Landau theory of a Fermi liquid

The free electron model = 'Fermi gas' is very successful at describing metals but it is not clear why this is so since electron-electron interactions are completely ignored.

Landau first considered the "normal modes" of an interacting electron system. The low lying excitations he called quasiparticles.

The quasiparticles have as many degrees of freedom as the electrons. They can be labeled by k.

Quasiparticles can be have the same spin, charge, and k vectors as the electrons.

It is not easy to calculate $E(k)$.

Concepts like the density of states refer to quasiparticles.

Landau theory of a Fermi liquid

If there are no electron-electron interactions, electrons have an infinite lifetime and the probability that a state is occupied is given by the Fermi function.

If there are interactions, quasiparticles have a finite lifetime. The lifetime can be calculated by Fermi's golden rule.

The occupation probability of a state depends on the occupation the other states. You solve for the probability distribution by solving a master equation. The occupation probability is not given by the Fermi function.

$$\Gamma_{k\to k'} = \frac{2\pi}{\hbar}\left|\langle\psi_k|H|\psi_{k'}\rangle\right|^2 \delta\left(E_k - E_{k'}\right)$$

Quasiparticles

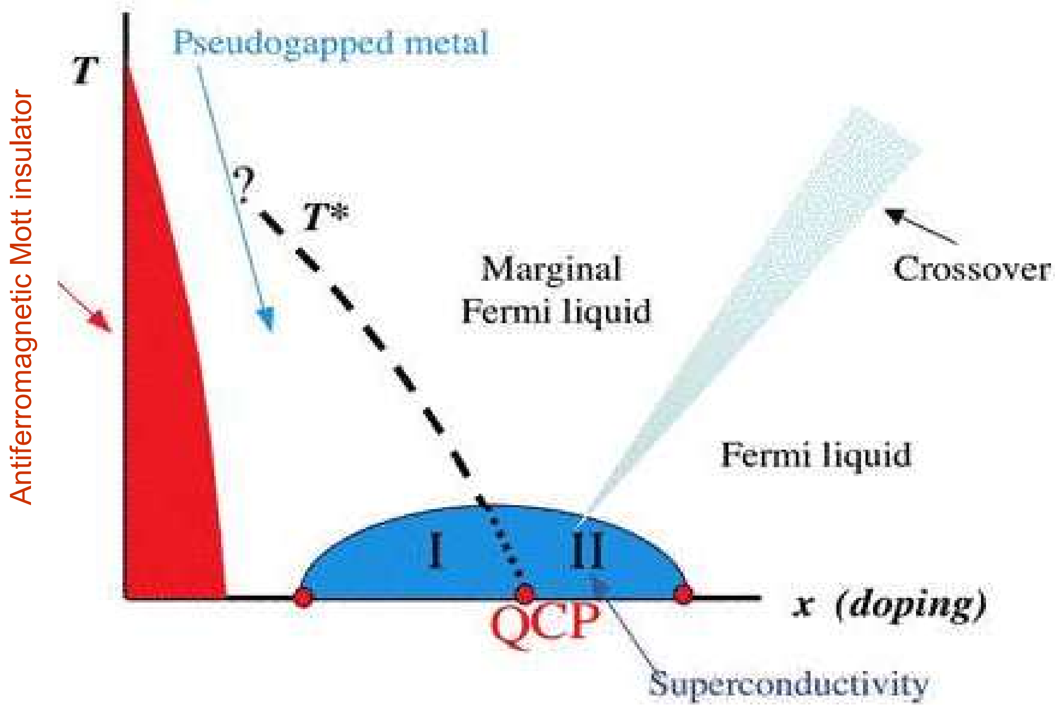
Systems of many interacting particles are very difficult to solve. The first task is to determine the ground state. This is the state that the system enters at zero temperature.

