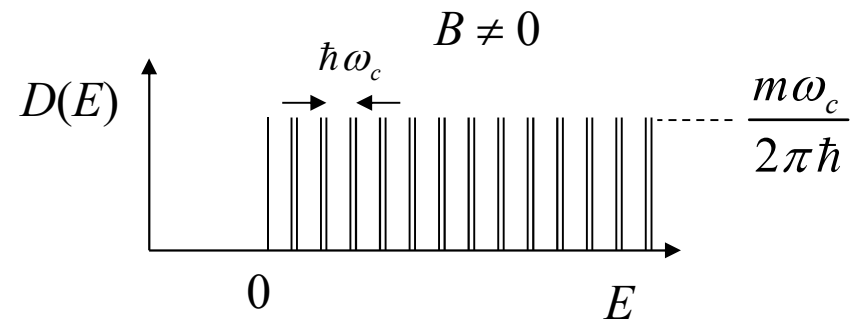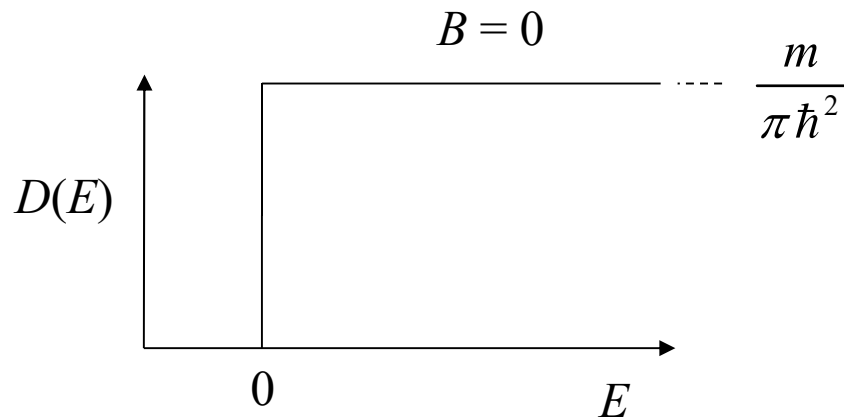# Spin

In a magnetic field, there is a shift of the energy of the electrons because of their spin.

$$E = -\vec{\mu} \cdot \vec{B} = \pm \frac{g}{2} \mu_B B$$
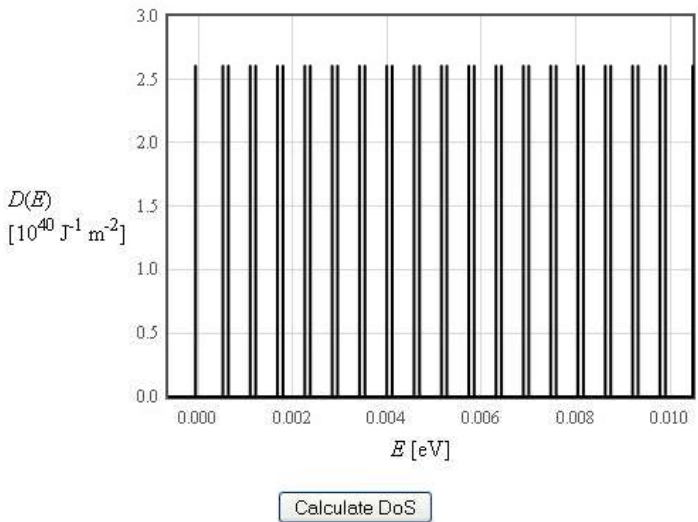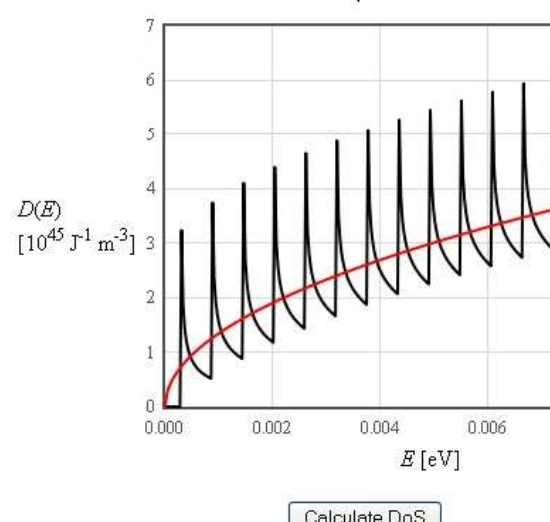
Bohr magneton $\quad \mu_B = \dfrac{e\hbar}{2m_e} \qquad$ g-factor $\quad g \approx 2 \qquad\qquad \hbar\omega_c = \dfrac{\hbar e B}{m} = 2\mu_B B$
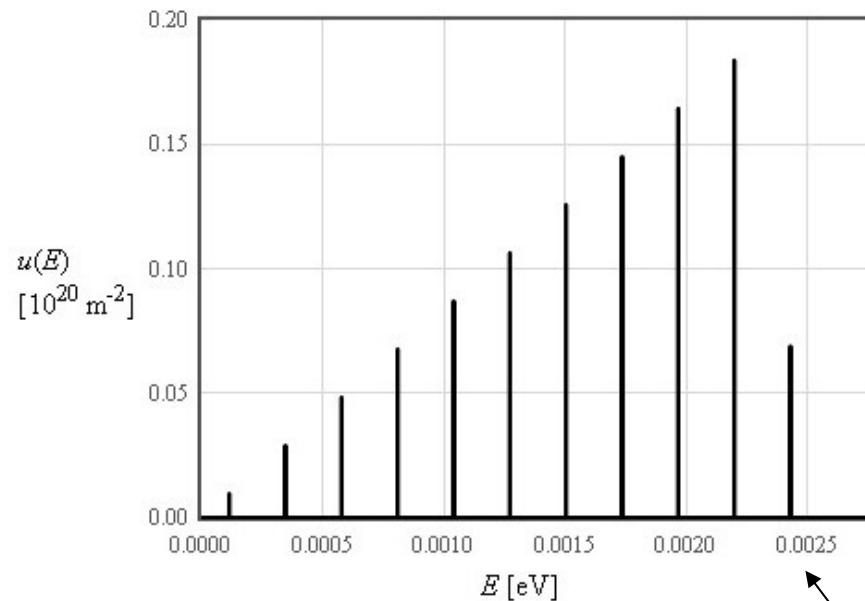

