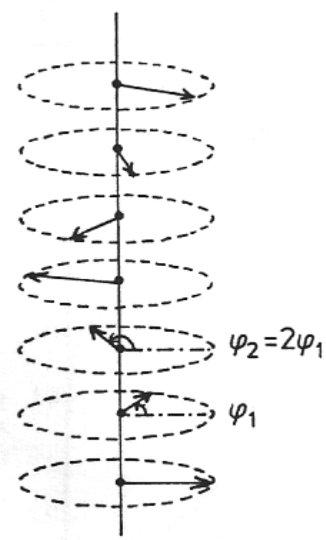
Magnetic ordering

Ferromagnetism ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

Ferrimagnetism ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Antiferromagnetism ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Helimagnetism



All ordered magnetic states have excitations called magnons

Ferrimagnets

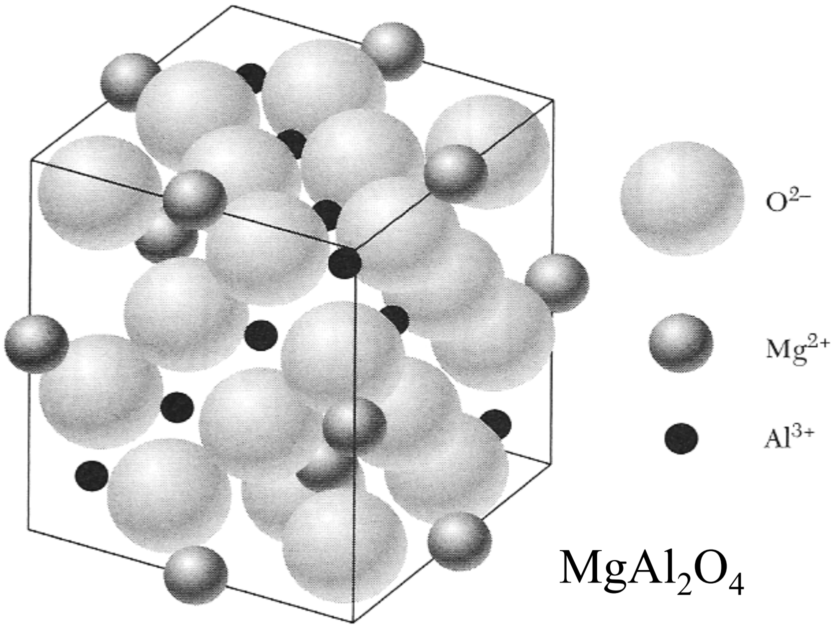
Magnetite Fe_3O_4
(Magnetstein)

Ferrites $MO{\cdot}Fe_2O_3$

M = Fe, Zn, Cd, Ni, Cu, Co, Mg

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Two sublattices A and B.



$MgAl_2O_4$

Spinel crystal structure XY_2O_4

8 tetrahedral sites A (surrounded by 4 O) $5\mu_B \uparrow$

16 octahedral sites B (surrounded by 6 O) $9\mu_B \downarrow$

per unit cell

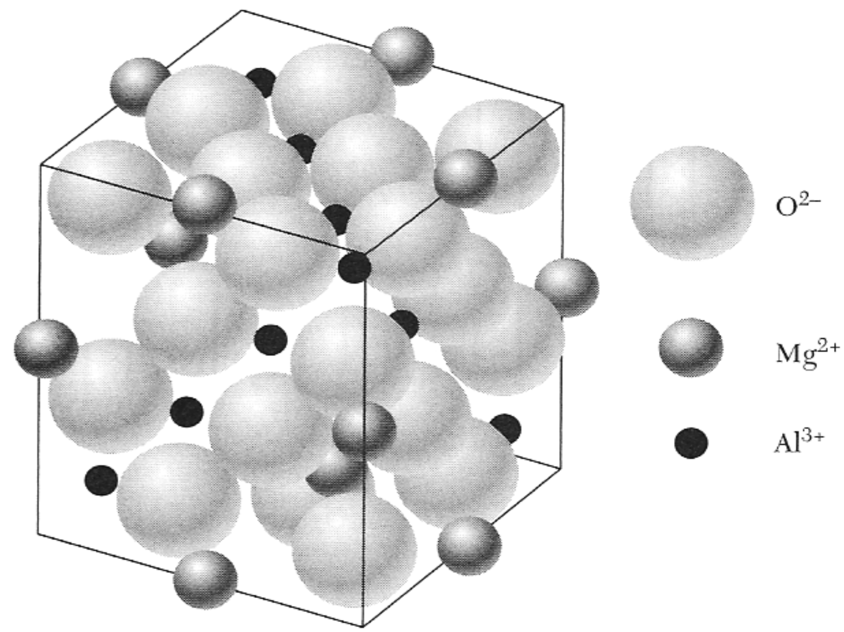
Ferrimagnets

Magnetite Fe_3O_4

Ferrites $MO{\cdot}Fe_2O_3$

M = Fe, Zn, Cd, Ni, Cu, Co, Mg

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓



Exchange integrals J_{AA}, J_{AB}, and J_{BB} are all negative (antiparallel preferred)

$|J_{AB}| > |J_{AA}|,|J_{BB}|$

Mean field theory (Ferrimagnetism and Antiferromagnetism)

Heisenberg Hamiltonian

$$H = -\sum_{i,j} J_{i,j} \vec{S}_i \cdot \vec{S}_j - g\mu_B B \sum_i \vec{S}_i$$

Exchange energy

Mean field approximation

$$\vec{B}_{MF,A} = \frac{1}{g\mu_B} \sum_\delta J_{i,AB} \left\langle \vec{S}_B \right\rangle + \frac{1}{g\mu_B} \sum_\delta J_{i,AA} \left\langle \vec{S}_A \right\rangle$$

$$\vec{B}_{MF,B} = \frac{1}{g\mu_B} \sum_\delta J_{i,AB} \left\langle \vec{S}_A \right\rangle + \frac{1}{g\mu_B} \sum_\delta J_{i,BB} \left\langle \vec{S}_B \right\rangle$$

$$\vec{M}_A = g\mu_B \frac{N}{V} \left\langle \vec{S}_A \right\rangle \qquad \vec{M}_B = g\mu_B \frac{N}{V} \left\langle \vec{S}_B \right\rangle$$

Mean field theory

The spins can take on two energies. These energies are different on the A sites and B because the A spins see a different environment as the B spins.

$$E_A = \pm \frac{1}{2} g \mu_B (B_{MF,A} + B_a) \qquad E_B = \pm \frac{1}{2} g \mu_B (B_{MF,B} + B_a)$$

Calculate the average magnetization with Boltzmann factors:

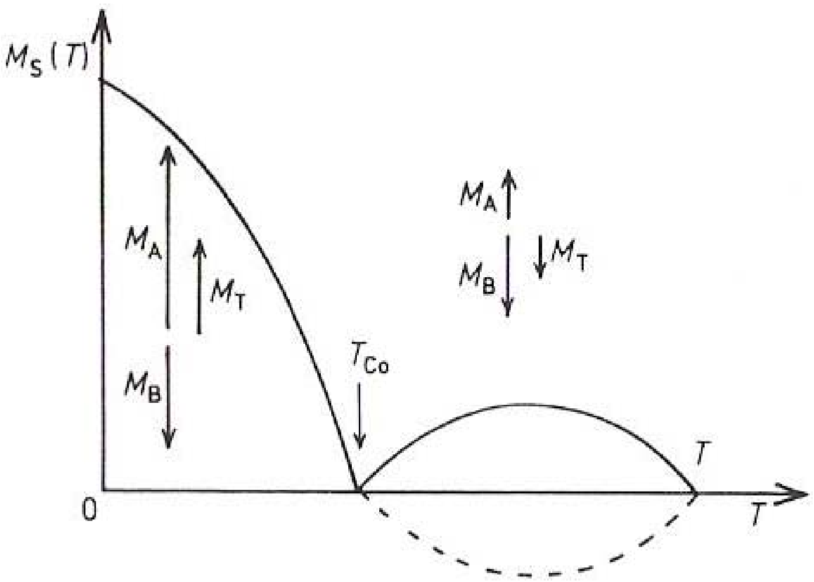
$$M_A = N\mu \tanh\left(\frac{\mu\left(B_{MF,A} + B_a\right)}{k_B T} \right) \qquad M_B = N\mu \tanh\left(\frac{\mu\left(B_{MF,B} + B_a\right)}{k_B T} \right)$$

$$M_A = M_{s,A} \tanh\left(\frac{\mu_0 \mu_{AB} M_B + \mu_0 \mu_{AA} M_A + \mu B_a}{k_B T} \right)$$

$$M_B = M_{s,B} \tanh\left(\frac{\mu_0 \mu_{AB} M_A + \mu_0 \mu_{BB} M_B + \mu B_a}{k_B T} \right)$$

