

Electrons

Free electron Fermi gas

1 - d

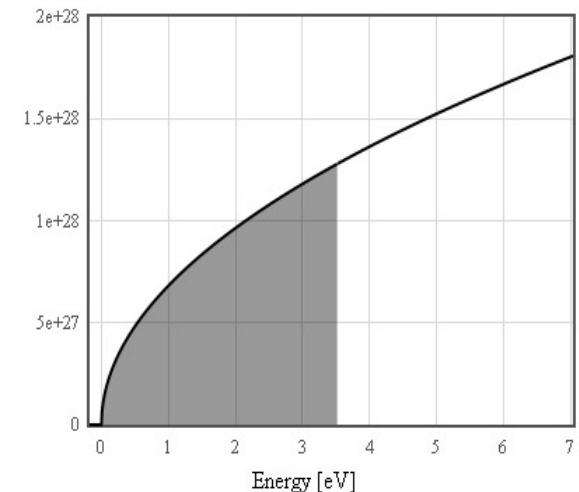
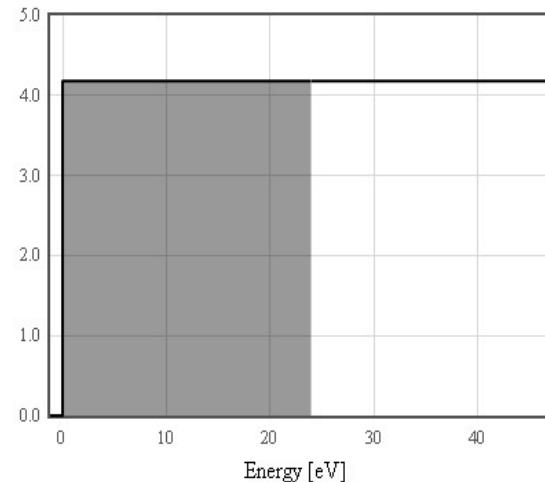
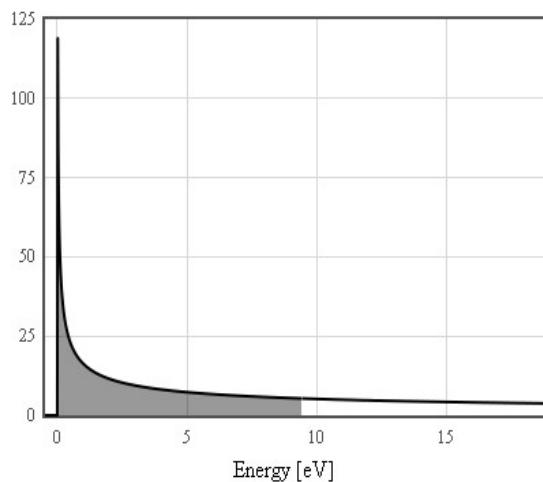
$$D(E) = \sqrt{\frac{2m}{\hbar^2 \pi^2 E}} = \frac{n}{2\sqrt{E_F E}} \quad \text{J}^{-1}\text{m}^{-1}$$

2 - d

$$D(E) = \frac{m}{\hbar^2 \pi} = \frac{n}{E_F} \quad \text{J}^{-1}\text{m}^{-2}$$

3 - d

$$D(E) = \frac{\pi}{2} \left(\frac{2m}{\hbar^2 \pi^2} \right)^{3/2} \sqrt{E} = \frac{3n}{2E_F^{3/2}} \sqrt{E} \quad \text{J}^{-1}\text{m}^{-3}$$



Internal energy density at $T = 0$

$$u = \int_{-\infty}^{\infty} ED(E)dE = \int_0^{E_f} ED(E)dE$$

$$D(E) = \frac{(2m)^{\frac{3}{2}}}{2\pi^2 \hbar^3} \sqrt{E} \quad \text{J}^{-1} \text{ m}^{-3}$$

$$u = \int_0^{E_f} \frac{(2m)^{\frac{3}{2}}}{2\pi^2 \hbar^3} E^{3/2} dE = \frac{(2m)^{\frac{3}{2}}}{5\pi^2 \hbar^3} E_f^{\frac{5}{2}}$$

$$E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \quad u(T=0) = \frac{3}{5} n E_F$$

$$u(T=0) = \frac{\pi^{\frac{4}{3}} \hbar^2}{10m} (3n)^{\frac{5}{3}} = \frac{\pi^{\frac{4}{3}} \hbar^2}{10m} \left(\frac{3N}{V} \right)^{\frac{5}{3}}$$

Pressure 3-D

$$P = - \left(\frac{\partial U}{\partial V} \right)_N$$

$$u(T=0) = \frac{\pi^{\frac{4}{3}} \hbar^2}{10m} (3n)^{\frac{5}{3}} = \frac{\pi^{\frac{4}{3}} \hbar^2}{10m} \left(\frac{3N}{V} \right)^{\frac{5}{3}}$$

$$U = Vu \propto V^{-2/3}$$

$$P = - \left(\frac{\partial U}{\partial V} \right)_N = \frac{2}{3} \frac{U}{V} = \frac{2}{5} n E_F = \frac{\hbar^2 (9\pi^4 n^5)^{\frac{1}{3}}}{5m}$$

Bulk modulus

$$B = -V \frac{\partial P}{\partial V}$$

$$P = - \left(\frac{\partial U}{\partial V} \right)_N = \frac{\hbar^2 \left(9\pi^4 N^5 / V^5 \right)^{1/3}}{5m}$$

$$P \propto V^{-5/3}$$

$$B = \frac{5}{3} P = \frac{10}{9} \frac{U}{V} = \frac{2}{3} n E_F = \frac{\hbar^2 \left(3\pi^4 n^5 \right)^{1/3}}{m} \quad \text{N/m}^2$$

See: Landau and Lifshitz, Statistical Physics 1
or Ashcroft and Mermin, Solid State Physics

Bulk modulus

Table 2.2

BULK MODULI IN 10^{10} DYNES/CM² FOR SOME TYPICAL METALS^a

METAL	FREE ELECTRON B	MEASURED B
Li	23.9	11.5
Na	9.23	6.42
K	3.19	2.81
Rb	2.28	1.92
Cs	1.54	1.43
Cu	63.8	134.3
Ag	34.5	99.9
Al	228	76.0

^a The free electron value is that for a free electron gas at the observed density of the metal, as calculated from Eq. (2.37).

Results of the quantization of the Schrödinger equation for free fermions in 1, 2, and 3 dimensions.

