# Semiconductors

Exam

HS B 10:15 – 11:15

1 A4 handwritten notes
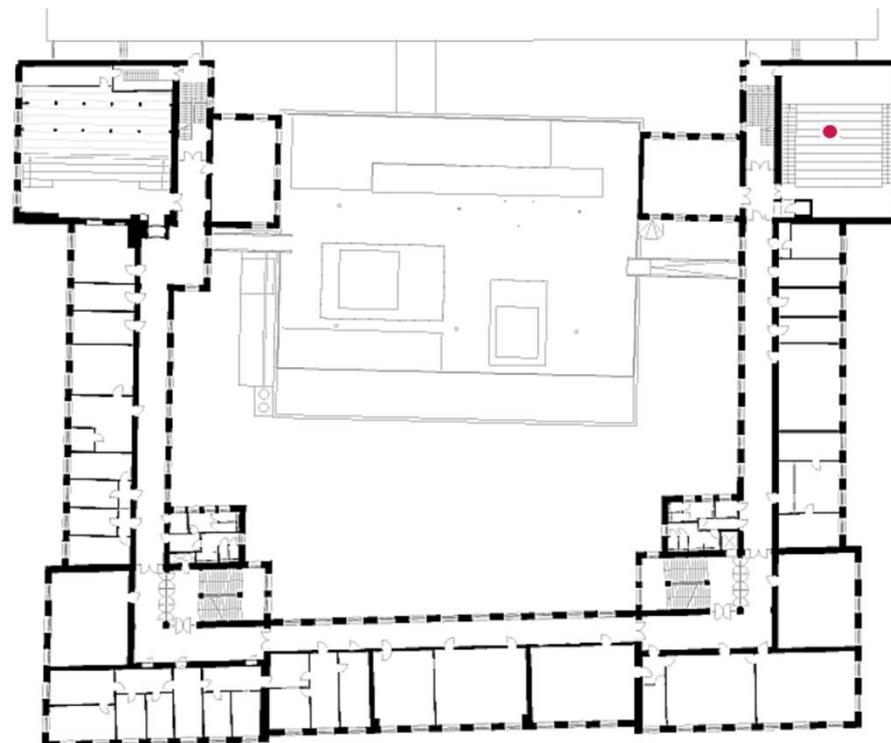
Boltzmann Approximation

Boltzmann approximation

of the valence band and the bottom of the conduction band the density of states of a semiconductor can be approximated as,

$$D(E) = \begin{cases} D_v\sqrt{E_v - E}, & \text{for } E < E_v \\ 0, & \text{for } E_v < E < E_c \\ D_c\sqrt{E - E_c}, & \text{for } E_c < E \end{cases}$$

ad D_c are constants that describe the form of the density of states near the band edges. Often in the literature, these constants are given in terms of the sses' m_h^* and m_e^* or the 'effective density of states at 300 K' $N_v(300)$ and $N_c(300)$. The relations to D_v and D_c are,

$$D_v = \frac{(2m_h^*)^{3/2}}{2\pi^2\hbar^3} = \frac{\sqrt{\pi}N_v(300)}{2(k_BT)^{3/2}}, \qquad D_c = \frac{(2m_e^*)^{3/2}}{2\pi^2\hbar^3} = \frac{\sqrt{\pi}N_c(300)}{2(k_BT)^{3/2}}.$$

ow shows the density of states of various semiconductors in this approximation. The Fermi function is plotted as well. At low energies the value of the tes are occupied. At high energies the Fermi function goes to zero and those states are unoccupied. In the limit of low temperture, the chemical potent ;ap, $\mu = E_g/2$. As the temperature increases, the chemical potential moves towards the band with the lower density of states.


$D_c =$	1.48E31	1/eV 1/cm³	Semiconductor
$D_v =$	5.25E30	1/eV 1/cm³	Si Ge GaAs
$E_g =$	1.166-4.73E-4*T*T/(T+636)	eV	InAs InP In:
$T_1 =$	300	K	
	Replot		

http://lampx.tugraz.at/~hadley/ss1/semiconductors/boltzmann.php
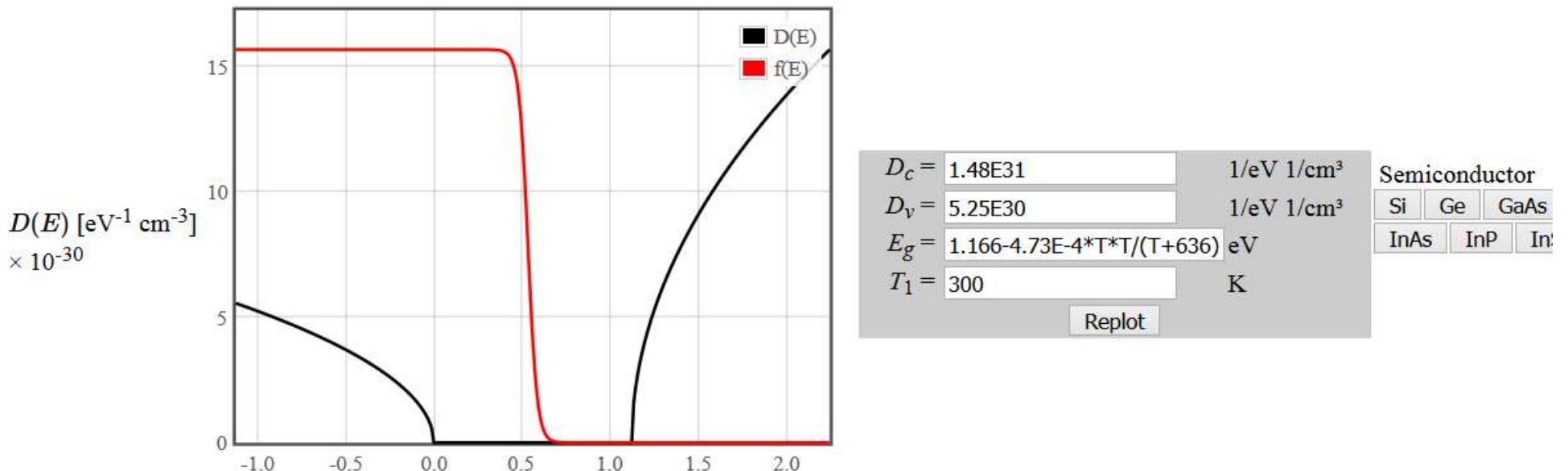
Law of mass action

$$np = N_c \exp\left(\frac{\mu - E_c}{k_B T}\right) N_v \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

E_c ———————————

E_g

$$np = N_c N_v \exp\left(\frac{-E_g}{k_B T}\right)$$

E_v ———————————
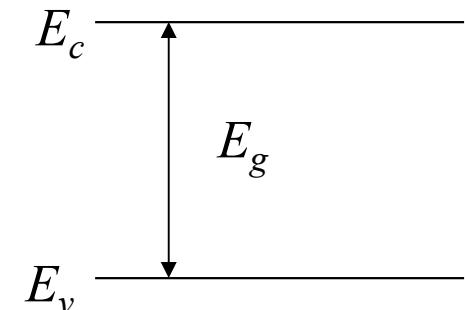
For intrinsic semiconductors (no impurities)

$$n = p = n_i = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2k_B T}\right)$$

intrinsic carrier density

Chemical potential of an intrinsic semiconductor

$$n = p = N_c \exp\left(\frac{\mu - E_c}{k_B T}\right) = N_v \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

$$\exp\left(\frac{\mu - E_c - E_v + \mu}{k_B T}\right) = \frac{N_v}{N_c}$$

$$\frac{2\mu}{k_B T} = \frac{E_c + E_v}{k_B T} + \ln\left(\frac{N_v}{N_c}\right)$$

$$\boxed{\mu = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln\left(\frac{N_v}{N_c}\right)}$$

E_c ——————————

μ - - - - - - - - - - - - -

E_v ——————————

Semiconductors   n,k

database   InGaAsP   Levels   Equivalents   Bibliografic database

NSM Archive - Physical Properties of Semiconductors

Si	- Silicon	Ge	- Germanium
GaP	- Gallium Phosphide	GaAs	- Gallium Arsenide
InAs	- Indium Arsenide	C	- Diamond
GaSb	- Gallium Antimonide	InSb	- Indium Antimonide
InP	- Indium Phosphide	$GaAs_{1-x}Sb_x$	- Gallium Arsenide Antimonide
$Al_xGa_{1-x}As$	- Aluminium Gallium Arsenide		
AlN	- Aluminium Nitride		
BN	- Boron Nitride		