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓ Ferrimagnetism

gauss = 10^{-4} T
oersted = $10^{-4}/4\pi x 10^{-7}$ A/m



D. Gignoux, magnetic
properties of Metallic systems

Table 33.3
SELECTED FERRIMAGNETS, WITH CRITICAL TEMPERATURES T_c AND SATURATION MAGNETIZATION M_0

MATERIAL	T_c (K)	M_0 (gauss)[a]
Fe_3O_4 (magnetite)	858	510
$CoFe_2O_4$	793	475
$NiFe_2O_4$	858	300
$CuFe_2O_4$	728	160
$MnFe_2O_4$	573	560
$Y_3Fe_5O_{12}$ (YIG)	560	195

[a] At $T = 0$(K).
Source: F. Keffer, *Handbuch der Physik*, vol. 18, pt. 2, Springer, New York, 1966.

Kittel

Magnetization of a Magnetite Single Crystal Near the Curie Point*

D. O. SMITH†
Laboratory for Insulation Research, Massachusetts, Institute of Technology, Cambridge, Massachusetts
(Received January 20, 1956)

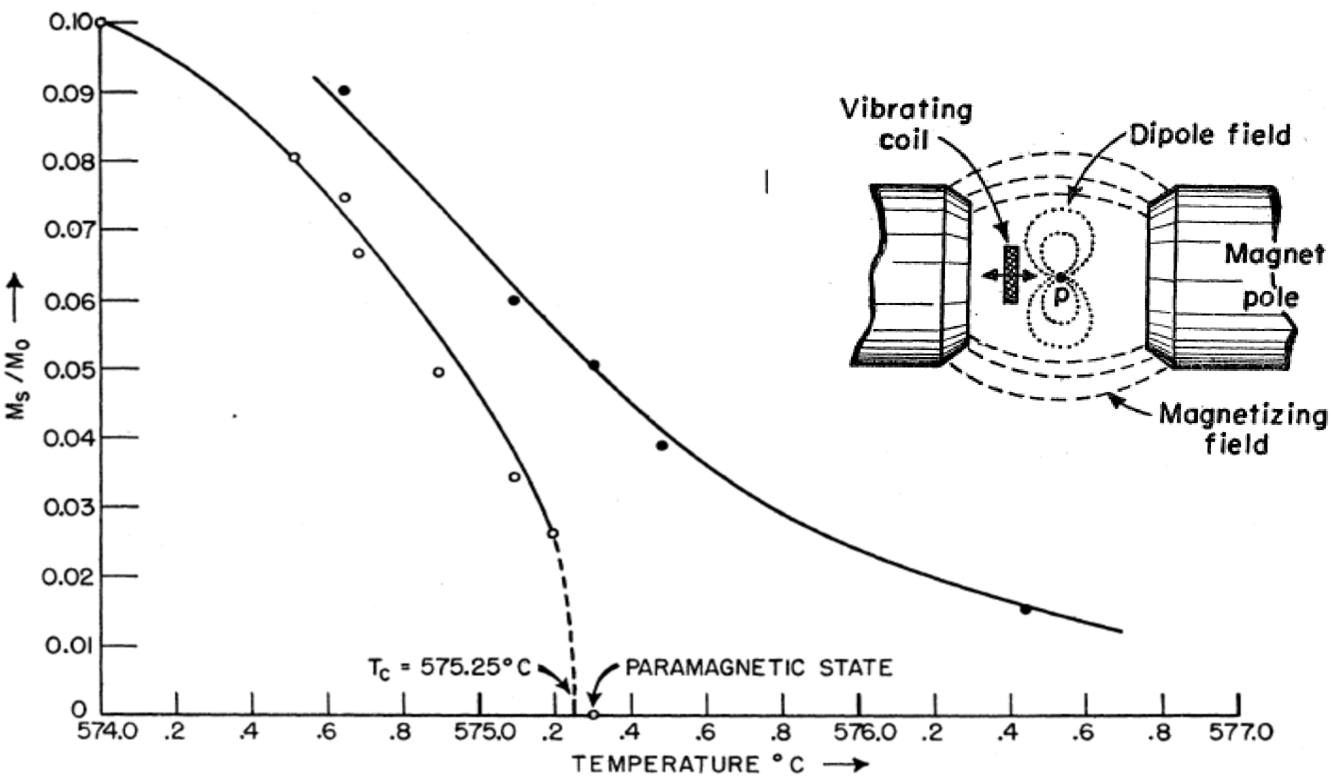


FIG. 9. M_s/M_0 *vs* T in the [111] direction near the Curie point for single-crystal magnetite.

FIG. 2. Principle of the vibrating-coil magnetometer.

Antiferromagnetism

Negative exchange energy $J_{AB} < 0$.

$$\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$$

At low temperatures, below the Neel temperature T_N, the spins are aligned antiparallel and the macroscopic magnetization is zero.

Spin ordering can be observed by neutron scattering.

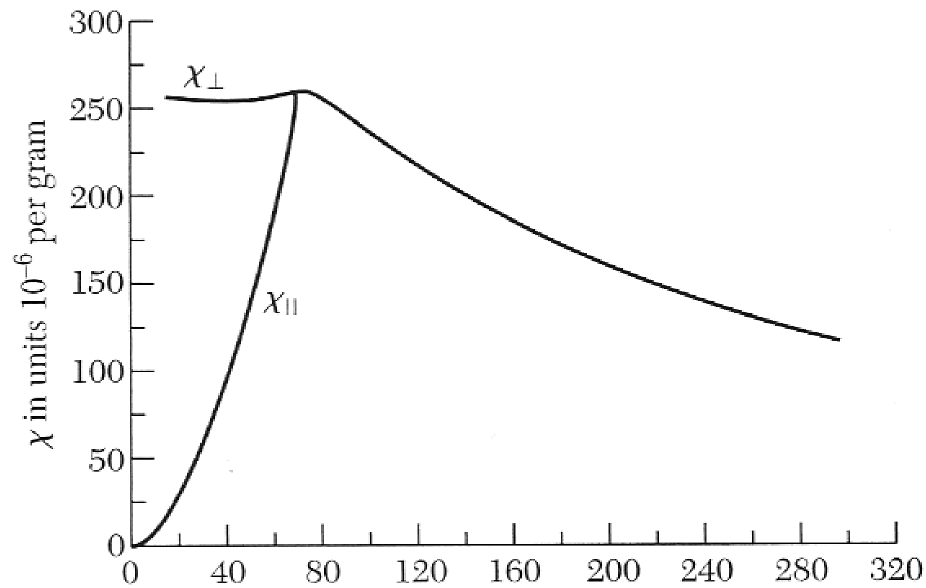
At high temperature antiferromagnets become paramagnetic. The macroscopic magnetization is zero and the spins are disordered in zero field.

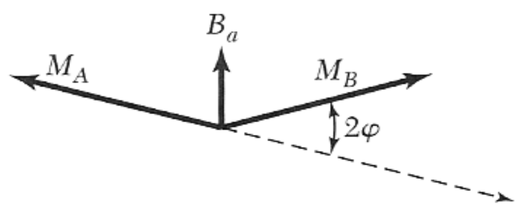
Curie-Weiss temperature

$$\chi \approx = \frac{C}{T + \Theta}$$

Antiferromagnetism ↑↓↑↓↑↓↑↓

Average spontaneous magnetization is zero at all temperatures.