A simple model for metals is the free electron model where the potential energy of the electrons is zero and the electron-electron interactions are ignored. This is equivalent to any system of noninteracting fermions with zero potential energy. In this model the thermodynamic properties only depend on one parameter, the particle density n . In the table below, n denotes the number of particles per meter in one-dimension, the number of particles per square meter in two-dimensions, and the number of particles per cubic meter in three dimensions.

	1-D Schrödinger equation for a free particle $i\hbar \frac{d\psi}{dx} = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2}$	2-D Schrödinger equation for a free particle $i\hbar \frac{d\psi}{dx} = -\frac{\hbar^2}{2m} \left(\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} \right)$	3-D Schrödinger equation for a free particle $i\hbar \frac{d\psi}{dx} = -\frac{\hbar^2}{2m} \left(\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} \right)$
Eigenfunction solutions	$A_k \exp(i(kx - \alpha t))$	$A_{\vec{k}} \exp(i(\vec{k} \cdot \vec{r} - \alpha t))$	$A_{\vec{k}} \exp(i(\vec{k} \cdot \vec{r} - \alpha t))$
Dispersion relation	$E = \hbar\omega = \frac{\hbar^2 k^2}{2m} \text{ J}$	$E = \hbar\omega = \frac{\hbar^2 k^2}{2m} \text{ J}$	$E = \hbar\omega = \frac{\hbar^2 k^2}{2m} \text{ J}$
Density of states	$D(k) = \frac{2}{\pi}$	$D(k) = \frac{k}{\pi} \text{ m}^{-1}$	$D(k) = \frac{k^2}{\pi^2} \text{ m}^{-2}$
Density of states $D(E) = D(k) \frac{dk}{dE}$	$D(E) = \frac{1}{\pi\hbar} \sqrt{\frac{2m}{E}} = \frac{n}{2\sqrt{E_F E}} \text{ J}^1 \text{m}^{-1}$	$D(E) = \frac{m}{\pi\hbar^2} = \frac{n}{E_F} \text{ J}^1 \text{m}^{-2}$	$D(E) = \frac{(2m)^{\frac{3}{2}}}{2\pi^2\hbar^3} \sqrt{E} = \frac{3n}{2E_F^{3/2}} \sqrt{E} \text{ J}^1 \text{m}^{-3}$
Fermi energy E_F $n = \int_{-\infty}^{E_F} D(E) dE$	$E_F = \frac{\pi^2 \hbar^2 n^2}{8m} \text{ J}$	$E_F = \frac{\pi \hbar^2 n}{m} \text{ J}$	$E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{\frac{2}{3}} \text{ J}$
$D(E_F)$	$D(E_F) = \frac{4m}{\pi^2 \hbar^2 n} \text{ J}^1 \text{m}^{-1}$	$D(E_F) = \frac{m}{\pi\hbar^2} \text{ J}^1 \text{m}^{-2}$	$D(E_F) = \frac{m(3n)^{\frac{1}{3}}}{\pi^{\frac{2}{3}}\hbar^2} \text{ J}^1 \text{m}^{-3}$
$D'(E_F) = \frac{dD}{dE} \Big _{E=E_F}$	$D'(E_F) = \frac{-16m^2}{\pi^4 \hbar^4 n^3} \text{ J}^2 \text{m}^{-1}$	$D'(E_F) = 0 \text{ J}^2 \text{m}^{-2}$	$D'(E_F) = \frac{m^2}{\hbar^4 \sqrt[3]{3\pi^8 n}} \text{ J}^2 \text{m}^{-3}$
Chemical potential μ $n = \int_{-\infty}^{\mu} D(E) f(E) dE$	$\mu \approx E_F - \frac{\pi^2}{6} (k_B T)^2 \frac{D'(E_F)}{D(E_F)} \text{ J}$ $\approx \frac{\pi^2 \hbar^2 n^2}{8m} + \frac{2m}{3\hbar^2 n^2} (k_B T)^2 \text{ J}$	$\mu = k_B T \ln \left(\exp \left(\frac{E_F}{k_B T} \right) - 1 \right) \text{ J}$ $= k_B T \ln \left(\exp \left(\frac{\pi \hbar^2 n}{mk_B T} \right) - 1 \right) \text{ J}$	$\mu \approx E_F - \frac{\pi^2}{6} (k_B T)^2 \frac{D'(E_F)}{D(E_F)} \text{ J}$ $\approx \frac{\hbar^2}{2m} (3\pi^2 n)^{\frac{2}{3}} - \frac{\pi^{\frac{2}{3}} m}{\zeta + 2 \zeta^{\frac{1}{2}}} (k_B T)^2 \text{ J}$



Sommerfeld Expansion

Arnold
Sommerfeld

We would like to perform integrals of the form

$$\int_{-\infty}^{\infty} H(E) f(E) dE$$

Examples:

$$n = \int_{-\infty}^{\infty} D(E) f(E) dE \quad u = \int_{-\infty}^{\infty} E D(E) f(E) dE$$

Integrate by parts (Partielle Integration)

$$\int_{-\infty}^{\infty} \frac{dK(E)}{dE} f(E) dE = K(\infty) f(\infty) - K(-\infty) f(-\infty) - \int_{-\infty}^{\infty} K(E) \frac{df(E)}{dE} dE$$

$$K(E) = \int_{-\infty}^E H(E') dE' \quad H(E) = \frac{dK(E)}{dE}$$

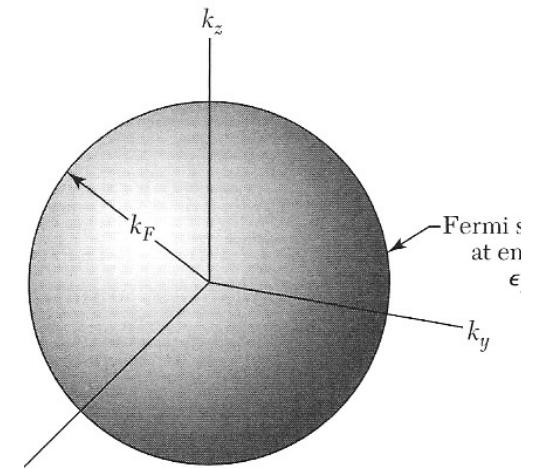
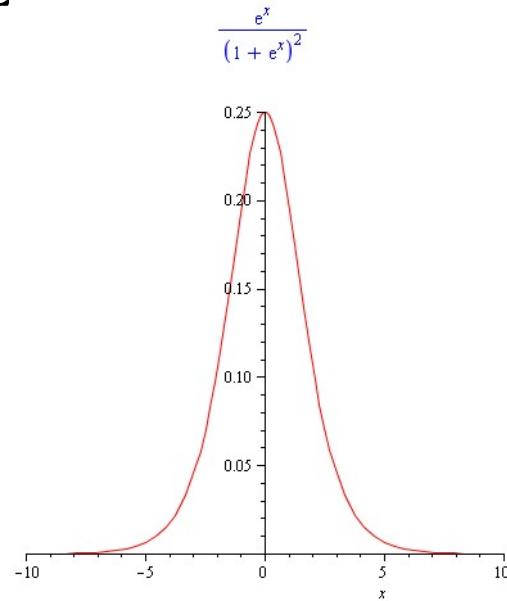
Sommerfeld Expansion

