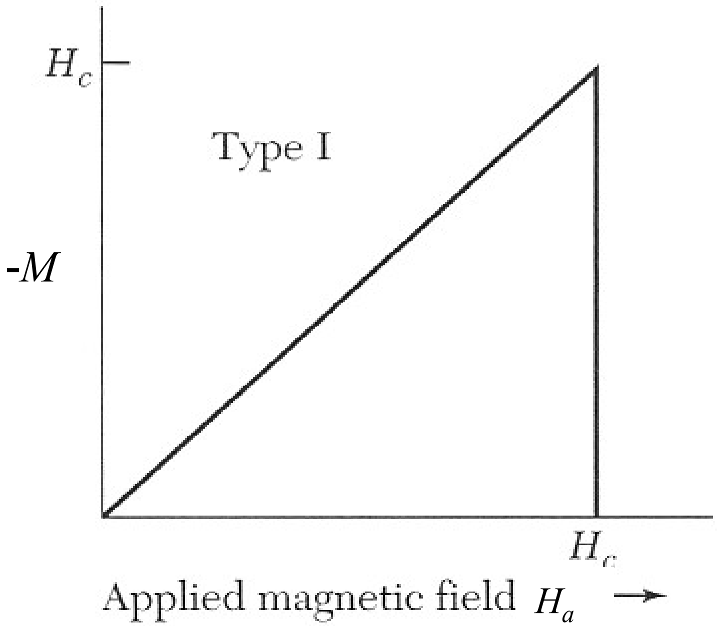
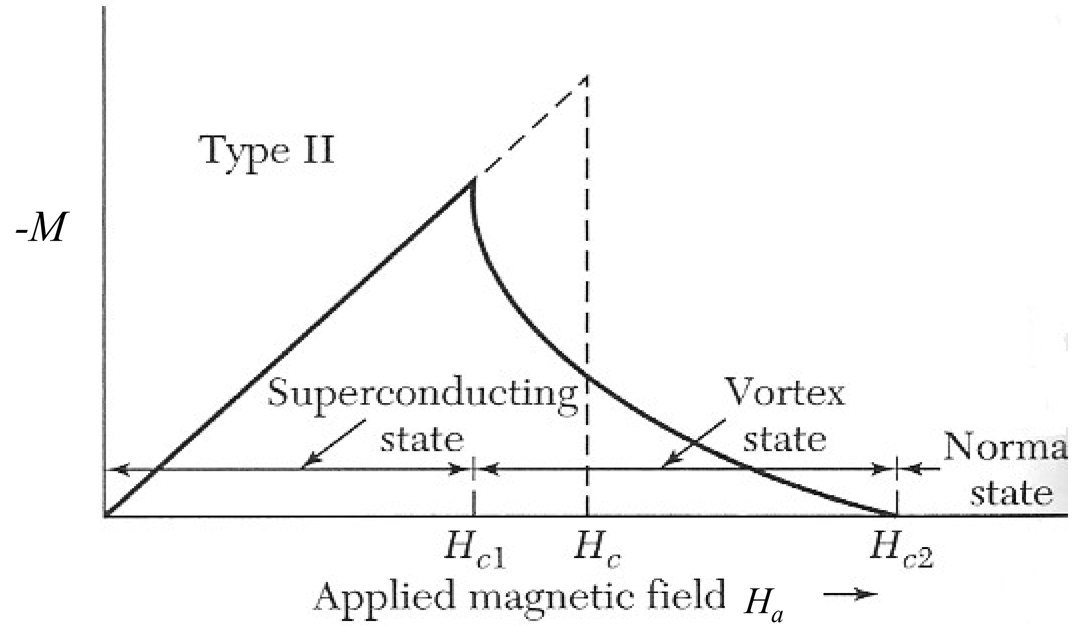
Type I and Type II