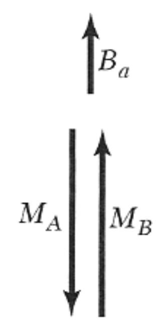
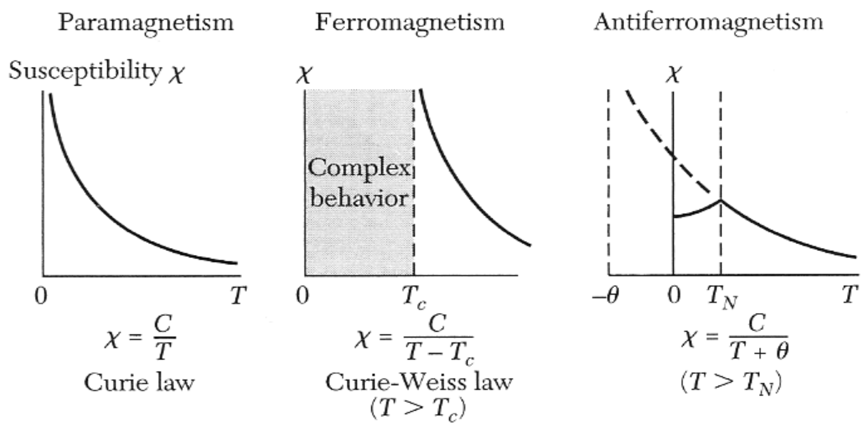


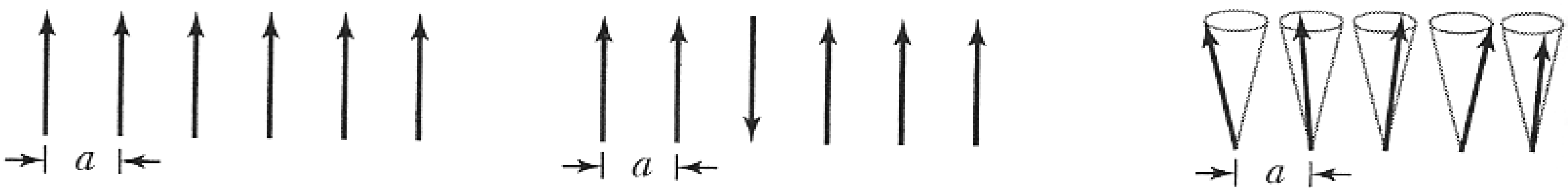
Table 2 Antiferromagnetic crystals ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Substance	Paramagnetic ion lattice	Transition temperature, T_N, in K	Curie-Weiss θ, in K	$\frac{\theta}{T_N}$	$\frac{\chi(0)}{\chi(T_N)}$
MnO	fcc	116	610	5.3	$\frac{2}{3}$
MnS	fcc	160	528	3.3	0.82
MnTe	hex. layer	307	690	2.25	
MnF_2	bc tetr.	67	82	1.24	0.76
FeF_2	bc tetr.	79	117	1.48	0.72
$FeCl_2$	hex. layer	24	48	2.0	<0.2
FeO	fcc	198	570	2.9	0.8
$CoCl_2$	hex. layer	25	38.1	1.53	
CoO	fcc	291	330	1.14	
$NiCl_2$	hex. layer	50	68.2	1.37	
NiO	fcc	525	~2000	~4	
Cr	bcc	308			

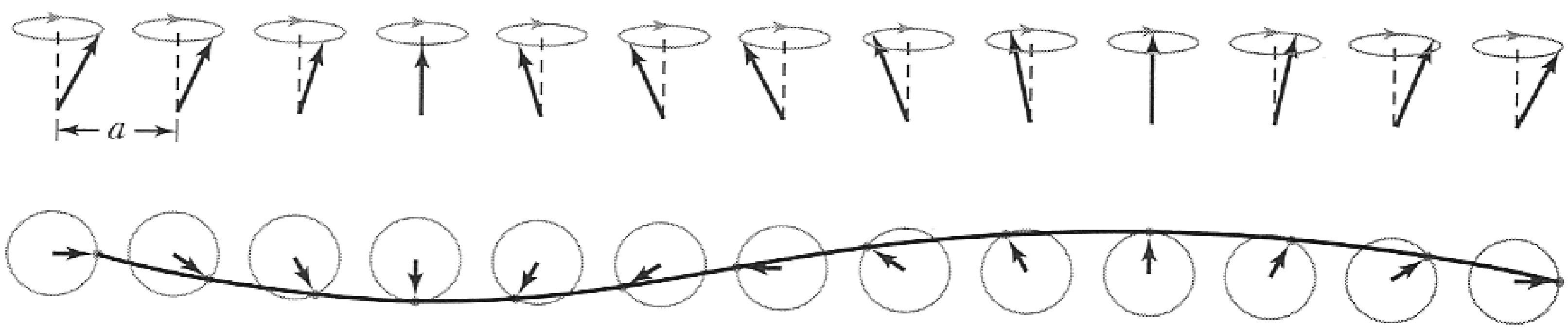


from Kittel

Magnons



Magnons are excitations of the ordered ferromagnetic state



Magnons

Energy of the Heisenberg term involving spin *p*

$$-2J\vec{S}_p \cdot \left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

The magnetic moment of spin *p* is

$$\vec{\mu}_p = -g\,\mu_B \vec{S}_p$$

$$-\vec{\mu}_p \cdot \left(\frac{-2J}{g\,\mu_B}\right)\left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

This has the form $-\mu_p \cdot B_p$ where B_p is

$$\vec{B}_p = \left(\frac{-2J}{g\,\mu_B}\right)\left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

Magnons

$$\vec{\mu}_p = -g\mu_B \vec{S}_p \qquad \vec{B}_p = \left(\frac{-2J}{g\mu_B}\right)\left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

The rate of change of angular momentum is the torque

$$\hbar \frac{d\vec{S}_p}{dt} = \vec{\mu}_p \times \vec{B}_p = 2J\left(\vec{S}_p \times \vec{S}_{p+1} + \vec{S}_p \times \vec{S}_{p-1}\right)$$

If the amplitude of the deviations from perfect alignment along the *z*-axis are small:

$$\hbar \frac{dS_p^x}{dt} = 2J\left|S\right|\left(S_{p+1}^y - 2S_p^y + S_{p-1}^y\right)$$

$$\hbar \frac{dS_p^y}{dt} = 2J\left|S\right|\left(S_{p+1}^x - 2S_p^x + S_{p-1}^x\right)$$

$$\hbar \frac{d\vec{S}_p^z}{dt} = 0$$

Magnons

$$\hbar \frac{dS_p^x}{dt} = 2J\left|S\right|\left(S_{p+1}^y - 2S_p^y + S_{p-1}^y\right)$$

$$\hbar \frac{dS_p^y}{dt} = 2J\left|S\right|\left(S_{p+1}^x - 2S_p^x + S_{p-1}^x\right)$$

$$\hbar \frac{dS_p^z}{dt} = 0$$

These are coupled linear differential equations. The solutions have the form:

$$\begin{pmatrix} S_p^x \\ S_p^y \end{pmatrix} = \begin{pmatrix} u_k^x \\ u_k^y \end{pmatrix} \exp\left[i\left(kpa - \omega t\right)\right]$$

$$-i\hbar\omega u_k^x e^{ikpa} = 2J\left|S\right|\left(-e^{ik(p+1)a} + 2e^{ikpa} - e^{-ik(p-1)a}\right)u_k^y$$

$$-i\hbar\omega u_k^y e^{ikpa} = -2J\left|S\right|\left(-e^{ik(p+1)a} + 2e^{ikpa} - e^{-ik(p-1)a}\right)u_k^x$$

Cancel a factor of e^{ikpa}.