300 K

$E_g = 1.12$ eV
$E_L = 2.0$ eV
$E_X = 1.2$ eV
$E_{so} = 0.044$ eV
$E_{\Gamma 1} = 3.4$ eV
$E_{\Gamma 2} = 4.2$ eV

http://www.matprop.ru/semicond
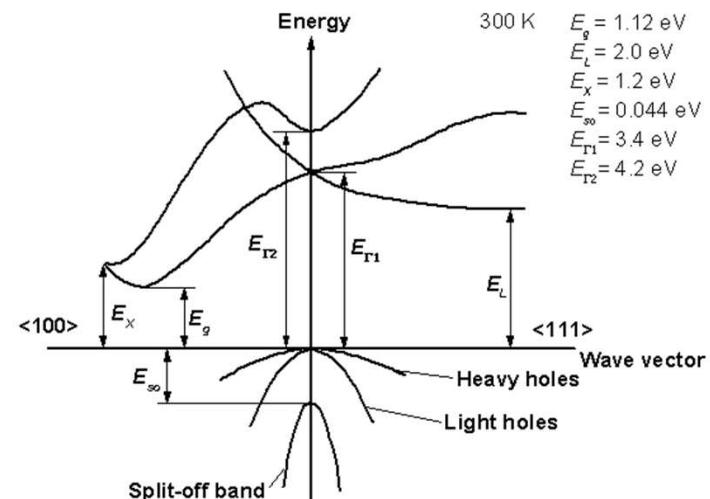
The electrical contribution to the thermodynamic properties of insulators depend on band edges

Boltzmann approximation

The table below gives the contribution of electrons in intrinsic semiconductors and insulators to some thermodynamic quantities. These results where calculated in the Boltzmann approximation where it is assumed that the chemical potential lies in the band gap more than $3k_BT$ from the band edge. The electronic contribution to the thermodynamic quantities are usually much smaller than the contribution of the phonons and thus the electronic components are often simply ignored.

	1-d	2-d
Density of states m_e^* and m_h^* are 'density of states' effective masses	$D(E) = \begin{cases} \dfrac{1}{\hbar\pi}\sqrt{\dfrac{2m_h^*}{(E_v - E)}} & E < E_v \\ 0 & E_v < E < E_c \\ \dfrac{1}{\hbar\pi}\sqrt{\dfrac{2m_e^*}{(E - E_c)}} & E > E_c \end{cases}$ $\mathrm{J^{-1}\,m^{-1}}$	$D(E) = \begin{cases} \dfrac{m_h^*}{\hbar^2\pi} H(E_v - E) & E < E_v \\ 0 & E_v < E < E_c \\ \dfrac{m_e^*}{\hbar^2\pi} H(E - E_c) & E > E_c \end{cases}$ $\mathrm{J^{-1}\,m^{-2}}$ $H(x) = 0$ for $x < 0$ and $H(x) = 1$ for $x > 0$
Density of states N_v and N_c are the effective densities of states	$D(E) = \begin{cases} N_v(300)\sqrt{\dfrac{2}{300\pi k_B (E_v - E)}} & E < E_v \\ 0 & E_v < E < E_c \\ N_c(300)\sqrt{\dfrac{2}{300\pi k_B (E - E_c)}} & E > E_c \end{cases}$ $\mathrm{J^{-1}\,m^{-1}}$	$D(E) = \begin{cases} \dfrac{N_v(300)}{300 k_B} H(E_v - E) & E < E_v \\ 0 & E_v < E < E_c \\ \dfrac{N_c(300)}{300 k_B} H(E - E_c) & E > E_c \end{cases}$ $\mathrm{J^{-1}\,m^{-2}}$
Density of electrons in the conduction band $n = \displaystyle\int_{E_c}^{\infty} D(E)f(E)dE$	$n = \sqrt{\dfrac{m_e^* k_B T}{\hbar^2 \pi}} \exp\left(\dfrac{\mu - E_c}{k_B T}\right)$ $\mathrm{m^{-1}}$ $= N_c \exp\left(\dfrac{\mu - E_c}{k_B T}\right)$	$n = \dfrac{m_e^* k_B T}{\hbar^2 \pi} \exp\left(\dfrac{\mu - E_c}{k_B T}\right)$ $\mathrm{m^{-2}}$ $= N_c \exp\left(\dfrac{\mu - E_c}{k_B T}\right)$
Density of holes in the valence band $p = \displaystyle\int_{-\infty}^{E_v} D(E)(1 - f(E))dE$	$p = \sqrt{\dfrac{m_h^* k_B T}{\hbar^2 \pi}} \exp\left(\dfrac{E_v - \mu}{k_B T}\right)$ $\mathrm{m^{-1}}$ $= N_v \exp\left(\dfrac{\mu - E_c}{k_B T}\right)$	$p = \dfrac{m_h^* k_B T}{\hbar^2 \pi} \exp\left(\dfrac{E_v - \mu}{k_B T}\right)$ $\mathrm{m^{-2}}$ $= N_v \exp\left(\dfrac{\mu - E_c}{k_B T}\right)$

Intrinsic semiconductors

In the Boltzmann approximation, the density of states of a semiconductor is,

$$D(E) = \begin{cases} \frac{(2m_h^*)^{3/2}}{2\pi^2\hbar^3}\sqrt{E_v - E}, & \text{if } E < E_v \\ 0, & \text{if } E_v < E < E_c \\ \frac{(2m_e^*)^{3/2}}{2\pi^2\hbar^3}\sqrt{E - E_c}, & \text{if } E_c < E \end{cases}$$

Here m_e^* and m_h^* are the 'density of states effective masses' for electrons and holes. Usually in the literature, effective density of states at 300 K is given instead of the 'density of states effective masses'. The relationship between the two is,

$$m_h^* = \frac{\pi\hbar^2}{300k_B}\left(\sqrt{2}N_v(300)\right)^{2/3}$$

$$m_e^* = \frac{\pi\hbar^2}{300k_B}\left(\sqrt{2}N_c(300)\right)^{2/3}$$

In an intrinsic semiconductor, the density of electrons equals the density of holes, $n = p = n_i = \sqrt{N_c\left(\frac{T}{300}\right)^{3/2}N_v\left(\frac{T}{300}\right)^{3/2}}\exp\left(\frac{-E_g}{2k_BT}\right)$.

By setting the concentration of electrons equal to the concentration of holes, it is possible to solve for the chemical potential. The bandgap of most semiconductors is temperature dependent. The form below lets you input the temperature dependance of the bandgap. The bandgaps for some semiconductors can be loaded into the form with the buttons on the right.

$$n = N_c(300)\left(\frac{T}{300}\right)^{3/2}\exp\left(\frac{\mu - E_c}{k_BT}\right) = p = N_v(300)\left(\frac{T}{300}\right)^{3/2}\exp\left(\frac{E_v - \mu}{k_BT}\right).$$

$$\mu = \frac{E_v + E_c}{2} + k_BT\ln\left(\frac{N_v(300)}{N_c(300)}\right).$$
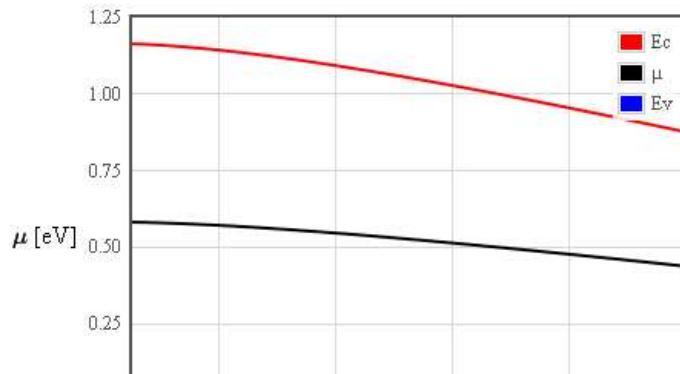
N_c(300 K) =	2.78E19	1/cm³
N_v(300 K) =	9.84E18	1/cm³
E_g =	1.166-4.73E-4*T*T/(T+636)	eV
T_1 =	50	K
T_2 =	1000	K

Semiconductor
[Si] [Ge] [GaAs]

[Replot]

http://lamp.tu-graz.ac.at/~hadley/ss1/semiconductors/intrinsic.php

# metals, semiconductors, and insulators

v = 17.024, b = 0.4951, n = 2

[v --] [v++] [b --] [b++] [n --] [n++]