$$\int_{-\infty}^{\infty} H(E) f(E) dE = - \int_{-\infty}^{\infty} K(E) \frac{df(E)}{dE} dE$$

$$\frac{-df(E)}{dE} = \frac{\frac{1}{k_B T} \exp\left(\frac{E-\mu}{k_B T}\right)}{\left(1 + \exp\left(\frac{E-\mu}{k_B T}\right)\right)^2}$$

Expand $K(E)$ around $E = \mu$

$$K(E) \approx K(\mu) + \frac{dK}{dE} \Bigg|_{E=\mu} (E - \mu) + \frac{1}{2} \frac{d^2 K}{dE^2} \Bigg|_{E=\mu} (E - \mu)^2 + \dots$$



Sommerfeld Expansion

$$x = \frac{E - \mu}{k_B T}$$

$$\int_{-\infty}^{\infty} \frac{e^x}{(1 + e^x)^2} dx = 1$$

$$\int_{-\infty}^{\infty} \frac{x e^x}{(1 + e^x)^2} dx = 0$$

$$\int_{-\infty}^{\infty} \frac{x^2 e^x}{(1 + e^x)^2} dx = \frac{\pi^2}{3}$$

$$\int_{-\infty}^{\infty} \frac{x^3 e^x}{(1 + e^x)^2} dx = 0$$

$$\int_{-\infty}^{\infty} \frac{x^4 e^x}{(1 + e^x)^2} dx = \frac{7\pi^4}{15}$$

Sommerfeld Expansion

$$\int_{-\infty}^{\infty} H(E)f(E)dE \approx K(\mu) + \frac{\pi^2}{6} (k_B T)^2 \left. \frac{dH(E)}{dE} \right|_{E=\mu} + \frac{7\pi^4}{360} (k_B T)^4 \left. \frac{d^3H(E)}{dE^3} \right|_{E=\mu} + \dots$$

$$K(\mu) = \int_{-\infty}^{\mu} H(E)dE \quad H(E) = \frac{dK(E)}{dE}$$

Sommerfeld Expansion: chemical potential

$$\int_{-\infty}^{\infty} H(E) f(E) dE = K(\mu) + \frac{\pi^2}{6} (k_B T)^2 \left. \frac{dH(E)}{dE} \right|_{E=\mu} + \frac{7\pi^4}{360} (k_B T)^4 \left. \frac{d^3 H(E)}{dE^3} \right|_{E=\mu} + \dots$$

For μ in 3-d:

$$n = \int_{-\infty}^{\infty} D(E) f(E) dE$$

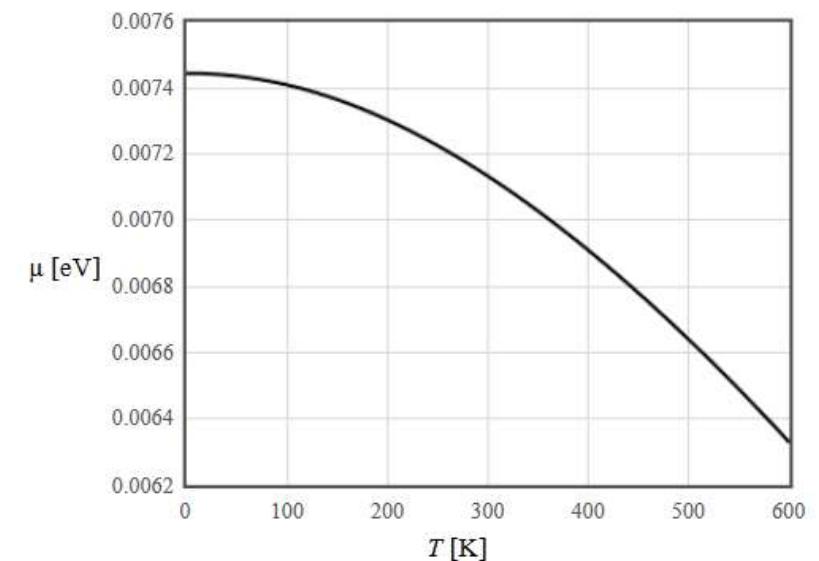
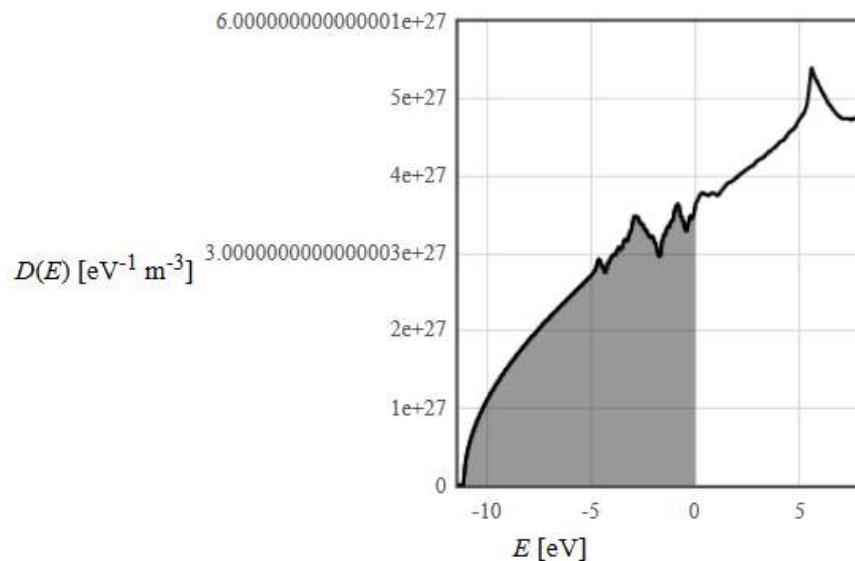
$$n = \int_{-\infty}^{\mu} D(E) dE + \frac{\pi^2}{6} (k_B T)^2 D'(E_F) + \dots$$


$$K(\mu) = \int_{-\infty}^{\mu} D(E) dE = \int_{-\infty}^{E_F} D(E) dE + \int_{E_F}^{\mu} D(E) dE \approx n + (\mu - E_F) D(E_F)$$

