu}{k_B T}\right) = \frac{2D_v}{\sqrt{\pi}} (k_B T)^{3/2} \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

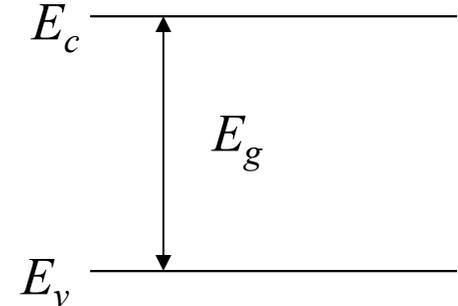
$$N_v = 2 \left( \frac{m_h^* k_B T}{2\pi \hbar^2} \right)^{3/2} = \text{Effective density of states in the valence band}$$

# Law of mass action

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$$np = N_c \exp\left(\frac{\mu - E_c}{k_B T}\right) N_v \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

$$np = N_c N_v \exp\left(\frac{-E_g}{k_B T}\right)$$



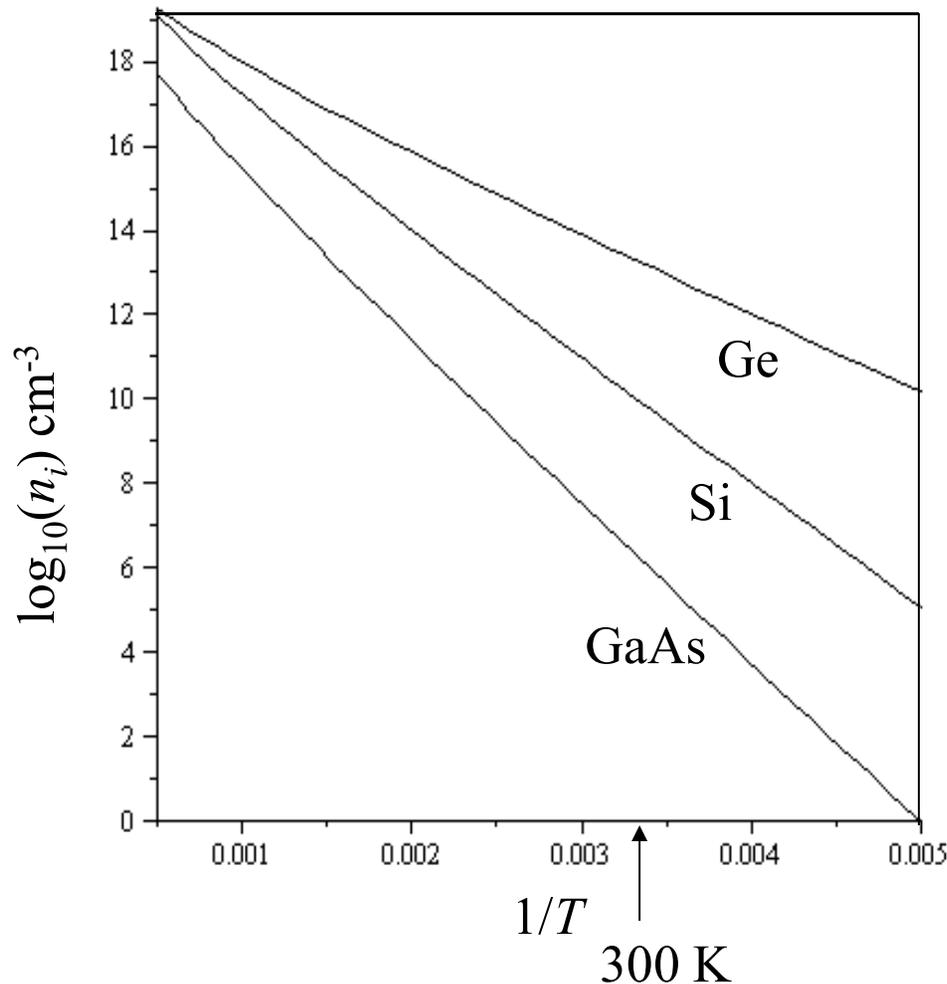
For intrinsic semiconductors (no impurities)

$$n = p = n_i = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2k_B T}\right)$$

intrinsic carrier density

# Intrinsic carrier concentration

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$$n_i = \sqrt{N_v N_c \left( \frac{T}{300} \right)^3 \exp\left( -\frac{E_g}{2k_B T} \right)}$$

$$\sim 5 \times 10^{22} \text{ atoms/cm}^3$$