Landau theory, susceptibility

Add a magnetic field

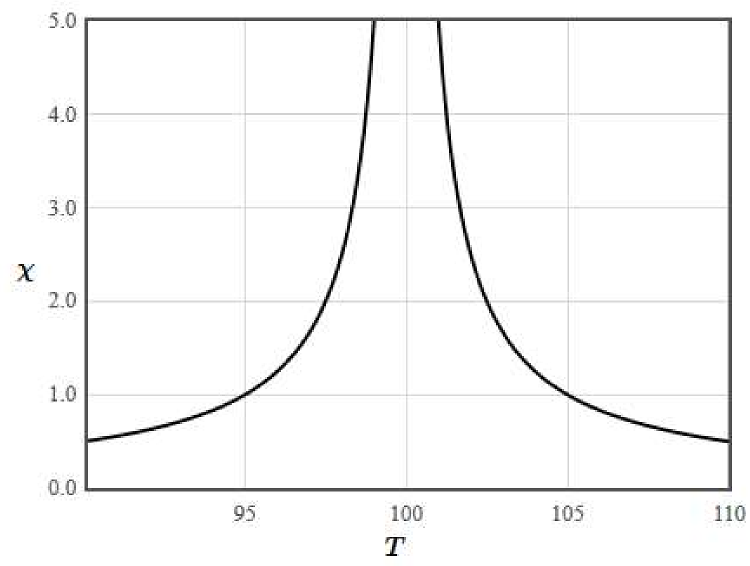
$$f = f_0 + \alpha_0 \left(T - T_c\right) m^2 + \tfrac{1}{2}\beta m^4 - mB$$

$$\frac{df}{dm} = 2\alpha_0 \left(T - T_c\right) m + 2\beta m^3 - B = 0$$

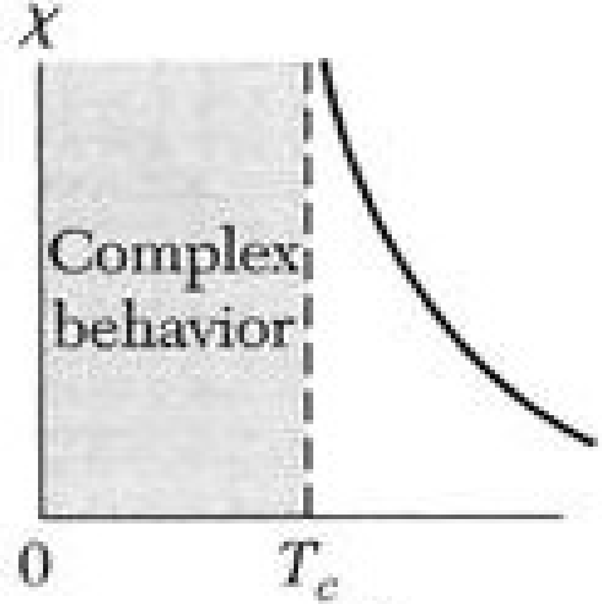
Above T_c, m is finite for finite B. For small m,

