

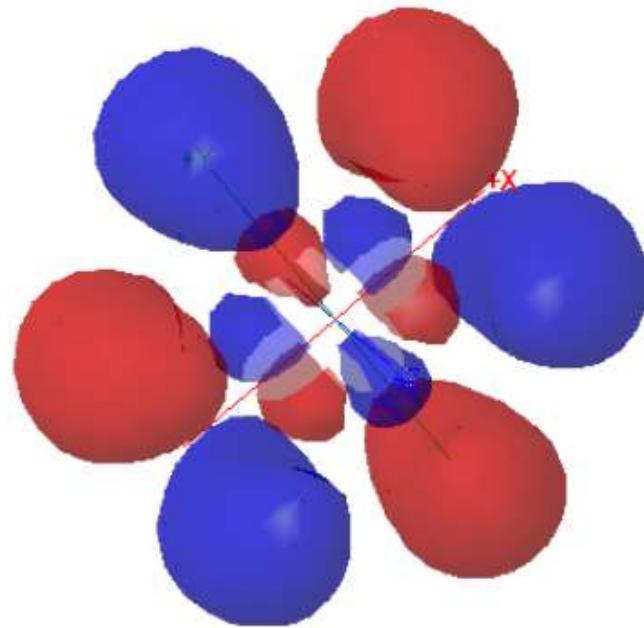
# Hydrogen atom, Atomic Orbitals

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# Atomic orbitals

<http://lampx.tugraz.at/~hadley/ss1/molecules/atoms/AOs.php>

5f



Atomic orbitals:

1s								
2s								
3s								
4s	3d xy	3d yz	3d xz	3d z <sup>2</sup>	3d x <sup>2</sup> -y <sup>2</sup>	4px	4py	4pz
5s	4d xy	4d yz	4d xz	4d z <sup>2</sup>	4d x <sup>2</sup> -y <sup>2</sup>	5px	5py	5pz
6s	5d xy	5d yz	5d xz	5d z <sup>2</sup>	5d x <sup>2</sup> -y <sup>2</sup>	6px	6py	6pz
4f	4f	4f	4f	4f	4f	4f	4f	4f
5f	5f	5f	5f	5f	5f	5f	5f	5f

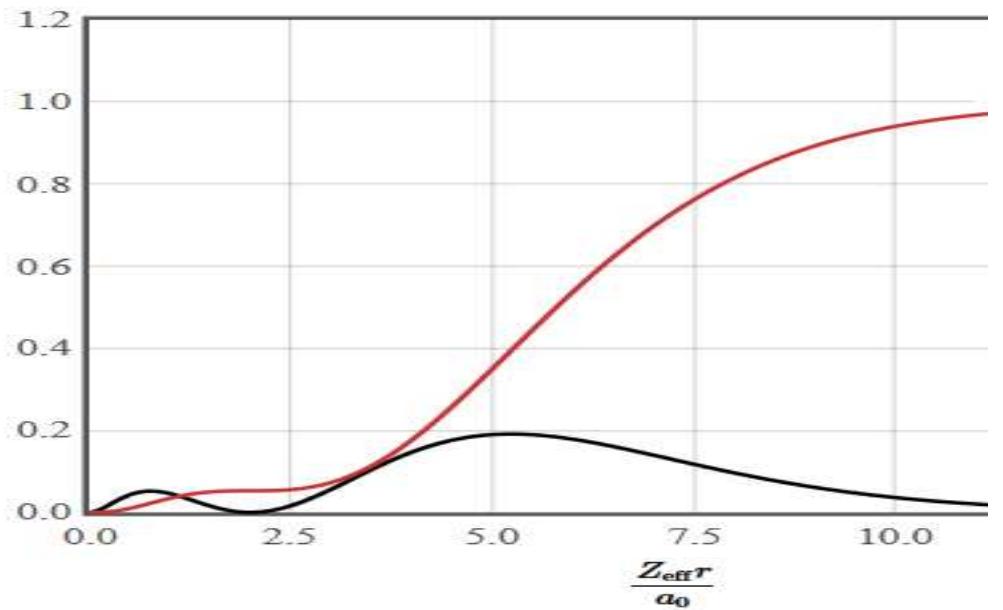
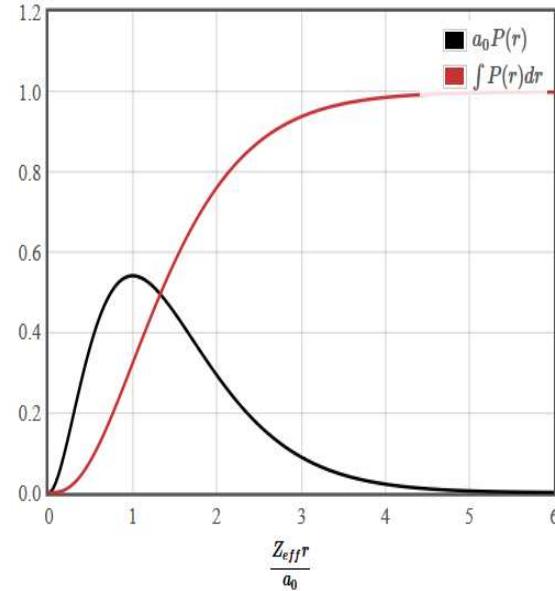
$$\langle \phi_m | H | \phi_n \rangle = \frac{-\hbar^2}{2m} \langle \phi_m | \nabla^2 | \phi_n \rangle - \frac{2e^2}{4\pi\epsilon_0} \langle \phi_m | \frac{1}{|\vec{r}|} | \phi_n \rangle$$

