Landau theory of phase transitions

A phase transition is associated with a broken symmetry.

magnetism	direction of magnetization
cubic - tetragonal	different point group
water - ice	translational symmetry
ferroelectric	direction of polarization
superconductivity	gauge symmetry

Lev Landau

Landau theory: order parameter

At a phase transition, an order parameter can be defined that is zero above the phase transition and nonzero below the phase transition.

Ferromagnetism	Magnetization
Ferroelectricity	Polarization
Superconductivity	Superconducting order parameter
Peierls Transition	amplitude of 2*a* distortion, gap
cubic-tetragonal	c/a-1
structural	diffraction peak

1st and 2nd order phase transitions

First order:
There is a latent heat
order parameter increases discontinuously

water - ice

$$L = T(S_A - S_B)$$

Second order:
No latent heat
order parameter increases continuously from zero

superfluidity
superconductivity
ferromagnetism
ferroelectricity
Peierls transition

Landau theory of 2nd order phase transitions

Since the order parameter goes to zero at the phase transition, expand the free energy in terms of the order parameter.

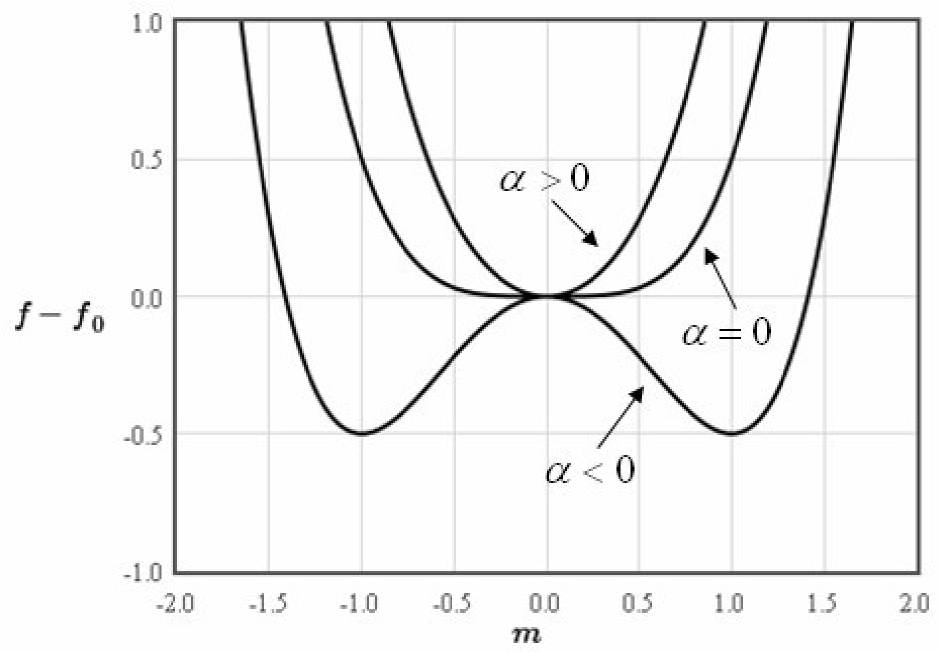
$$f = f_0 + \alpha m^2 + \tfrac{1}{2}\beta m^4 + \cdots \qquad \beta > 0$$

Minimize the free energy.

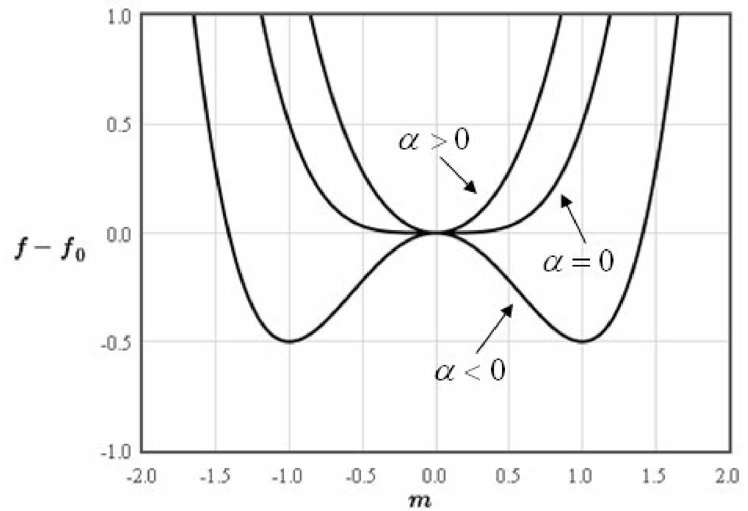
$$\frac{df}{dm} = 2\alpha m + 2\beta m^3 + \cdots = 0$$

$$m = \pm\sqrt{\frac{-\alpha}{\beta}} \qquad T < T_c$$

$$m = 0 \qquad T > T_c$$



Temperature dependence of the order parameter



At $T=T_c$ $\alpha = 0$

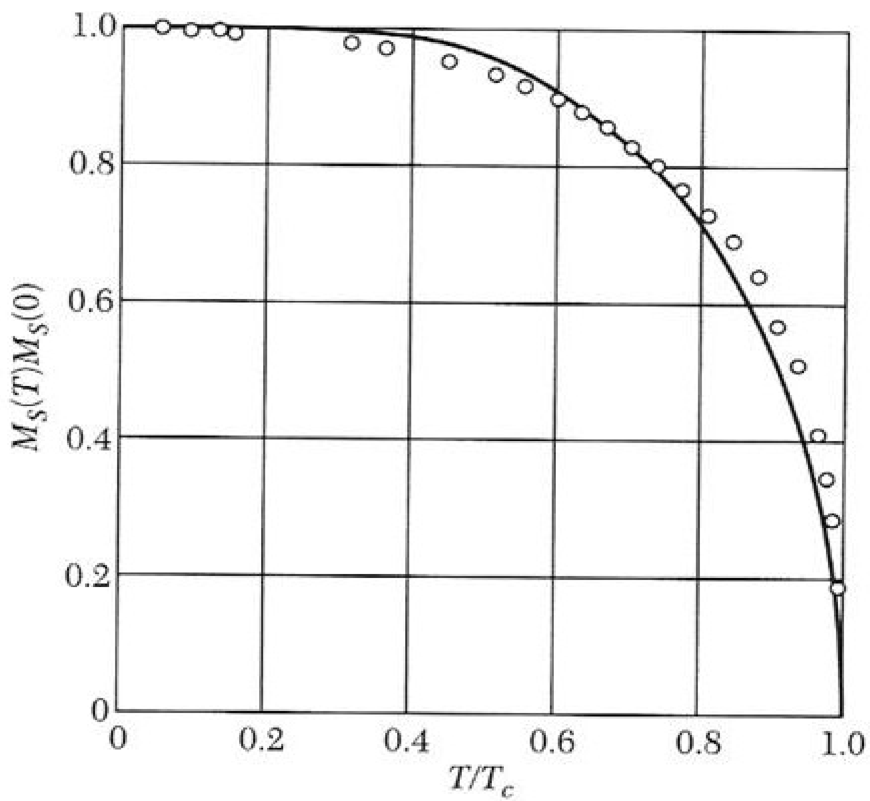
Expand α interms of T - T_c. Keep only the linear term. m and T - T_c are both small near Tc.