$$D(E) = \frac{m\omega_c}{2\pi\hbar} \sum_{v=0}^{\infty} \delta\left(E - \hbar\omega_c\left(v + \tfrac{1}{2} - \tfrac{g}{4}\right)\right) + \delta\left(E - \hbar\omega_c\left(v + \tfrac{1}{2} + \tfrac{g}{4}\right)\right)$$

ization of the Schrödinger equation for free electrons a magnetic field in 2 and 3 dimensions.

	2-D Schrödinger equation $$i\hbar\frac{d\psi}{dt} = \frac{1}{2m}\left(-i\hbar\nabla - |e|\vec{A}\right)^2\psi$$	3-D Schrödinger equation $$i\hbar\frac{d\psi}{dt} = \frac{1}{2m}\left(-i\hbar\nabla - |e|\vec{A}\right)^2\psi$$
Eigenfunction solutions	$\psi = g_\nu(x)\exp\left(ik_y y\right)$ $g_\nu(x)$ is a harmonic oscillator wavefunction	$\psi = g_\nu(x)\exp\left(ik_y y\right)\exp\left(ik_z z\right)$ $g_\nu(x)$ is a harmonic oscillator wavefun
Energy eigenvalues	$E = \hbar\omega_c\left(\nu+\tfrac{1}{2}\right)$ J $\nu = 0,1,2,\cdots \qquad \omega_c = \dfrac{|eB_z|}{m}$	$E = \dfrac{\hbar^2 k_z^2}{2m} + \hbar\omega_c\left(\nu+\tfrac{1}{2}\right)$ J $\nu = 0,1,2,\cdots \qquad \omega_c = \dfrac{|eB_z|}{m}$
Density of states	$D(E) = \dfrac{m\omega_c}{2\pi\hbar}\sum_{\nu=0}^{\infty}\delta\left(E - \hbar\omega_c\left(\nu+\tfrac{1}{2}\right) - \dfrac{g\mu_B}{2}B\right) + \delta\left(E - \hbar\omega_c\left(\nu+\tfrac{1}{2}\right) + \dfrac{g\mu_B}{2}B\right)$ J^{-1}m^{-2} $D(E)$ $[10^{40}$ J^{-1} m$^{-2}]$ Calculate DoS	$D(E) = \dfrac{(2m)^{3/2}\omega_c}{4\pi^2\hbar^2}\sum_{\nu=0}^{\infty}\dfrac{H\left(E - \hbar\omega_c\left(\nu+\tfrac{1}{2}\right)\right)}{\sqrt{E - \hbar\omega_c\left(\nu+\tfrac{1}{2}\right)}}$ $D(E)$ $[10^{45}$ J^{-1} m$^{-3}]$ Calculate DoS
	$E_F = \hbar\omega_c\left(\text{Int}\left(\dfrac{\pi\hbar n}{m\omega_c}\right) + \dfrac{1}{2}\right)$	