Magnons

$$-i\hbar\omega u_k^x = 2J\left|S\right|\left(-e^{ika}+2-e^{-ika}\right)u_k^y$$

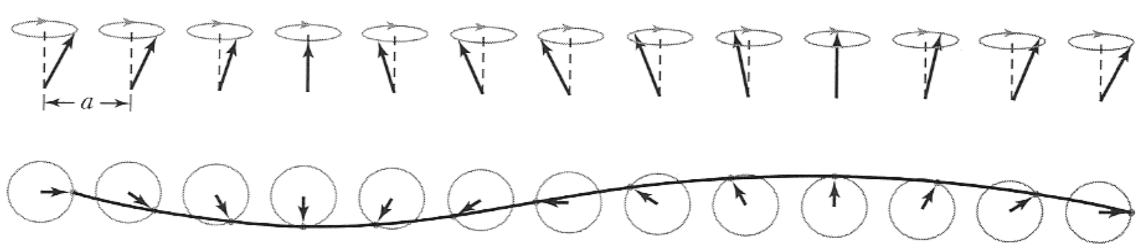
$$-i\hbar\omega u_k^y = -2J\left|S\right|\left(-e^{ika}+2-e^{-ika}\right)u_k^x$$

These equations will have solutions when,

$$\begin{vmatrix} i\hbar\omega & 4J\left|S\right|\left(1-\cos(ka)\right) \\ -4J\left|S\right|\left(1-\cos(ka)\right) & i\hbar\omega \end{vmatrix} = 0$$

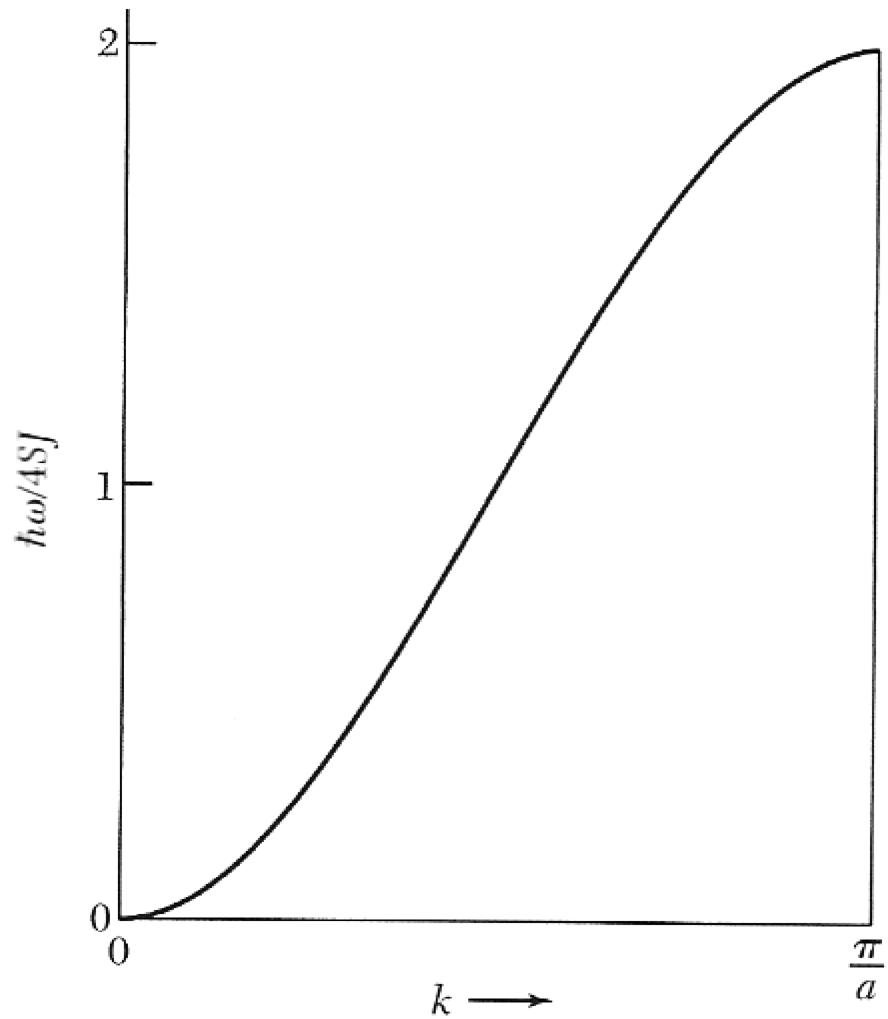
The dispersion relation is:

$$\hbar\omega = 4J\left|S\right|\left(1-\cos(ka)\right)$$



Magnon dispersion relation

$$\hbar\omega = 4JS\left(1 - \cos(ka)\right)$$

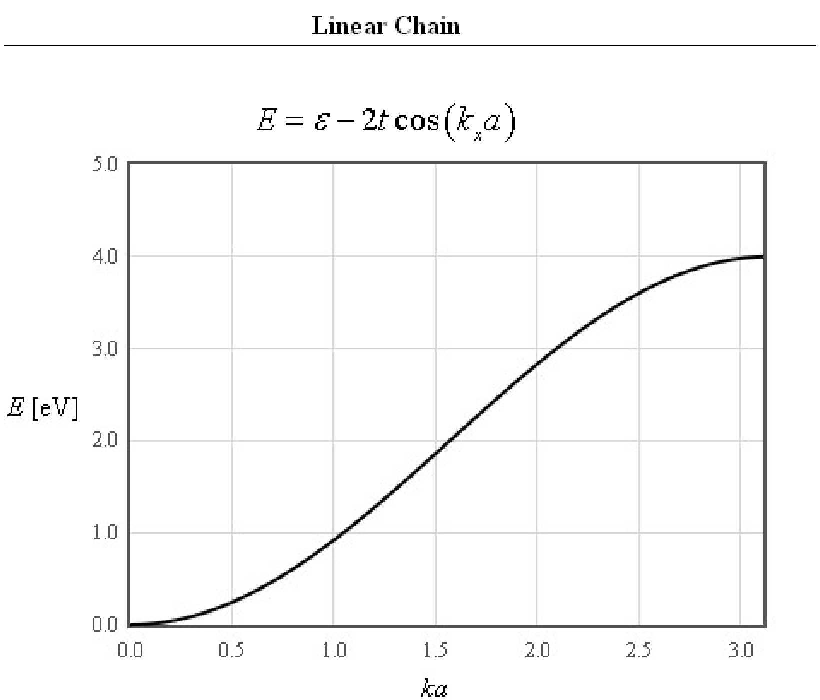


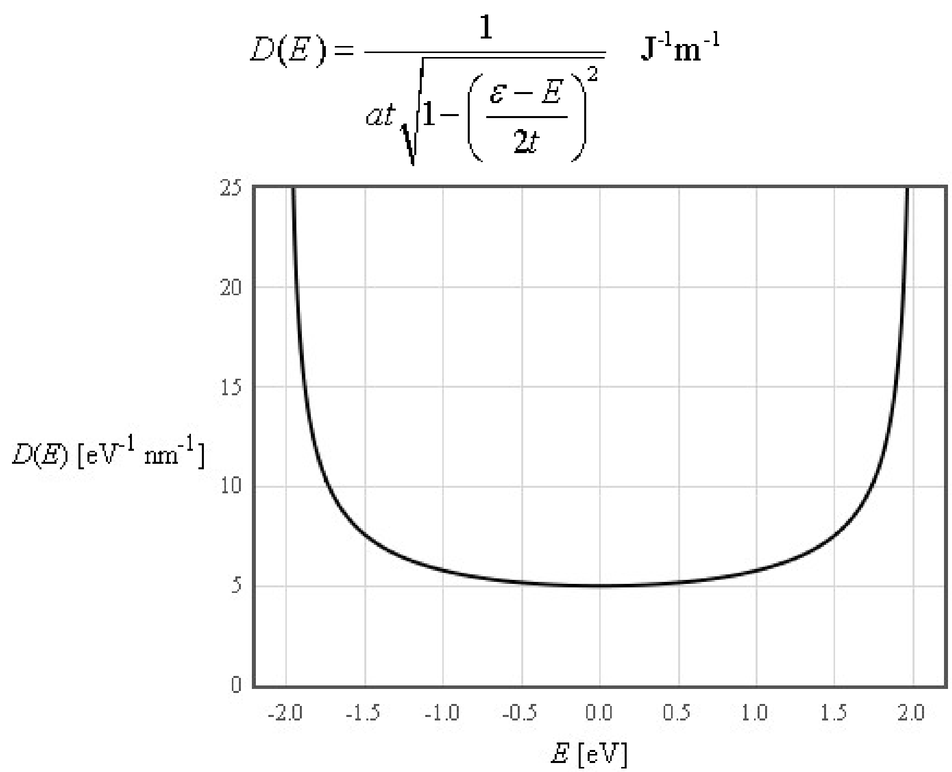
A phonon dispersion relation would be linear at the origin

Magnon density of states

$$\hbar\omega = 4JS\left(1-\cos(ka)\right)$$

Mathematically this is the same problem as the tight binding model for electrons on a one-dimensional chain.