# Radial distribution function

$$P(r) = 4\pi r^2 |\psi|^2$$

$$\phi_{1s}^Z = \sqrt{\frac{Z^3}{\pi a_0^3}} \exp\left(-\frac{Zr}{a_0}\right)$$

$$\phi_{2s}^Z = \frac{1}{4} \sqrt{\frac{Z^3}{2\pi a_0^3}} \left(2 - \frac{Zr}{a_0}\right) \exp\left(-\frac{Zr}{2a_0}\right)$$



## Expectation energy

Often in molecular or solid state physics we know the Hamiltonian but we can't solve the Schrödinger equation associated with this Hamiltonian. In these cases we often guess a solution and then calculate the corresponding energy.

Consider the Hamiltonian for a hydrogen atom. In spherical coordinates it is,

$$H\psi = \frac{-\hbar^2}{2m} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \varphi^2} \right] - \frac{e^2}{4\pi\epsilon_0 r} \psi = E\psi.$$

Find the expectation value of the energy  $E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle}$  for the following wavefunction,

$$\psi = \exp\left(\frac{-r^2}{a^2}\right),$$

where  $a$  is a parameter. Note that this wavefunction is not an eigenfunction of the Hamiltonian. Determine the value of  $a$  that minimizes the energy. Compare  $a$  to the Bohr radius  $a_0 = 5.3 \times 10^{-11}$  m.

$$E = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \frac{\iiint \Psi^*(r, \theta, \varphi) H \Psi(r, \theta, \varphi) r^2 \sin \theta dr d\theta d\varphi}{\iiint \Psi^*(r, \theta, \varphi) \Psi(r, \theta, \varphi) r^2 \sin \theta dr d\theta d\varphi}$$

# Helium

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# Helium atom

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$$\frac{-\hbar^2}{2m} \left( \nabla_1^2 \Psi + \nabla_2^2 \Psi \right) - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} \Psi - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} \Psi + \frac{e^2}{4\pi\epsilon_0 |\vec{r}_1 - \vec{r}_2|} \Psi = E\Psi$$

$|\Psi(\vec{r}_1, \vec{r}_2)|^2$  is the probability to find one of the electrons at  $r_1$  and the other one at  $r_2$ .

# Helium atom

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neglect the electron-electron interaction term

$$H_{red}^{He} = \frac{-\hbar^2}{2m} (\nabla_1^2 \Psi + \nabla_2^2 \Psi) - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} \Psi - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} \Psi + \cancel{\frac{e^2}{4\pi\epsilon_0 |\vec{r}_1 - \vec{r}_2|}} \Psi = E\Psi$$

assume a product wave function

$$\Psi(\vec{r}_1, \vec{r}_2) = \phi_1(\vec{r}_1) \phi_2(\vec{r}_2)$$

# Separation of variables (Trennung der Veränderlichen)

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$$\frac{-\hbar^2}{2m} \nabla_1^2 \phi_1 \phi_2 - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} \phi_1 \phi_2 = \frac{\hbar^2}{2m} \nabla_2^2 \phi_1 \phi_2 + \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} \phi_1 \phi_2 + E \phi_1 \phi_2$$

