Quantum Hall Effect

$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{xy} = \frac{E_y}{j_x}$$
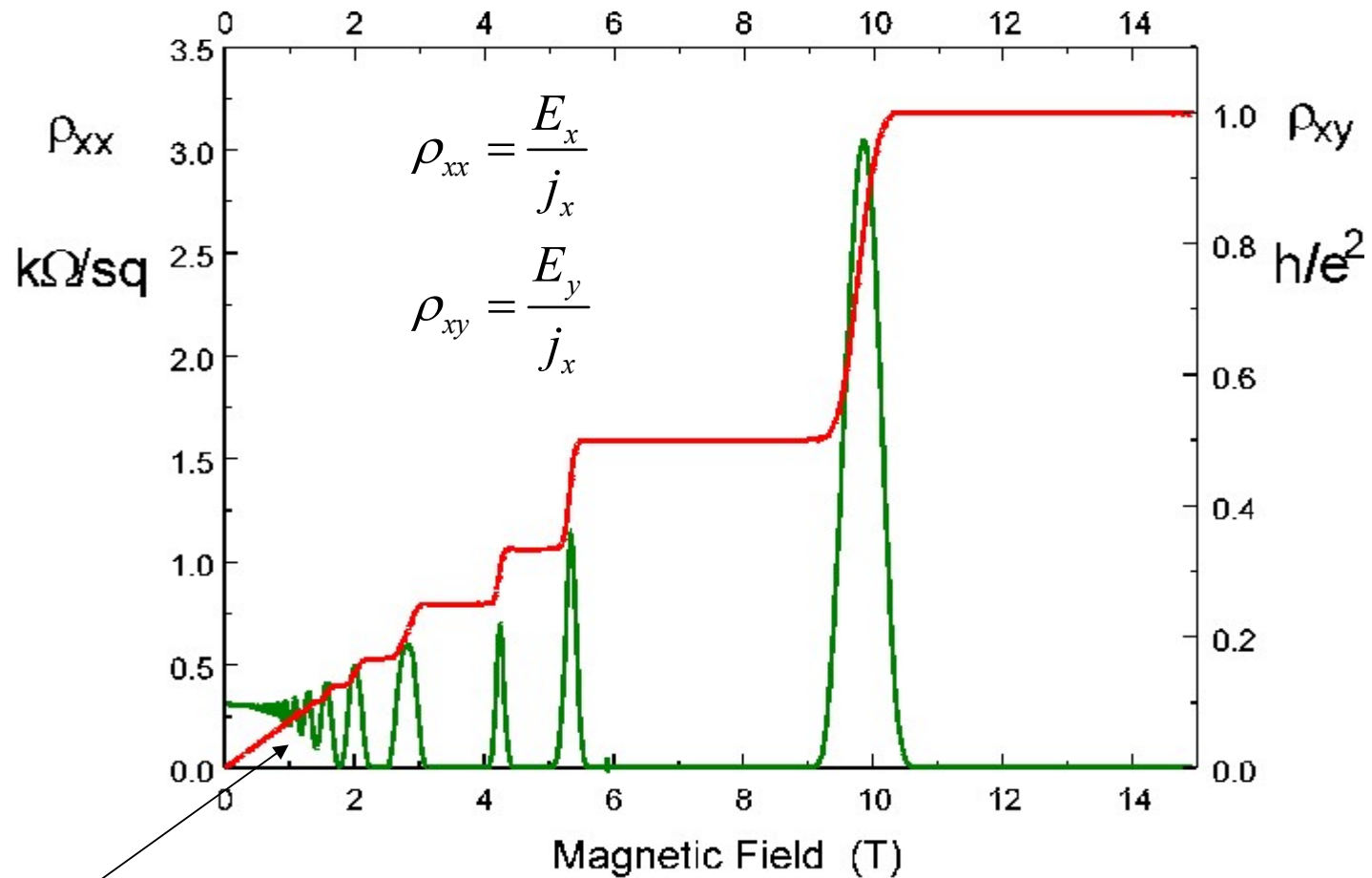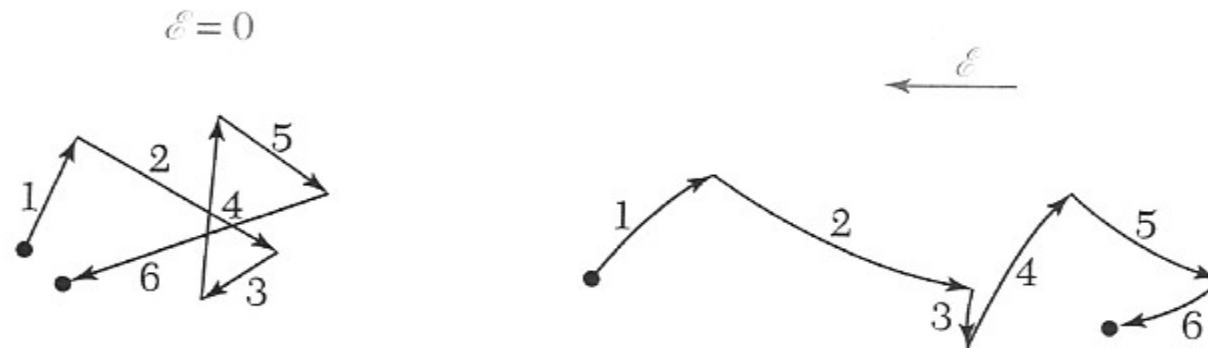
Shubnikov-De Haas oscillations