Density of states → Specific heat

The specific heat is the derivative of the internal energy with respect to the temperature.

$$c_v = \left(\frac{\partial u}{\partial T}\right)_{V,N}$$

This can be expressed in terms of an integral over the frequency ω.

$$c_v = \frac{\partial}{\partial T}\int u(\omega)d\omega = \frac{\partial}{\partial T}\int \hbar\omega D(\omega)\frac{1}{e^{\frac{\hbar\omega}{k_BT}}-1}d\omega$$

The Leibniz integral rule can be used to bring the differentiation inside the integral. If the phonon density of states $D(\omega)$ is temperature independent, the result is,

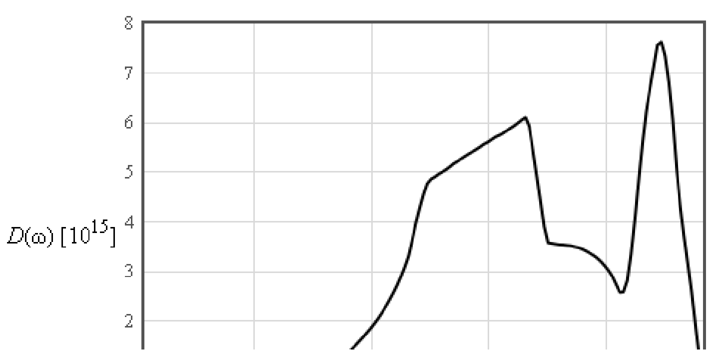
$$c_v = \int \hbar\omega D(\omega)\frac{\partial}{\partial T}\left(\frac{1}{e^{\frac{\hbar\omega}{k_BT}}-1}\right)d\omega$$

Since only the Bose-Einstein factor depends on temperature, the differentiation can be performed analytically and the expression for the specific heat is,

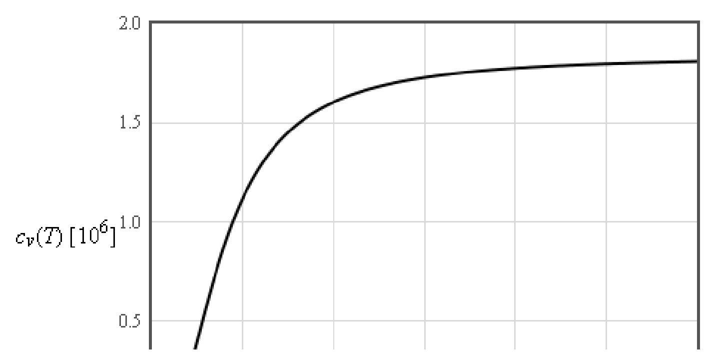
$$c_v = \int \left(\frac{\hbar\omega}{T}\right)^2 \frac{D(\omega)e^{\frac{\hbar\omega}{k_BT}}}{k_B\cdot\left(e^{\frac{\hbar\omega}{k_BT}}-1\right)^2}d\omega$$

The form below uses this formula to calculate the temperature dependence of the specific heat from tabulated data for the density of states. The density of states data is input as two columns in the textbox at the lower left. The first column is the angular-frequency ω in rad/s. The second column is the density of states. The units of the density of states depends on the dimensionality: s/m for 1d, s/m² for 2d, and s/m³ for 3d.

After the 'DoS → cv(T)' button is pressed, the density of states is plotted on the left and $c_V(T)$ is plotted from temperature $T_{\min}$ to temperature $T_{\max}$ on the right. The data for the $c_V(T)$ plot also appear in tabular form in the lower right textbox. The first column is the temperature in Kelvin and the second column is the specific heat in units of J K^{-1} m^{-1}, J K^{-1} m^{-3}, or J K^{-1} m^{-3} depending on the dimensionality.



DoS → cv(T)



Ferromagnetic magnons - simple cubic

The dispersion relation in one dimension:

$$\hbar\omega = 4J\left|S\right|\left(1-\cos(ka)\right)$$

The dispersion relation for a cubic lattice in three dimensions:

$$\hbar\omega = 2J\left|S\right|\left(z-\sum_{\vec{\delta}}\cos(\vec{k}\cdot\vec{\delta})\right)$$

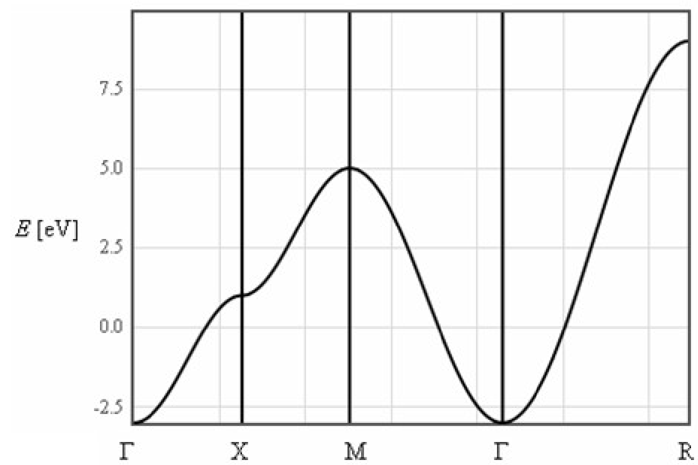
The magnon contribution to thermodynamic properties can be calculated similar to the phonon contribution to the thermodynamic properties.

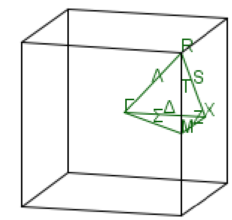
Magnons

simple cubic 3-D

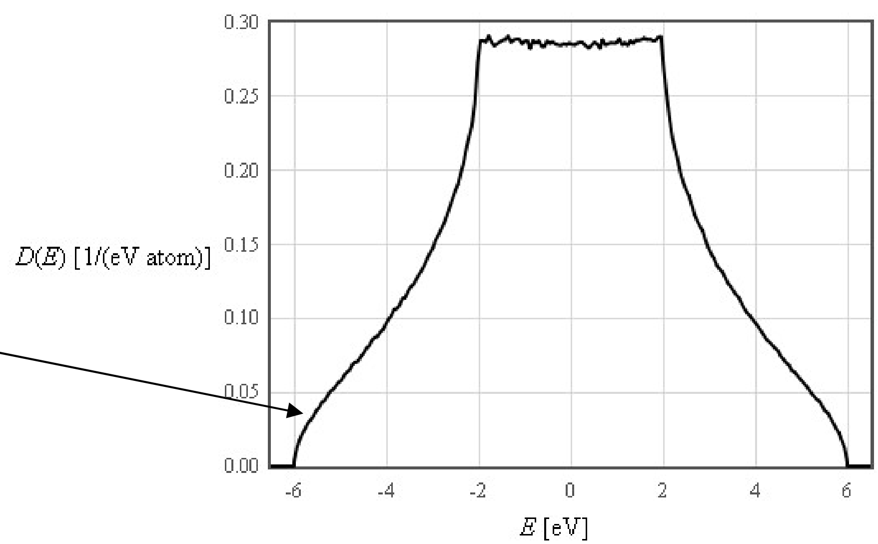
$$E = \varepsilon - 2t\left(\cos(k_x a) + \cos(k_y a) + \cos(k_z a)\right)$$

$$\hbar\omega = 2J\left|S\right|\left(z - \sum_{\vec{\delta}} \cos(\vec{k}\cdot\vec{\delta})\right)$$





Dispersion relation is mathematically equivalent to tight binding model for electrons.