$$m = \frac{B}{2\alpha_0 \left(T - T_c\right)} \qquad T > T_c$$

$$\chi = \frac{1}{2\alpha_0 \left(T - T_c\right)} \qquad \text{Curie-Weiss}$$



Ferromagnetism

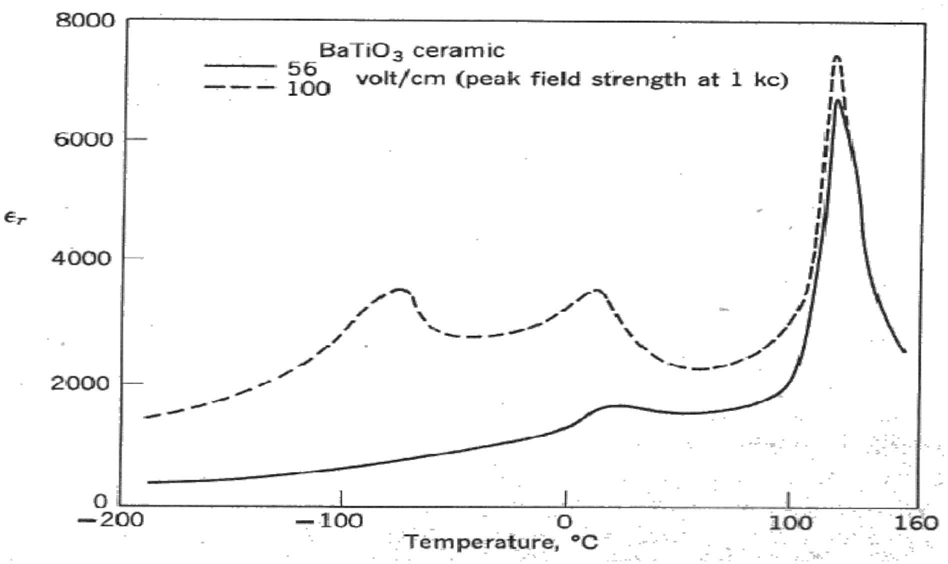


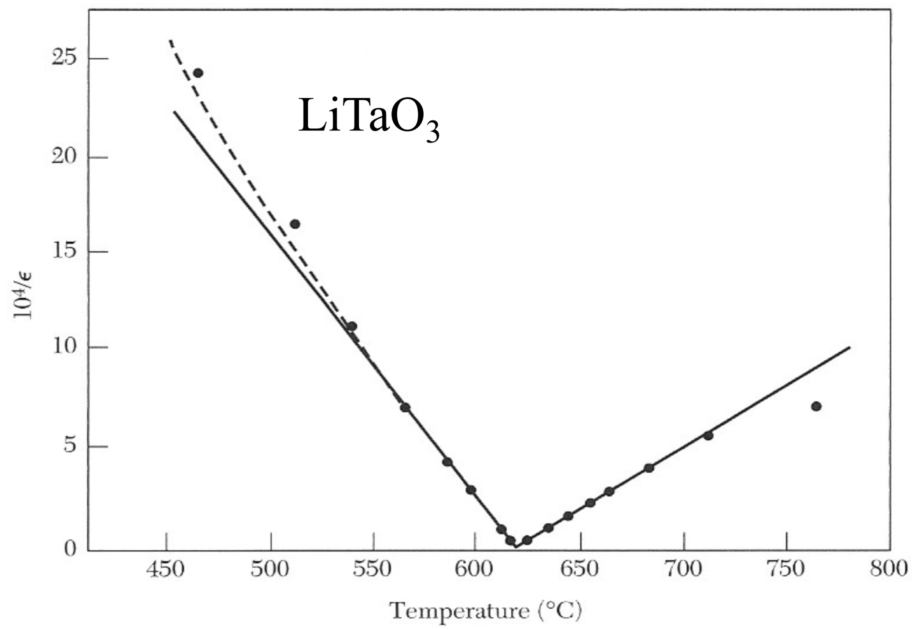
$$\chi = \frac{C}{T - T_c}$$

Curie-Weiss law

$(T > T_c)$

Landau theory of phase transitions