Energy spectral density 2d

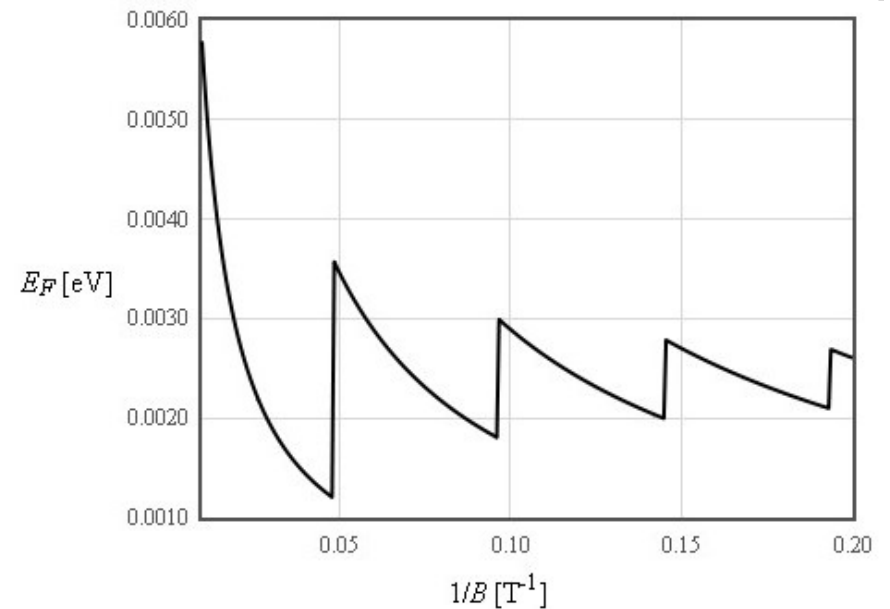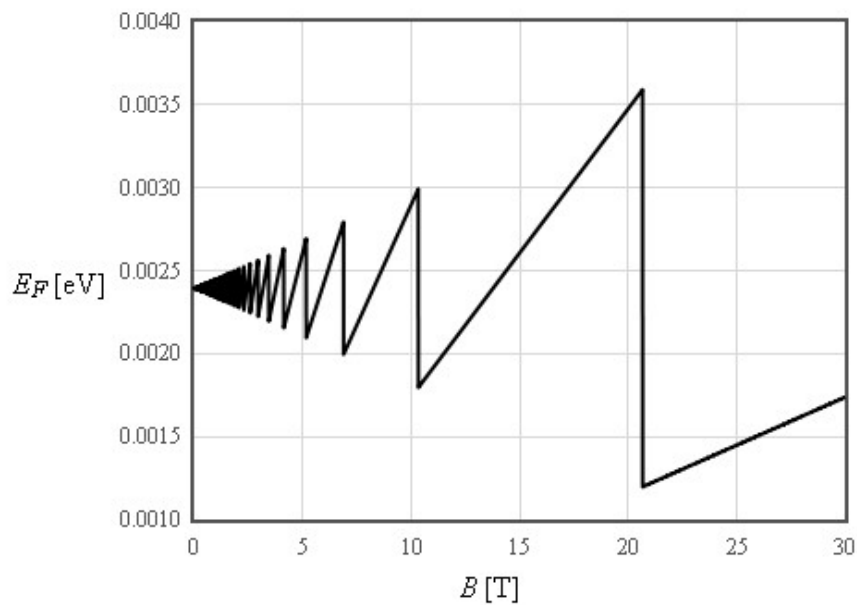At $T = 0$