$\sqrt{\omega}$

Long wavelength / low temperature limit

Dispersion relation: $\hbar\omega \approx 2JSk^2a^2$

The density of states: $D(\omega) \propto \sqrt{\omega}$

Magnons are bosons: $\langle n_k \rangle = \dfrac{1}{\exp\left(\dfrac{\hbar\omega_k}{k_B T}\right) - 1}$

$$u = \int_0^\infty \frac{\hbar\omega D(\omega) d\omega}{\exp\left(\dfrac{\hbar\omega_k}{k_B T}\right) - 1} \propto T^{5/2}$$

$$c_v \propto T^{3/2}$$

Magnons

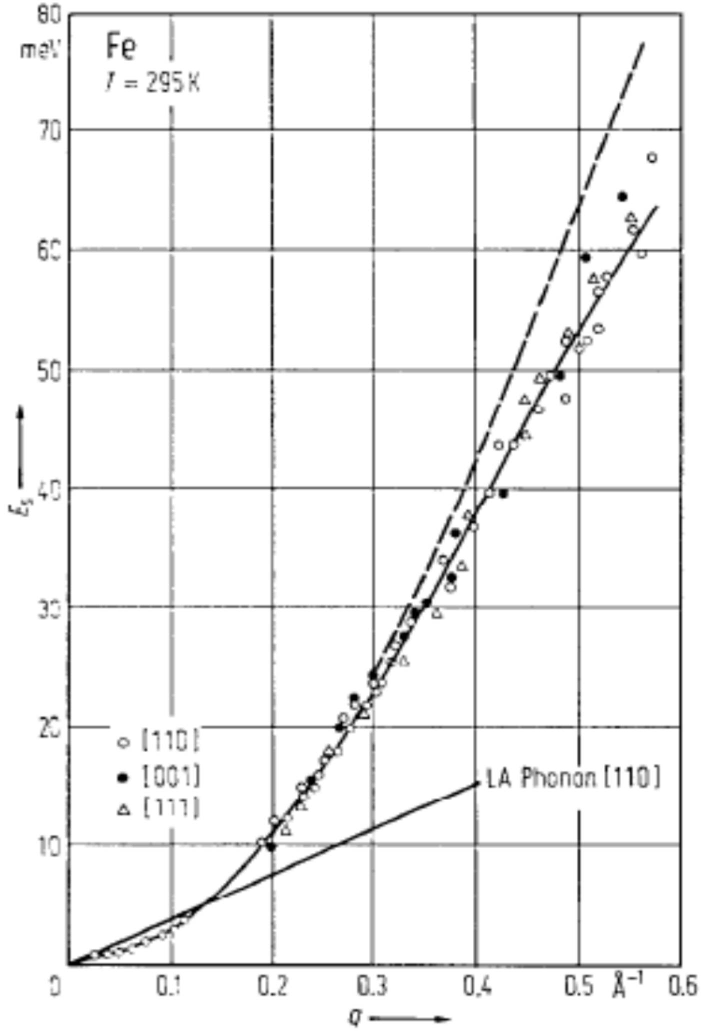


Fig. 1. Constant-E scan TAS-measured spin wave dispersion relation for various directions in a single crystal of Fe at 295 K. The dashed line corresponds to the Heisenberg model with $D = 281$ meV Å^2 and $\beta = 1.0$ Å^2 [68 S 3], see also [73 M 1].

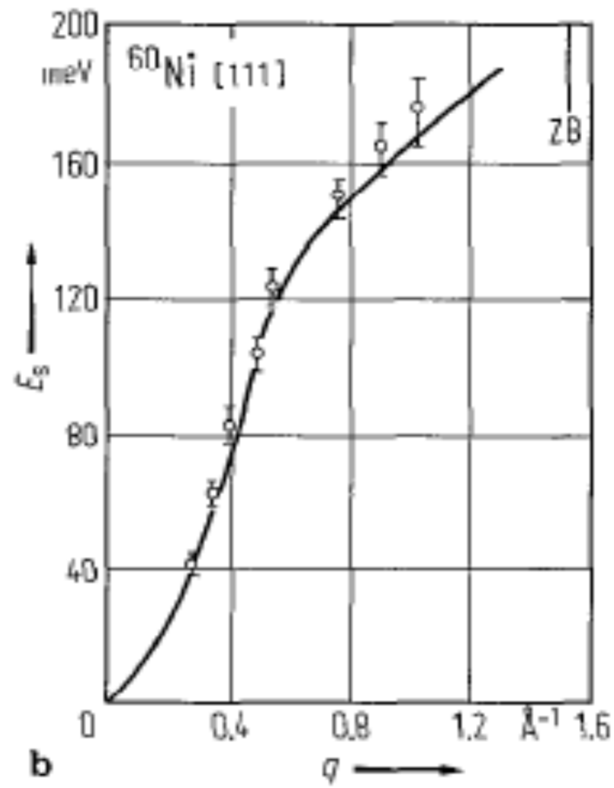
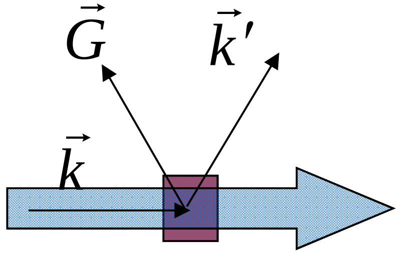


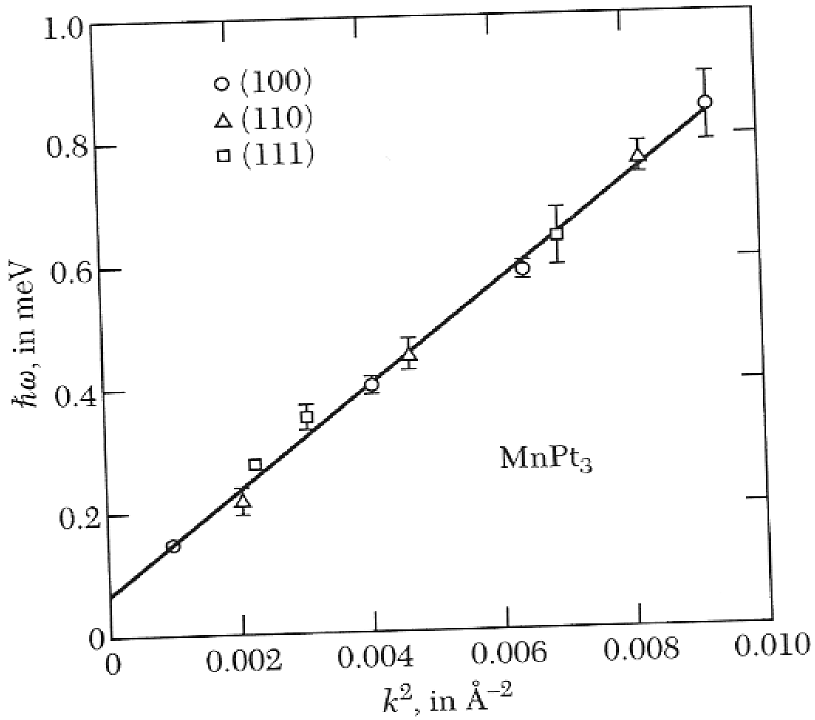
Fig. 6b. Room-temperature spin wave dispersion curve for the [111] direction of ^{60}Ni. ZB shows the position of the zone boundary [85 M 1]. The solid curve is from calculations [85 C 1, 83 C 1].