Dispersion relation

Density of states

http://lampx.tugraz.at/~hadley/ss1/separablecrystals/thermo.html
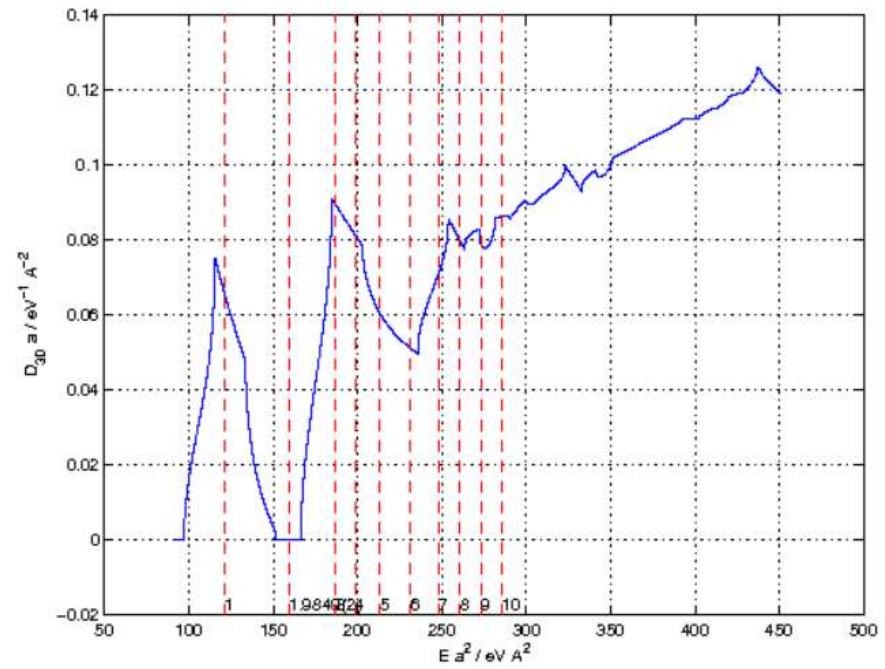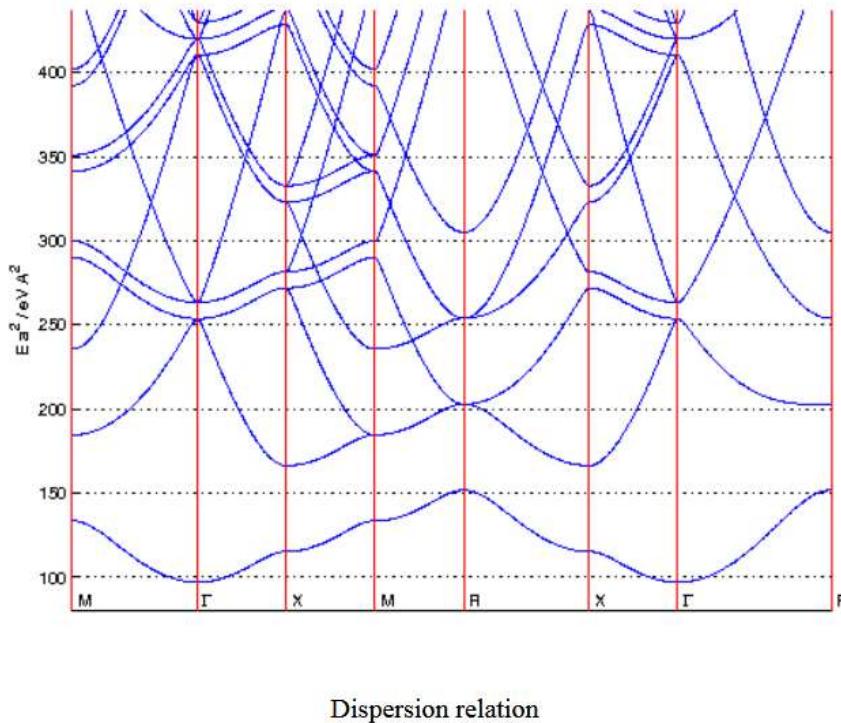
Narrow bandgap semiconductors
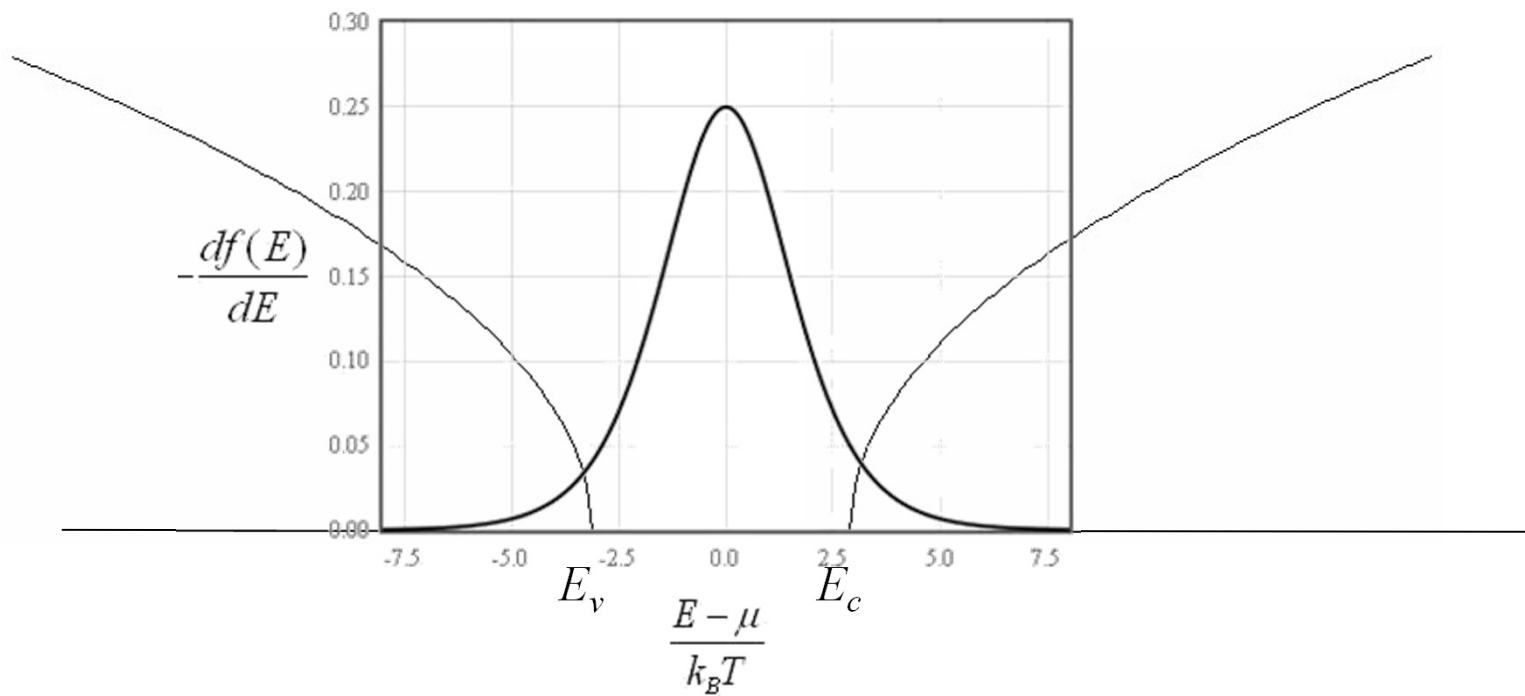
Use the programs for metals for small bandgap semiconductors.