$$f = f_0 + \alpha_0 \left(T - T_c\right) m^2 + \tfrac{1}{2} \beta m^4 + \cdots$$

The temperature dependence of the magnetization is

$$m = \pm \sqrt{\frac{\alpha_0 \left(T_c - T\right)}{\beta}} \qquad T < T_c$$

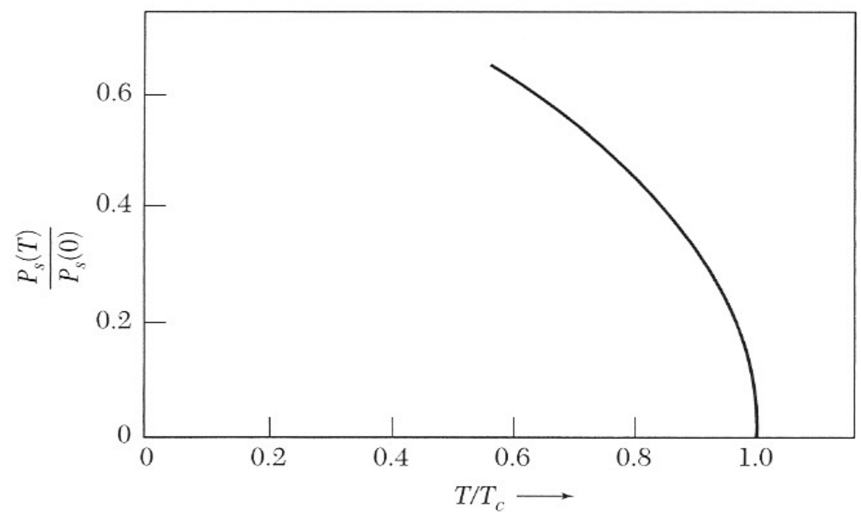
Landau theory of phase transitions

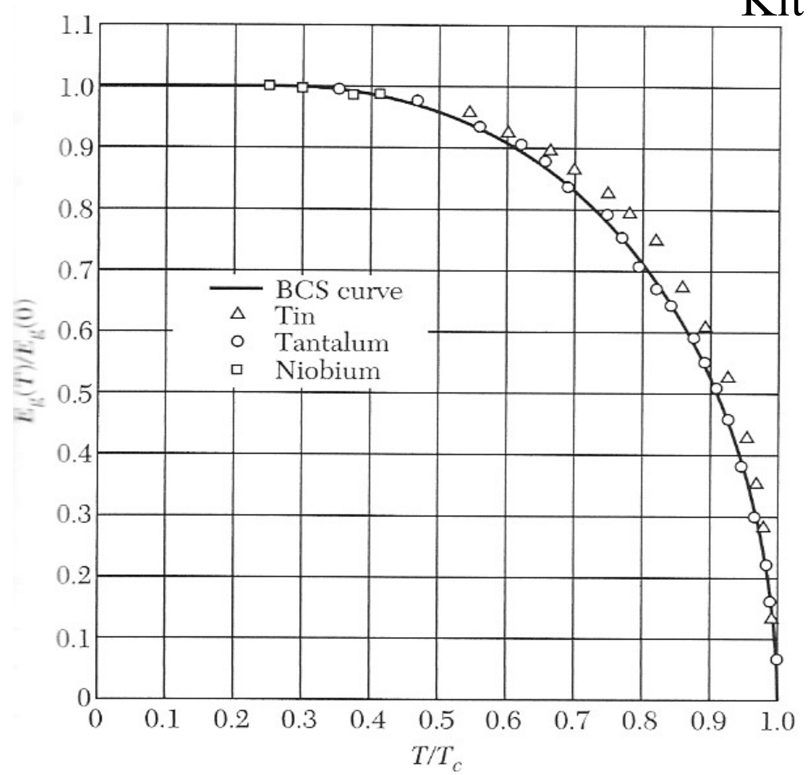


$$m = \sqrt{\frac{\alpha_0 \left(T_c - T\right)}{\beta}} \qquad T < T_c$$

Landau theory of phase transitions

Kittel





$$P = \sqrt{\frac{\alpha_0 \left(T_c - T\right)}{\beta}} \qquad T < T_c$$

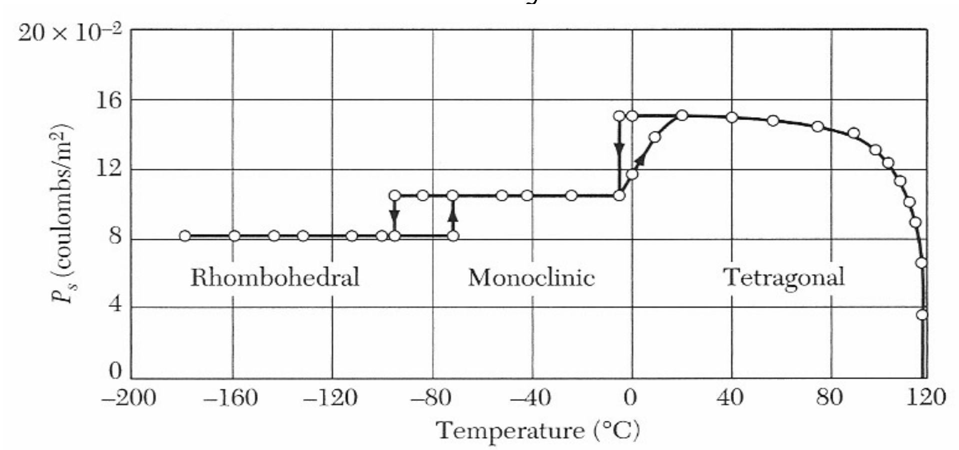
ferroelectricity

superconductivity

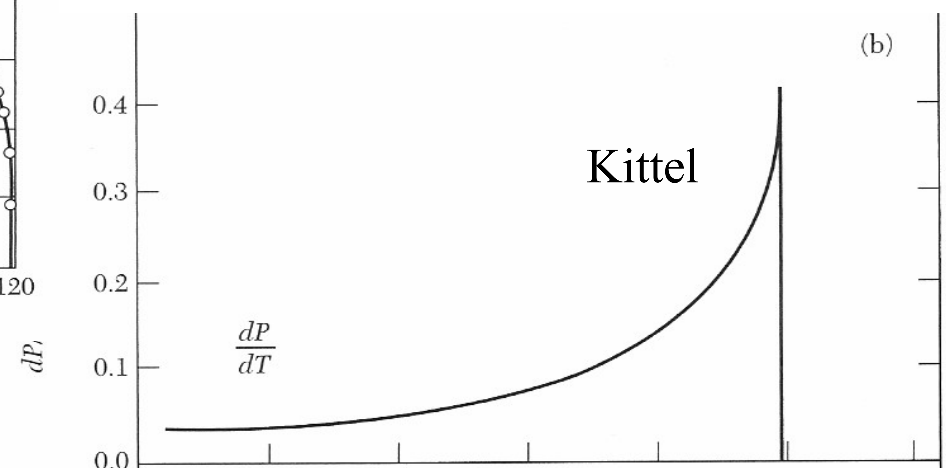