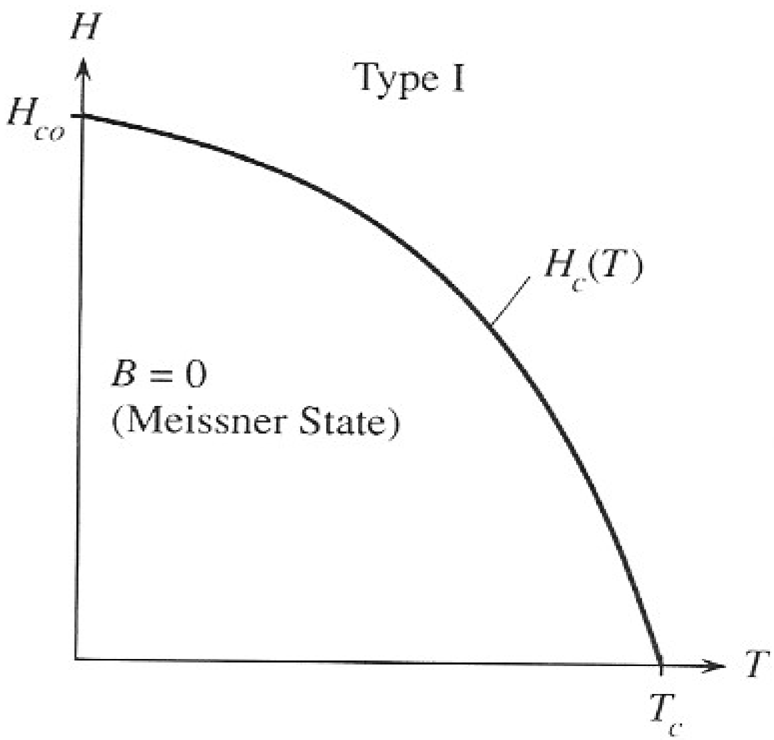


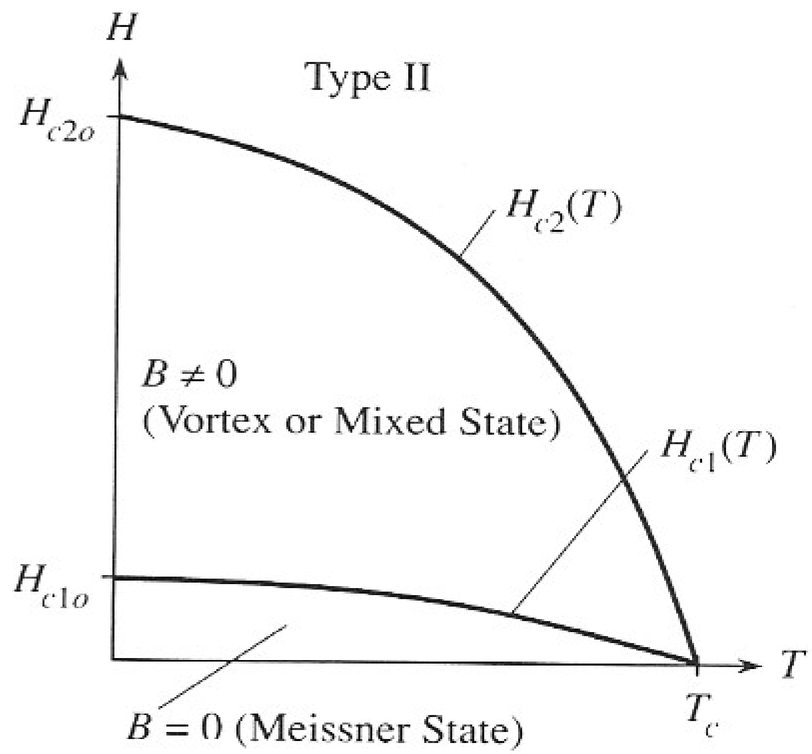


$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right)$$

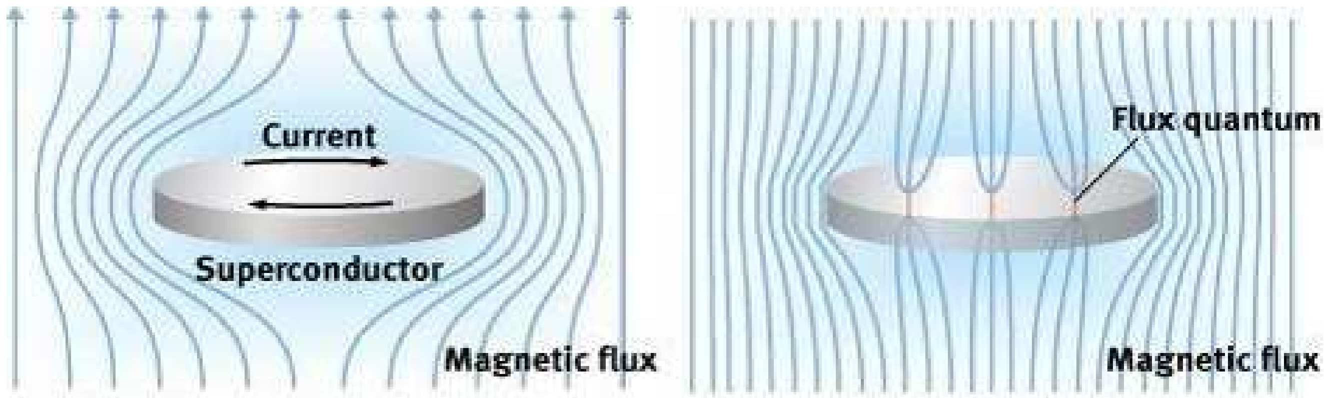
Superconductors are perfect diamagnets at low fields.
B=0 inside a bulk superconductor.

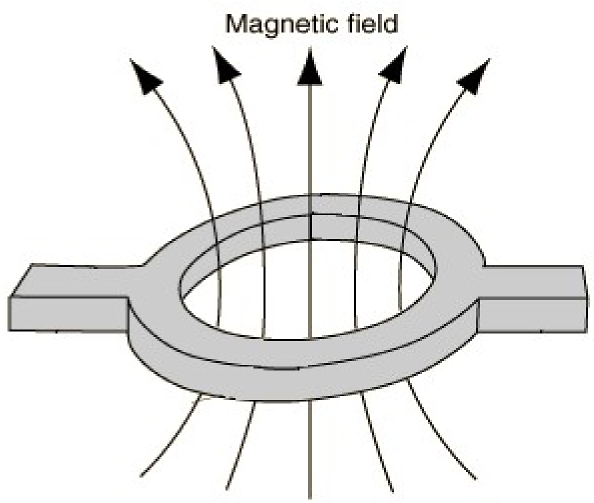
Type I and Type II





Flux quantization





$$\Phi = n\Phi_0 = \frac{nh}{2e}$$

Flux is quantized through a superconducting ring.