Resistance standard
25812.807557(18) Ω

Review of the Hall effect: Diffusive transport

$$\mathscr{E} = 0$$

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{sc}}$$

$$-\frac{e\tau_{sc}}{m}\vec{E} = \vec{v}_d$$

mobility: $\quad -\mu_e\vec{E} = \vec{v}_d$

$$\vec{j} = -n|e|\vec{v}_d = n|e|\mu_e\vec{E} = \frac{ne^2\tau_{sc}}{m}\vec{E} = \sigma\vec{E}$$

Review of the Hall effect

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{sc}} \quad \longleftarrow \quad \text{diffusive regime}$$

$$\vec{F} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right) = m\frac{\vec{v}_d}{\tau_{sc}}$$

If B is in the z-direction, the three components of the force are

$$-e\left(E_x + v_{dy}B_z\right) = m\frac{v_{dx}}{\tau_{sc}}$$

$$-e\left(E_y - v_{dx}B_z\right) = m\frac{v_{dy}}{\tau_{sc}}$$

$$-e\left(E_z\right) = m\frac{v_{dz}}{\tau_{sc}}$$

Magnetic field (diffusive regime)
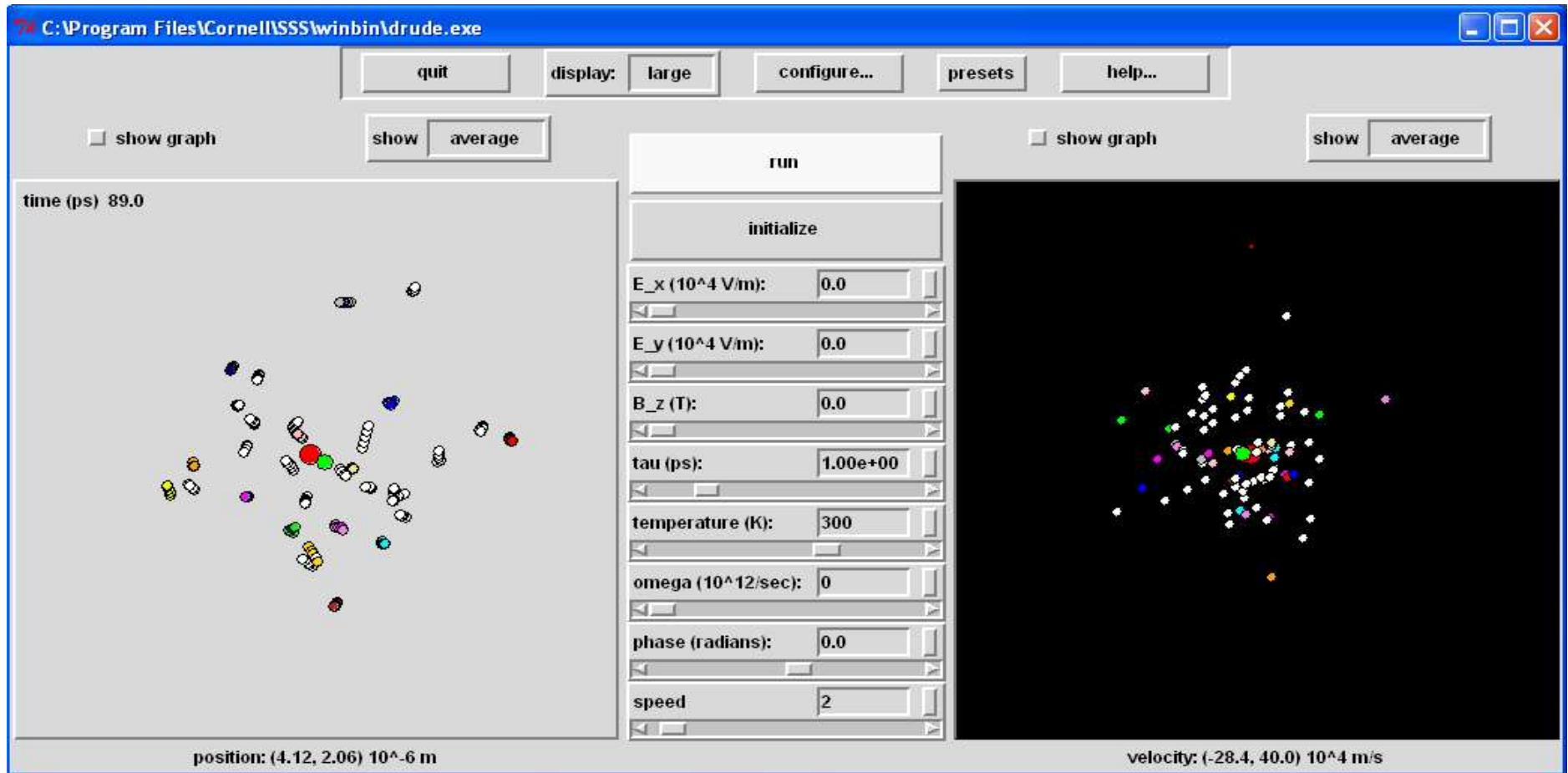
For a magnetic field in the *z*-direction