Neutron magnetic scattering

Neutrons can scatter inelastically from magnetic material and create or annihilate magnons



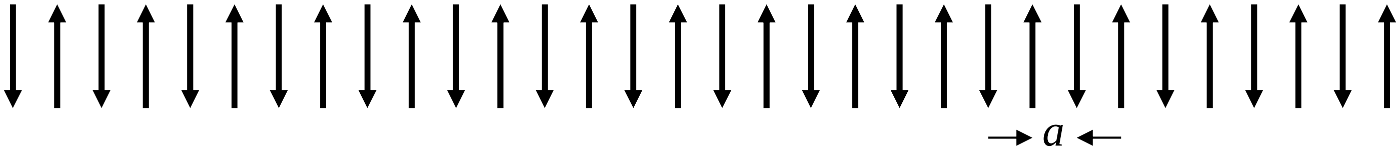
$$\vec{k}_n = \vec{k}_n{}' + \vec{k}_{magnon} + \vec{G}$$



$$\frac{\hbar^2 k'^2}{2m_n} \pm \hbar\omega_{magnon} = \frac{\hbar^2 k^2}{2m_n} + \frac{\hbar^2 G^2}{2m_{crystal}}$$

0

Antiferromagnet magnons



$$\hbar \frac{dS_p^{Ax}}{dt} = 2J\left|S\right|\left(-S_p^{By} - 2S_p^{Ay} - S_{p-1}^{By}\right)$$

$$\hbar \frac{dS_p^{Ay}}{dt} = -2J\left|S\right|\left(-S_p^{Bx} - 2S_p^{Ax} - S_{p-1}^{Bx}\right)$$

$$\hbar \frac{dS_p^{Bx}}{dt} = 2J\left|S\right|\left(S_{p+1}^{Ay} + 2S_p^{By} + S_p^{Ay}\right)$$

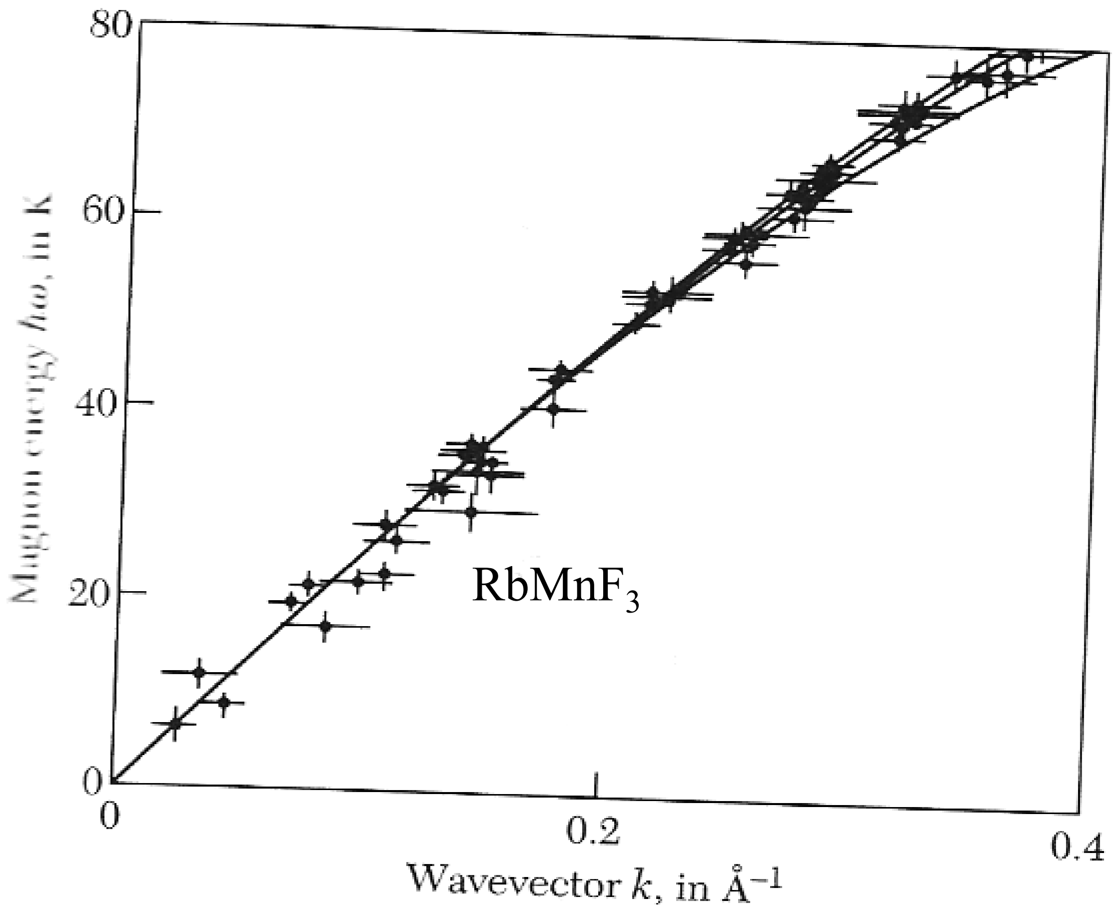
$$\hbar \frac{dS_p^{By}}{dt} = -2J\left|S\right|\left(S_{p+1}^{Ax} + 2S_p^{Bx} + S_p^{Ax}\right)$$

$$\hbar \frac{dS_p^{Az}}{dt} = 0 \qquad \hbar \frac{dS_p^{Bz}}{dt} = 0$$

$$\begin{pmatrix} S_p^{Ax} \\ S_p^{Ay} \\ S_p^{Bx} \\ S_p^{By} \end{pmatrix} = \begin{pmatrix} u_k^{Ax} \\ u_k^{Ay} \\ u_k^{Bx} \\ u_k^{By} \end{pmatrix} \exp\left[i\left(2kpa - \omega t\right)\right]$$

Antiferromagnet magnons

$$\hbar\omega = 4|J|S|\sin(ka)|$$



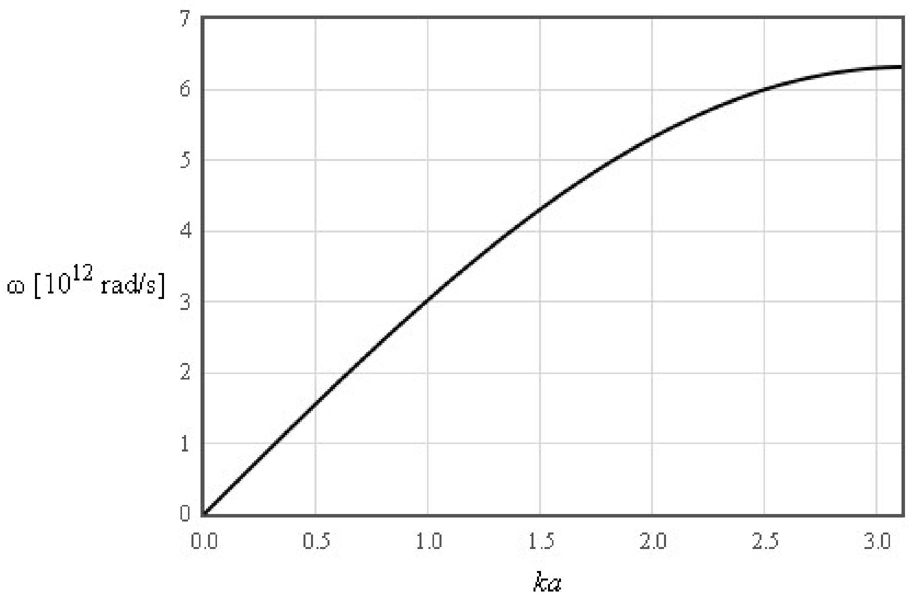
Brillouin zone boundary is at $k = \pi/2a$