Probability current

Schrödinger equation for a charged particle in an electric and magnetic field is

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}(-i\hbar\nabla-qA)^2\psi+V\psi$$

write out the $(-i\hbar\nabla-qA)^2\psi$ term

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}\left(-\hbar^2\nabla^2+i\hbar qA\nabla+ihq\nabla A+q^2A^2\right)\psi+V\psi$$

write the wave function in polar form

$$\psi=|\psi|e^{i\theta}$$

$$\nabla\psi=\nabla|\psi|e^{i\theta}+i\nabla\theta|\psi|e^{i\theta}$$

$$\nabla^2\psi=\nabla^2|\psi|e^{i\theta}+2i\nabla\theta\nabla|\psi|e^{i\theta}+i\nabla^2\theta|\psi|e^{i\theta}-\left(\nabla\theta\right)^2|\psi|e^{i\theta}$$

Probability current

Schrödinger equation becomes:

$$i\hbar\frac{\partial|\psi|}{\partial t}-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(\nabla^2|\psi|+2i\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)$$
$$+i\hbar qA\left(\nabla|\psi|+i\nabla\theta|\psi|\right)+ihq\nabla A|\psi|+q^2A^2|\psi|\Big]+V|\psi|$$

Real part:

$$-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{-\hbar^2}{2m}\left(\nabla^2-\left(\nabla\theta-\frac{q}{\hbar}\vec{A}\right)^2\right)|\psi|+V|\psi|$$

Imaginary part:

$$\hbar\frac{\partial|\psi|}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(2\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)+2\hbar qA\nabla|\psi|+\hbar q|\psi|\nabla A\Big]$$

Probability current

Imaginary part:

$$\hbar \frac{\partial |\psi|}{\partial t} = \frac{1}{2m}\left[-\hbar^2 \left(2\nabla \theta \nabla |\psi| + i\nabla^2 \theta |\psi| - (\nabla \theta)^2 |\psi| \right) + 2\hbar q A \nabla |\psi| + \hbar q |\psi| \nabla A \right]$$

Multiply by $|\psi|$ and rearrange

$$\frac{\partial}{\partial t}|\psi|^2 + \nabla \cdot \left[\frac{\hbar}{m}|\psi|^2 \left(\nabla \theta - \frac{q}{\hbar}\vec{A} \right) \right] = 0$$

This is a continuity equation for probability

$$\frac{\partial P}{\partial t} + \nabla \cdot \vec{S} = 0$$

The probability current: $$\vec{S} = \frac{\hbar}{m}|\psi|^2 \left(\nabla \theta - \frac{q}{\hbar}\vec{A} \right)$$

Probability current / supercurrent

The probability current: $$\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$$

This result holds for all charged particles in a magnetic field.

In superconductivity the particles are Cooper pairs $q = -2e$, $m = 2m_e$, $|\psi|^2 = n_{cp}$.

All superconducting electrons are in the same state so

$$\vec{j} = -2en_{cp}\vec{S}$$

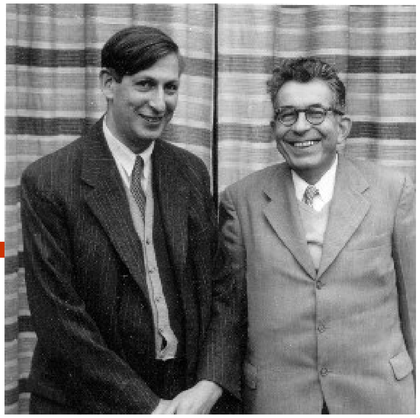
$$\boxed{\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)}$$

London gauge $\nabla\theta = 0$

$$\vec{j} = \frac{-2n_{cp}e^2}{m_e}\vec{A} = \frac{-n_s e^2}{m_e}\vec{A}$$

$$n_s = 2n_{cp}$$

1st London equation



Heinz & Fritz

$$\vec{j} = \frac{-n_s e^2}{m_e}\vec{A}$$

$$\frac{d\vec{j}}{dt} = \frac{-n_s e^2}{m_e}\frac{d\vec{A}}{dt} = \frac{n_s e^2}{m_e}\vec{E} \qquad \frac{d\vec{A}}{dt} = -\vec{E}$$

First London equation:

$$\boxed{\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e}\vec{E}}$$

Classical derivation:

$$-e\vec{E} = m\frac{d\vec{v}}{dt} = -\frac{m}{n_s e}\frac{d\vec{j}}{dt}$$

$$\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e}\vec{E}$$

2nd London equation

$$\vec{j} = \frac{-n_s e^2}{m_e} \vec{A}$$

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \nabla \times \vec{A}$$

Second London equation:

$$\boxed{\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B}}$$

Meissner effect

Combine second London equation with Ampere's law

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B} \qquad \nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla \times \nabla \times \vec{B} = \frac{-n_s e^2 \mu_0}{m_e} \vec{B}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \left(\nabla \cdot \vec{B} \right) - \nabla^2 \vec{B}$$