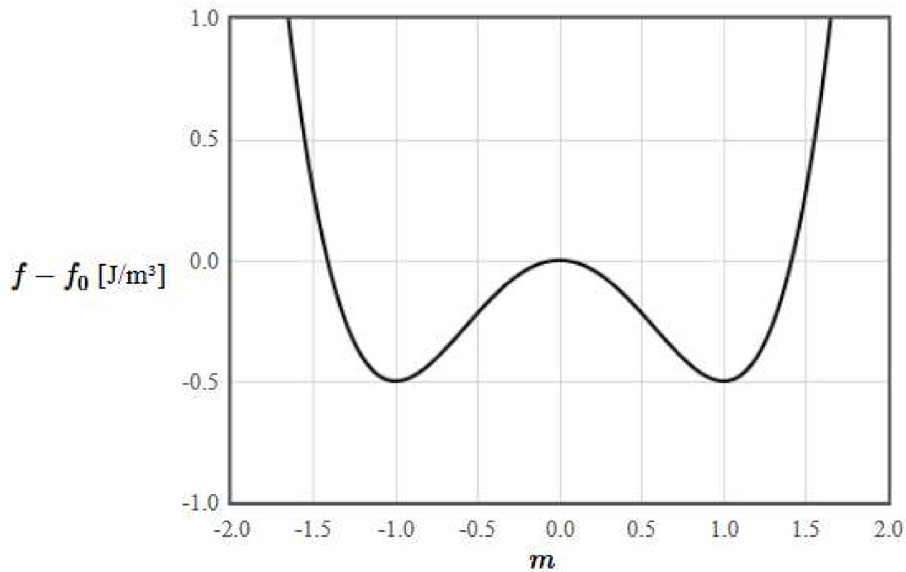


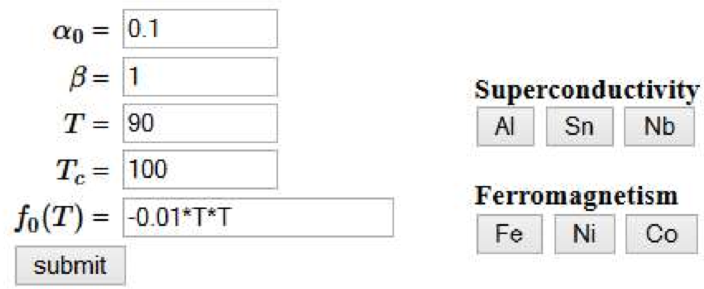
$$\varepsilon_r = 1 + \chi$$

$$\chi = \frac{1}{2\alpha_0 \left(T - T_c\right)}$$

Curie-Weiss law

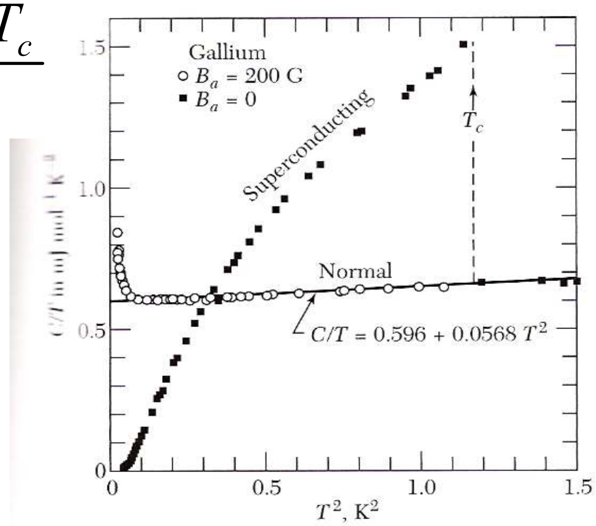
Fitting the α_0 and β parameters

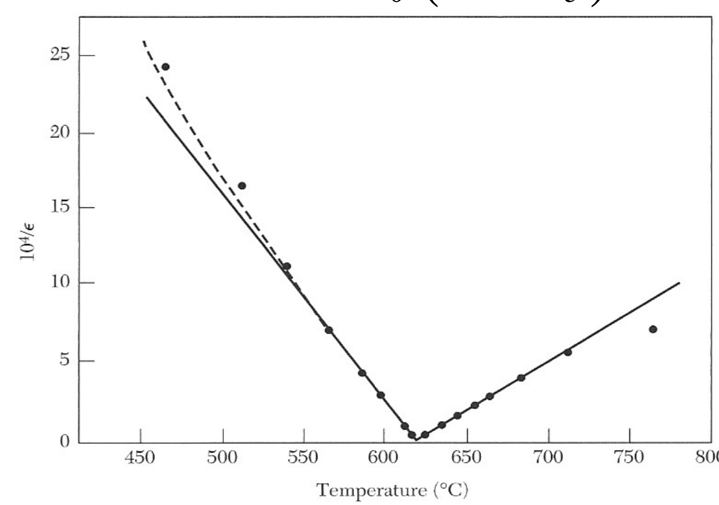




$$\chi = \frac{1}{2\alpha_0\left(T - T_c\right)}$$