$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$
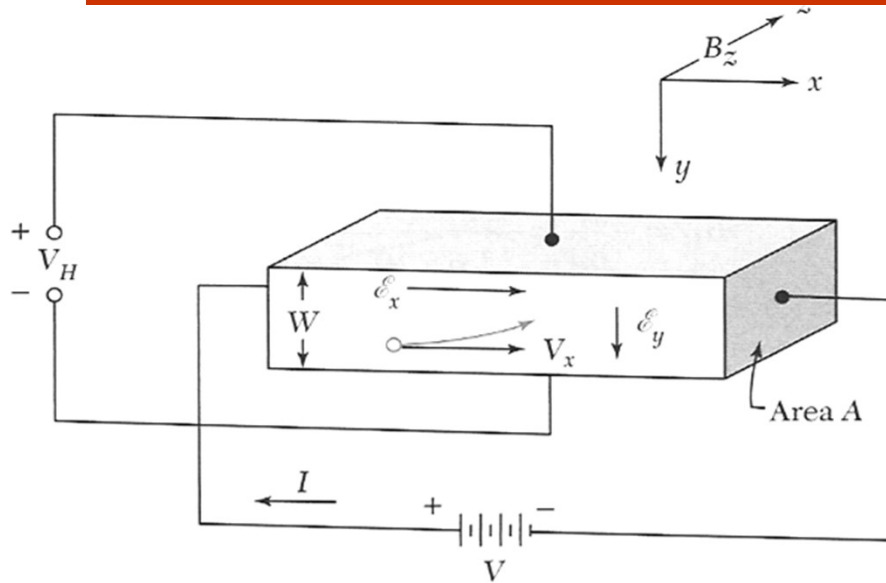
$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

If $E_y = 0$,

$$v_{d,y} = -\frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$\tan\theta_H = -\frac{eB_z}{m}\tau_{sc}$$

If no forces are applied, the electrons diffuse.
The average velocity moves against an electric field.
In just a magnetic field, the average velocity is zero.
In an electric and magnetic field, the electrons move in a straight line at the Hall angle.
The drift velocity decreases as the B field increases.

The Hall Effect (diffusive regime)