Large gap -> large effective mass

$$E_k = \varepsilon - 2t\cos(ka)$$

$$m^* = \frac{\hbar^2}{\dfrac{d^2 E}{dk^2}}$$

$$\frac{d^2 E}{dk^2} = 2ta^2$$

$$m^* \sim \frac{1}{t}$$
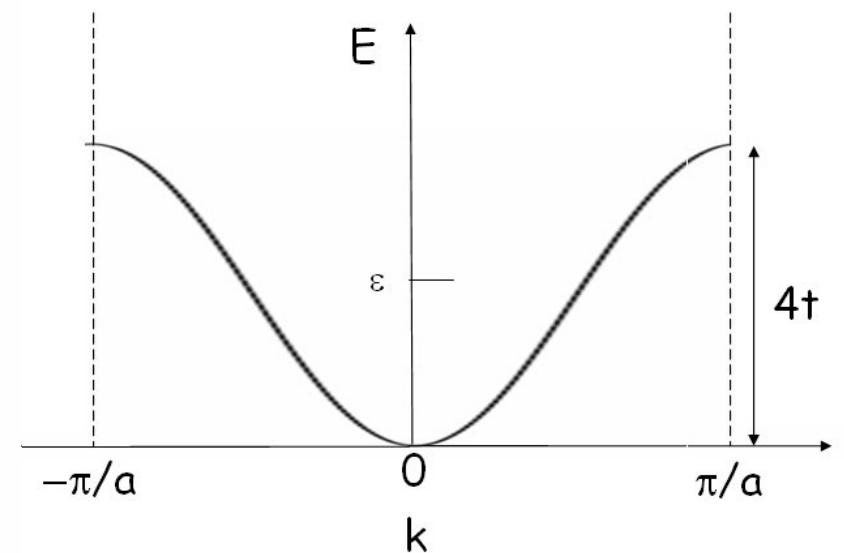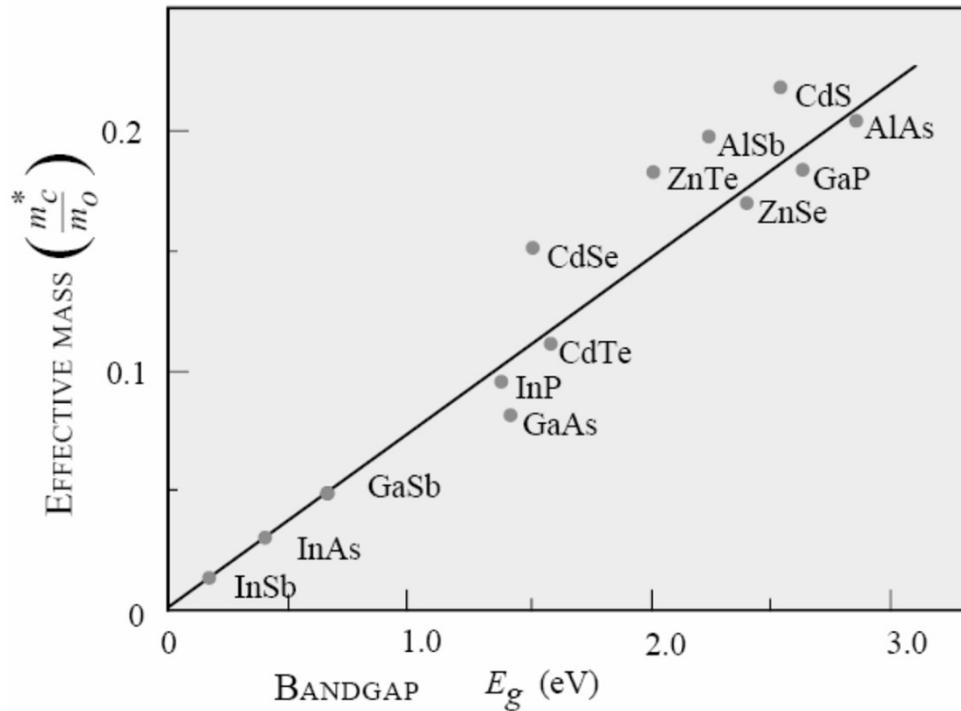
narrow bands -> large effective mass

Measuring the effective mass

Cyclotron resonance $\qquad \omega_c = \dfrac{eB}{m^*}$

Resonant absorption occurs when rf waves with the cyclotron resonance frequency are applied. This can be used to experimentally determine the effective mass.

Knowing the effective mass, the scattering time can be calculated from the measured conductivity.

$$\sigma = \frac{ne^2 \tau_{sc}}{m^*}$$

Extrinsic Semiconductors

Extrinsic semiconductors

The introduction of impurity atoms that can and electrons or holes is called doping.

n-type : donor atoms contribute electrons to the conduction band. Examples: P, As in Si.

p-type : acceptor atoms contribute holes to the valence band. Examples: B, Ga, Al in Si.

	IIIA	IVA	VA	VIA
	5 B	6 C	7 N	8 O
	13 Al	14 Si	15 P	16 S
IIB				
30 Zn	31 Ga	32 Ge	33 As	34 Se
48 Cd	49 In	50 Sn	51 Sb	52 Te

Ionization of dopants
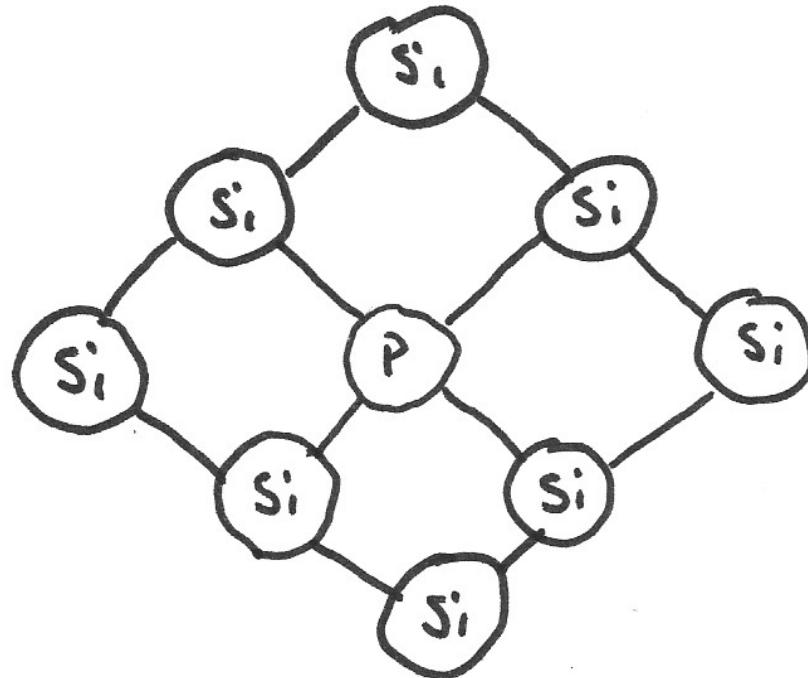
Easier to ionize a P atom in Si than a free P atom

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}$$

Ionization energy is smaller by a factor: $\dfrac{m^*}{m}\left(\dfrac{\varepsilon_0}{\varepsilon_r \varepsilon_0}\right)^2$

Ionization energy ~ 25 meV

Direct Observation of Friedel Oscillations around Incorporated Si_{Ga} Dopants in GaAs by Low-Temperature Scanning Tunneling Microscopy

M. C. M. M. van der Wielen, A. J. A. van Roij, and H. van Kempen

Research Institute for Materials, University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands
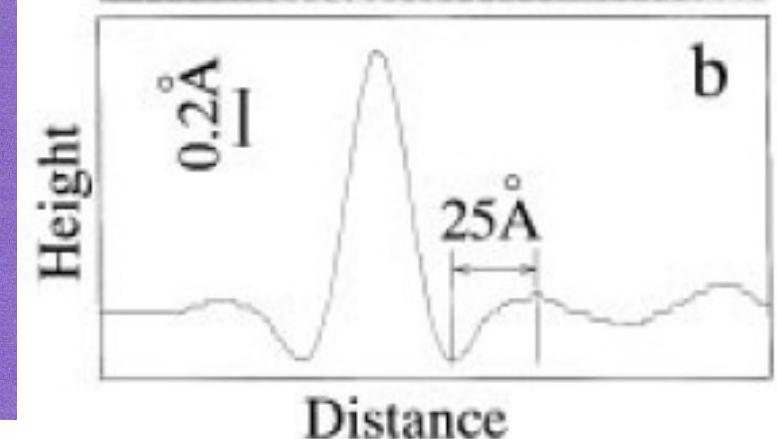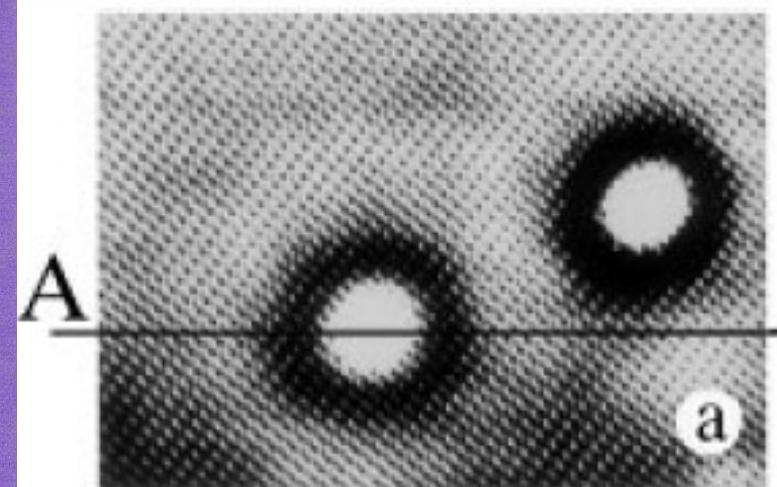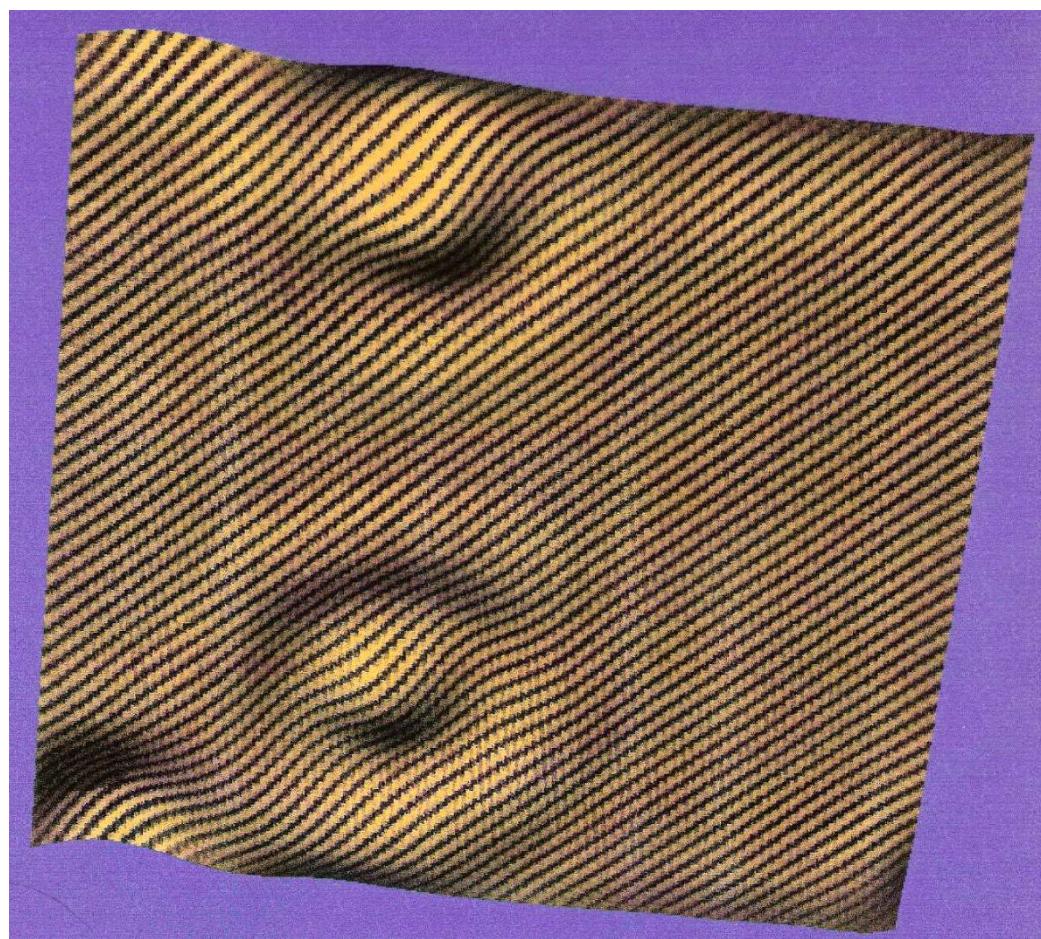
Crystal growth