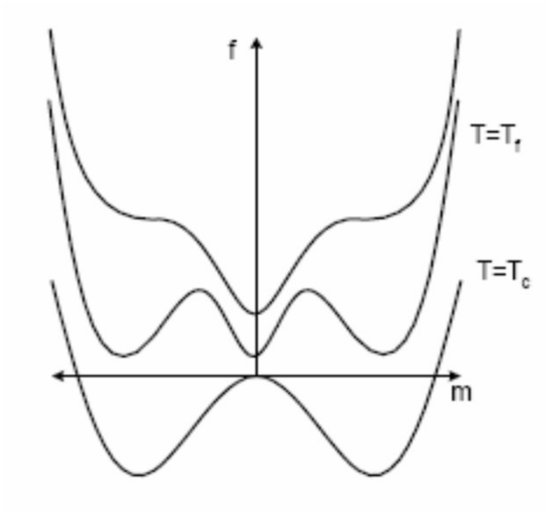
$$\Delta c_v = \frac{2\alpha_0^2 T_c}{\beta}$$





First order transitions

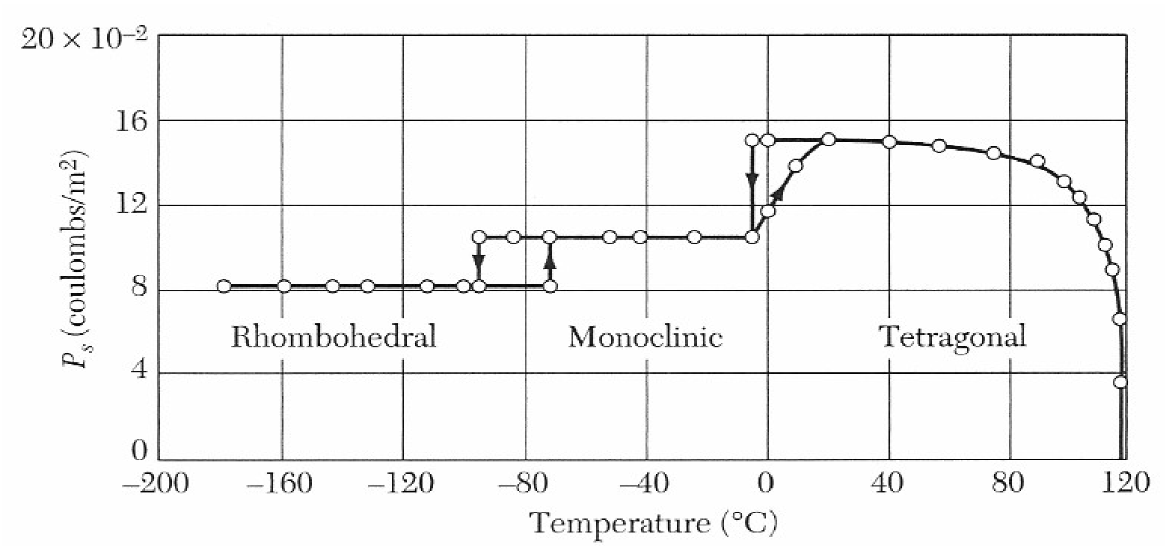
$$f = f_0 + \alpha_0 \left(T - T_c\right) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0$$

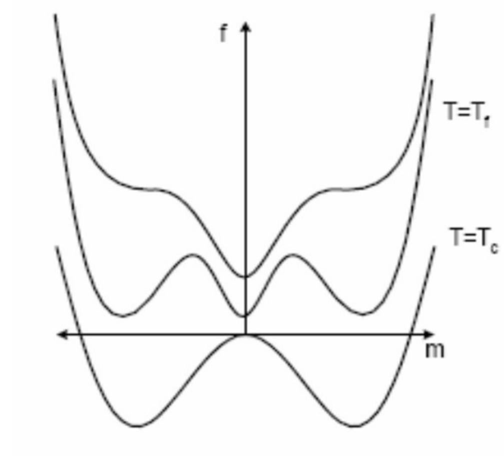


There is a jump in the order parameter at the phase transition.