divide by  $\phi_1 \phi_2$

$$\frac{-\hbar^2}{2m\phi_1} \nabla_1^2 \phi_1 - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} = C = \frac{\hbar^2}{2m\phi_2} \nabla_2^2 \phi_2 + \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} + E$$

$$\frac{-\hbar^2}{2m} \nabla_1^2 \phi_1 - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} \phi_1 = C \phi_1$$

$$\frac{-\hbar^2}{2m} \nabla_2^2 \phi_2 - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} \phi_2 = (E - C) \phi_2$$

# Reduced Hamiltonian

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$$H_{red}^{He} = \frac{-\hbar^2}{2m} \left( \nabla_1^2 \Psi + \nabla_2^2 \Psi \right) - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} \Psi - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} \Psi = E\Psi$$

any product of atomic orbitals with  $Z = 2$  solves the reduced Hamiltonian

$$\Psi(\vec{r}_1, \vec{r}_2) = \phi_{n1}(\vec{r}_1) \phi_{n2}(\vec{r}_2) \quad E = -\frac{13.6(2)^2}{n_1^2} - \frac{13.6(2)^2}{n_2^2}$$

symmetric:  $\Psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} (\phi_{n1}(\vec{r}_1) \phi_{n2}(\vec{r}_2) + \phi_{n1}(\vec{r}_2) \phi_{n2}(\vec{r}_1))$

antisymmetric:  $\Psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} (\phi_{n1}(\vec{r}_1) \phi_{n2}(\vec{r}_2) - \phi_{n1}(\vec{r}_2) \phi_{n2}(\vec{r}_1))$

# Indistinguishable particles

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$$|\Psi(\vec{r}_1, \vec{r}_2)|^2 = |\Psi(\vec{r}_2, \vec{r}_1)|^2$$

$$\Psi(\vec{r}_1, \vec{r}_2) = \pm \Psi(\vec{r}_2, \vec{r}_1)$$

bosons       $\Psi(\vec{r}_1, \vec{r}_2) = \Psi(\vec{r}_2, \vec{r}_1)$

integer spin:  
photons, phonons,  
 ${}^4\text{He}$

fermions       $\Psi(\vec{r}_1, \vec{r}_2) = -\Psi(\vec{r}_2, \vec{r}_1)$

half integer spin:  
electrons, neutrons,  
protons,  ${}^3\text{He}$

# Spin

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Spin appears naturally in the relativistic formulation of quantum mechanics but in the nonrelativistic formulation, we just add the spin states.

$\uparrow$  = spin up  
 $\downarrow$  = spin down

Spin orbitals:  $\phi_{1s} \uparrow, \phi_{1s} \downarrow, \phi_{2s} \uparrow, \phi_{2s} \downarrow, \dots$

# Slater determinants

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The antisymmetric solution can be written as a determinant,

$$\Psi_0^{\text{He}}(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_{1s}^{\text{He}} \uparrow (\vec{r}_1) & \phi_{1s}^{\text{He}} \downarrow (\vec{r}_1) \\ \phi_{1s}^{\text{He}} \uparrow (\vec{r}_2) & \phi_{1s}^{\text{He}} \downarrow (\vec{r}_2) \end{vmatrix} = \frac{1}{\sqrt{2}} \phi_{1s}^{\text{He}}(\vec{r}_1) \phi_{1s}^{\text{He}}(\vec{r}_2) (\uparrow(\vec{r}_1) \downarrow(\vec{r}_2) - \uparrow(\vec{r}_2) \downarrow(\vec{r}_1))$$

Exchanging two columns changes the sign of the determinant.

If two columns are the same, the determinant is zero = Pauli exclusion.

# Slater determinants

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The antisymmetric  $N$  electron wave function can be written,

$$\Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_{1s} \uparrow(\vec{r}_1) & \phi_{1s} \downarrow(\vec{r}_1) & \cdots & \phi_N \uparrow(\vec{r}_1) \\ \phi_{1s} \uparrow(\vec{r}_2) & \phi_{1s} \downarrow(\vec{r}_2) & \cdots & \phi_N \uparrow(\vec{r}_2) \\ \vdots & \vdots & & \vdots \\ \phi_{1s} \uparrow(\vec{r}_N) & \phi_{1s} \downarrow(\vec{r}_N) & \cdots & \phi_N \uparrow(\vec{r}_N) \end{vmatrix}.$$

Exchanging two columns changes the sign of the determinant.