Sommerfeld Expansion: chemical potential

$$n = n + (\mu - E_F) D(E_F) + \frac{\pi^2}{6} (k_B T)^2 D'(E_F)$$

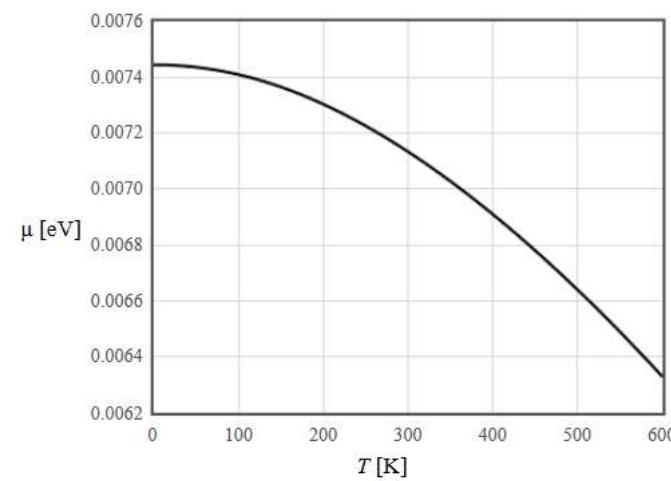
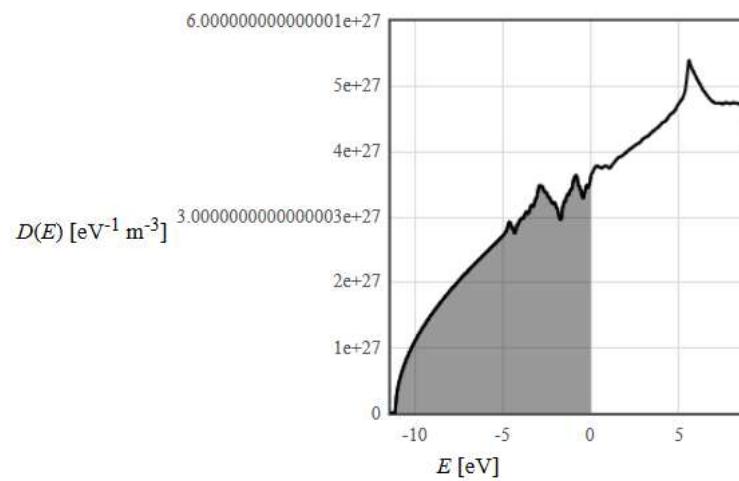
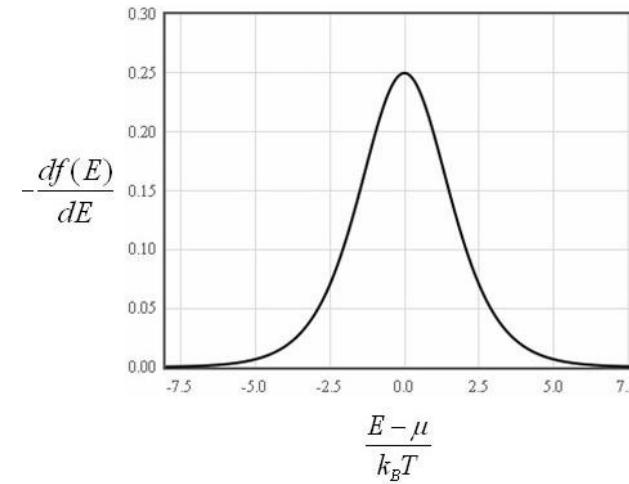
$$\mu \approx E_F - \frac{\pi^2}{6} (k_B T)^2 \frac{D'(E_F)}{D(E_F)}$$



Aluminum

Sommerfeld Expansion: chemical potential

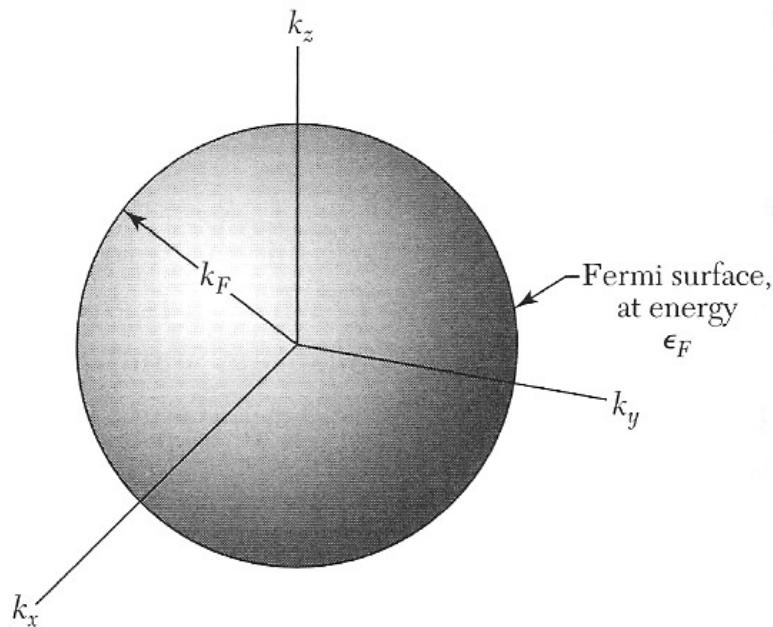
$$n = - \int_{-\infty}^{\infty} K(E) \frac{df(E)}{dE} dE = \int_{-\infty}^{\infty} \frac{K(E) \exp\left(\frac{E-\mu}{k_B T}\right) dE}{k_B T \left(\exp\left(\frac{E-\mu}{k_B T}\right) + 1 \right)^2}.$$