The gap in a Peierls transition has this form

Landau theory of phase transitions

ferroelectricity



Pyroelectric coefficient



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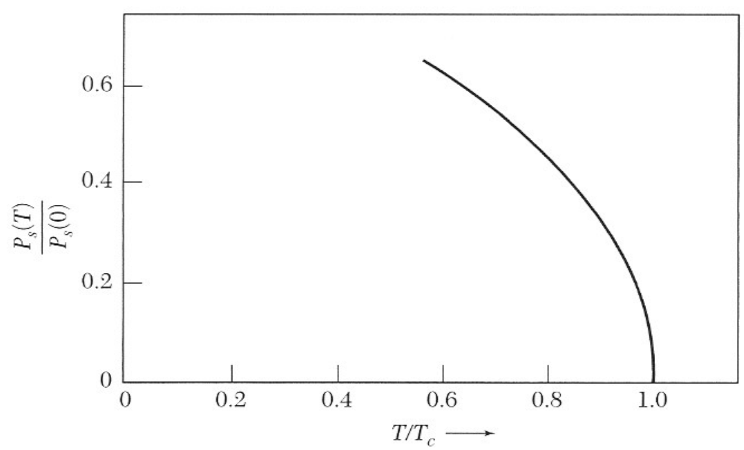
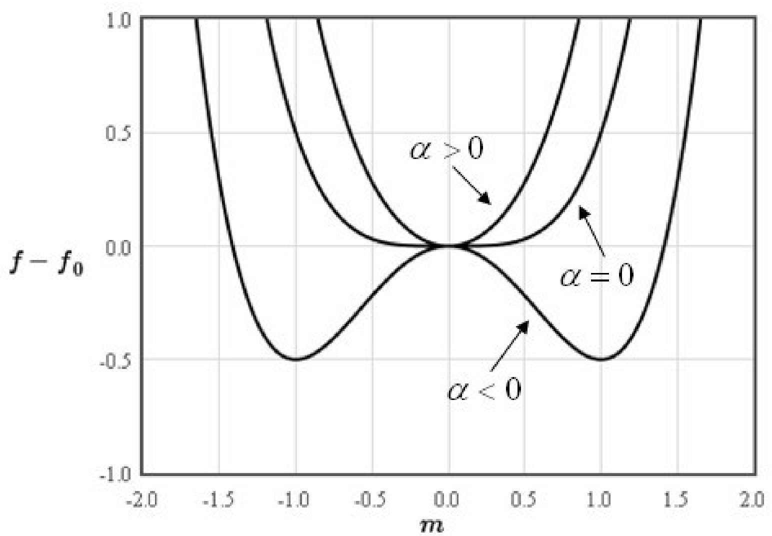


$$P = \sqrt{\frac{\alpha_0 (T_c - T)}{\beta}} \qquad T < T_c$$

$$\frac{dP}{dT} = \frac{1}{2}\sqrt{\frac{\alpha_0}{\beta(T_c - T)}} \qquad T < T_c$$

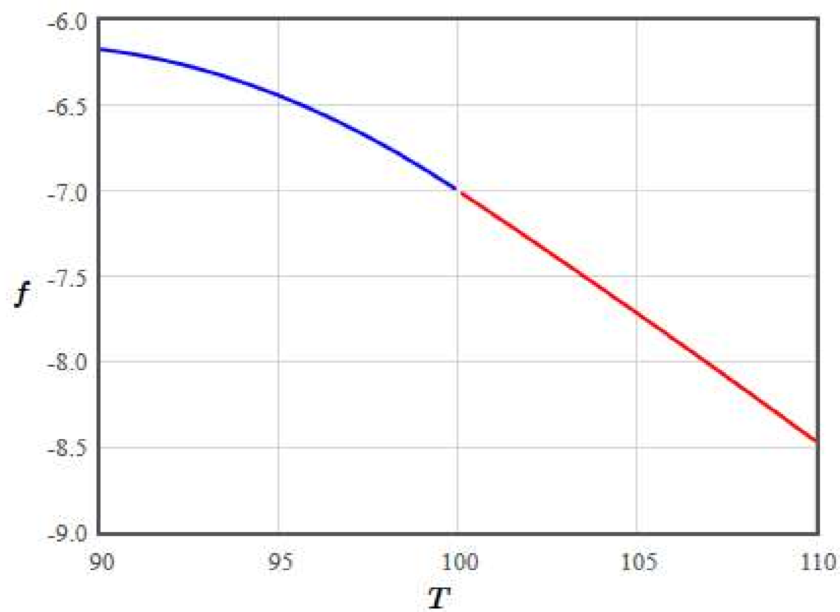
can also calculate the pyromagnetic coefficient

Free energy



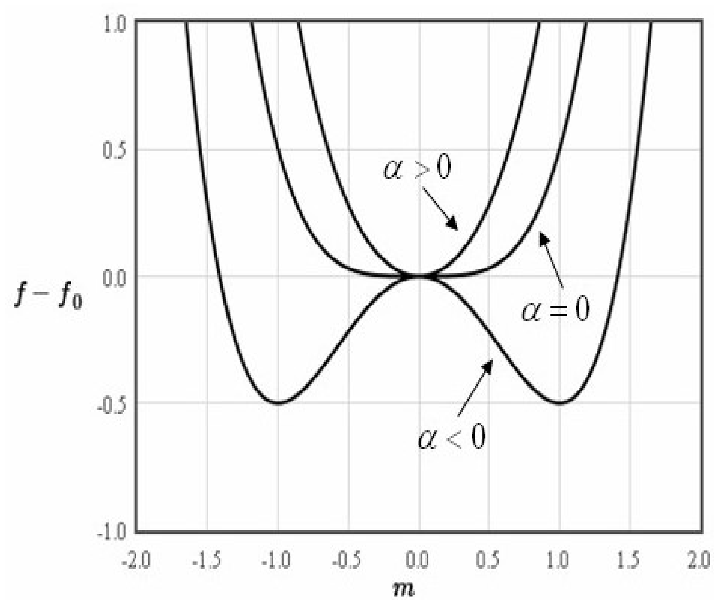
$$f = f_0 + \alpha_0 \left(T - T_c\right) m^2 + \tfrac{1}{2} \beta m^4 + \cdots$$