Helmholtz equation: $\lambda^2 \nabla^2 \vec{B} = \vec{B}$

London penetration depth: $\lambda = \sqrt{\dfrac{n_s e^2 \mu_0}{m_e}}$

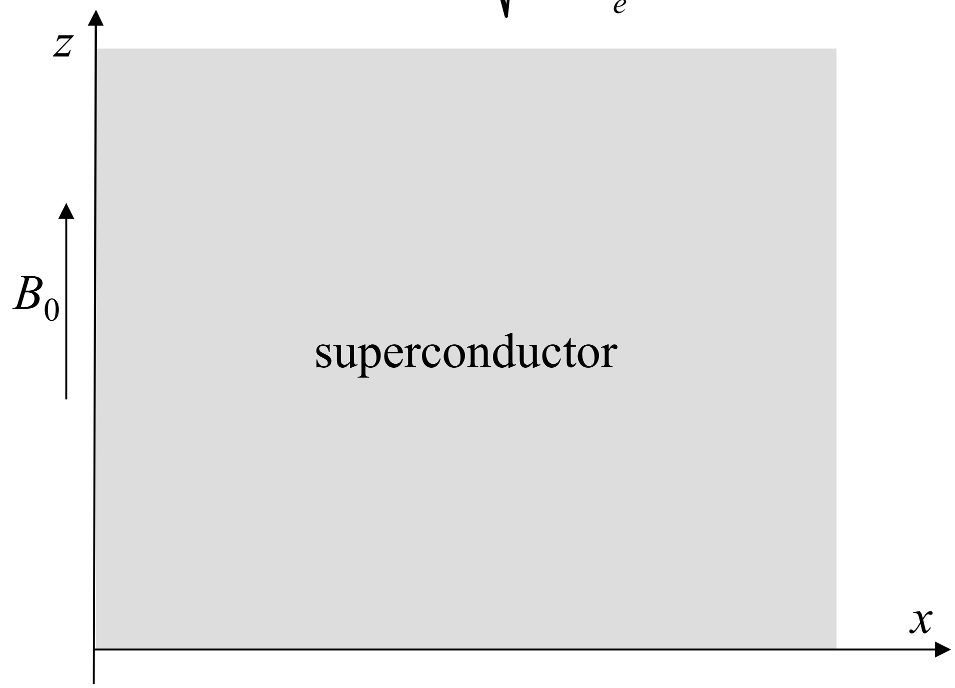
Meissner effect

$$\lambda^2 \nabla^2 \vec{B} = \vec{B} \qquad \lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$$

solution to Helmholtz equation:

$$\vec{B} = \vec{B}_0 \exp\left(\frac{-x}{\lambda}\right)\hat{z}$$

Al	λ = 50 nm
In	λ = 65 nm
Sn	λ = 50 nm
Pb	λ = 40 nm
Nb	λ = 85 nm



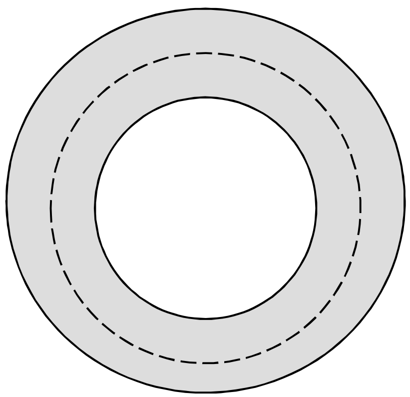
$$\nabla \times \vec{B} = \mu_0 \vec{j} \qquad \vec{j} = \frac{\vec{B}_0}{\mu_0 \lambda}\exp\left(\frac{-x}{\lambda}\right)\hat{y}$$

Flux quantization

$$\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$

For a ring much thicker than the penetration depth, $j = 0$ along the dotted path.

$$0 = \left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$



Integrate once along the dotted path.

$$\oint \nabla\theta \cdot d\vec{l} = -\frac{2e}{\hbar}\oint \vec{A}\cdot d\vec{l} = -\frac{2e}{\hbar}\int_S \nabla\times\vec{A}\cdot d\vec{s} = -\frac{2e}{\hbar}\int_S \vec{B}\cdot d\vec{s} = -\frac{2e}{\hbar}\Phi$$

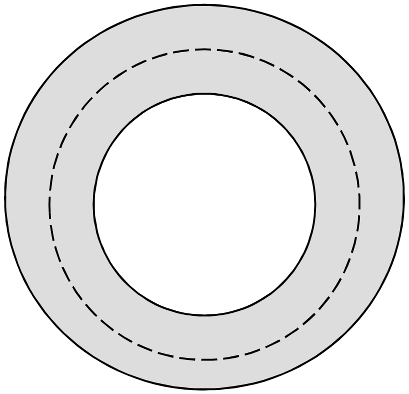
Stokes' theorem

magnetic flux

Flux quantization

$$\oint \nabla \theta \cdot d\vec{l} = 2\pi n = -\frac{2e}{\hbar}\Phi$$

n = ... -2, -1, 0, 1, 2 ...