Antiferromagnet magnons

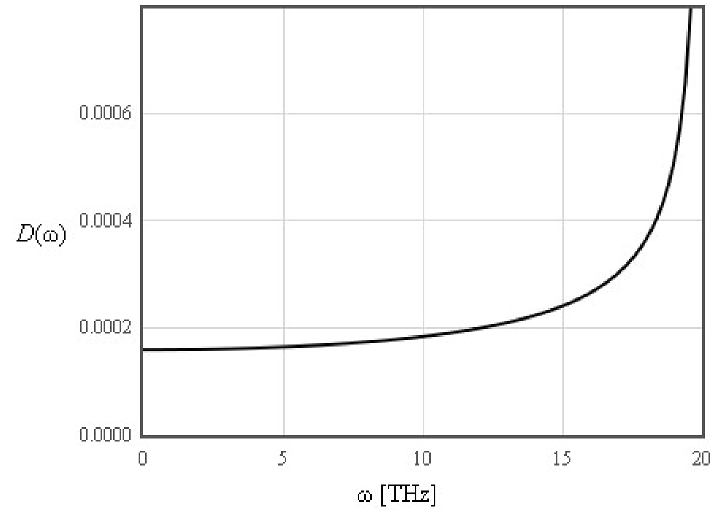
$$\hbar\omega = 4\left|J\right|S\left|\sin(ka)\right|$$

Mathematically equivalent to phonons in 1-d

$$\omega = \sqrt{\frac{4C}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|$$



$$D(\omega) = \frac{1}{\pi a\sqrt{\frac{C}{m}}\sqrt{1-\frac{\omega^2 m}{4C}}}$$



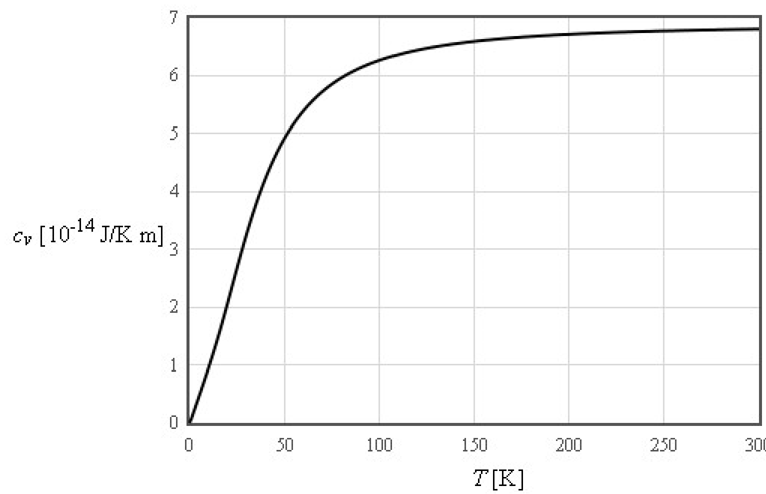
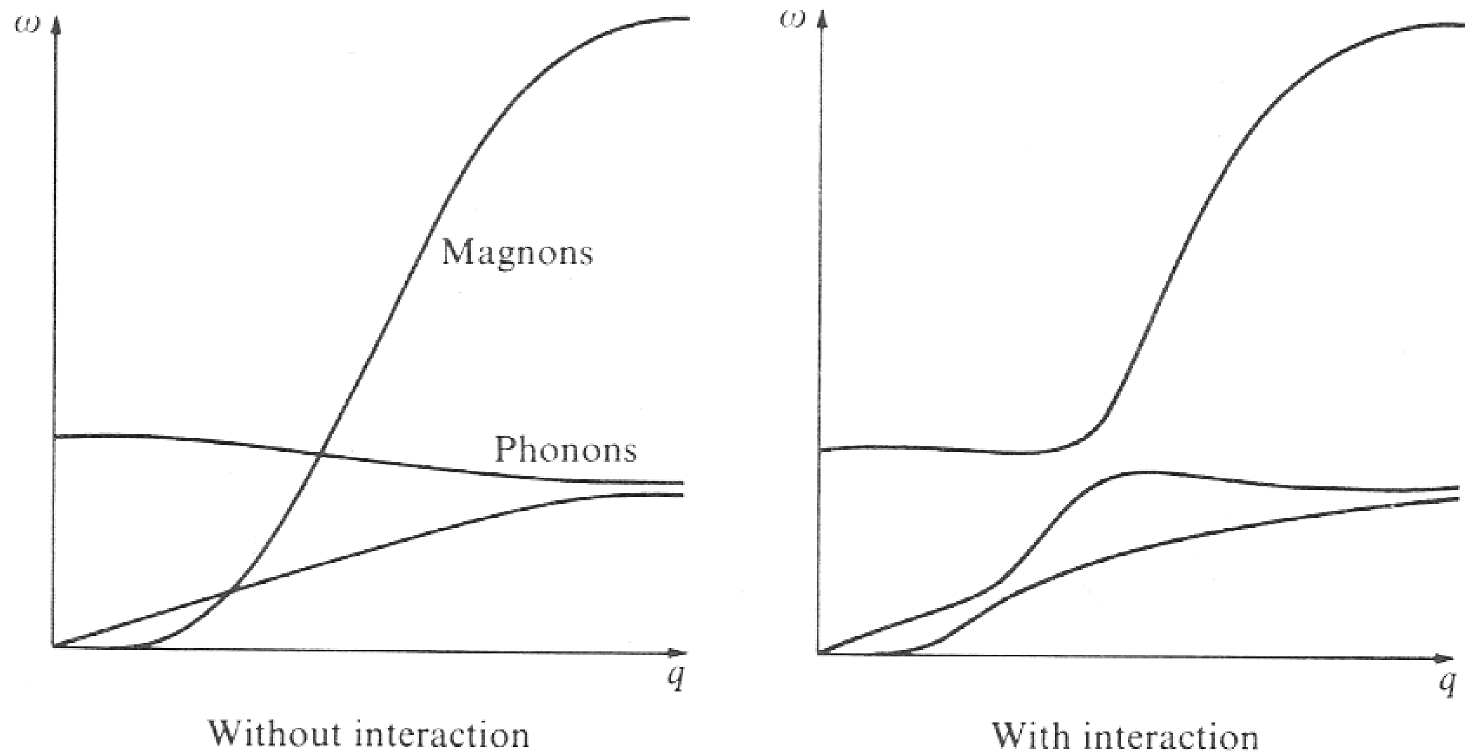


Fig. 5.7 Schematic magnon and phonon dispersion curves. The magnon curve has been compressed by a factor of order 10 for illustrative purposes.

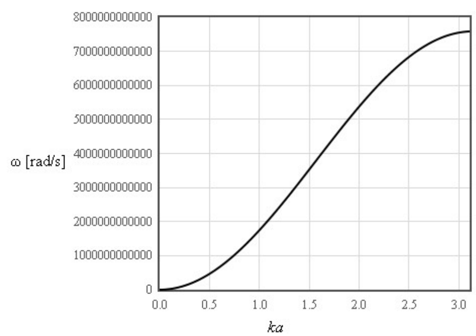
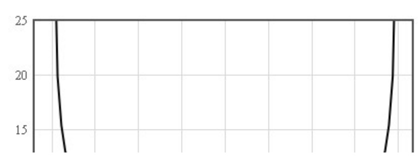


From: *Solid State Theory*, Harrison

Student project

Make a table of magnon properties like the table of phonon properties

Magnons

	1-D ferromagnetic magnons	1-D antiferromagnetic magnons	3-D low temperature limit	
Equations of motion in mean field theory				
Eigenfunction solutions	$\begin{pmatrix} S_p^x \\ S_p^y \end{pmatrix} = \begin{pmatrix} u_k^x \\ u_k^y \end{pmatrix} \exp\left[i\left(kpa - \omega t\right)\right]$			
Dispersion relation	$\hbar\omega = 4JS\left(1-\cos(ka)\right)$  Calculate ω(k)			
				$D(E)$

Fe bcc
Ni fcc
Co hcp

1 student / column