$u(E)$
$[10^{20} \, m^{-2}]$

unfilled Landau level

analog to the Planck radiation law

Fermi energy 2d

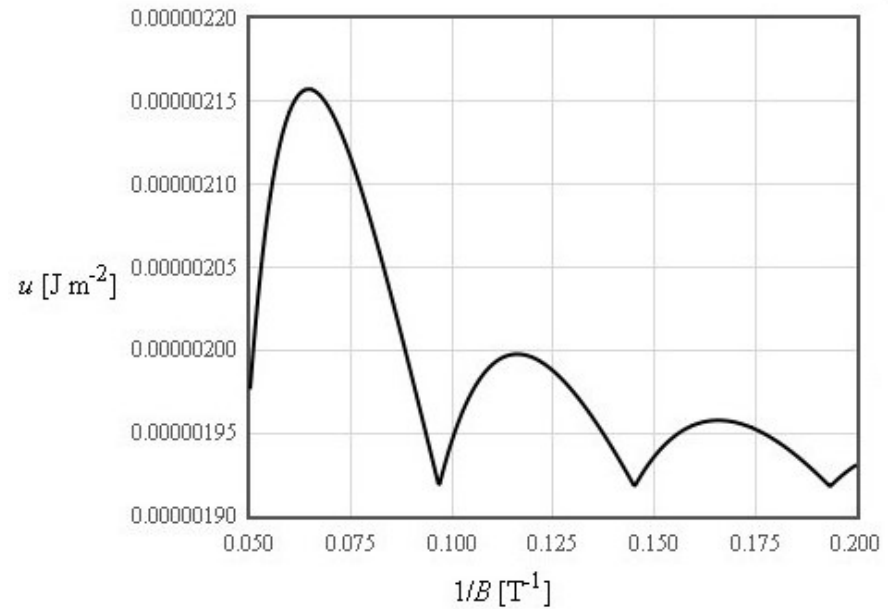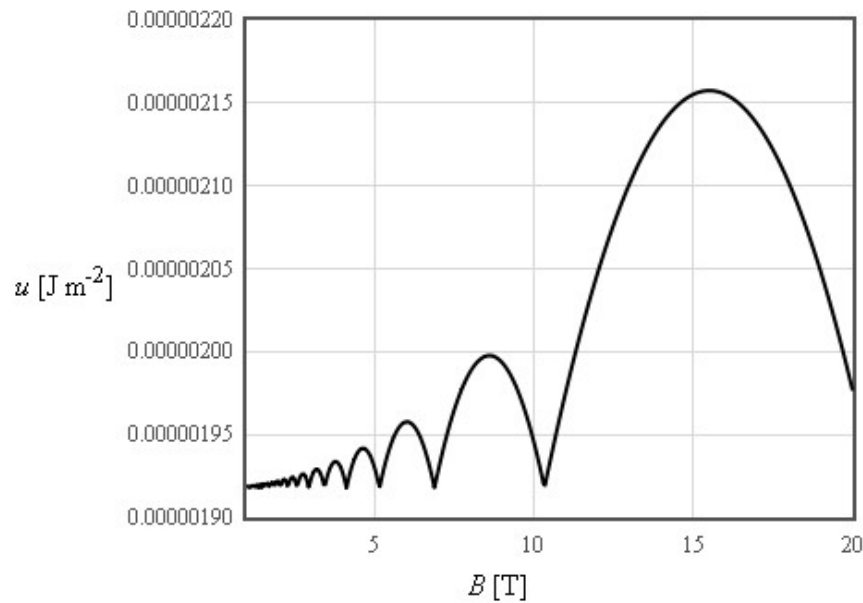$$n = \int_{-\infty}^{E_F} D(E)dE$$

When there is only one Landau level, the Fermi energy rises linearly with field.

Periodic in $1/B$

Large field limit $\longrightarrow$ $E_F = \dfrac{\hbar\omega_c}{2} = \dfrac{\hbar eB}{2m}$

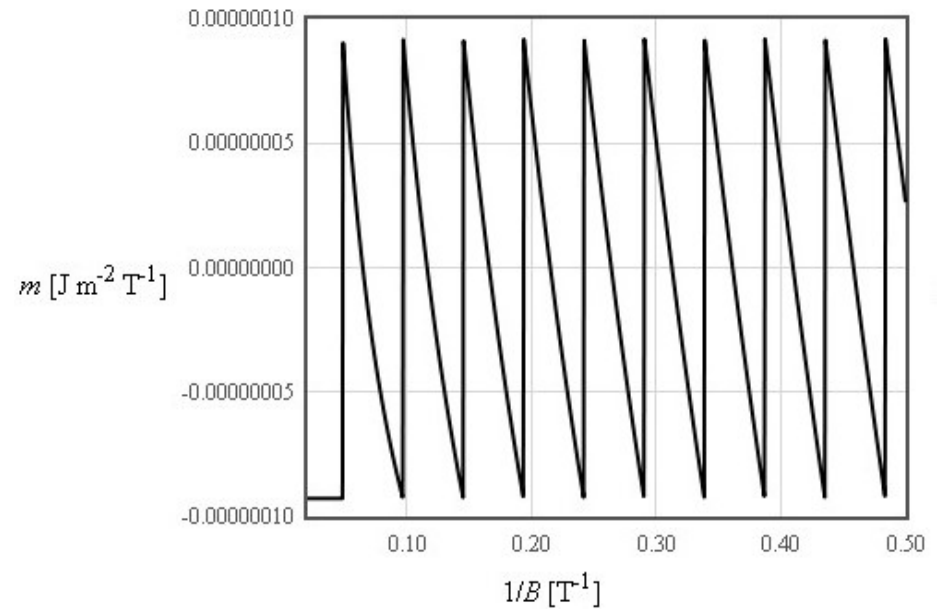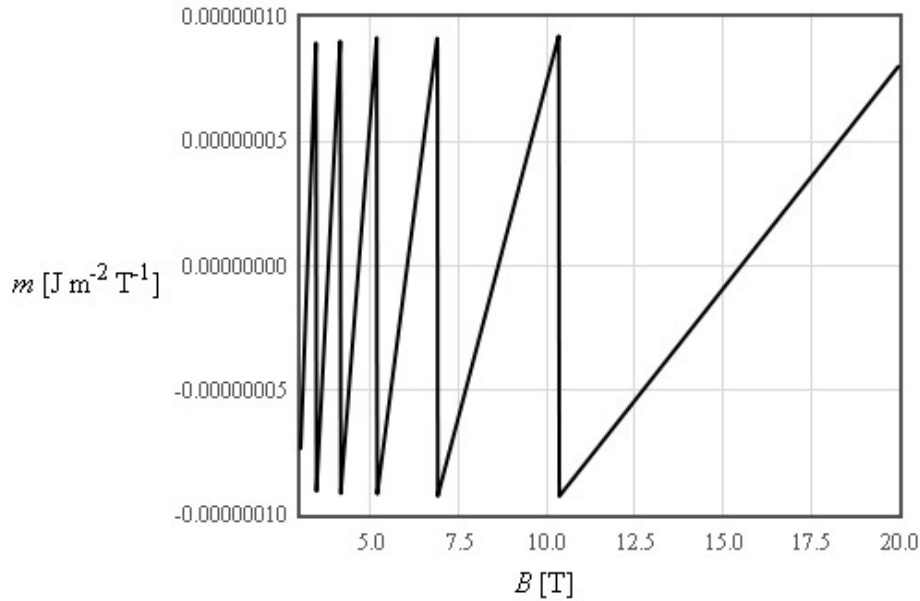# Internal energy 2d