Czochralski Process

Melting of polysilicon, doping · Introduction of the seed crystal · Beginning of the crystal growth · Crystal pulling · Formed crystal with a residue of melted silicon

add dopants to the melt
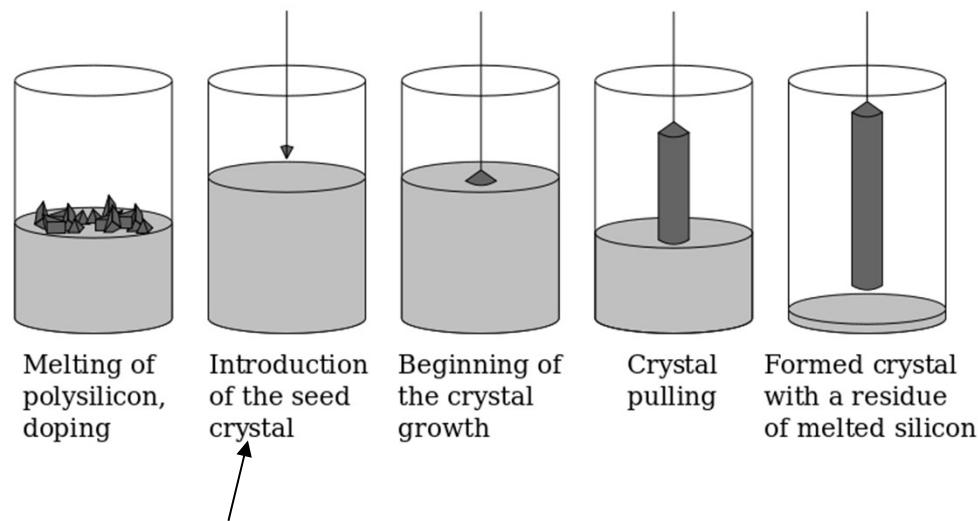
images from wikipedia

Crystal growth

Float zone Process

Neutron transmutation

$$^{30}\text{Si} + \text{n} \rightarrow\ ^{31}\text{Si} + \gamma$$
$$^{31}\text{Si} \rightarrow\ ^{31}\text{P} + \beta$$
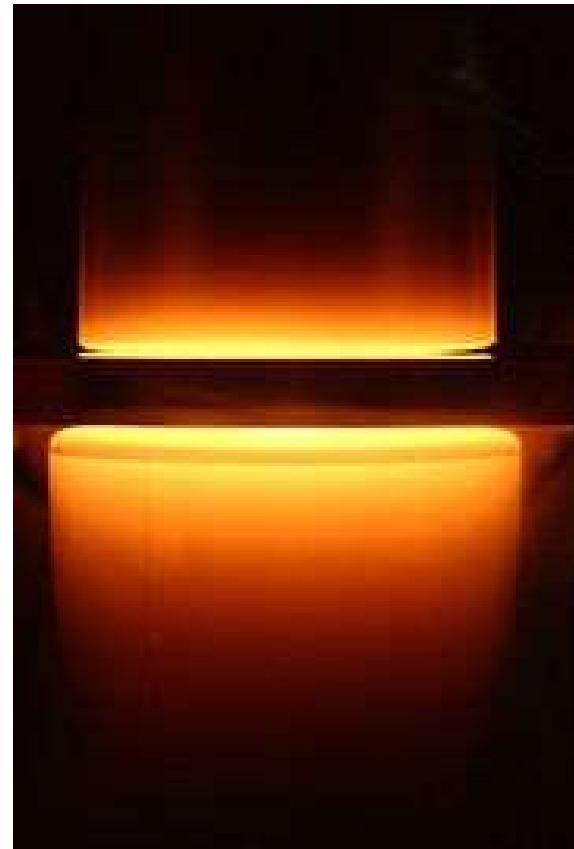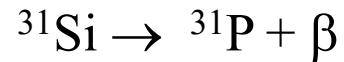
image from wikipedia

Chemical vapor deposition
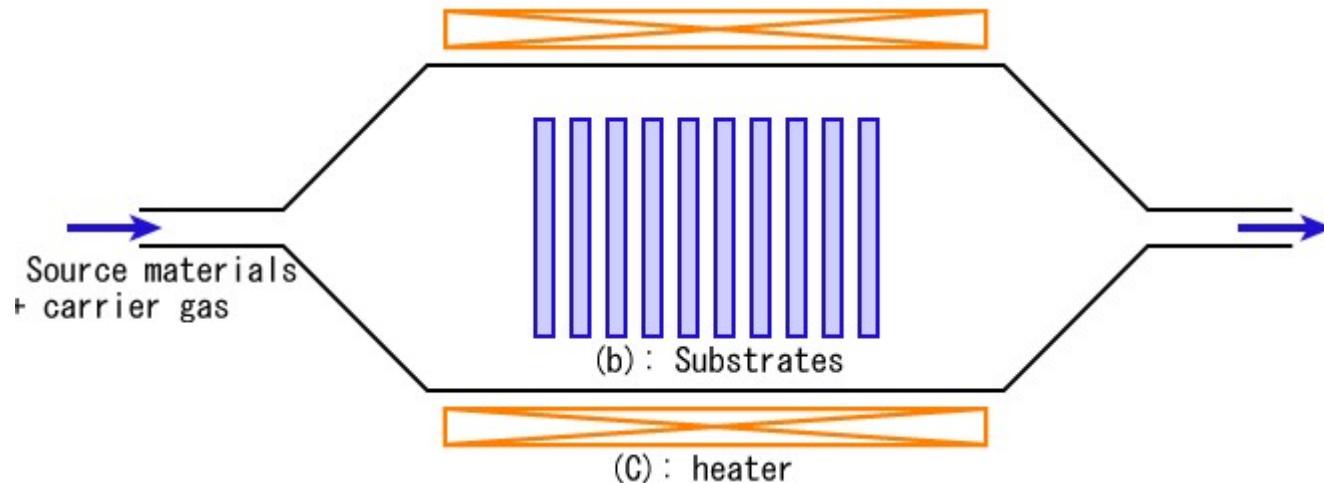

Epitaxial silicon CVD SiH_4 (silane) or SiH_2Cl_2 (dichlorosilane)
PH_3 (phosphine) for n-doping or B_2H_6 (diborane) for p-doping.

image from wikipedia

Gas phase diffusion

AsH_3 (Arsine) or PH_3 (phosphine) for n-doping
B_2H_6 (diborane) for p-doping.