<http://lampx.tugraz.at/~hadley/ss1/materials/thermo/dos2mu.html>

Internal energy

$$u = \int_{-\infty}^{\infty} ED(E)f(E)dE$$



Sommerfeld Expansion: internal energy

$$\int_{-\infty}^{\infty} H(E)f(E)dE = K(\mu) + \frac{\pi^2}{6} (k_B T)^2 \left. \frac{dH(E)}{dE} \right|_{E=\mu} + \frac{7\pi^4}{360} (k_B T)^4 \left. \frac{d^3 H(E)}{dE^3} \right|_{E=\mu} + \dots$$

$$u = \int_{-\infty}^{\infty} ED(E)f(E)dE$$

$$H(E) = ED(E)$$

$$K(\mu) = \int_{-\infty}^{\mu} ED(E)dE$$

$$\left. \frac{dH}{dE} \right|_{E=\mu} = D(\mu) + \mu \frac{dD(\mu)}{dE} \approx D(E_F) + E_F \frac{dD(E_F)}{dE}$$

$$u = \int_{-\infty}^{\mu} ED(E)dE + \frac{\pi^2}{6} (k_B T)^2 (D(E_F) + E_F D'(E_F)) + \dots$$

Sommerfeld Expansion: internal energy

$$u = \int_{-\infty}^{\mu} ED(E)dE + \frac{\pi^2}{6} (k_B T)^2 (D(E_F) + E_F D'(E_F))$$

$$u = \int_{-\infty}^{E_F} ED(E)dE + (\mu - E_F) E_F D(E_F) + \frac{\pi^2}{6} (k_B T)^2 (D(E_F) + E_F D'(E_F))$$

$$\mu \approx E_F - \frac{\pi^2}{6} (k_B T)^2 \frac{D'(E_F)}{D(E_F)}$$

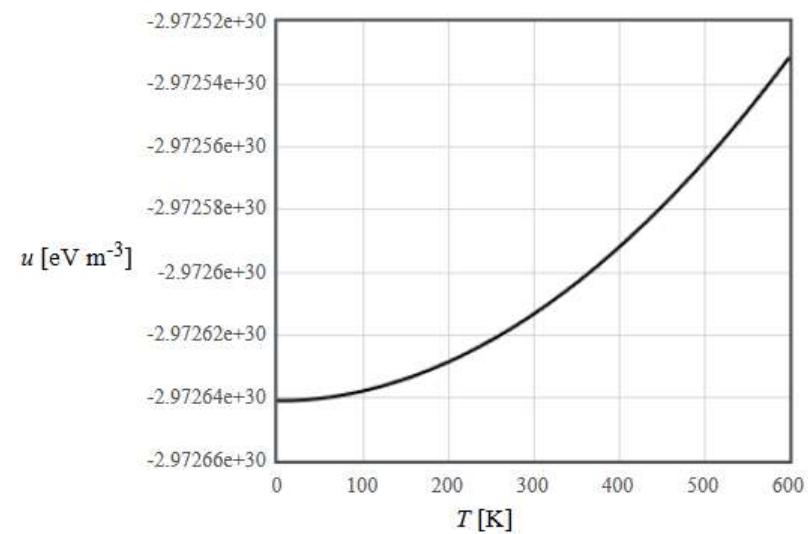
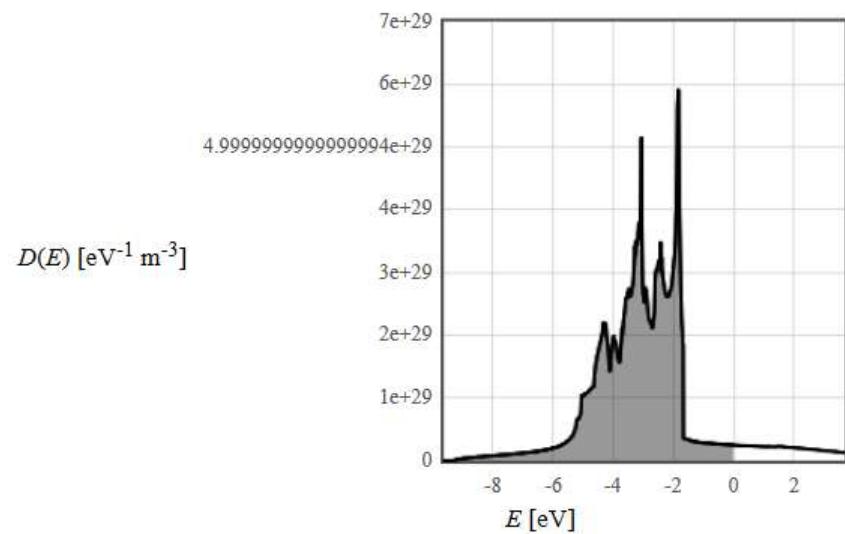
$$u = \int_{-\infty}^{E_F} ED(E)dE + (E_F - \frac{\pi^2}{6} (k_B T)^2 \frac{D'(E_F)}{D(E_F)} - E_F) E_F D(E_F) + \frac{\pi^2}{6} (k_B T)^2 (D(E_F) + E_F D'(E_F))$$

$$u \approx \int_{-\infty}^{E_F} ED(E)dE + \frac{\pi^2}{6} (k_B T)^2 D(E_F)$$

Free electrons: $u \approx \frac{3}{5} n E_F + \frac{\pi^2 D(E_F)}{6} (k_B T)^2 = \frac{\hbar^2}{10m} (\pi^4 3^5 n^5)^{\frac{1}{3}} + \frac{m (3\pi^2 n)^{\frac{1}{3}}}{6\hbar^2} (k_B T)^2 \quad \text{J m}^{-3}$