$$m = \pm\sqrt{\frac{\alpha_0 \left(T_c - T\right)}{\beta}} \qquad T < T_c$$



$$f = f_0 - \frac{\alpha_0^2 \left(T - T_c\right)^2}{\beta} + \cdots$$

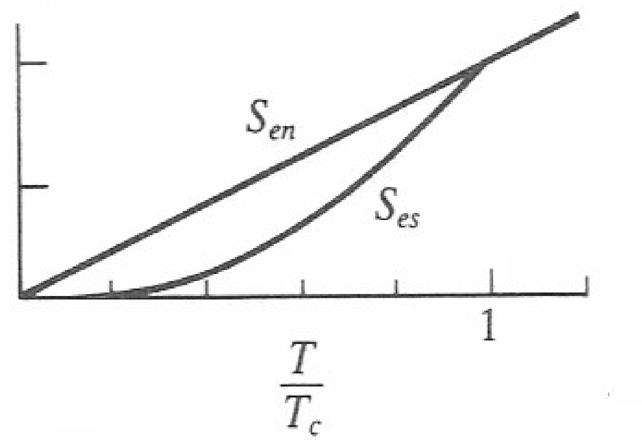
Entropy



$$f = f_0(T) - \frac{\alpha_0^2 (T - T_c)^2}{\beta} + \cdots$$

$$s = -\frac{\partial f}{\partial T} = s_0(T) + \frac{2\alpha_0^2 (T - T_c)}{\beta} + \cdots$$