http://www.microfab.de/foundry/services/diffusion/index.html

Ion implantation
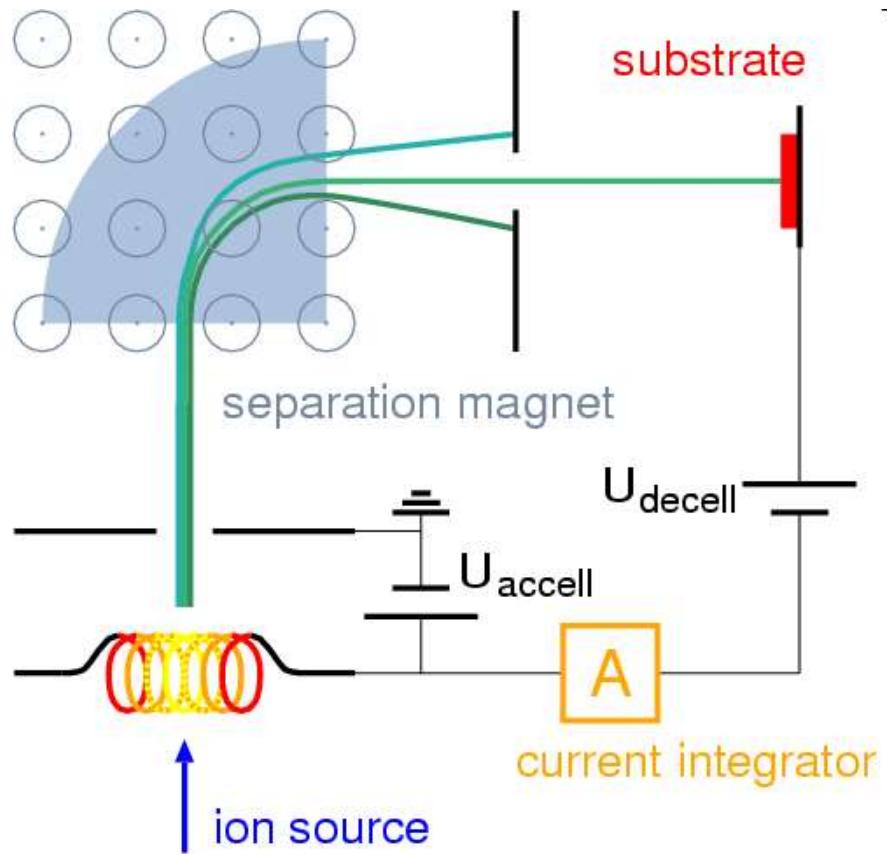
Implant at 7° to avoid channeling

Donors

Five valence electrons: P, As

States are added in the band gap just below the conduction band

E_c

E_D

E_g

E_v

$\mu(T=0)$

$D(E)$
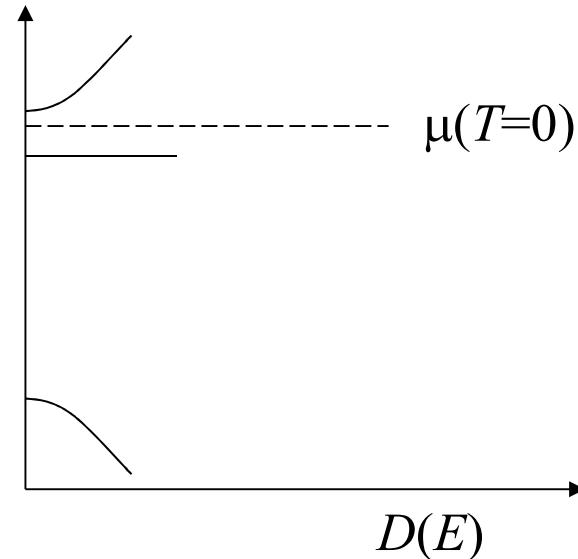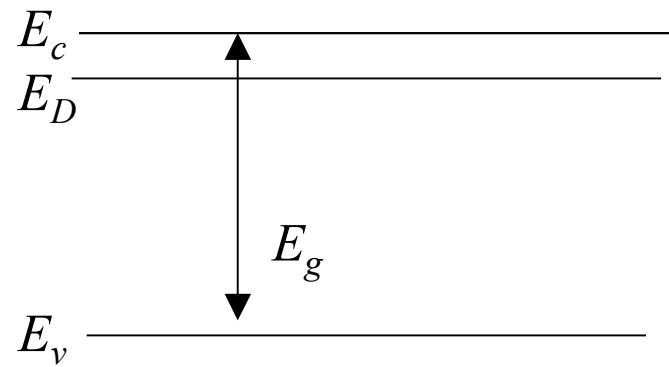
n-type: $n \sim N_D$ Many more electrons in the conduction band than holes in the valence band.

majority carriers: electrons; minority carriers: holes

Acceptors

Three valence electrons: B, Al, Ga

States are added in the band gap just above the valence band

E_c

E_g

E_A
E_v

$\mu(T{=}0)$

D(E)
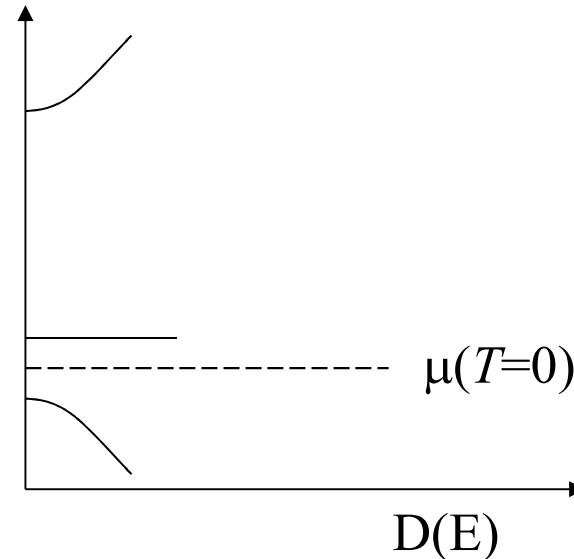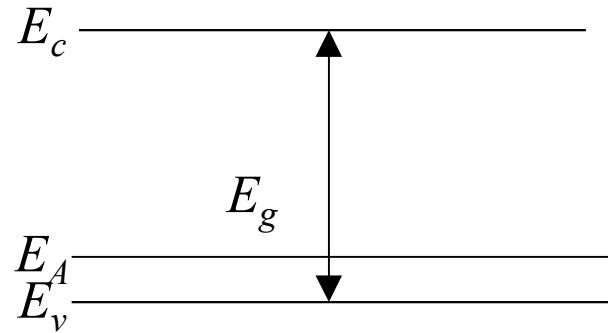
p-type: $p \sim N_A$ Many more holes in the valence band than electrons in the conduction band.

majority carriers: holes; minority carriers: electrons

Source: Semiconductor Devices Physics and Technology, S.M. Sze, 1985

Donor and Acceptor Energies

Semiconductor	Donor	Energy (meV)
Si	Li	33
	Sb	39
	P	45
	As	54
Ge	Li	9.3
	Sb	9.6
	P	12
	As	13
GaAs	Si	5.8
	Ge	6.0
	S	6.0
	Sn	6.0

Semiconductor	Acceptor	Energy (meV)
Si	B	45
	Al	67
	Ga	72
	In	160
Ge	B	10
	Al	10
	Ga	11
	In	11
GaAs	C	26
	Be	28
	Mg	28
	Si	35

Energy below the conduction band   Energy above the valence band

n-type (extrinsic)