Sommerfeld Expansion: internal energy

$$u \approx \int_{-\infty}^{E_F} ED(E)dE + \frac{\pi^2}{6} (k_B T)^2 D(E_F)$$

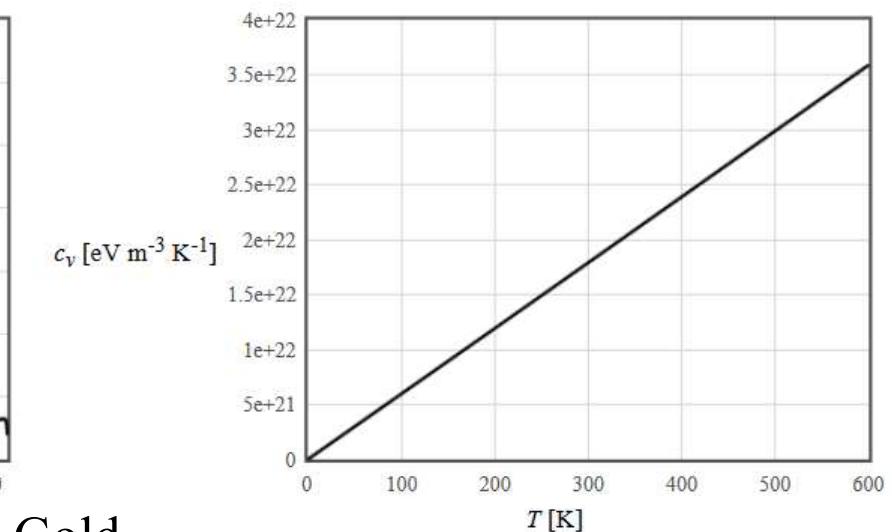
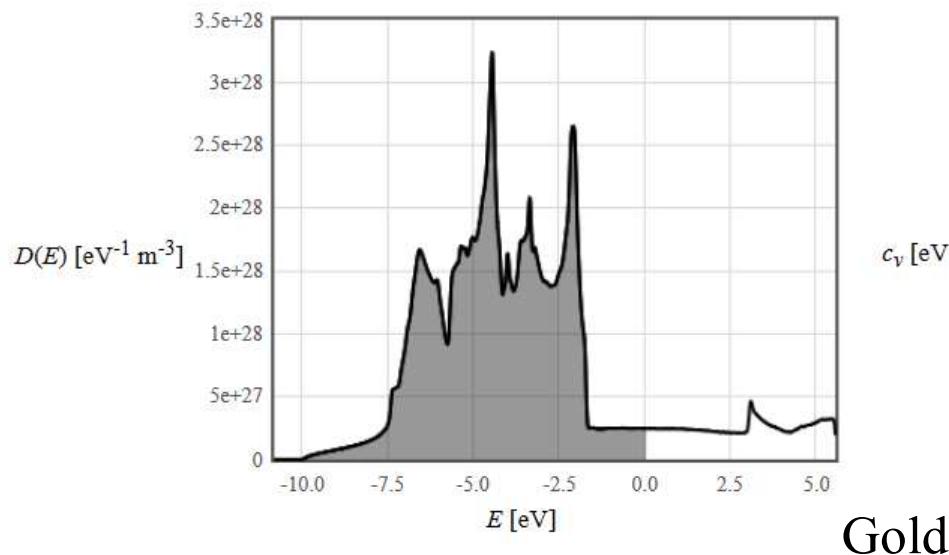


Copper

Sommerfeld Expansion: specific heat

$$c_v = \frac{du}{dT} \approx \frac{d}{dT} \left(u(T=0) + \frac{\pi^2}{6} (k_B T)^2 D(E_F) \right) = \frac{\pi^2 D(E_F)}{3} k_B^2 T \quad \text{J K}^{-1} \text{m}^{-3}$$

free electrons: $\approx \left(\frac{\pi}{3} \right)^{\frac{2}{3}} \frac{mn^{\frac{1}{3}}}{\hbar^2} k_B^2 T \quad \text{J K}^{-1} \text{m}^{-3}$



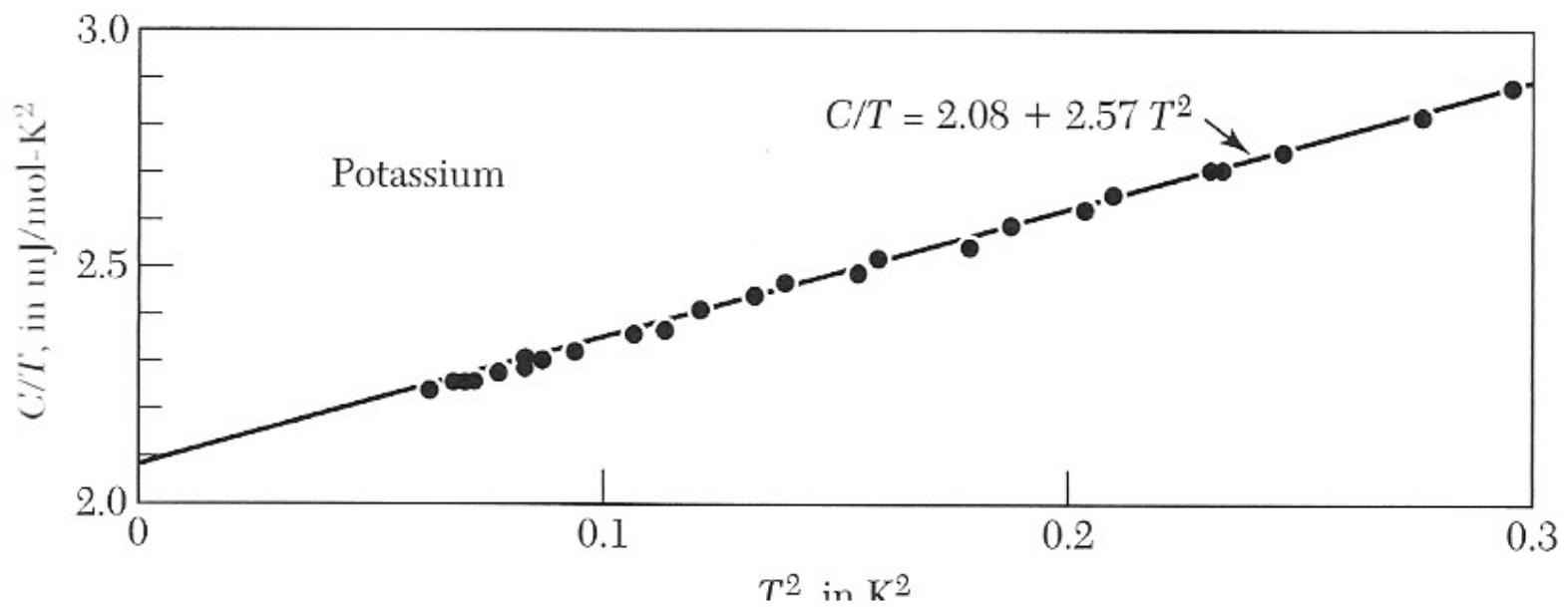
The electronic specific heat is linear in temperature

Electronic specific heat

$$c_{v,electrons} = \frac{du}{dT} \approx \left(\frac{\pi}{3} \right)^{\frac{2}{3}} \frac{mn^{\frac{1}{3}}}{\hbar^2} k_B^2 T \quad \text{J K}^{-1} \text{m}^{-3}$$

$$c_{v,total} = \gamma T + AT^3$$

electrons phonons



Effective mass

$$c_{v,electrons} = \frac{du}{dT} \approx \left(\frac{\pi}{3} \right)^{\frac{2}{3}} \frac{mn^{\frac{1}{3}}}{\hbar^2} k_B T \quad \text{J K}^{-1} \text{m}^{-3}$$

$$C_v = \gamma T + AT^3$$

$$\frac{m^*}{m} = \frac{\gamma_{observed}}{\gamma} = \frac{D(E_F)_{observed}}{D(E_F)_{free electron}}$$