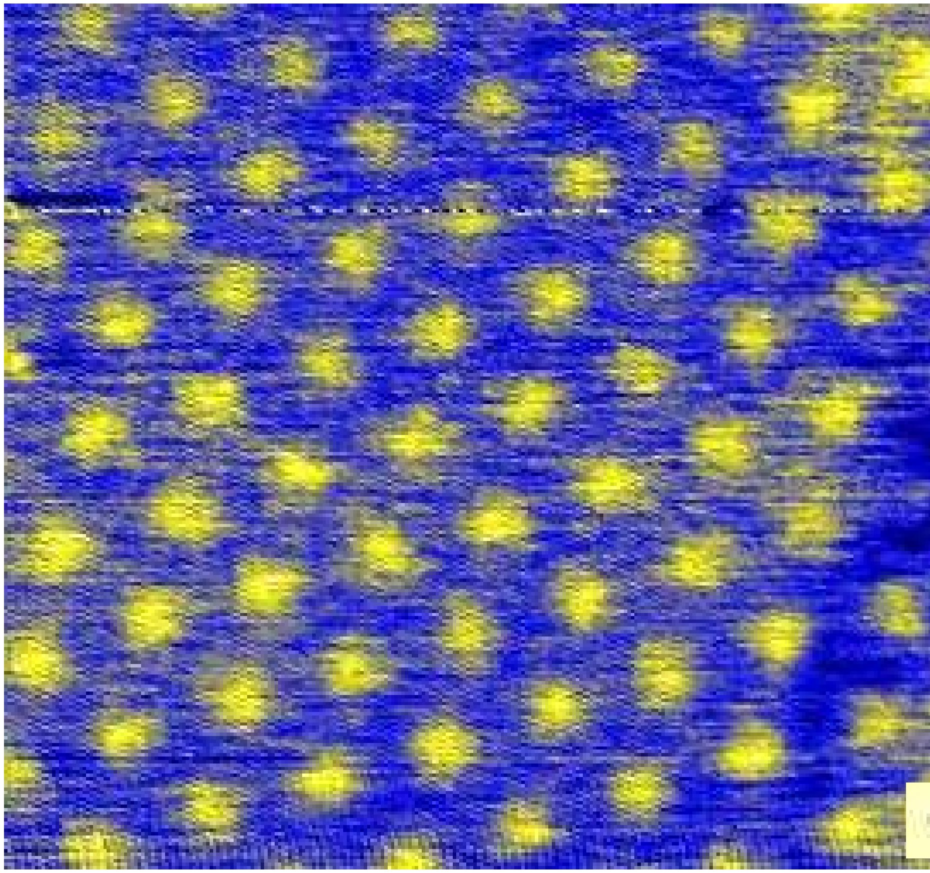
$$2\pi n = \frac{2e}{\hbar}\Phi = \frac{\Phi}{\Phi_0}$$

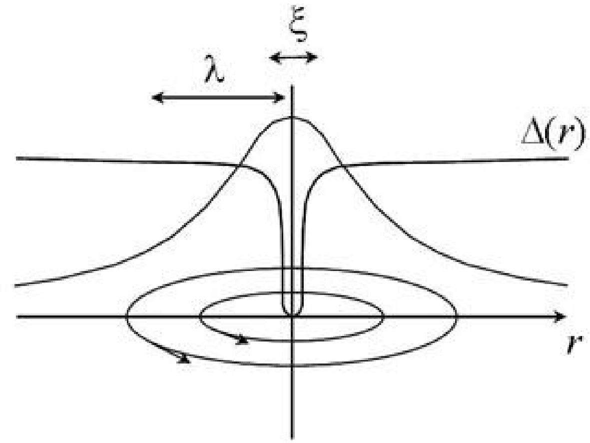
Flux quantization: $\boxed{\Phi = n\Phi_0}$

$$\Phi_0 = \frac{h}{2e} = 2.0679\times 10^{-15}\ [\mathrm{W = T\ m^2}]$$

Superconducting flux quantum

Vortices in Superconductors





$$\Phi_0 = h/2e \simeq 2 \times 10^{-15} \text{ Tesla m}^2$$

STS image of the vortex lattice in $NbSe_2$.
(630 nm x 500 nm, B = .4 Tesla, T = 4 K)

http://www.insp.upmc.fr/axe1/Dispositifs%20quantiques/AxeI2_more/VORTICES/vortexHD.htm