At $T = 0$ $u = \int_{-\infty}^{E_F} ED(E)dE$

$$u = n\frac{\hbar\omega_c}{2} = n\frac{\hbar eB}{2m}$$

Large field limit
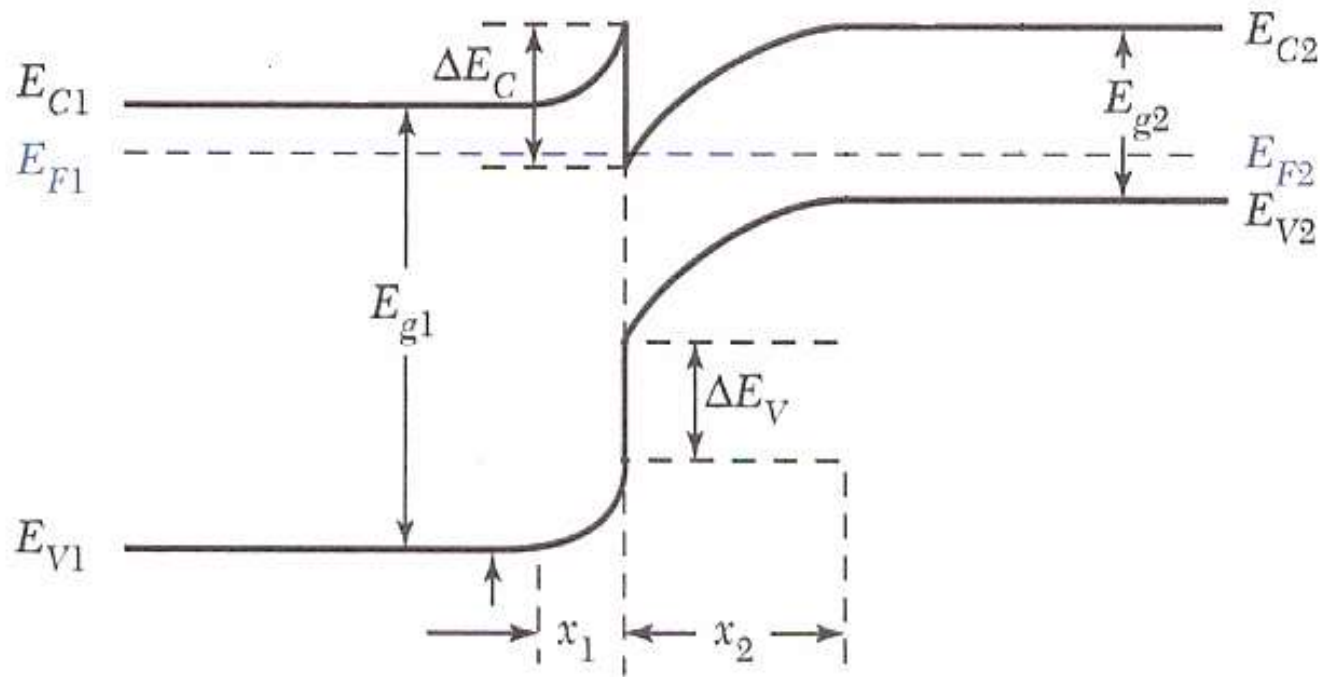
Magnetization   2d

At $T = 0$

$$m = -\frac{du}{dB} = -n\frac{\hbar e}{2m}$$

Large field limit
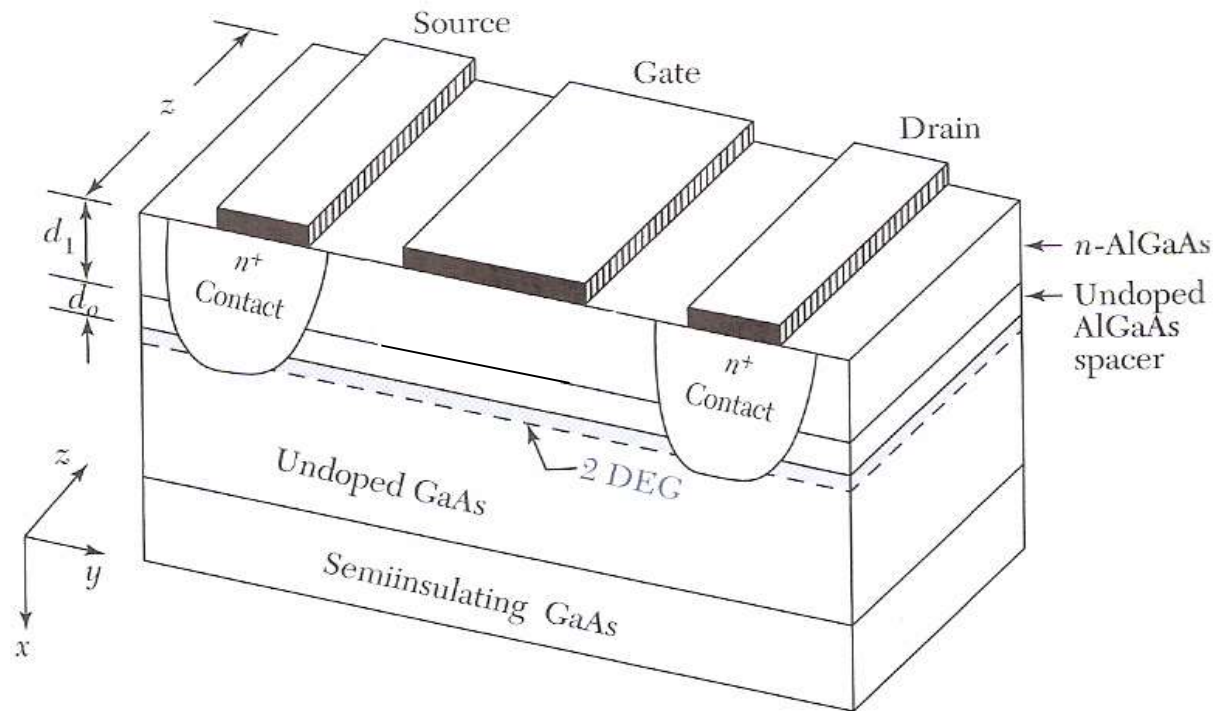
de Haas - van Alphen oscillations

Heterostructure

pn junction formed from two semiconductors with different band gaps