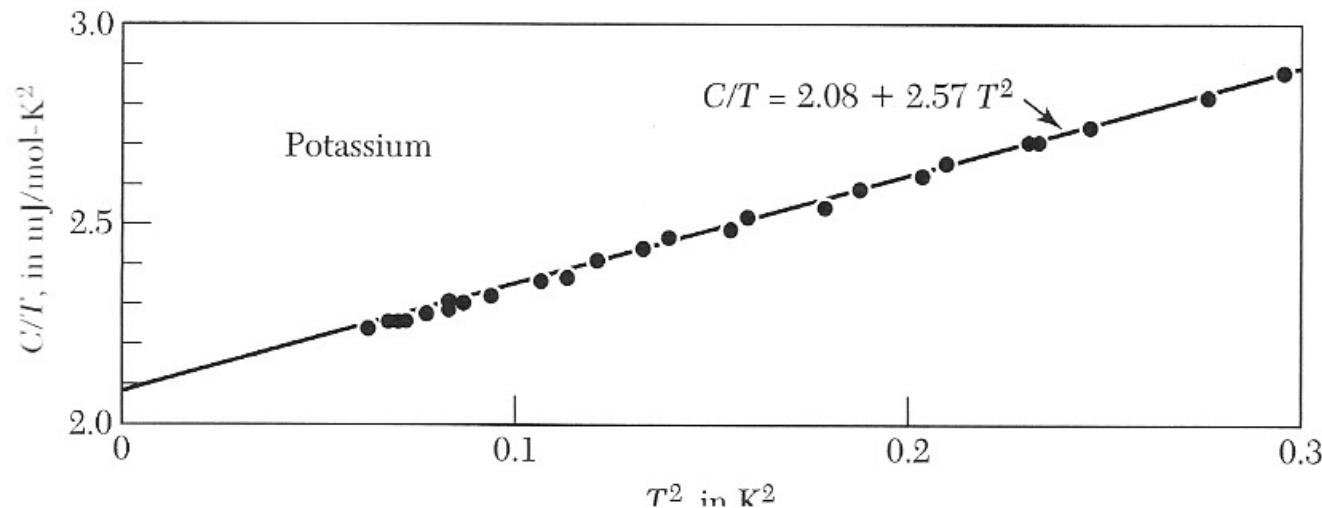


Table 2 Experimental and free electron values of electronic heat capacity constant γ of metals

(From compilations kindly furnished by N. Phillips and N. Pearlman. The thermal effective mass is defined by Eq. (38).)

Li	Be	B	C	N										
1.63	0.17													
0.749	0.500													
2.18	0.34													
Na	Mg	Al	Si	P										
1.38	1.3	1.35												
1.094	0.992	0.912												
1.26	1.3	1.48												
Observed γ in $\text{mJ mol}^{-1} \text{K}^{-2}$.														
Calculated free electron γ in $\text{mJ mol}^{-1} \text{K}^{-2}$														
$m_{\text{th}}/m = (\text{observed } \gamma)/(\text{free electron } \gamma)$.														
K	Ca	Sc	Ti	V	Cr	Mn(γ)	Fe	Co	Ni	Cu	Zn	Ga	Ge	As
2.08	2.9	10.7	3.35	9.26	1.40	9.20	4.98	4.73	7.02	0.695	0.64	0.596		0.19
1.668	1.511									0.505	0.753	1.025		
1.25	1.9									1.38	0.85	0.58		
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn (w)	Sb
2.41	3.6	10.2	2.80	7.79	2.0	—	3.3	4.9	9.42	0.646	0.688	1.69	1.78	0.11
1.911	1.790									0.645	0.948	1.233	1.410	
1.26	2.0									1.00	0.73	1.37	1.26	
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg(α)	Tl	Pb	Bi
3.20	2.7	10.	2.16	5.9	1.3	2.3	2.4	3.1	6.8	0.729	1.79	1.47	2.98	
2.238	1.937									0.642	0.952	1.29	1.509	
1.43	1.4									1.14	1.88	1.14	1.97	

from Kittel

Heavy Fermions

$$\frac{m^*}{m} = \frac{\gamma_{observed}}{\gamma}$$

Heavy fermions are materials that have effective masses 100 - 1000 times larger than the value expected from the free-electron theory. Examples are CeCu₆, the UBe₁₃, and CeAl₃. The last two are superconductors.

Something goes seriously wrong with the free electron model in these materials.

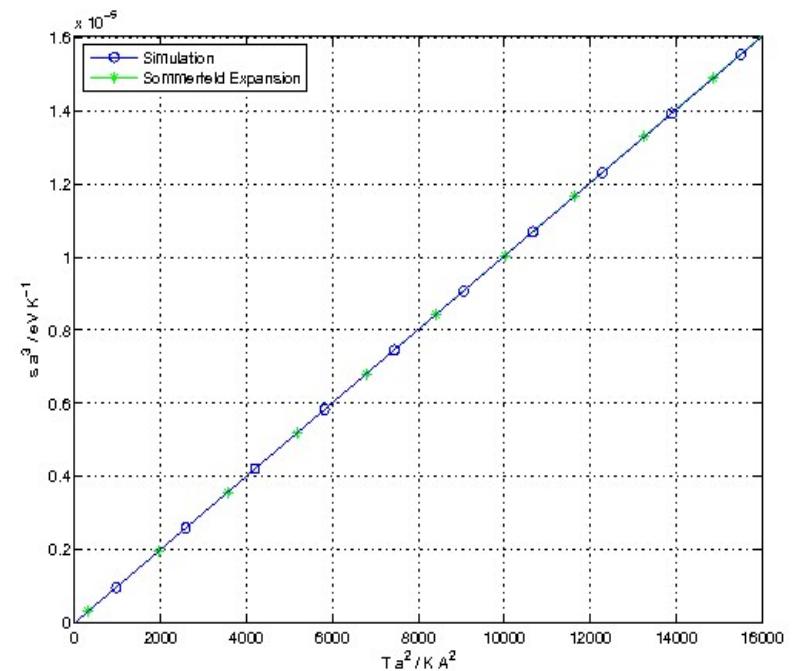
Entropy

$$c_v = \frac{du}{dT} \approx \frac{\pi^2 D(E_F)}{3} k_B^2 T \quad \text{J K}^{-1} \text{m}^{-3}$$

$$\frac{c_v}{T} = \left. \frac{\partial s}{\partial T} \right|_{N,V} \approx \frac{\pi^2 D(E_F)}{3} k_B^2 \quad \text{J K}^{-1} \text{m}^{-3}$$

free electrons: $s \approx \left(\frac{\pi}{3} \right)^{\frac{2}{3}} \frac{mn^{\frac{1}{3}}}{\hbar^2} k_B^2 T \quad \text{J K}^{-1} \text{m}^{-3}$