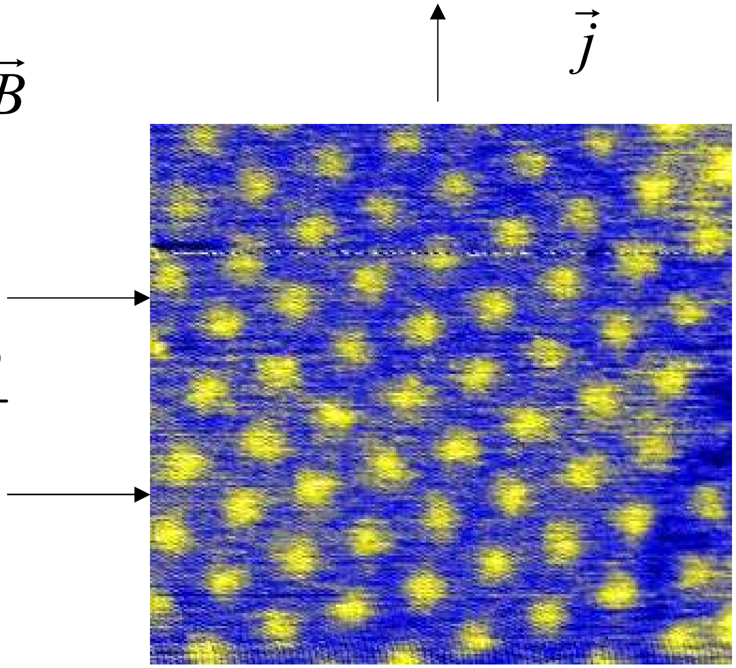
Vortices in Superconductors

Lorentz force $$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{j} = nq\vec{v}$$