MODFET (HEMT)

Modulation doped field effect transistor (MODFET)
High electron mobility transistor (HEMT)

The magnetic field can be at an angle to the 2DEG. The Landau splitting experiences the component perpendicular to the plane. The Zeeman splitting experiences the full field.

MOSFETs

Inversion
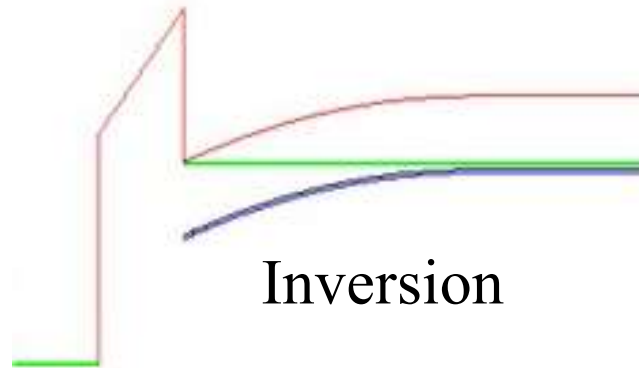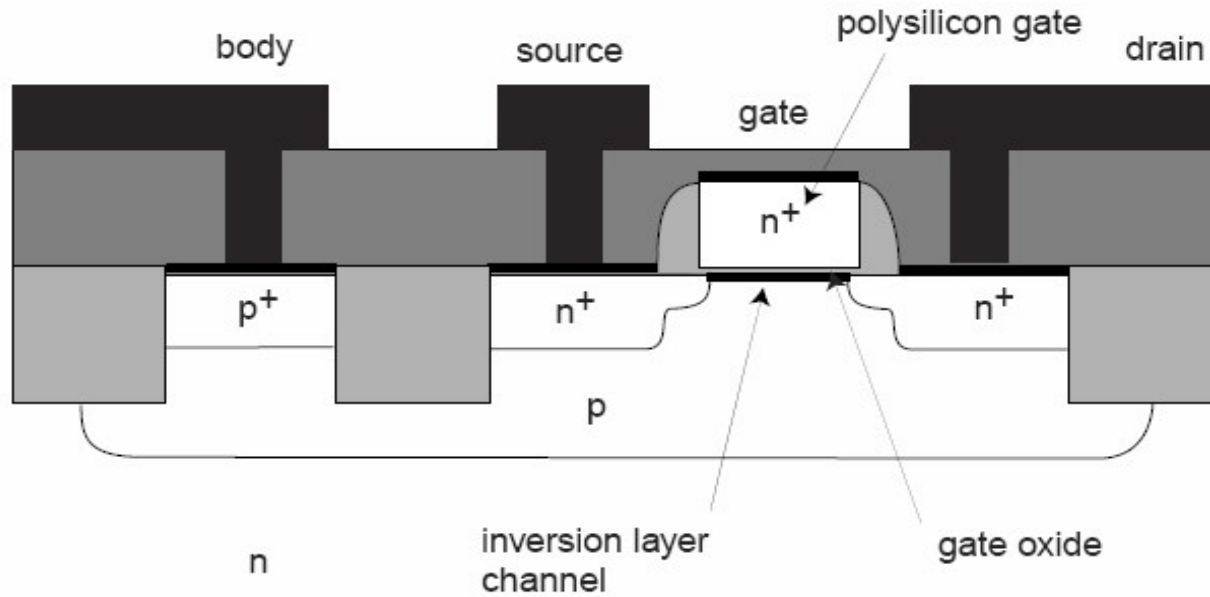
Scattering at the Fermi surface