If two columns are the same, the determinant is zero = Pauli exclusion.

Dirac notation:  $\Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) = |\phi_{1s} \uparrow, \phi_{1s} \downarrow, \dots, \phi_N \uparrow\rangle$

# Helium ground state

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$$H_{total} = \frac{-\hbar^2}{2m} \left( \nabla_1^2 + \nabla_2^2 \right) - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} + \frac{e^2}{4\pi\epsilon_0 |\vec{r}_1 - \vec{r}_2|}$$

Approximate antisymmetrized wave function (neglecting electron-electron interactions)

$$\Psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_{1s}^{He}(\vec{r}_1) \uparrow & \phi_{1s}^{He}(\vec{r}_1) \downarrow \\ \phi_{1s}^{He}(\vec{r}_2) \uparrow & \phi_{1s}^{He}(\vec{r}_2) \downarrow \end{vmatrix} = \frac{\phi_{1s}^{He}(\vec{r}_1)\phi_{1s}^{He}(\vec{r}_2)}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$

Energy neglecting  
e-e interactions  $\longrightarrow E = 2 \times \frac{-13.6Z^2}{n^2} = -108.8 \text{ eV}$

Approximate ground state  
evaluated with the total  $\longrightarrow E = \frac{\langle \Psi | H_{total} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = -74.83 \text{ eV}$   
Hamiltonian

# Matrix elements

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$$E_0^{\text{He}} \approx \frac{\langle \Psi_0^{\text{He}} | H_{\text{total}}^{\text{He}} | \Psi_0^{\text{He}} \rangle}{\langle \Psi_0^{\text{He}} | \Psi_0^{\text{He}} \rangle}$$

$$\frac{\iiint \iiint \Psi^*(x_1, y_1, z_1, x_2, y_2, z_2) H \Psi(x_1, y_1, z_1, x_2, y_2, z_2) dx_1 dy_1 dz_1 dx_2 dy_2 dz_2}{\iiint \iiint \Psi^*(x_1, y_1, z_1, x_2, y_2, z_2) \Psi(x_1, y_1, z_1, x_2, y_2, z_2) dx_1 dy_1 dz_1 dx_2 dy_2 dz_2}$$

# Helium ground state

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Try other wave functions in the full Hamiltonian

$$H_{total} = \frac{-\hbar^2}{2m} \left( \nabla_1^2 + \nabla_2^2 \right) - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_1|} - \frac{2e^2}{4\pi\epsilon_0 |\vec{r}_2|} + \frac{e^2}{4\pi\epsilon_0 |\vec{r}_1 - \vec{r}_2|}$$