$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$
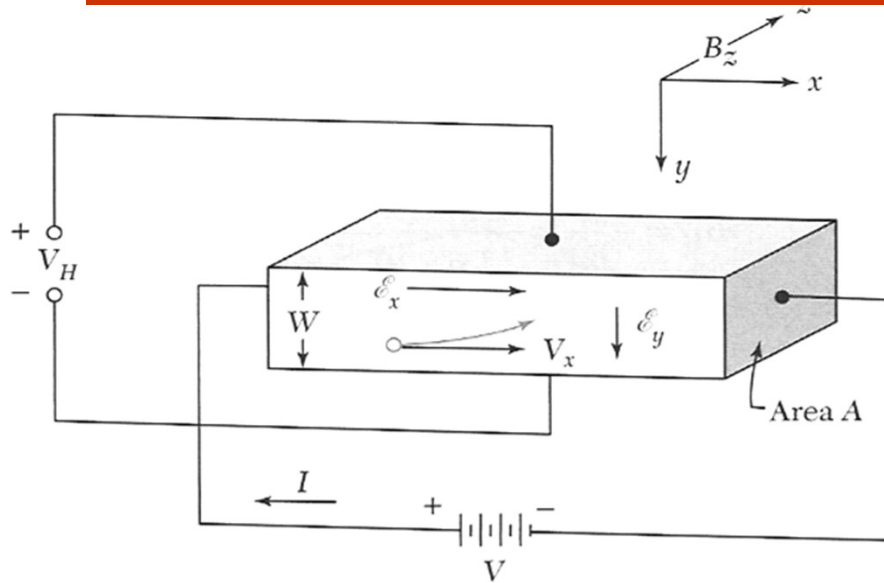
$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

If $v_{d,y} = 0$,

$E_y = v_{d,x}B_z = V_H/W = R_H j_x B_z$ $\qquad$ V_H = Hall voltage, R_H = Hall Constant

$v_{d,x} = -j_x/ne$

$$\boxed{R_H = E_y/j_x B_z = -1/ne}$$

The Hall Effect (diffusive regime)