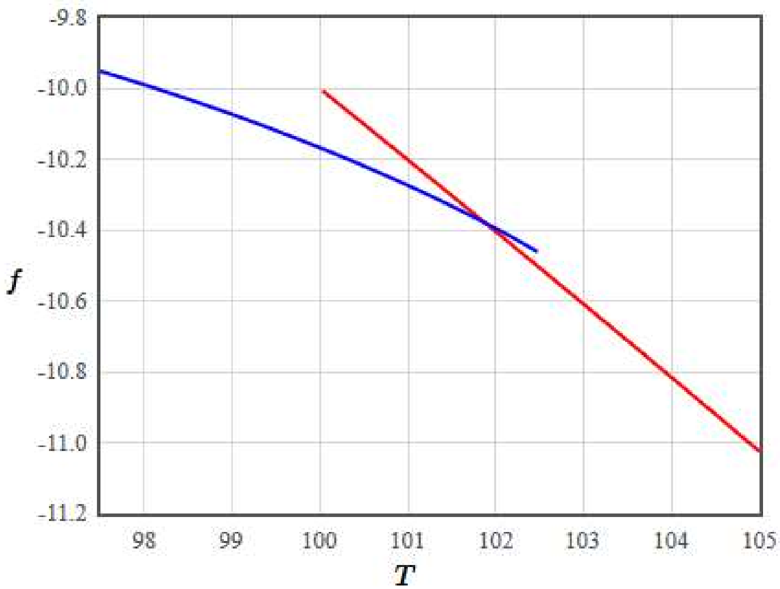
Kink in the entropy

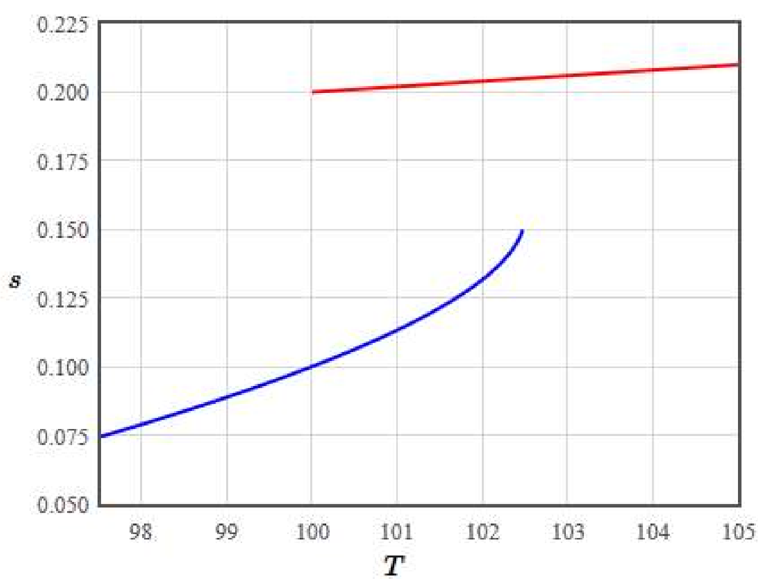


$$L = T\left(S_A - S_B\right) = 0$$

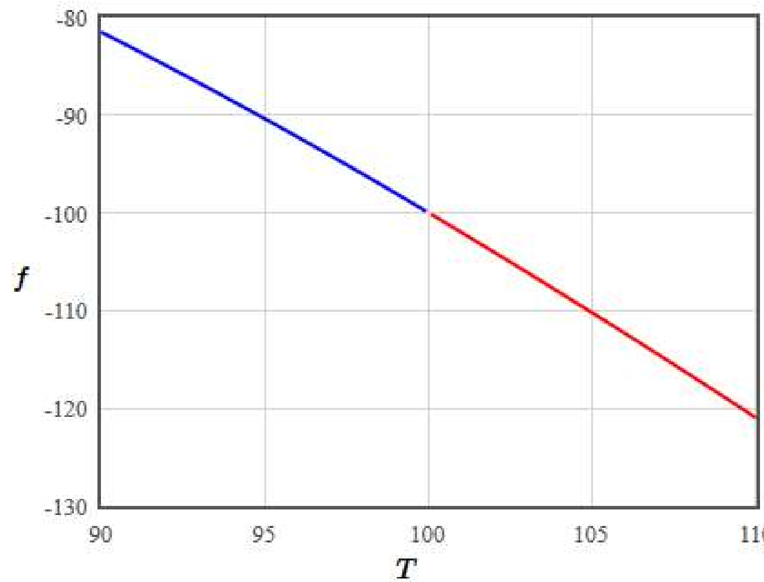
This is a second order phase transition

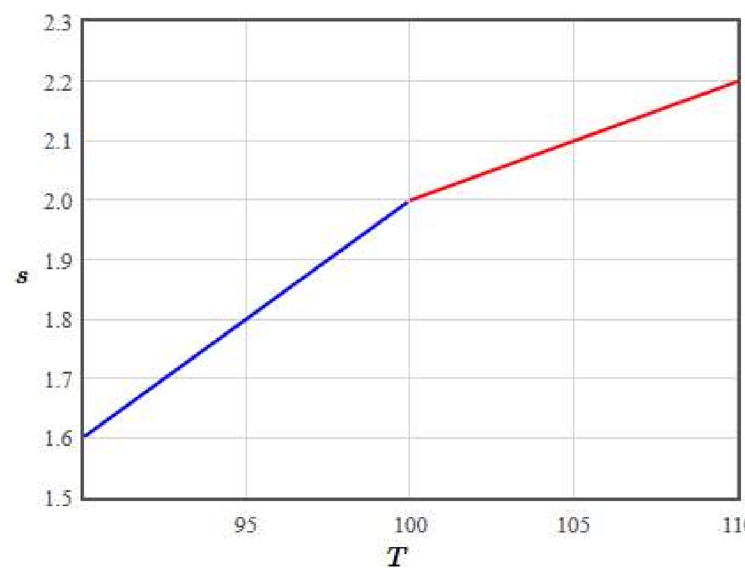
1st order



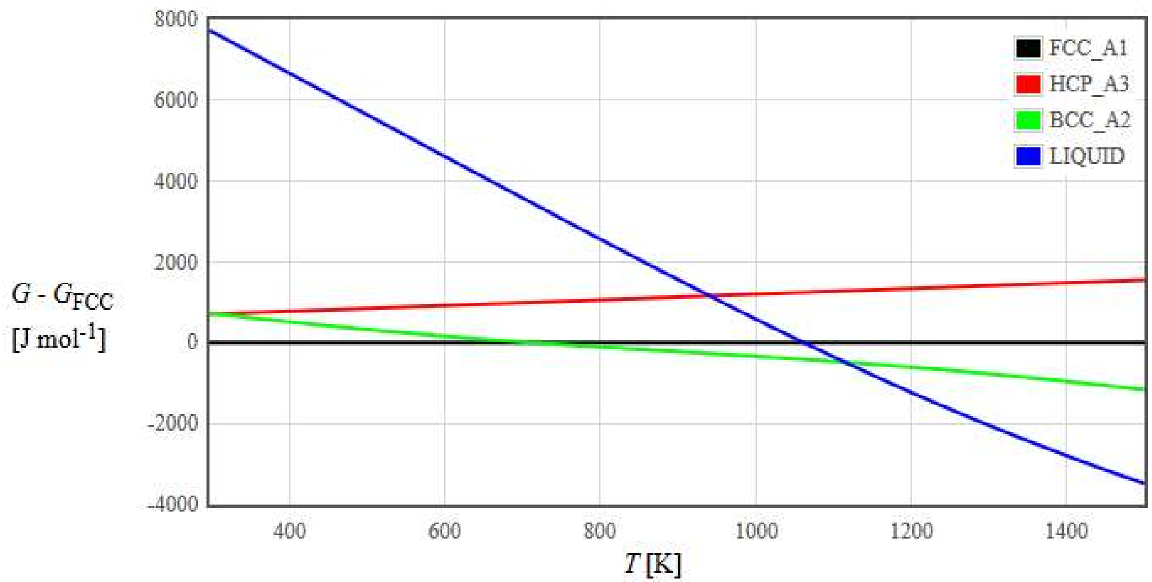


2nd order

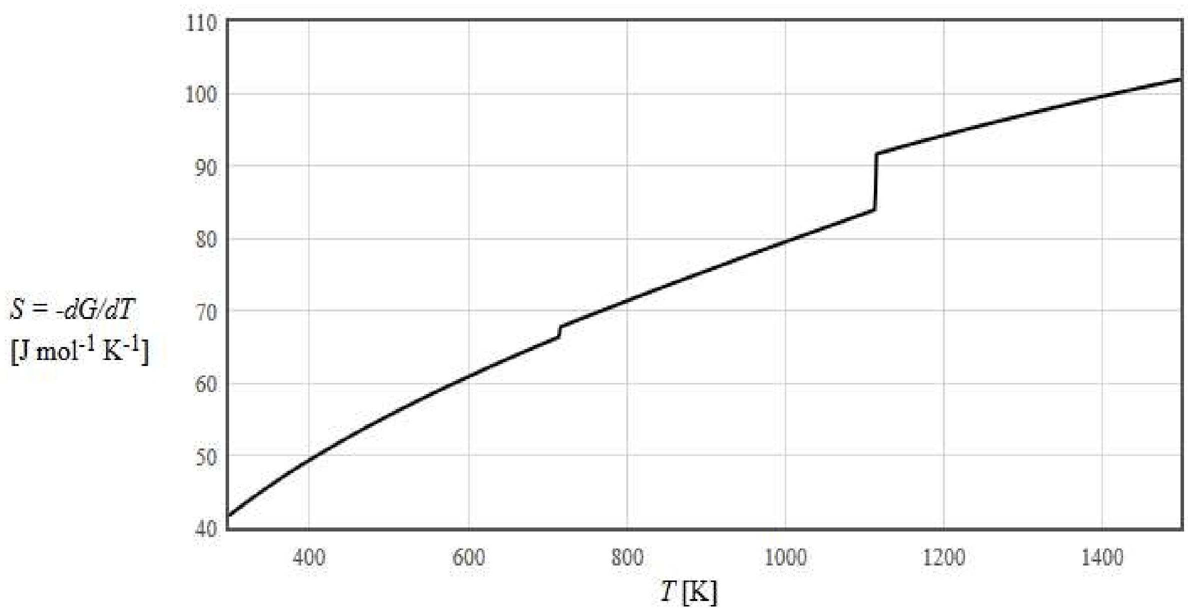




Ca



FCC_A1
HCP_A3
BCC_A2
LIQUID
8000
6000
4000
2000
0
-2000
-4000
400
600
800
1000
1200
1400
G - G_FCC
[J mol-1]
T [K]