First order transitions

$BaTiO_3$





First order transitions

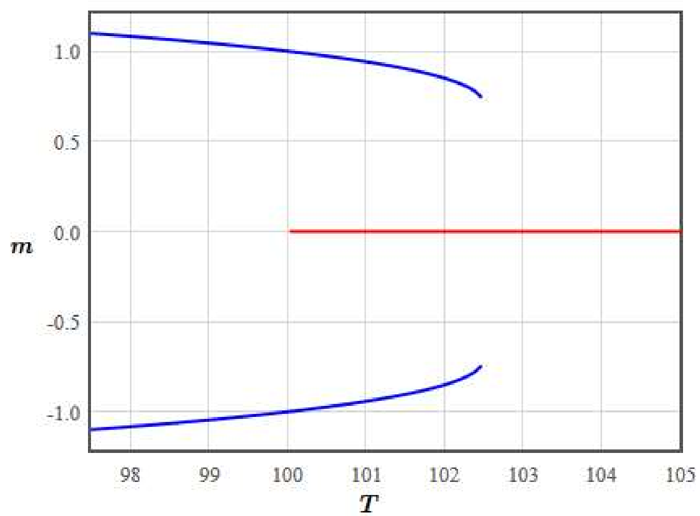
$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \beta < 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 = 0$$

One solution for $m = 0$.

$$\alpha_0 (T - T_c) + \beta m^2 + \gamma m^4 = 0$$

$$m^2 = 0, \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}$$



There will be a minimum at finite m as long as m^2 is real

$$T_1 = \frac{\beta^2}{4\alpha_0\gamma} + T_c$$

Jump in the order parameter

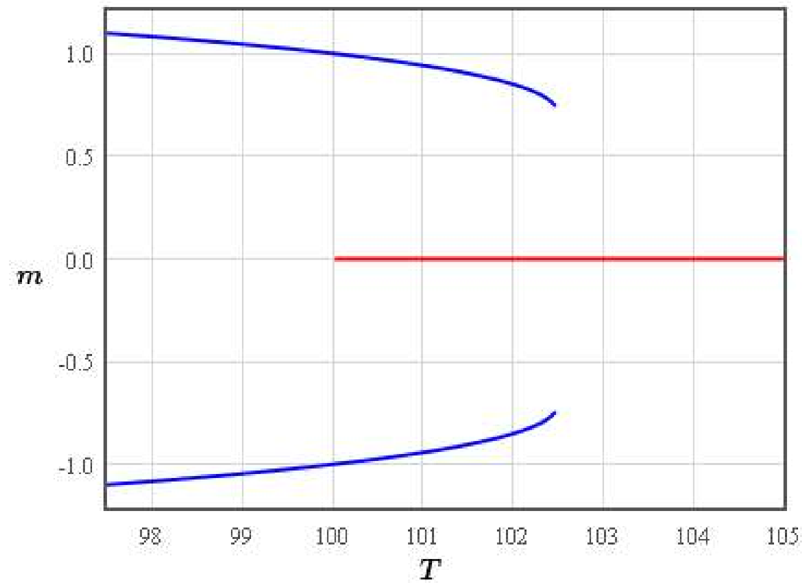
$$m^2 = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 \left(T - T_c\right)\gamma}}{2\gamma}$$

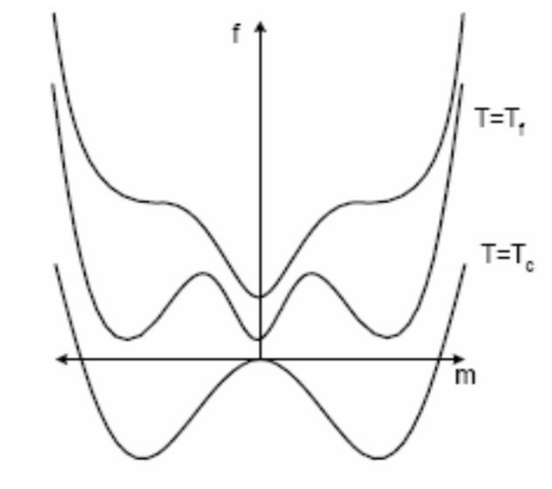
At T_c

$$\Delta m = \sqrt{\frac{-\beta}{\gamma}}$$

At T_1

$$\Delta m = \sqrt{\frac{-\beta}{2\gamma}}$$





First order transitions, entropy, c_v

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \beta < 0$$

$$m = 0, \pm\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}}$$

$$s = -\frac{\partial f}{\partial T} = \frac{\alpha_0}{2\gamma}\left(\beta - \sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}\right)$$