$$\rho_{xx} = \frac{E_x}{j_x}$$

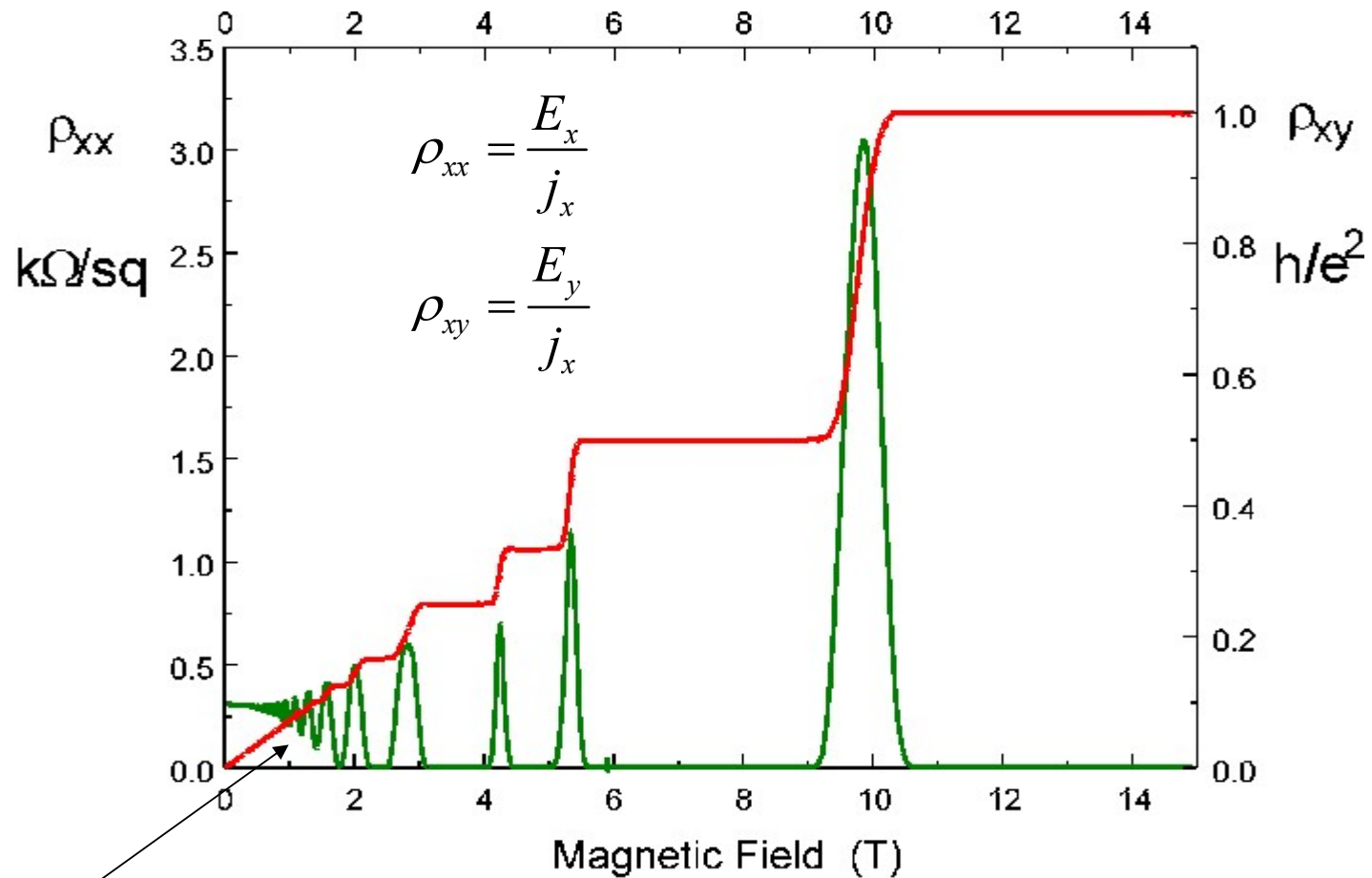$$\rho_{xy} = \frac{E_y}{j_x}$$

$$R_H = E_y/j_x B_z = -1/ne$$

multiply both sides by B_z

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$

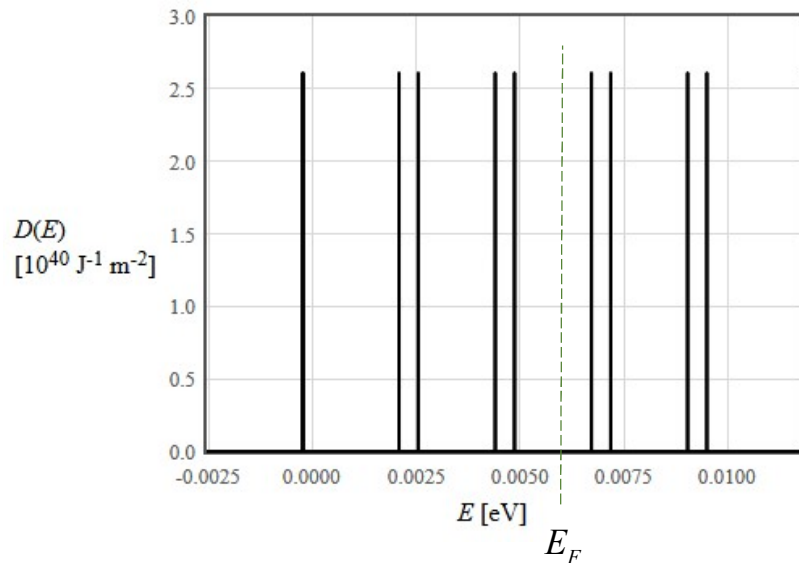The Hall resistivity is proportional to the magnetic field.

Quantum Hall Effect

$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{xy} = \frac{E_y}{j_x}$$

Shubnikov-De Haas oscillations

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Quantum hall effect

If the Fermi energy is between Landau levels, the electron density n is an integer v times the degeneracy of the Landau level $n = D_0 v$