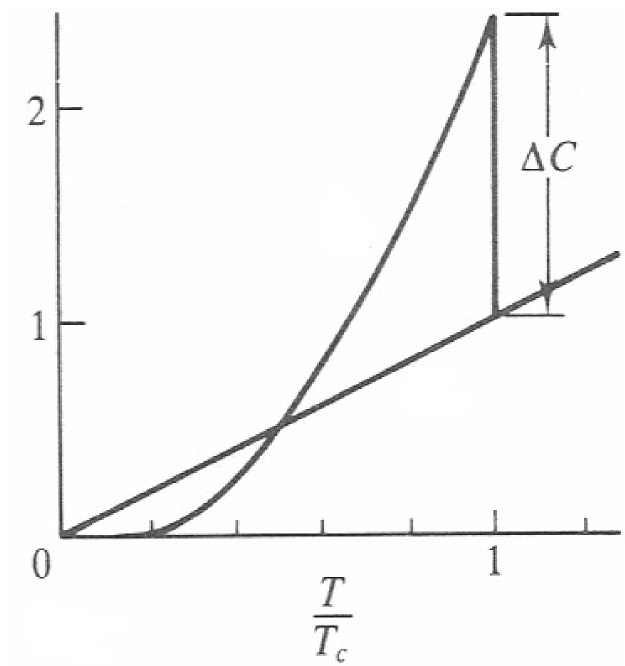


110
100
90
80
70
60
50
40
400
600
800
1000
1200
1400
S = -dG/dT
[J mol-1 K-1]
T [K]

Specific heat

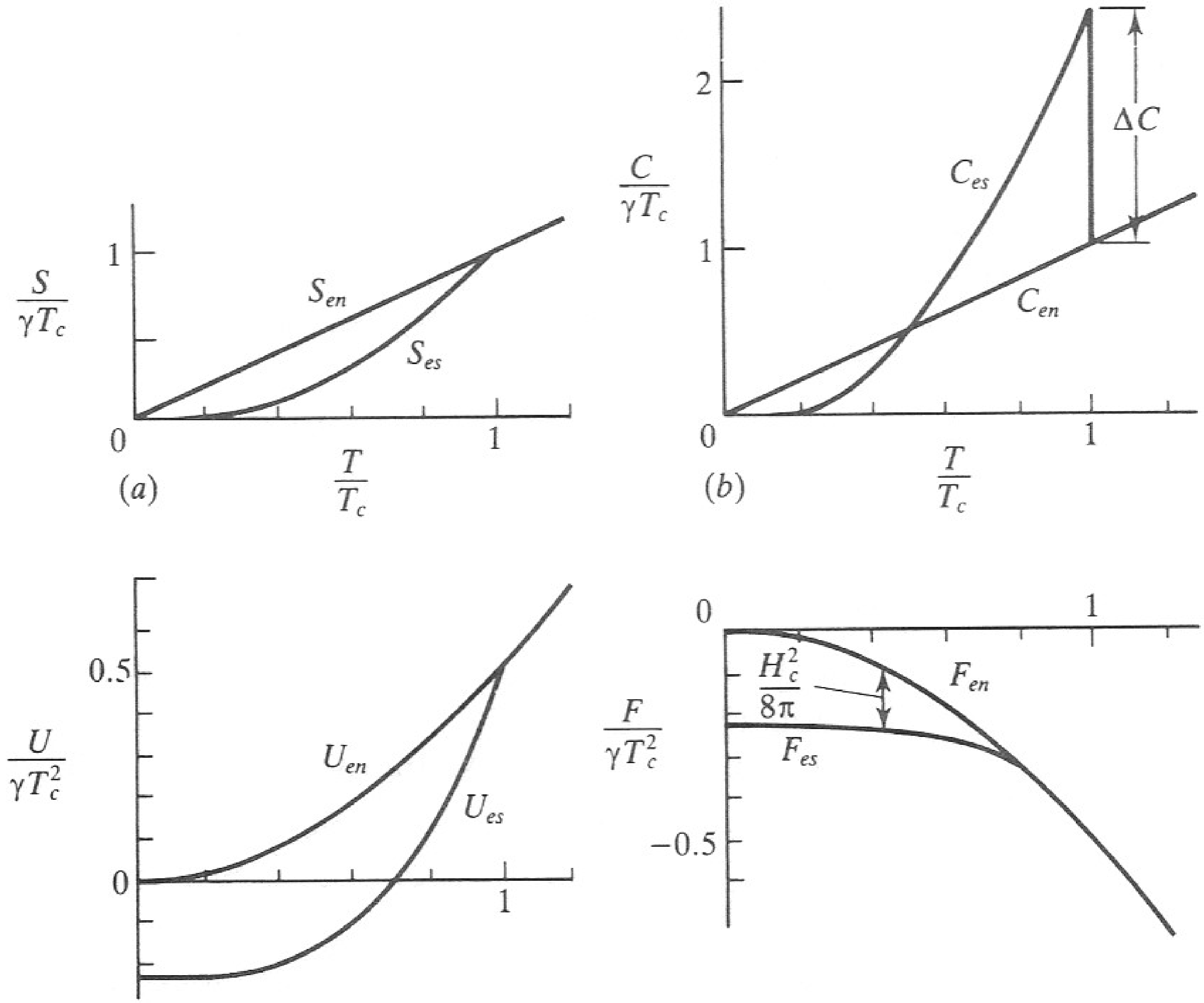
Entropy $$s = -\frac{\partial f}{\partial T} = s_0\left(T\right) + \frac{2\alpha_0^2\left(T - T_c\right)}{\beta} + \cdots$$

Specific heat $$c_v = T\frac{\partial s}{\partial T} = c_0\left(T\right) + \frac{2\alpha_0^2 T}{\beta} + \cdots \qquad T < T_c$$



There is a jump in the specific heat at the phase transition and then a linear dependence after the jump.

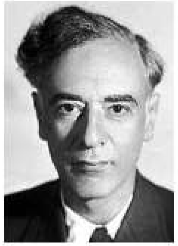
$$\Delta c_v = \frac{2\alpha_0^2 T_c}{\beta}$$



Introduction to Superconductivity, Tinkham

Outline
Quantization
Photons
Electrons
Magnetic effects and Fermi surfaces
Linear response
Transport
Crystal Physics
Electron-electron interactions
Quasiparticles
Structural phase transitions
Landau theory of second order phase transitions
Superconductivity
Exam questions
Appendices
Lectures
Books
Course notes
TUG students
Making presentations

Landau theory of second order phase transitions



Lev Landau

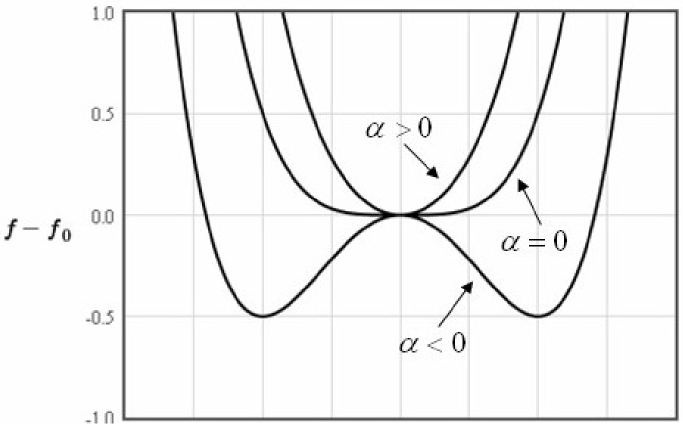
Normally, to calculate thermodynamic properties like the free energy, the entropy, or the specific heat, it is necessary to determine the microscopic states of system by solving the Schrödinger equation. For crystals, the microscopic states are labeled by k and the solutions of the Schrödinger equation are typically expressed as a dispersion relation where the energy is given for each k. The dispersion relation can be used to calculate the density of states and the density of states can be used to calculate the thermodynamic properties. This is typically a long and numerically intensive calculation.