$$\Psi(\vec{r}_1, \vec{r}_2) = \exp\left(\frac{-\alpha(r_1 + r_2)}{a_0}\right)$$

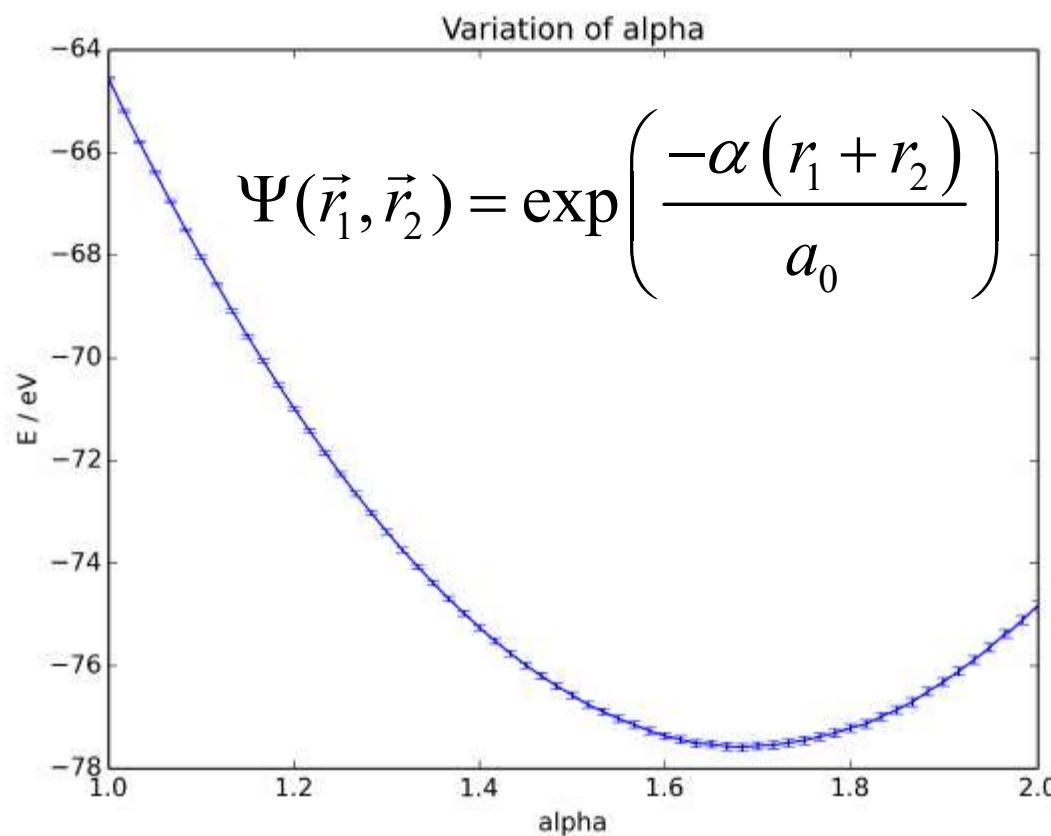
Electron screening makes the wave function larger

$$E = \frac{\langle \Psi | H_{total} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

## 4.1 Helium

The results of the total energy of the helium ground state for different effective nuclear charges  $\alpha$  are plotted in 1. The minimum (and therefore best estimate) lies at

$$\begin{aligned}\alpha &= (1.685 \pm 0.005) \\ E &= (-77.50 \pm 0.03) \text{ eV}\end{aligned}\tag{14}$$



# Helium ground state

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$e^{-2r_1/a_0} e^{-2r_2/a_0}$	-74.83 eV
$e^{-\alpha r_1/a_0} e^{-\alpha r_2/a_0}$	-77.4885 eV
$e^{-\alpha(r_1+r_2)/a_0} (1 + c  \vec{r}_1 - \vec{r}_2 )$	-78.6714 eV
1078 parameters	-79.0142 eV

The true wave function cannot be written as a product of two one-electron wave functions.

# Slater's rules

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**Effective Nuclear Charge  $Z_{\text{eff}}$**

	1s	2s,2p
H	1	
He	1.7	
Li	2.7	1.3
Be	3.7	1.95
B	4.7	2.6
C	5.7	3.25
N	6.7	3.9
O	7.7	4.55
Cl	8.7	5.2
Ne	9.7	5.85

Slater, J. C., Atomic Shielding Constants, Phys. Rev. 36, pp. 57–64, 1930. doi:10.1103/PhysRev.36.57

# Helium excited states

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One electron in 1s and one in 2s,  $\uparrow\uparrow, \downarrow\downarrow, \downarrow\uparrow$ , and  $\uparrow\downarrow$

$$\Psi_I = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_{1s}^{\text{He}} \uparrow(\vec{r}_1) & \phi_{2s}^{\text{He}} \uparrow(\vec{r}_1) \\ \phi_{1s}^{\text{He}} \uparrow(\vec{r}_2) & \phi_{2s}^{\text{He}} \uparrow(\vec{r}_2) \end{vmatrix} = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1)\phi_{2s}^{\text{He}}(\vec{r}_2) - \phi_{2s}^{\text{He}}(\vec{r}_1)\phi_{1s}^{\text{He}}(\vec{r}_2)) \uparrow\uparrow,$$

$$\Psi_{II} = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_{1s}^{\text{He}} \downarrow(\vec{r}_1) & \phi_{2s}^{\text{He}} \downarrow(\vec{r}_1) \\ \phi_{1s}^{\text{He}} \downarrow(\vec{r}_2) & \phi_{2s}^{\text{He}} \downarrow(\vec{r}_2) \end{vmatrix} = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1)\phi_{2s}^{\text{He}}(\vec{r}_2) - \phi_{2s}^{\text{He}}(\vec{r}_1)\phi_{1s}^{\text{He}}(\vec{r}_2)) \downarrow\downarrow,$$