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$

Each Landau level can hold the same number of electrons.
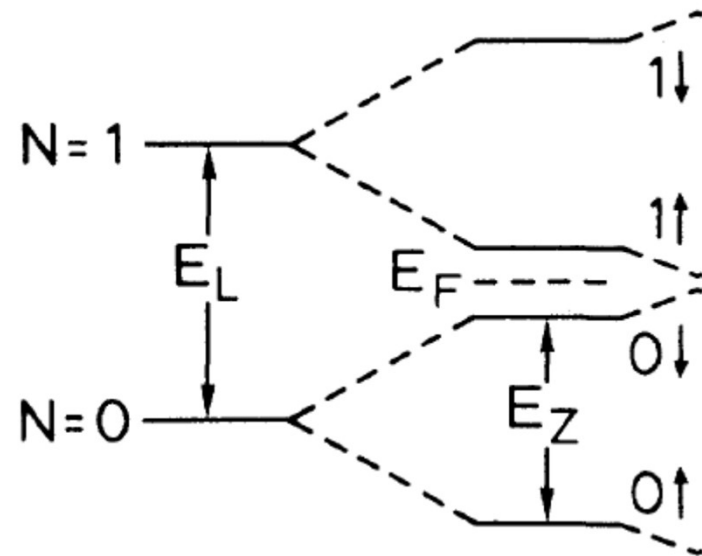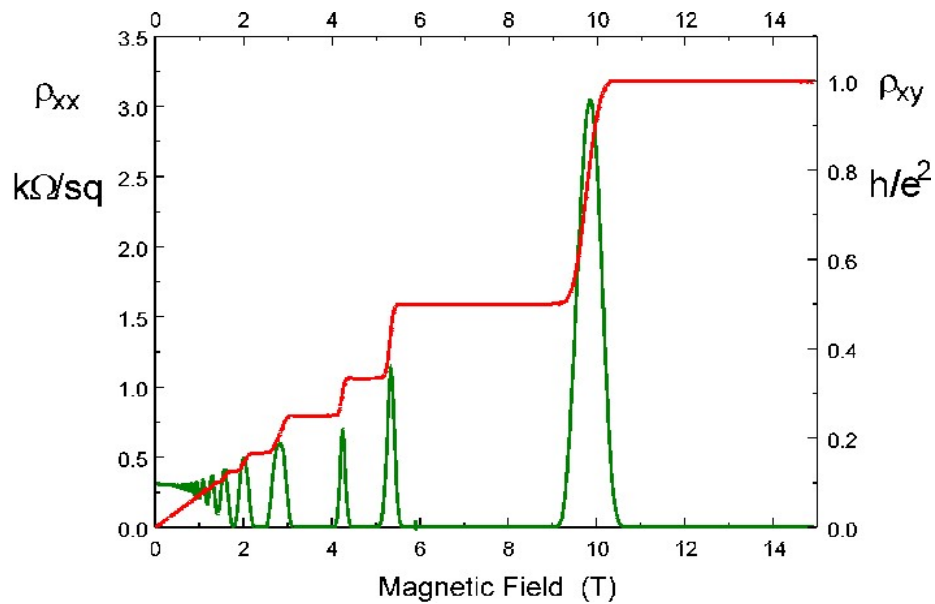
$$\rho_{xy} = \frac{-B_z}{ne} = \frac{-hD_0}{ve^2 D_0} = \frac{-h}{ve^2}$$