n-type $N_D > N_A$, $p \sim 0$

$$n = N_D = N_c \exp\left(\frac{\mu - E_c}{k_B T}\right)$$

$$\boxed{\mu = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)}$$

For n-type, $n \sim$ density of donors,
$p = n_i^2/n$
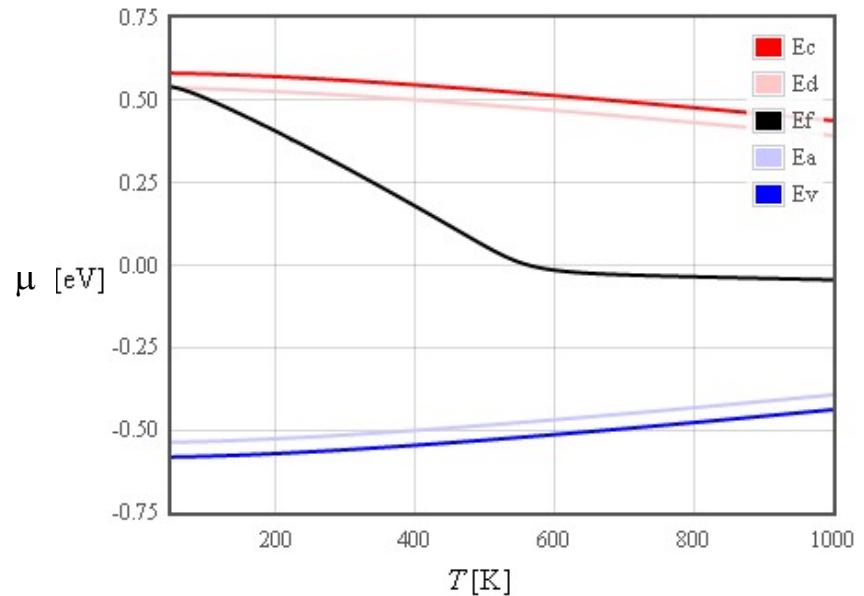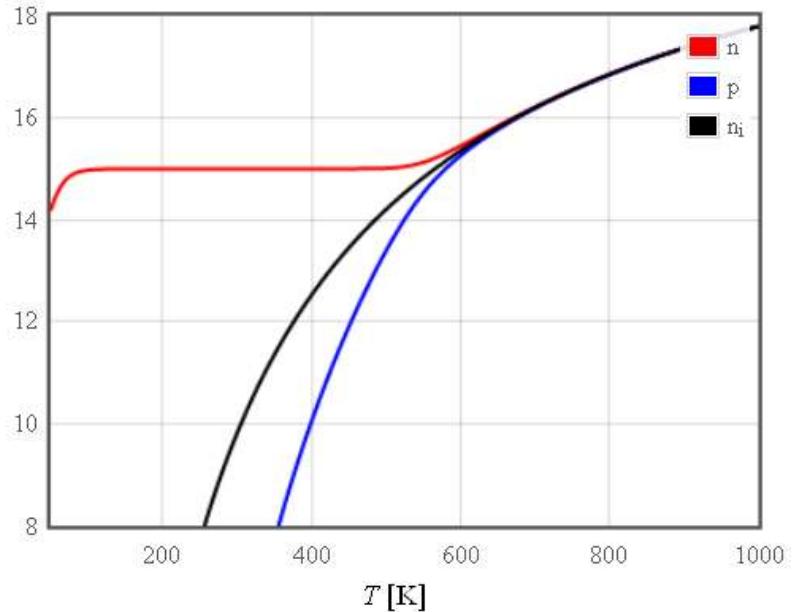
p-type (extrinsic)

p-type $N_A > N_D$, $n \sim 0$

$$p = N_A = N_v \exp\left(\frac{E_v - \mu}{k_B T}\right)$$

$$\mu = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)$$

For p-type, $p \sim$ density of acceptors, $n = n_i^2/p$
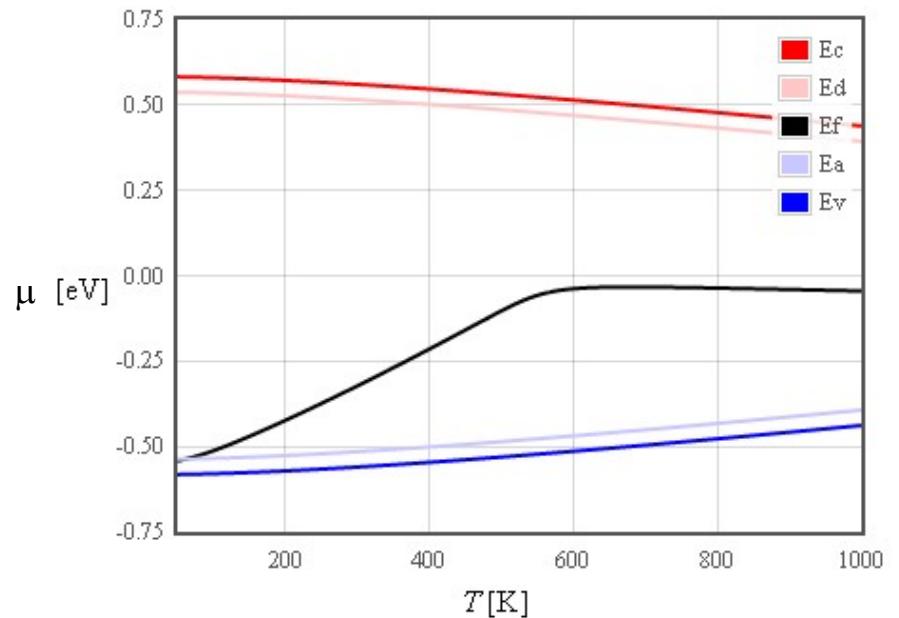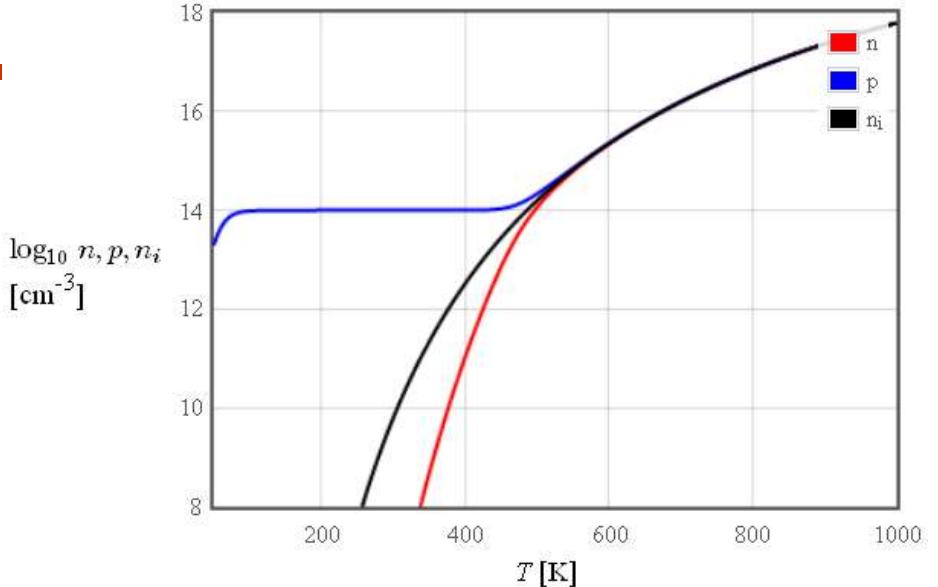
Intrinsic semiconductors
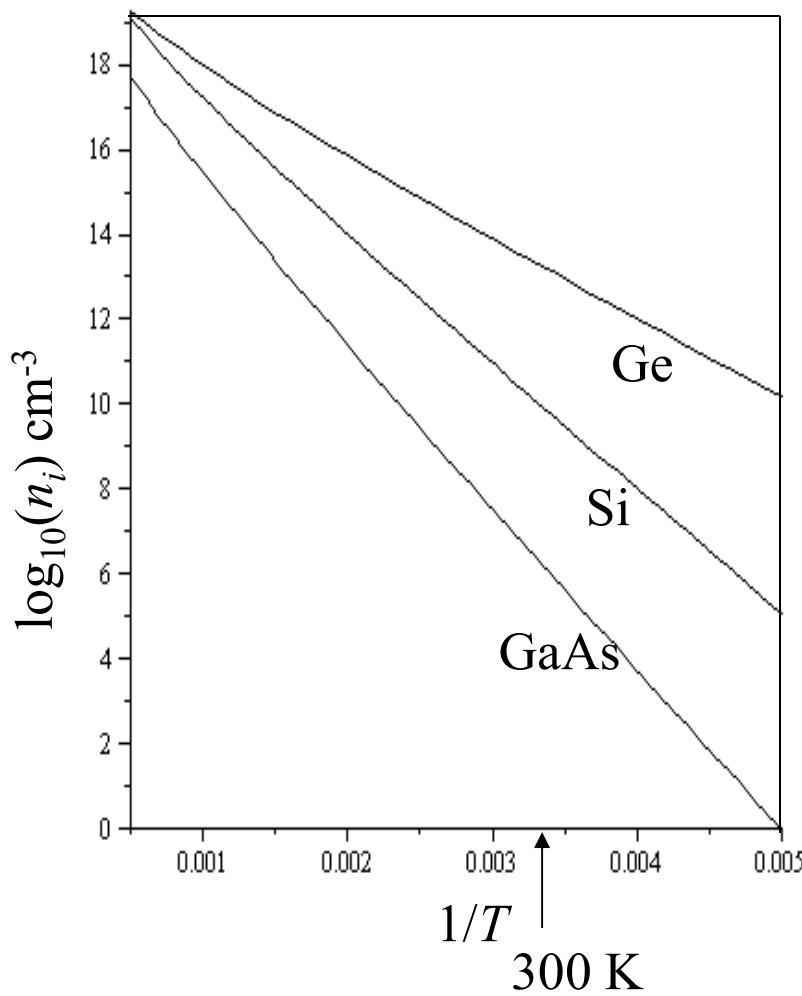
$$n_i = \sqrt{N_v N_c}\, \exp\left(-\frac{E_g}{2k_B T}\right)$$