$$c_v = T\frac{\partial s}{\partial T} = \frac{\alpha_0^2 T}{\sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}}$$

branch where the order parameter is nonzero

First order transitions, susceptibility

$$f = f_0 + \alpha_0 \left(T - T_c\right) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 - mB \qquad \beta < 0$$

$$\frac{df}{dm} = 2\alpha_0 \left(T - T_c\right) m + 2\beta m^3 + 2\gamma m^5 - B = 0$$

At the minima $$B = 2\alpha_0 \left(T - T_c\right) m + 2\beta m^3 + 2\gamma m^5$$

For small m,

$$\chi = \left.\frac{dm}{dB}\right|_{m=0} = \frac{1}{2\alpha_0 \left(T - T_c\right)}$$ Curie - Weiss

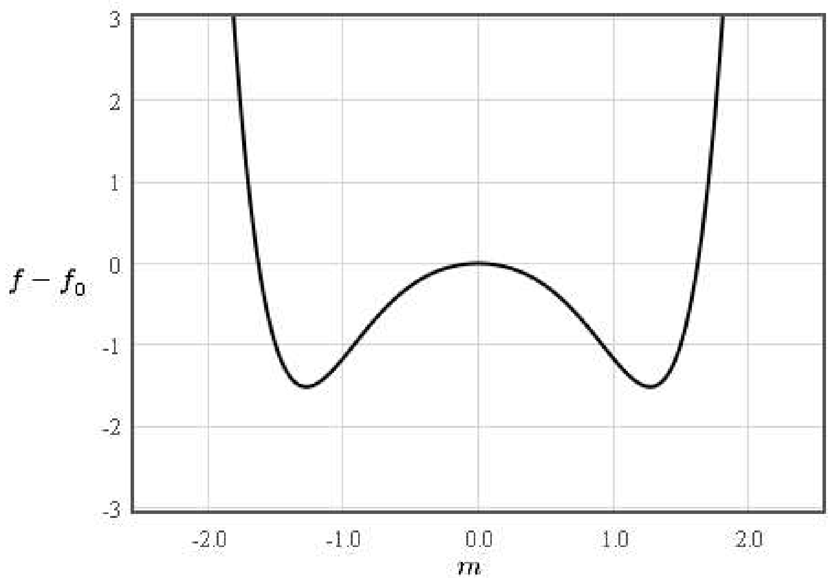
$$\chi = \left.\frac{dm}{dB}\right|_{m=\sqrt{\frac{-\beta}{2\gamma}}} = \frac{1}{2\alpha_0 \left(T - T_1\right)}$$

Landau theory of a first order phase transition

The free energy for a first order transition in Landau theory is,

$$f(T) = f_0(T) + \alpha_0(T - T_c)m^2 + \frac{1}{2}\beta m^4 + \frac{1}{3}\gamma m^6 \qquad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0.$$

Here $f_0(T)$ describes the temperature dependence of the high temperature phase near the phase transition.