At room temperature, phonon energies are much less than the Fermi energy. The energy of electrons hardly changes as they scatter from phonons. Electrons scatter from a point close to the Fermi surface to another point close to the Fermi surface.
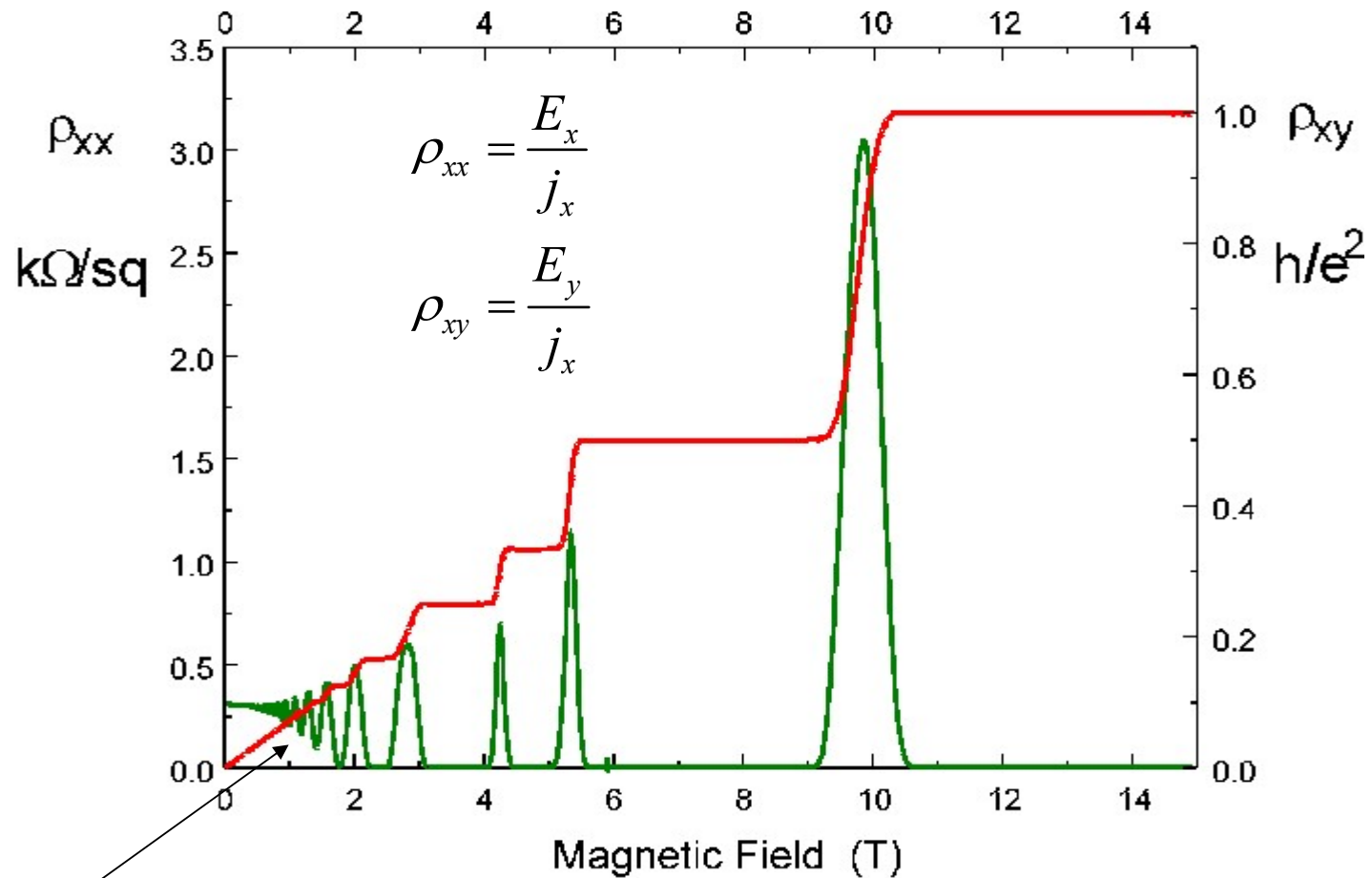
Changing the magnetic field changes the number of states at the Fermi energy.