$$\Psi_{III} = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_{1s}^{\text{He}} \uparrow(\vec{r}_1) & \phi_{2s}^{\text{He}} \downarrow(\vec{r}_1) \\ \phi_{1s}^{\text{He}} \uparrow(\vec{r}_2) & \phi_{2s}^{\text{He}} \downarrow(\vec{r}_2) \end{vmatrix} = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1) \uparrow \phi_{2s}^{\text{He}}(\vec{r}_2) \downarrow - \phi_{2s}^{\text{He}}(\vec{r}_1) \downarrow \phi_{1s}^{\text{He}}(\vec{r}_2) \uparrow),$$

$$\Psi_{IV} = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_{1s}^{\text{He}} \downarrow(\vec{r}_1) & \phi_{2s}^{\text{He}} \uparrow(\vec{r}_1) \\ \phi_{1s}^{\text{He}} \downarrow(\vec{r}_2) & \phi_{2s}^{\text{He}} \uparrow(\vec{r}_2) \end{vmatrix} = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1) \downarrow \phi_{2s}^{\text{He}}(\vec{r}_2) \uparrow - \phi_{2s}^{\text{He}}(\vec{r}_1) \uparrow \phi_{1s}^{\text{He}}(\vec{r}_2) \downarrow).$$

$$E = \frac{\langle \Psi | H_{red}^{He} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = -\frac{13.6 * 2^2}{1^2} - \frac{13.6 * 2^2}{2^2} = -68 \text{ eV}$$

The antisymmetric solution  $\Psi = 0$  for  $\vec{r}_1 = \vec{r}_2$ .

# Construct the Hamiltonian matrix

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Schrödinger equation

$$H_{total}^{He} (c_I \Psi_I + c_{II} \Psi_{II} + c_{III} \Psi_{III} + c_{IV} \Psi_{IV}) = E (c_I \Psi_I + c_{II} \Psi_{II} + c_{III} \Psi_{III} + c_{IV} \Psi_{IV})$$

Multiply from left by  $\Psi_i$ .

$$\begin{bmatrix} H_{I,I} & H_{I,II} & H_{I,III} & H_{I,IV} \\ H_{II,I} & H_{II,II} & H_{II,III} & H_{II,IV} \\ H_{III,I} & H_{III,II} & H_{III,III} & H_{III,IV} \\ H_{IV,I} & H_{IV,II} & H_{IV,III} & H_{IV,IV} \end{bmatrix} \begin{bmatrix} c_I \\ c_{II} \\ c_{III} \\ c_{IV} \end{bmatrix} = E \begin{bmatrix} c_I \\ c_{II} \\ c_{III} \\ c_{IV} \end{bmatrix}$$

$$H_{i,j} = \langle \Psi_i | H_{total}^{He} | \Psi_j \rangle$$

Student project: determine this matrix

# Transform to symmetric and antisymmetric orbital solutions

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$$\Psi_I = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1) \phi_{2s}^{\text{He}}(\vec{r}_2) - \phi_{2s}^{\text{He}}(\vec{r}_1) \phi_{1s}^{\text{He}}(\vec{r}_2)) \uparrow\uparrow,$$

$$\Psi_{II} = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1) \phi_{2s}^{\text{He}}(\vec{r}_2) - \phi_{2s}^{\text{He}}(\vec{r}_1) \phi_{1s}^{\text{He}}(\vec{r}_2)) \downarrow\downarrow,$$