$\alpha_0 =$ 0.1

$\beta =$ -1

$\gamma =$ 1

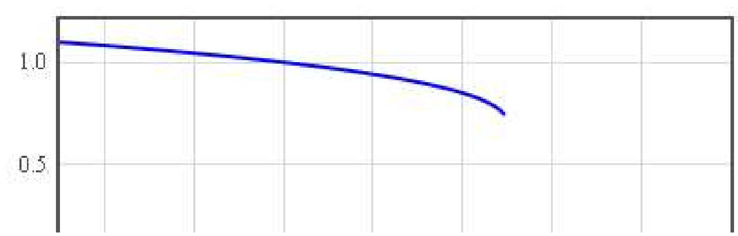
$T =$ 90

$T_c =$ 100

$f_0(T) =$ -0.01*T*T

submit

Order parameter



Superconductivity

Primary characteristic: zero resistance at dc

There is a critical temperature T_c above which superconductivity disappears

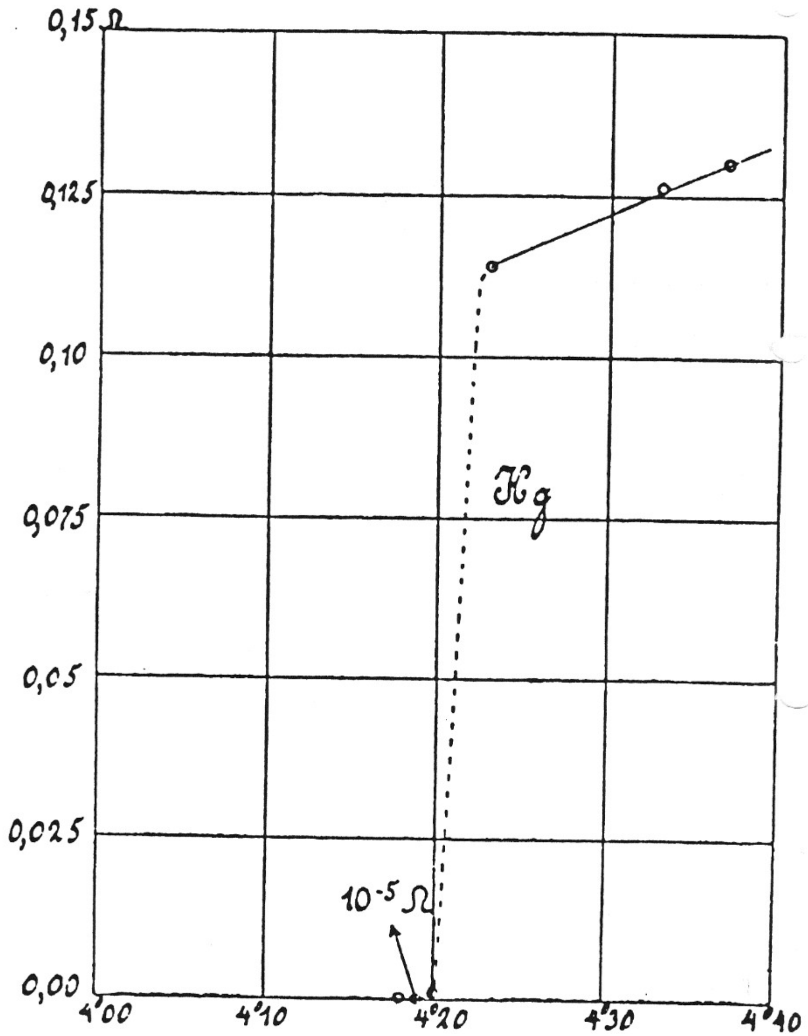
About 1/3 of all metals are superconductors