There are oscillations in the electrical conductivity as a function of magnetic field.

Shubnikov-De Haas oscillations
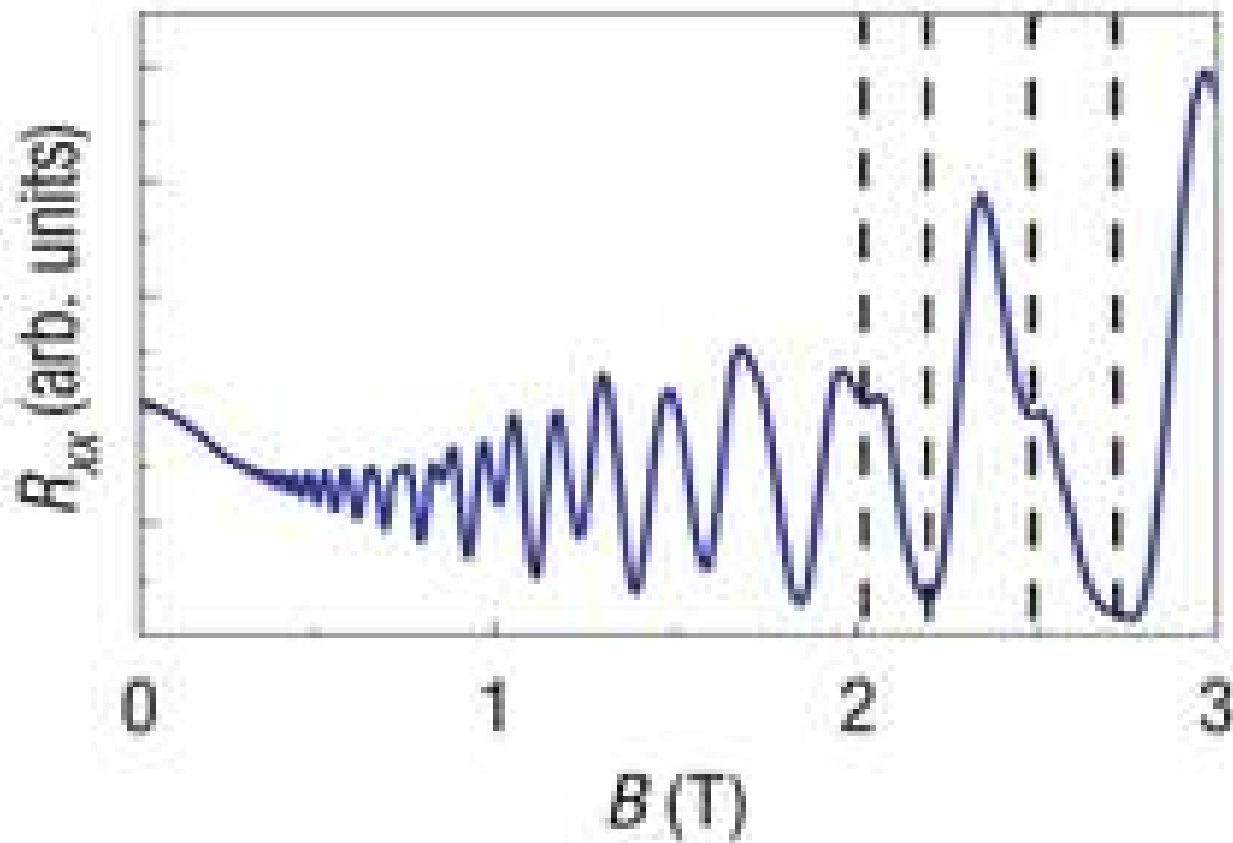
Quantum Hall Effect

$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{xy} = \frac{E_y}{j_x}$$

Shubnikov-De Haas oscillations

Resistance standard
25812.807557(18) Ω