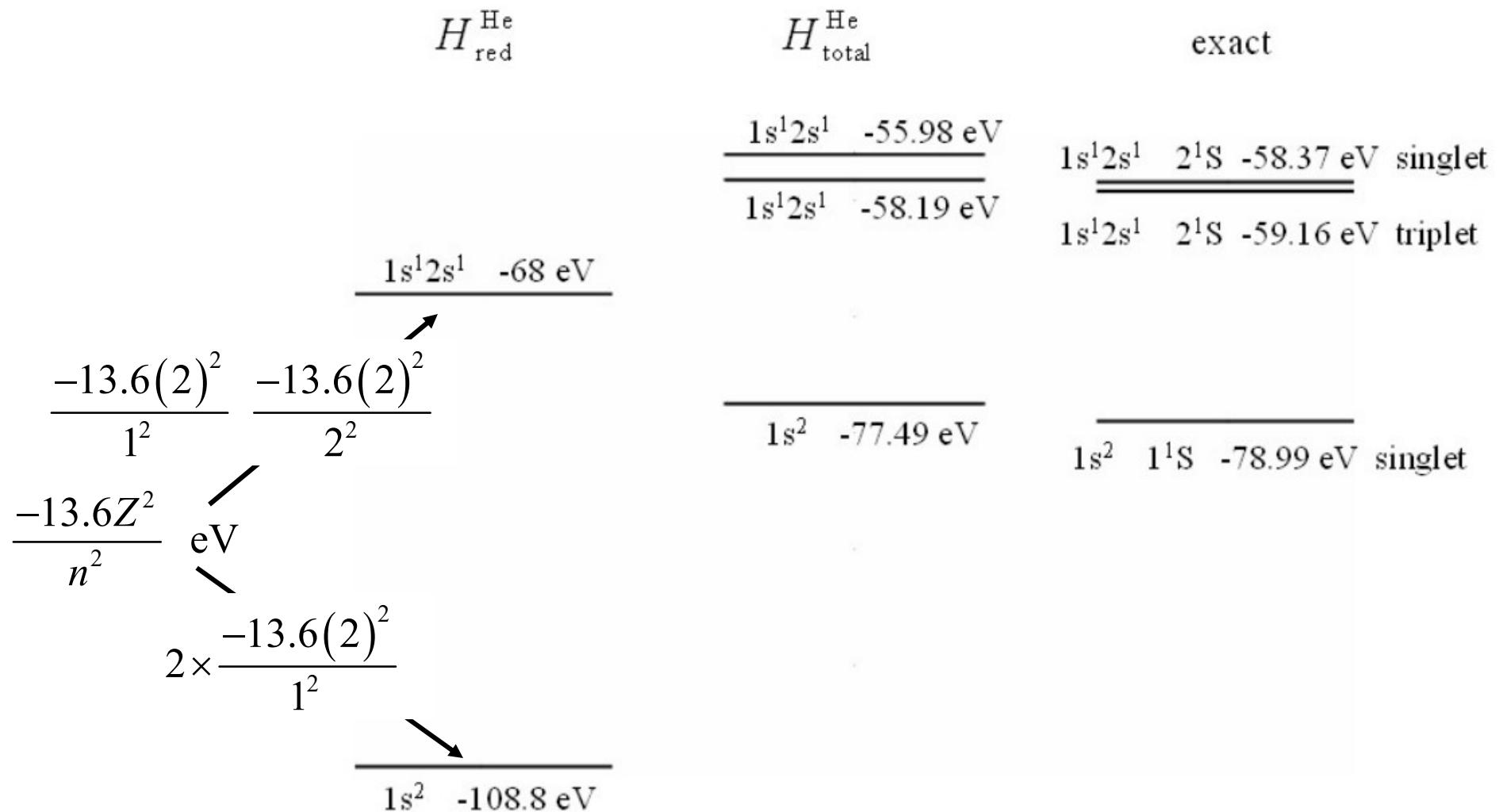
$$\Psi_{III} = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1) \uparrow \phi_{2s}^{\text{He}}(\vec{r}_2) \downarrow - \phi_{2s}^{\text{He}}(\vec{r}_1) \downarrow \phi_{1s}^{\text{He}}(\vec{r}_2) \uparrow),$$

$$\Psi_{IV} = \frac{1}{\sqrt{2}} (\phi_{1s}^{\text{He}}(\vec{r}_1) \downarrow \phi_{2s}^{\text{He}}(\vec{r}_2) \uparrow - \phi_{2s}^{\text{He}}(\vec{r}_1) \uparrow \phi_{1s}^{\text{He}}(\vec{r}_2) \downarrow).$$

$$\Psi_V = \frac{1}{\sqrt{2}} (\Psi_{III} + \Psi_{IV}) = \frac{1}{2} ((\phi_{1s}^{\text{He}}(\vec{r}_1) \phi_{2s}^{\text{He}}(\vec{r}_2) - \phi_{2s}^{\text{He}}(\vec{r}_1) \phi_{1s}^{\text{He}}(\vec{r}_2))(\uparrow\downarrow + \downarrow\uparrow),$$

$$\Psi_{VI} = \frac{1}{\sqrt{2}} (\Psi_{III} - \Psi_{IV}) = \frac{1}{2} ((\phi_{1s}^{\text{He}}(\vec{r}_1) \phi_{2s}^{\text{He}}(\vec{r}_2) + \phi_{2s}^{\text{He}}(\vec{r}_1) \phi_{1s}^{\text{He}}(\vec{r}_2))(\uparrow\downarrow - \downarrow\uparrow).$$