Landau realized that near a phase transition an approximate form for the free energy can be constructed without first calculating the microscopic states. He recognized it is always possible to identify an order parameter the is zero on the high temperature side of the phase transition and nonzero on the low temperature side of the phase transition. For instance, the magnetization can be considered the order parameter at a ferromagnetic - paramagnetic phase transition. For a structural phase transition from a cubic phase to a tetragonal phase, the order parameter can be taken to be c/a - 1 where c is the length of the long side of the tetragonal unit cell and a is the length of the short side of the tetragoal unit cell.

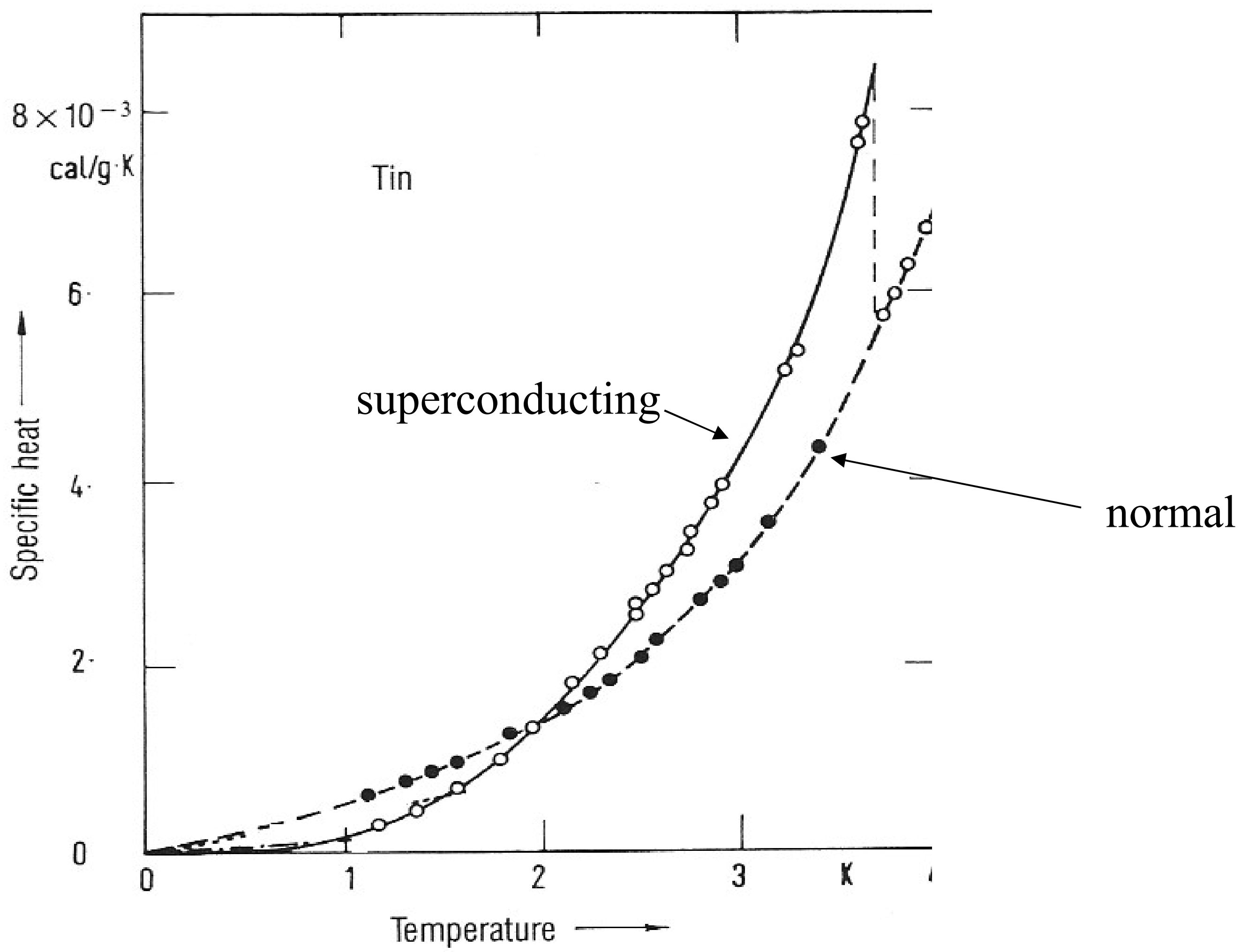
At a second order phase transition, the order parameter increases continuously from zero starting at the critical temperature of the phase transition. An example of this is the continuous increase of the magnetization at a ferromagnetic - paramagnetic phase transition. Since the order parameter is small near the phase transition, to a good approximation the free energy of the system can be approximated by the first few terms of a Taylor expansion of the free energy in the order parameter.

$$f(T) = f_0(T) + \alpha m^2 + \tfrac{1}{2}\beta m^4 \qquad \alpha_0 > 0, \quad \beta > 0.$$