Metals are usually superconductors OR magnetic, not both

Good conductors are bad superconductors

Kittel chapter 10

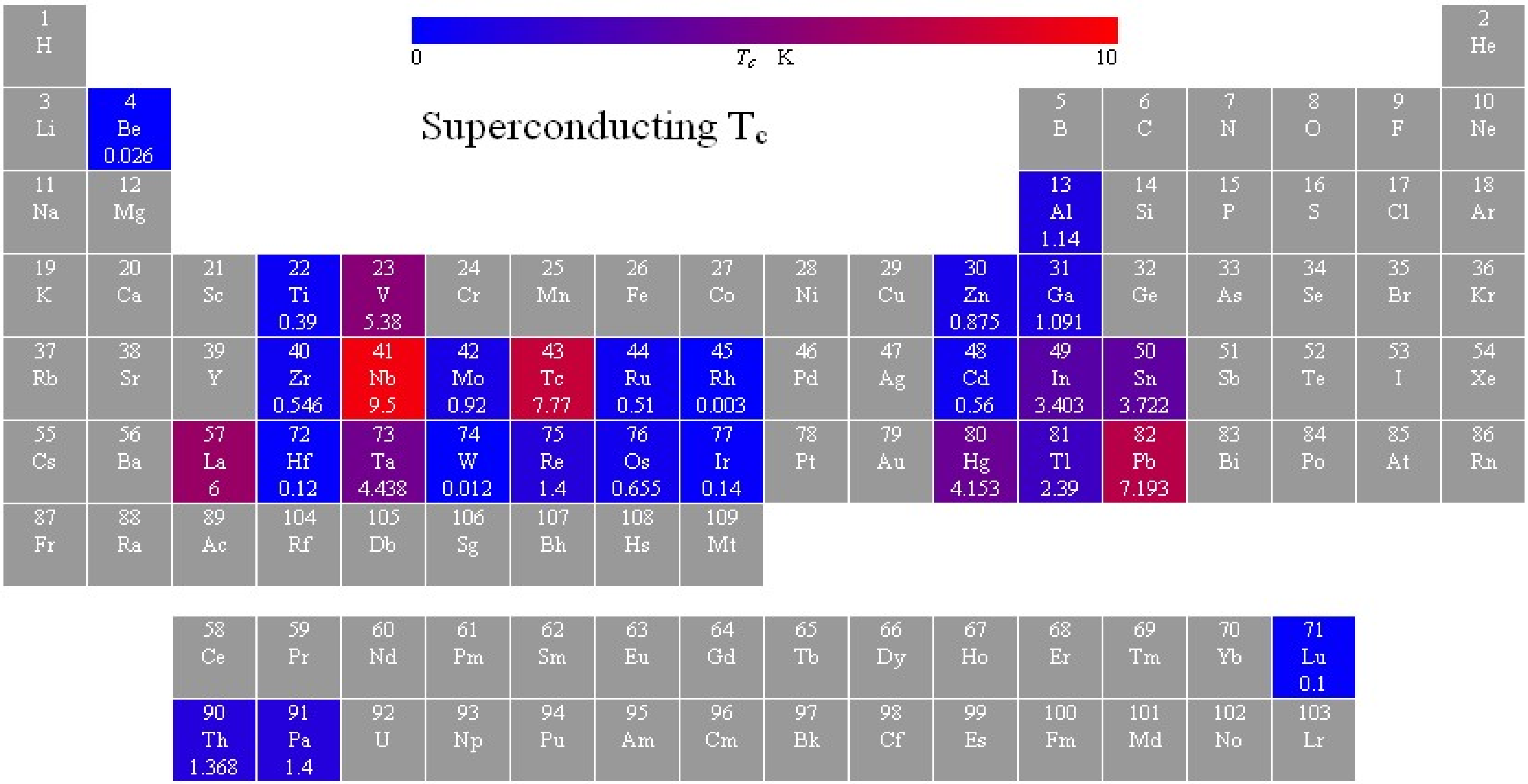
Superconductivity



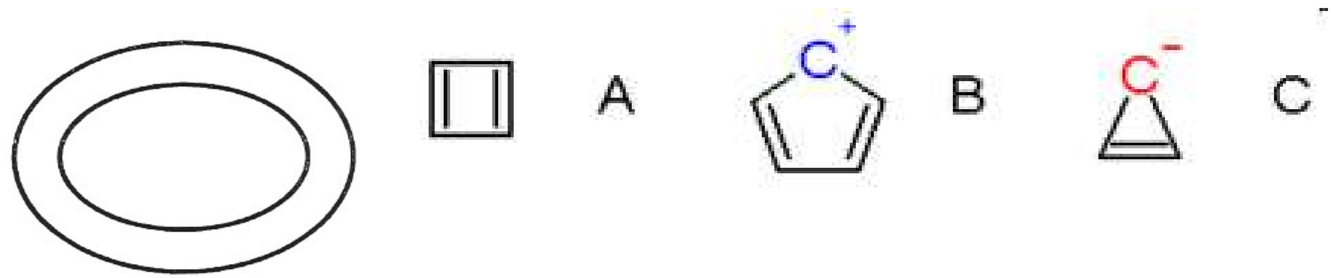
Heike Kammerling-Onnes

Superconductivity was discovered in 1911

Critical temperature



Superconductivity



Superconducting ring

Molecule with magnetic moment

Antiaromatic molecules are unstable and highly reactive

No measurable decay in current after 2.5 years. $\rho < 10^{-25}\ \Omega\mathrm{m}$.