$$D_0 = \frac{m\omega_c}{2\pi\hbar} = \frac{eB_z}{h}$$

$$\omega_c = \frac{eB_z}{m} \qquad\qquad B_z = \frac{hD_0}{e}$$

Quantum hall effect

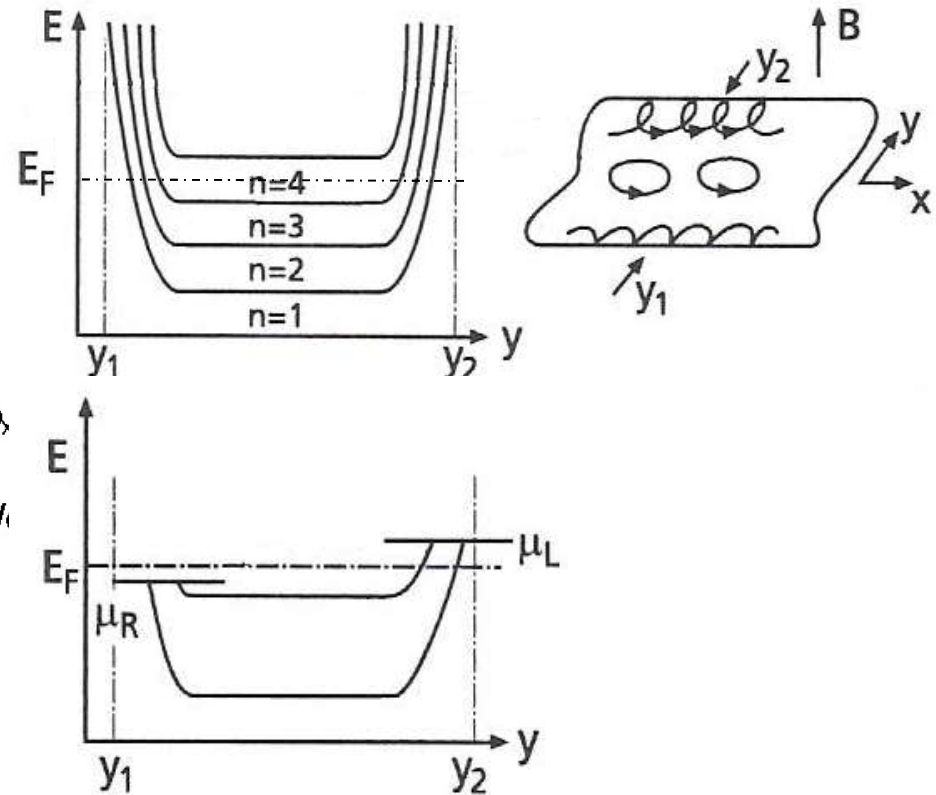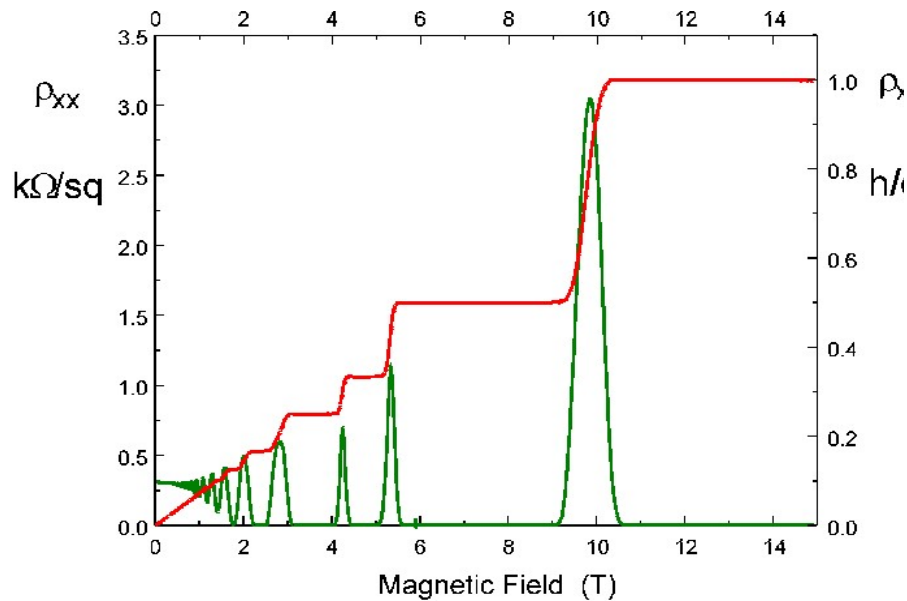$$\rho_{xy} = \frac{h}{ve^2}$$