Entropy density



Helmholtz free energy

$$s \approx \frac{\pi^2 D(E_F)}{3} k_B^2 T \quad \text{J K}^{-1} \text{m}^{-3}$$

$$\text{free electrons: } \approx \left(\frac{\pi}{3} \right)^{\frac{2}{3}} \frac{mn^{\frac{1}{3}}}{\hbar^2} k_B^2 T \quad \text{J K}^{-1} \text{m}^{-3}$$

Helmholtz free energy density

$$f = u - Ts \approx \int_{-\infty}^{E_F} ED(E) dE - \frac{\pi^2 D(E_F)}{6} (k_B T)^2 \quad \text{J m}^{-3}$$

$$\text{free electrons: } f \approx \frac{\hbar^2}{10m} \left(\pi^4 3^5 n^5 \right)^{\frac{1}{3}} - \frac{m (3\pi^2 n)^{\frac{1}{3}}}{6\hbar^2} (k_B T)^2 \quad \text{J m}^{-3}$$

Results of the quantization of the Schrödinger equation for free fermions in 1, 2, and 3 dimensions.

A simple model for metals is the free electron model where the potential energy of the electrons is zero and the electron-electron interactions are ignored. This is equivalent to any system of noninteracting fermions with zero potential energy. In this model the thermodynamic properties only depend on one parameter, the particle density n . In the table below, n denotes the number of particles per meter in one-dimension, the number of particles per square meter in two-dimensions, and the number of particles per cubic meter in three dimensions.

	1-D Schrödinger equation for a free particle $i\hbar \frac{d\psi}{dx} = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2}$	2-D Schrödinger equation for a free particle $i\hbar \frac{d\psi}{dx} = -\frac{\hbar^2}{2m} \left(\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} \right)$	3-D Schrödinger equation for a free particle $i\hbar \frac{d\psi}{dx} = -\frac{\hbar^2}{2m} \left(\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} \right)$
Eigenfunction solutions	$A_k \exp(i(kx - \alpha t))$	$A_{\vec{k}} \exp(i(\vec{k} \cdot \vec{r} - \alpha t))$	$A_{\vec{k}} \exp(i(\vec{k} \cdot \vec{r} - \alpha t))$
Dispersion relation	$E = \hbar\omega = \frac{\hbar^2 k^2}{2m} \text{ J}$	$E = \hbar\omega = \frac{\hbar^2 k^2}{2m} \text{ J}$	$E = \hbar\omega = \frac{\hbar^2 k^2}{2m} \text{ J}$
Density of states	$D(k) = \frac{2}{\pi}$	$D(k) = \frac{k}{\pi} \text{ m}^{-1}$	$D(k) = \frac{k^2}{\pi^2} \text{ m}^{-2}$
Density of states $D(E) = D(k) \frac{dk}{dE}$	$D(E) = \frac{1}{\pi\hbar} \sqrt{\frac{2m}{E}} = \frac{n}{2\sqrt{E_F E}} \text{ J}^1 \text{m}^{-1}$	$D(E) = \frac{m}{\pi\hbar^2} = \frac{n}{E_F} \text{ J}^1 \text{m}^{-2}$	$D(E) = \frac{(2m)^{\frac{3}{2}}}{2\pi^2\hbar^3} \sqrt{E} = \frac{3n}{2E_F^{3/2}} \sqrt{E} \text{ J}^1 \text{m}^{-3}$
Fermi energy E_F $n = \int_{-\infty}^{E_F} D(E) dE$	$E_F = \frac{\pi^2 \hbar^2 n^2}{8m} \text{ J}$	$E_F = \frac{\pi \hbar^2 n}{m} \text{ J}$	$E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{\frac{2}{3}} \text{ J}$
$D(E_F)$	$D(E_F) = \frac{4m}{\pi^2 \hbar^2 n} \text{ J}^1 \text{m}^{-1}$	$D(E_F) = \frac{m}{\pi\hbar^2} \text{ J}^1 \text{m}^{-2}$	$D(E_F) = \frac{m(3n)^{\frac{1}{3}}}{\pi^{\frac{2}{3}}\hbar^2} \text{ J}^1 \text{m}^{-3}$
$D'(E_F) = \frac{dD}{dE} \Big _{E=E_F}$	$D'(E_F) = \frac{-16m^2}{\pi^4 \hbar^4 n^3} \text{ J}^2 \text{m}^{-1}$	$D'(E_F) = 0 \text{ J}^2 \text{m}^{-2}$	$D'(E_F) = \frac{m^2}{\hbar^4 \sqrt[3]{3\pi^8 n}} \text{ J}^2 \text{m}^{-3}$
Chemical potential μ $n = \int_{-\infty}^{\mu} D(E) f(E) dE$	$\mu \approx E_F - \frac{\pi^2}{6} (k_B T)^2 \frac{D'(E_F)}{D(E_F)} \text{ J}$ $\approx \frac{\pi^2 \hbar^2 n^2}{8m} + \frac{2m}{3\hbar^2 n^2} (k_B T)^2 \text{ J}$	$\mu = k_B T \ln \left(\exp \left(\frac{E_F}{k_B T} \right) - 1 \right) \text{ J}$ $= k_B T \ln \left(\exp \left(\frac{\pi \hbar^2 n}{mk_B T} \right) - 1 \right) \text{ J}$	$\mu \approx E_F - \frac{\pi^2}{6} (k_B T)^2 \frac{D'(E_F)}{D(E_F)} \text{ J}$ $\approx \frac{\hbar^2}{2m} (3\pi^2 n)^{\frac{2}{3}} - \frac{\pi^{\frac{2}{3}} m}{\zeta + 2 \zeta^{\frac{1}{2}}} (k_B T)^2 \text{ J}$

Band Theory, Kittel chapter 7

Calculate the dispersion relation for electrons in a crystal