Extrinsic semiconductors
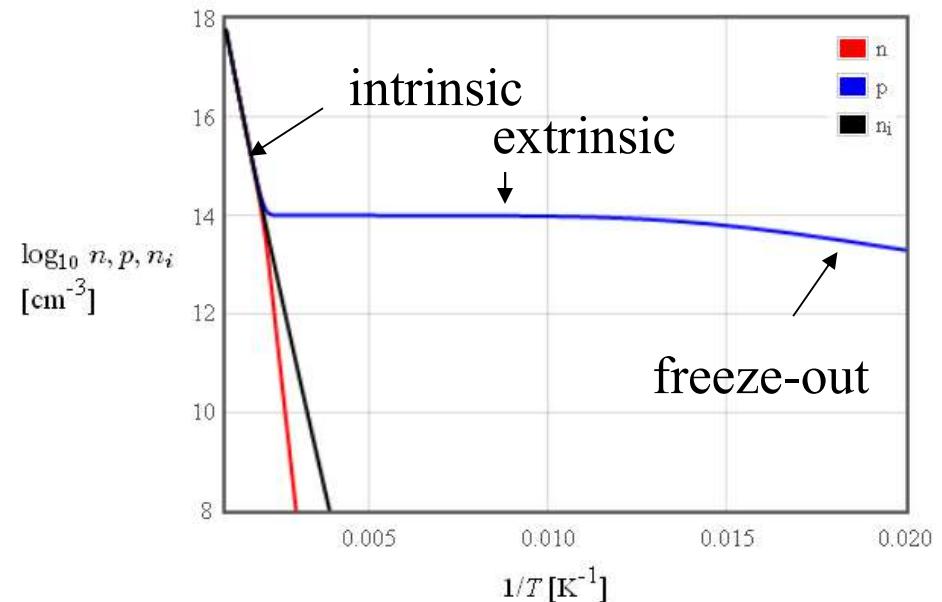
At high temperatures, extrinsic semiconductors have the same temperature dependence as intrinsic semiconductors.

Ionized donors and acceptors

For $E_v + 3k_BT < \mu < E_c - 3k_BT$ — Boltzmann approximation

$$N_D^+ = \frac{N_D}{1 + 2\exp\left(\dfrac{\mu - E_D}{k_BT}\right)}$$

$$N_A^- = \frac{N_A}{1 + 4\exp\left(\dfrac{E_A - \mu}{k_BT}\right)}$$

4 for materials with light
holes and heavy holes (Si)
2 otherwise

N_D = donor density cm^{-3} N_D^+ = ionized donor density cm^{-3}

N_A = donor density cm^{-3} N_A^- = ionized donor density cm^{-3}

Mostly, $N_D^+ = N_D$ and $N_A^- = N_A$

Charge neutrality

$$n + N_A^- = p + N_D^+$$

Carrier concentration vs. Fermi energy
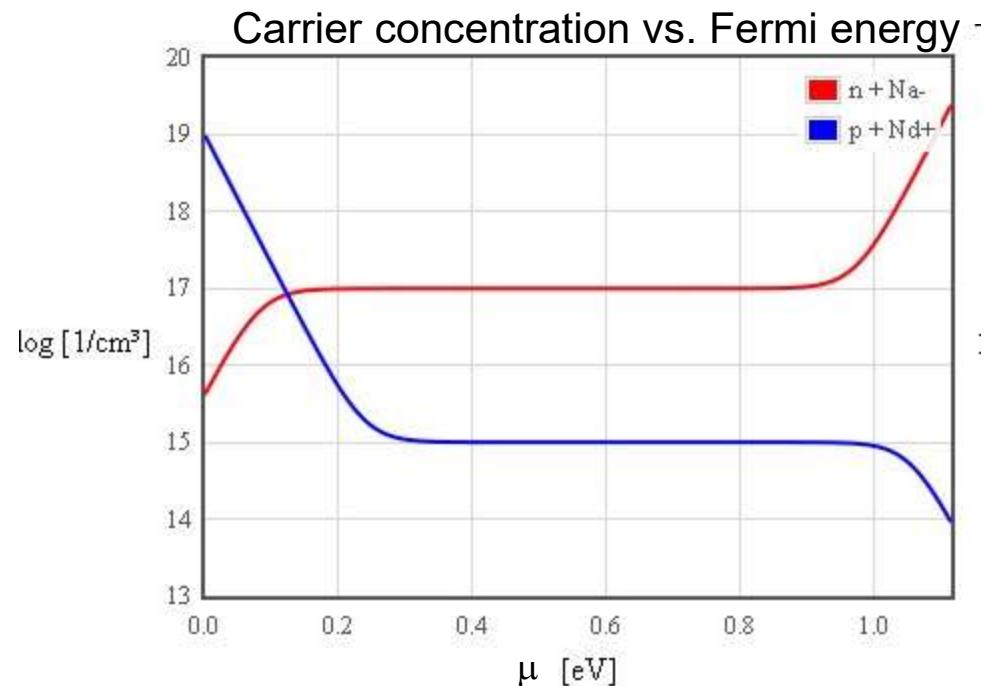

```
for ($i=0; $i<500; $i++) {
  $Ef = $i*$Eg/500;
  $n=$Nc*pow($T/300,1.5)*exp(1.6022E-19*($Ef-$Eg)/(1.38E-23*$T));
  $p=$Nv*pow($T/300,1.5)*exp(1.6022E-19*(-$Ef)/(1.38E-23*$T));
  $Namin = $Na/(1+4*exp(1.6022E-19*($Ea-$Ef)/(1.38E-23*$T)));
  $Ndplus = $Nd/(1+2*exp(1.6022E-19*($Ef-$Ed)/(1.38E-23*$T)));
}
```

E_f	n	p	N_d^+	N_a^-	$\log(n+N_a^-)$	$\log(p+N_d^+)$
0	4.16629283405	9.84E+18	1E+15	4.19743393218E+15	15.622983869	18.9930392318
0.00224	4.54358211887	9.0229075682E+18	1E+15	4.56020949614E+15	15.6589847946	18.9553946382
0.00448	4.95503779816	8.27366473417E+18	1E+15	4.95271809535E+15	15.694843609	18.9177504064
0.00672	5.40375389699	7.58663741327E+18	1E+15	5.37710747619E+15	15.7305487171	18.8801065693
0.00896	5.89210462781	6.95665926315E+18	1E+15	5.8256900025E+15	15.7660876057	18.8424631625

Fermi energy vs. temperature

Fermi energy of an extrinsic semiconductor is plotted as a function of temperature. At each temperature the Fermi energy was calculated by requiring that charge neutrality be satisfied.
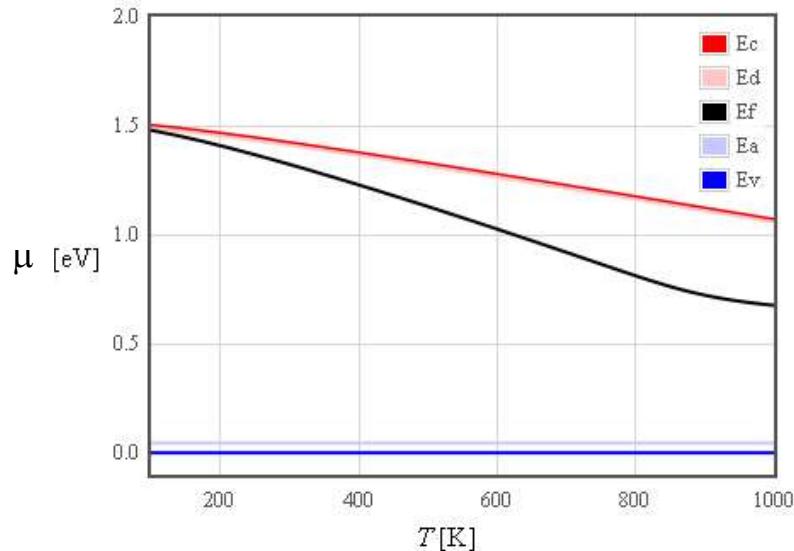
$N_c(300 \text{ K}) =$ 4.45E17	1/cm³	Semiconductor
$N_v(300 \text{ K}) =$ 7.72E18	1/cm³	Si Ge GaAs
$E_g =$ 1.519-5.41E-4*T*T/(T+204)	eV	
$N_d =$ 1E16	1/cm³	Donor
E_c - $E_d =$ 0.012	eV	P in Si P in Ge Si in GaAs
$N_a =$ 1E12	1/cm³	Acceptor
E_a - $E_v =$ 0.045	eV	B in Si B in Ge Si in GaAs
$T_1 =$ 100	K	
$T_2 =$ 1000	K	

Replot

Once the Fermi energy is known, the carrier densities n and p can be calculated from the formulas, $n = N_c \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{E_F - E_c}{k_B T}\right)$ and $p = N_v \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{E_v - E_F}{k_B T}\right)$.

The intrinsic carrier density is $n_i = \sqrt{N_c \left(\frac{T}{300}\right)^{3/2} N_v \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{-E_g}{2 k_B T}\right)}$.