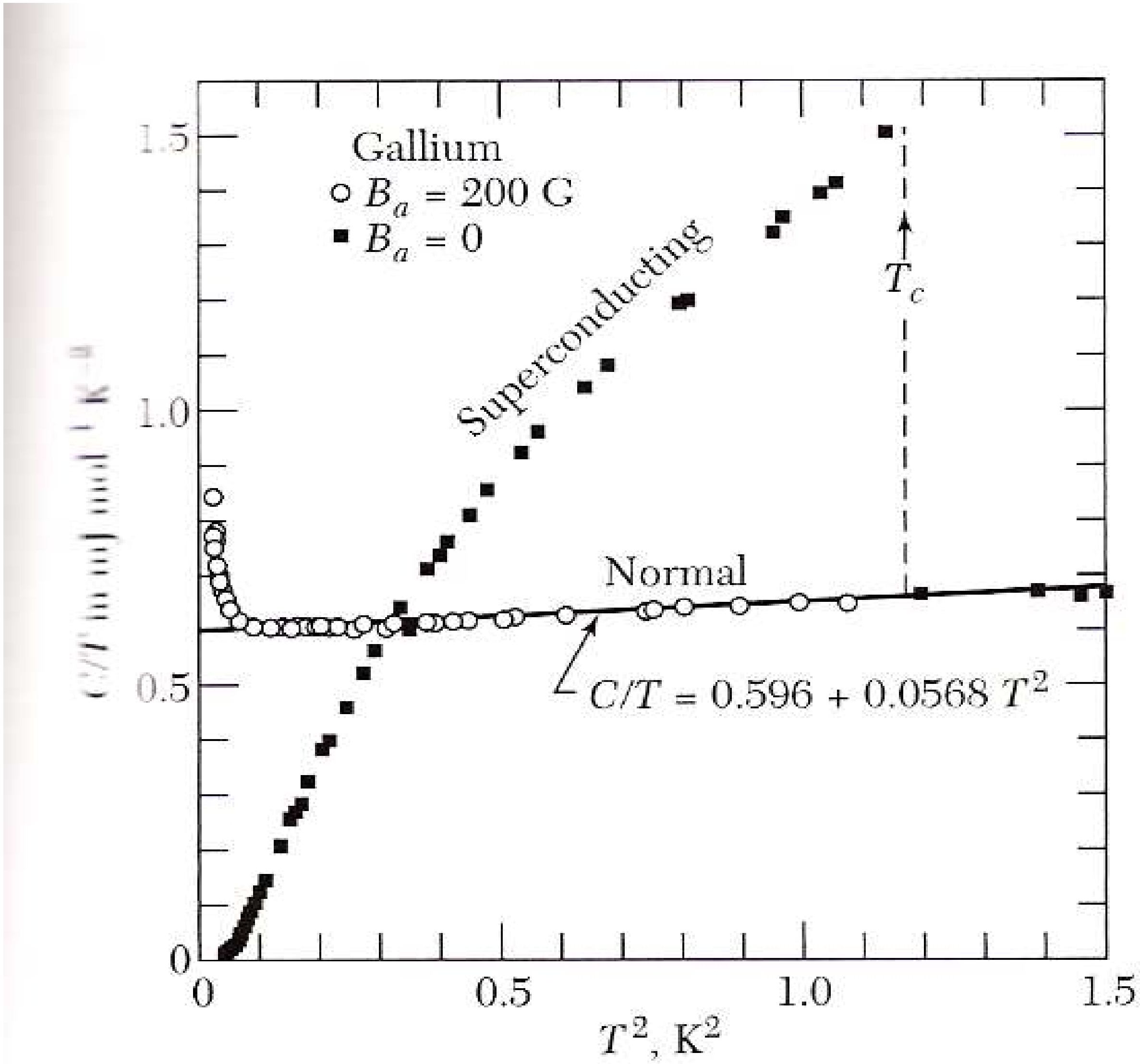
Here m is the order parameter, α and β are parameters, and $f_0(T)$ describes the temperature dependence of the high temperature phase near the phase transition. It is assumed that $\beta > 0$ so that the free energy has a minimum for finite values of the order parameter. When $\alpha > 0$, there is only one minimum at $m = 0$. When $\alpha < 0$ there are two minima with $m \neq 0$.



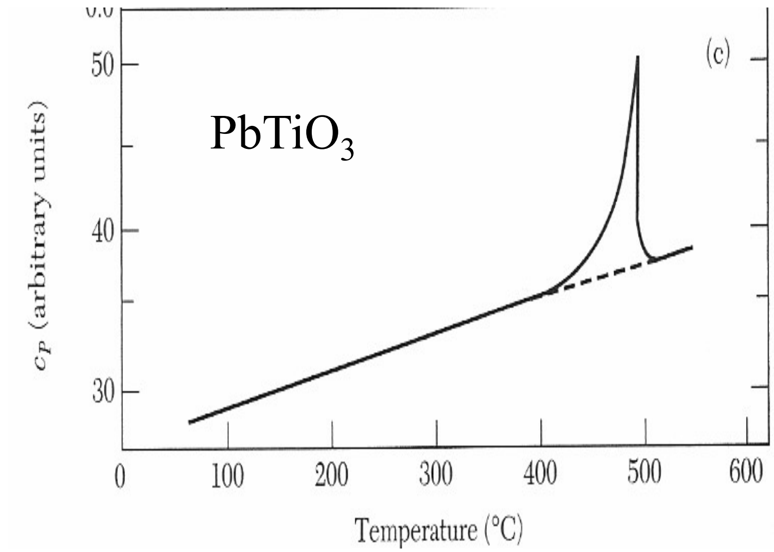
Specific heat

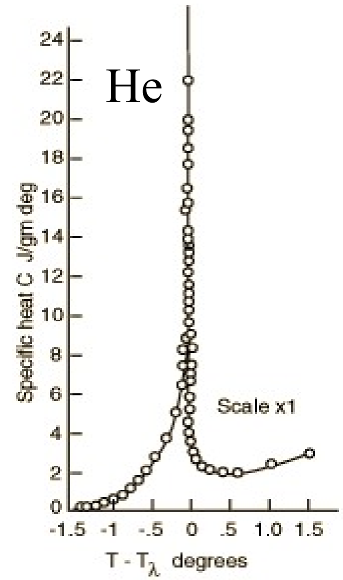


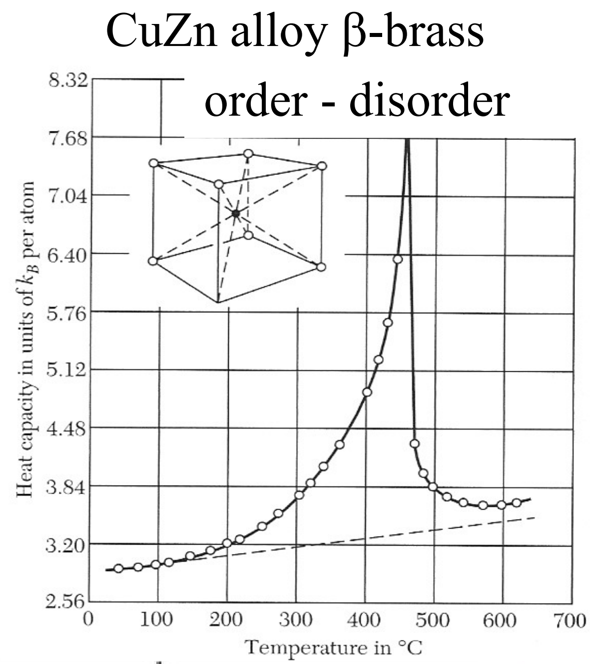
Specific heat



Specific heat







BaTiO$_3$. Heat capacity vs. temperature [76H].