# Helium excited states



## Energy Levels of Neutral Helium (He I)

Configuration	Term	<i>J</i>	Level( $\text{cm}^{-1}$ )	Ref.
$1s^2$	$^1S$	0	0.000	M02
$1s2s$	$^3S$	1	159855.9745	M02
$1s2s$	$^1S$	0	166277.4403	M02
$1s2p$	$^3P^{\circ}$	2	169086.7666	M02
	$^3P^{\circ}$	1	169086.8430	M02
	$^3P^{\circ}$	0	169087.8309	M02
$1s2p$	$^1P^{\circ}$	1	171134.8970	M02
$1s3s$	$^3S$	1	183236.7918	M02
$1s3s$	$^1S$	0	184864.8294	M02
$1s3p$	$^3P^{\circ}$	2	185564.5620	M02
	$^3P^{\circ}$	1	185564.5840	M02
	$^3P^{\circ}$	0	185564.8547	M02
$1s3d$	$^3D$	3	186101.5463	M02
	$^3D$	2	186101.5488	M02
	$^3D$	1	186101.5930	M02
$1s3d$	$^1D$	2	186104.9668	M02
$1s3p$	$^1P^{\circ}$	1	186209.3651	M02
$1s4p$	$^1P^{\circ}$	1	191492.7120	M02
He II ( $^2S_{1/2}$ )	<i>Limit</i>		198310.6691	M02



### Basic Atomic Spectroscopic Data

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$^1H$																			$^2He$
$^3Li$	$^4Be$																		
$^{11}Na$	$^{12}Mg$																		
$^{19}K$	$^{20}Ca$	$^{21}Sc$	$^{22}Ti$	$^{23}V$	$^{24}Cr$	$^{25}Mn$	$^{26}Fe$	$^{27}Co$	$^{28}Ni$	$^{29}Cu$	$^{30}Zn$	$^{31}Ga$	$^{32}Ge$	$^{33}As$	$^{34}Se$	$^{35}Br$	$^{36}Kr$		
$^{37}Rb$	$^{38}Sr$	$^{39}Y$	$^{40}Zr$	$^{41}Nb$	$^{42}Mo$	$^{43}Tc$	$^{44}Ru$	$^{45}Rh$	$^{46}Pd$	$^{47}Ag$	$^{48}Cd$	$^{49}In$	$^{50}Sn$	$^{51}Sb$	$^{52}Te$	$^{53}I$	$^{54}Xe$		
$^{55}Cs$	$^{56}Ba$	*	$^{72}Hf$	$^{73}Ta$	$^{74}W$	$^{75}Re$	$^{76}Os$	$^{77}Ir$	$^{78}Pt$	$^{79}Au$	$^{80}Hg$	$^{81}Tl$	$^{82}Pb$	$^{83}Bi$	$^{84}Po$	$^{85}At$	$^{86}Rn$		
$^{87}Fr$	$^{88}Ra$	+																	
* Lanthanides																			
+ Actinides																			
$^{57}La$	$^{58}Ce$	$^{59}Pr$	$^{60}Nd$	$^{61}Pm$	$^{62}Sm$	$^{63}Eu$	$^{64}Gd$	$^{65}Tb$	$^{66}Dy$	$^{67}Ho$	$^{68}Er$	$^{69}Tm$	$^{70}Yb$	$^{71}Lu$					
$^{89}Ac$	$^{90}Th$	$^{91}Pa$	$^{92}U$	$^{93}Np$	$^{94}Pu$	$^{95}Am$	$^{96}Cm$	$^{97}Bk$	$^{98}Cf$	$^{99}Es$									

$$E = hf = hc/\lambda$$

Names refer to  
approximate  
solutions