S. Koch, R. J. Haug, and K. v. Klitzing,
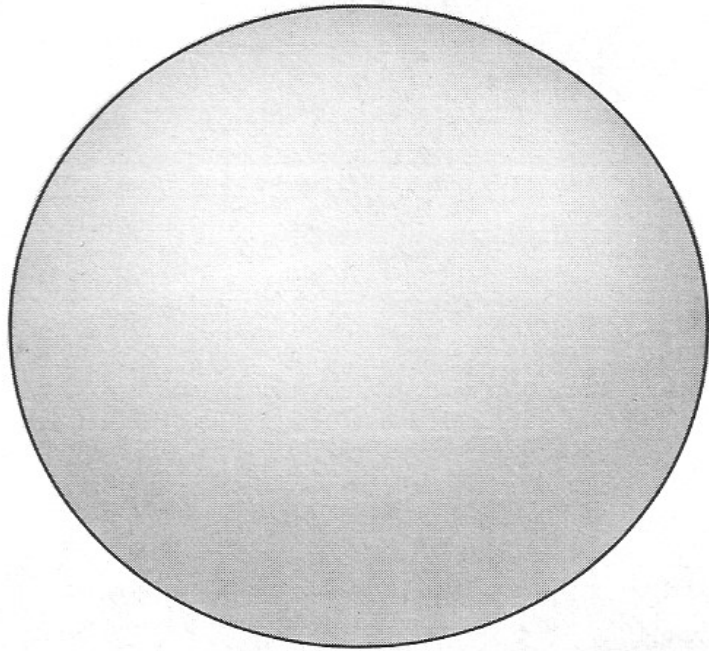Phys. Rev. B 47, 4048–4051 (1993)

Quantum Hall effect

Edge states are responsible for the zero resistance in ρ_{xx}

On the plateaus, resistance goes to zero because there are no states to scatter into.
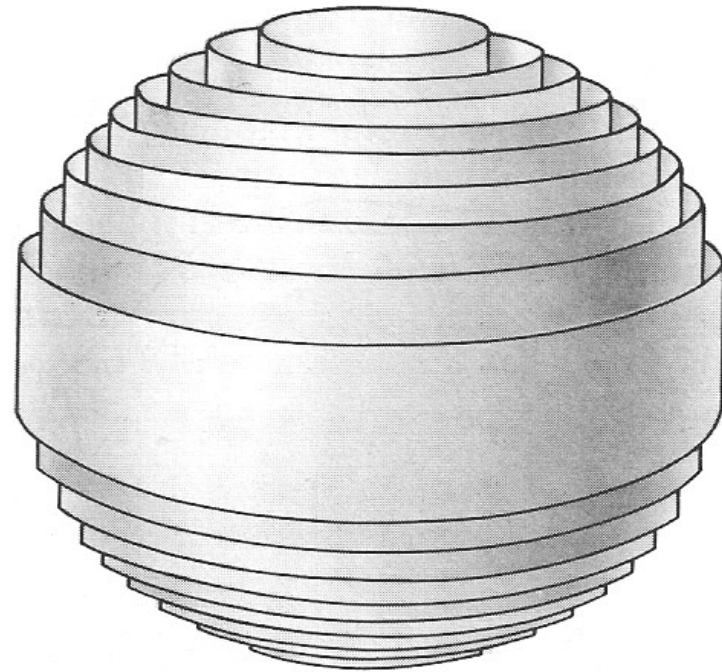

Ibach & Lueth (modified)
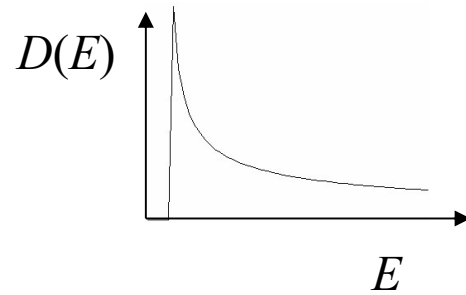
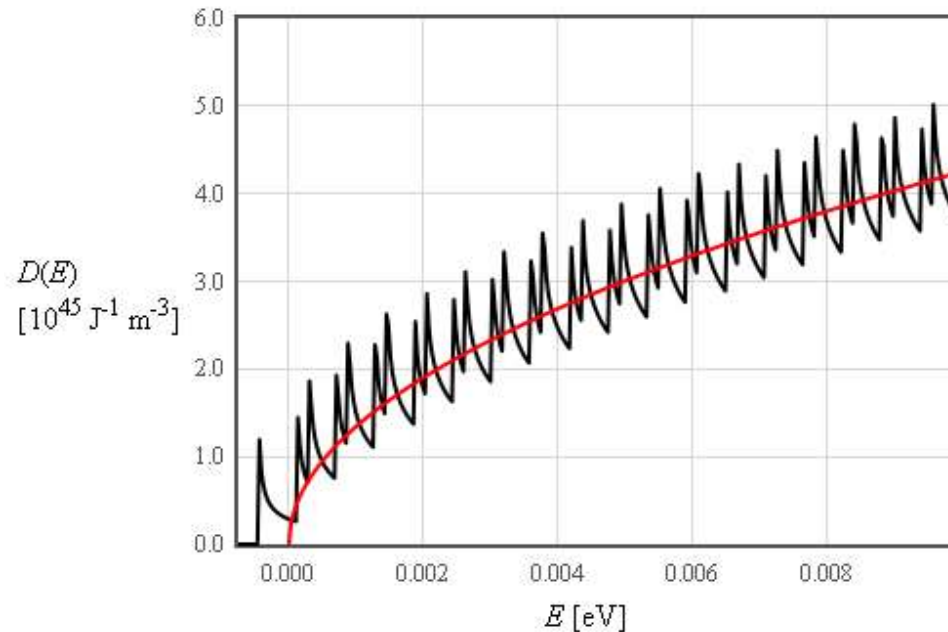# Fermi sphere in a magnetic field

$B = 0$

$B \neq 0$

Landau cylinders

Density of states 3d

convolution of

$D(E)$

E

and

$D(E)$

$\hbar\omega_c$

$\dfrac{m\omega_c}{2\pi\hbar}$

0   E

$D(E)$
$[10^{45}\ \mathrm{J^{-1}\ m^{-3}}]$

$$D(E) = \frac{(2m)^{3/2}\,\omega_c}{8\pi^2\hbar^2}\left(\sum_{v=0}^{\infty} \frac{H\left(E - \hbar\omega_c\left(v+\tfrac{1}{2}-g/4\right)\right)}{\sqrt{E - \hbar\omega_c\left(v+\tfrac{1}{2}-g/4\right)}} + \frac{H\left(E - \hbar\omega_c\left(v+\tfrac{1}{2}+g/4\right)\right)}{\sqrt{E - \hbar\omega_c\left(v+\tfrac{1}{2}+g/4\right)}} \right) \quad \mathrm{J^{-1}m^{-3}}$$