$$\vec{F} = \frac{1}{n}\vec{j} \times \vec{B}$$

$$\vec{j}$$



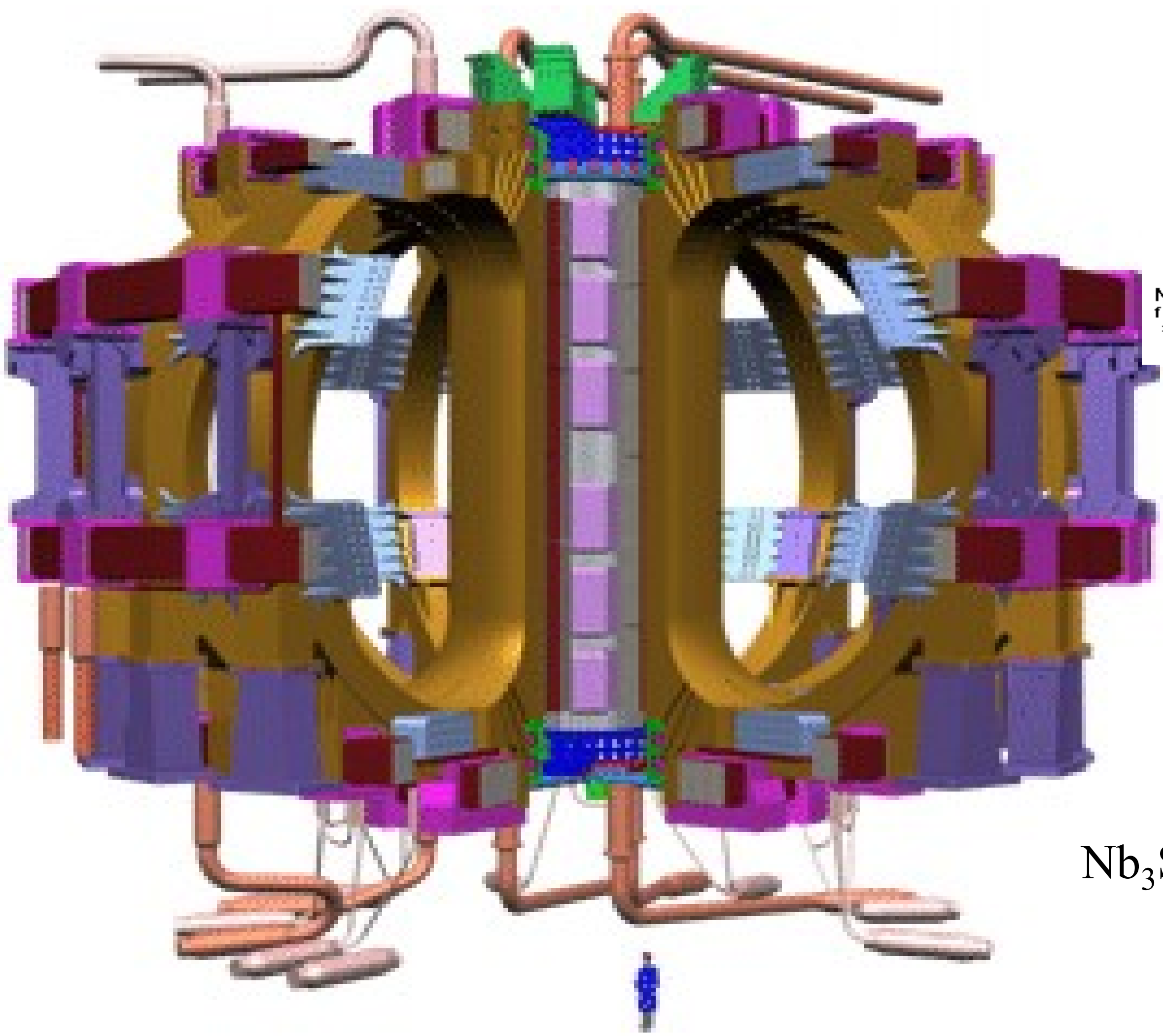
Faraday's law $$V = -\frac{d\Phi}{dt}$$

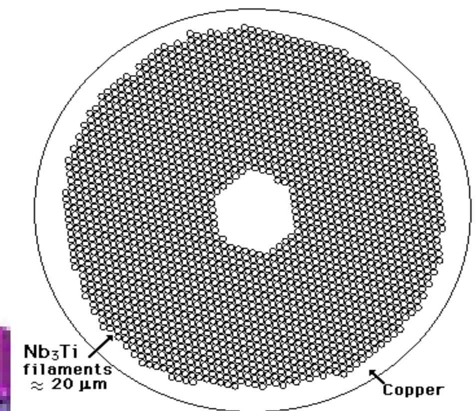
Defects are used to pin the vortices

Superconducting Magnets

Whole body MRI

ITER

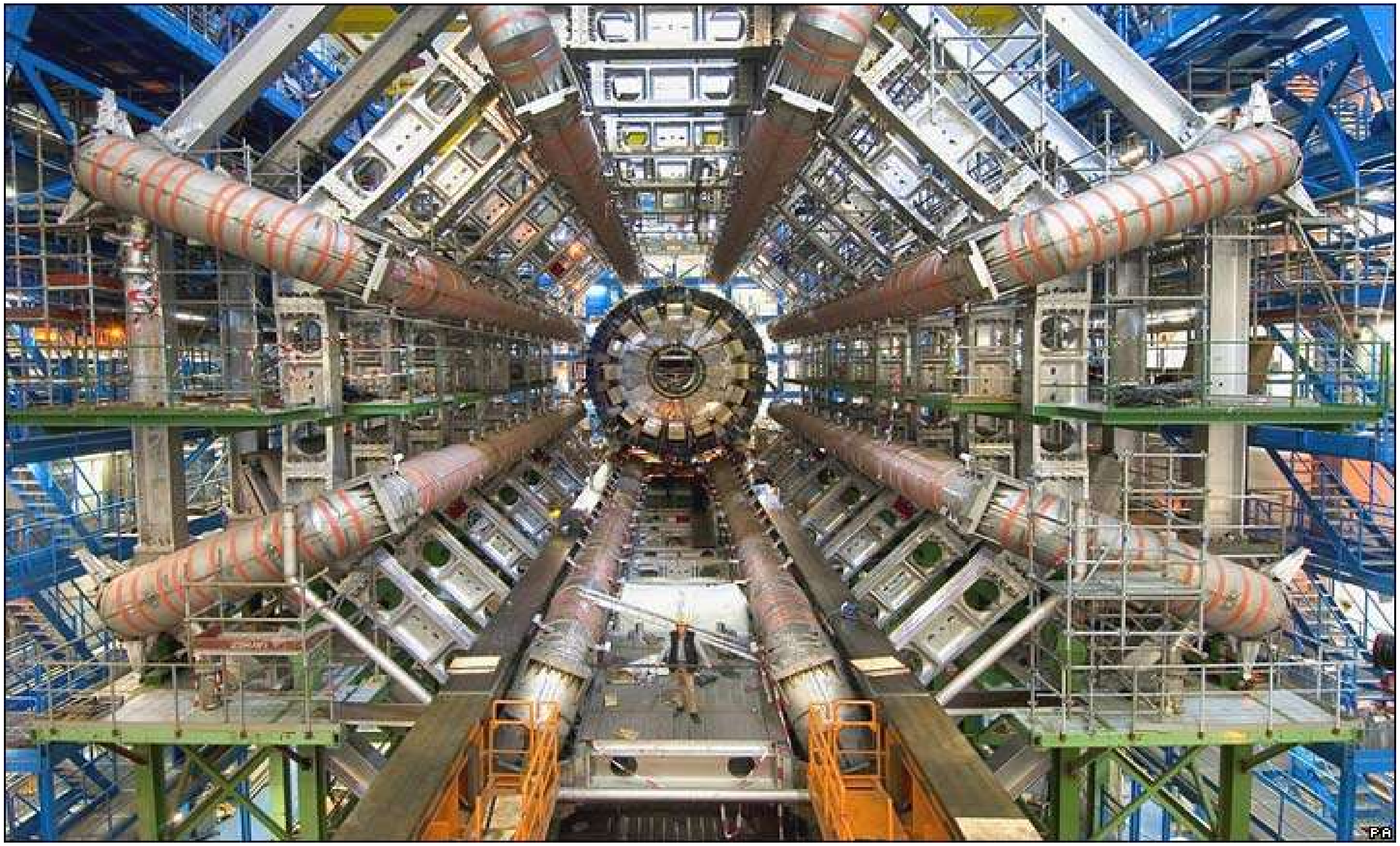




Magnet wire

Nb_3Sn Magnet

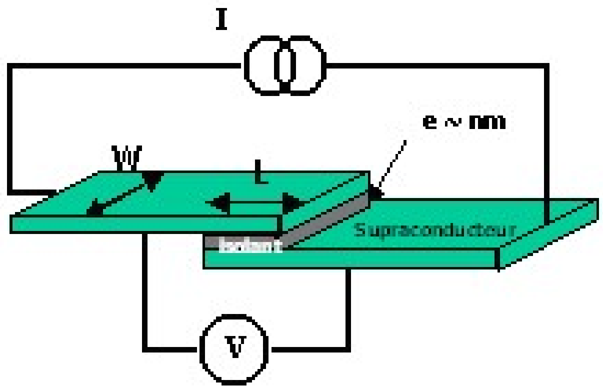
Superconducting magnets



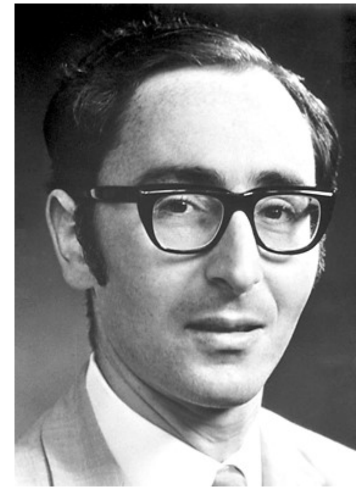
Largest superconducting magnet, CERN
21000 Amps

http://www.lne.eu/en/r_and_d/electrical_metrology/josephson-effect-ej.asp

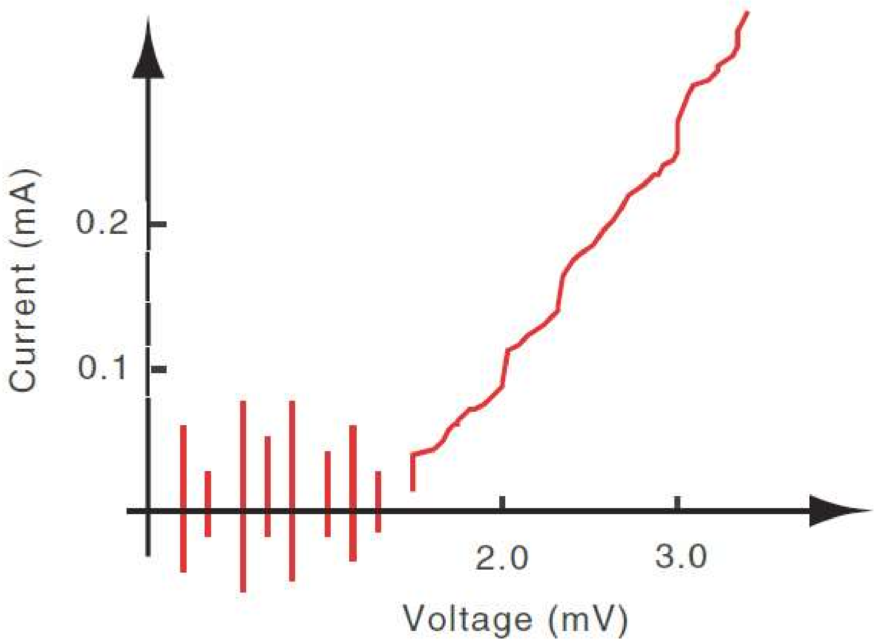
ac - Josephson effect



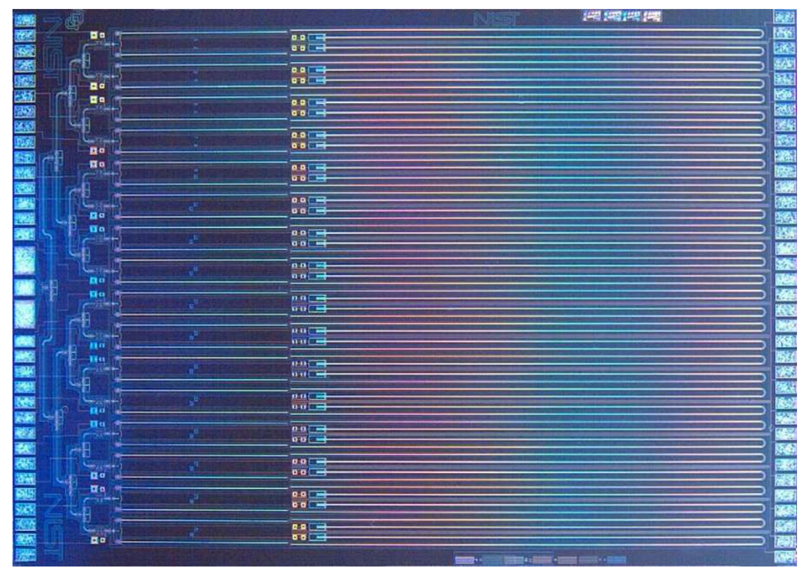
$$V = -\frac{d\Phi}{dt} = n\Phi_0 f = \frac{nhf}{2e}$$



Brian Josephson



DOI: 10.1140/epjst/e2009-01050-6



10 V standard

http://www.nist.gov/pml/history-volt/superconductivity_2000s.cfm