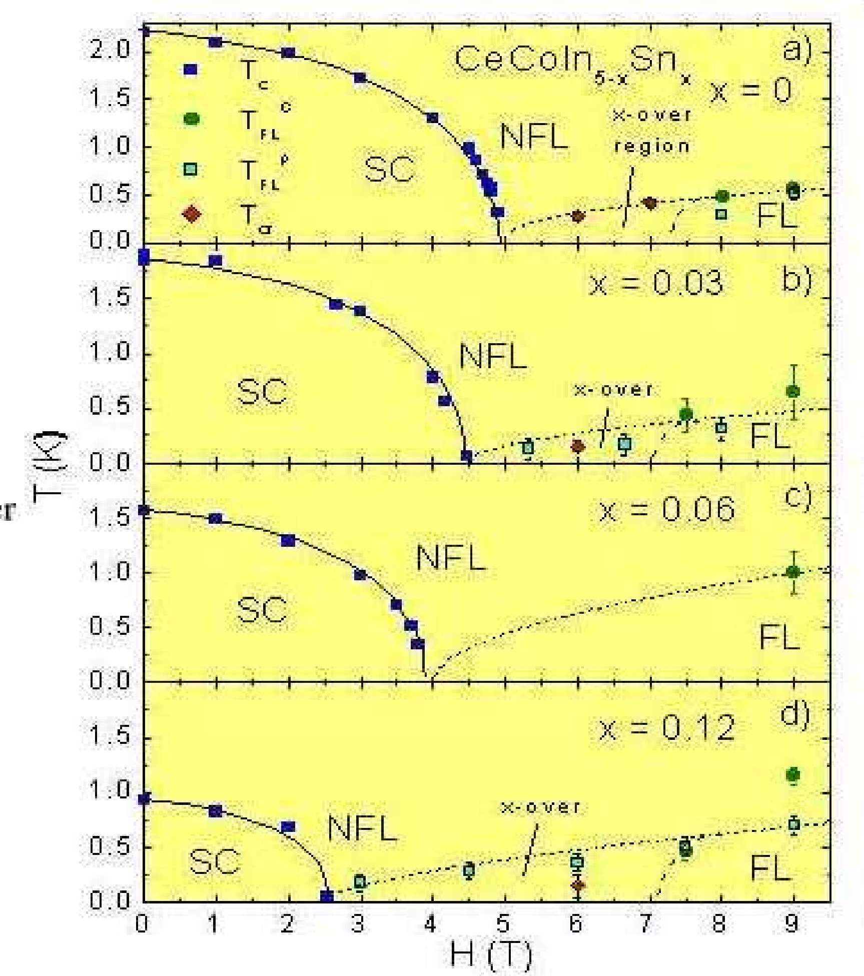
Next we consider the lowest energy excitations above the ground state by linearizing the equations of motion around the ground state. These are called the elementary excitations or quasiparticles.

Phonons, magnons, plasmons, polaritons, and excitions are examples of quasiparticles.

Instabilities Fermi liquid

Some metals cannot be described as a Fermi liquid.





Heavy Fermion $CeCu_2Si_2$

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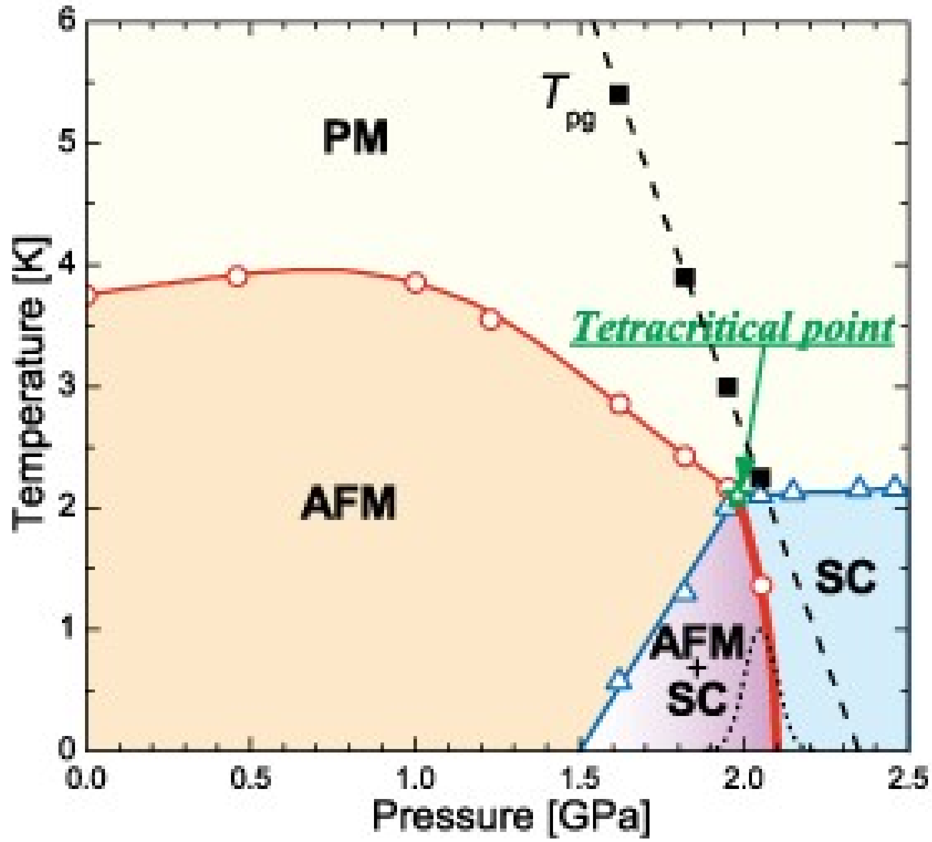
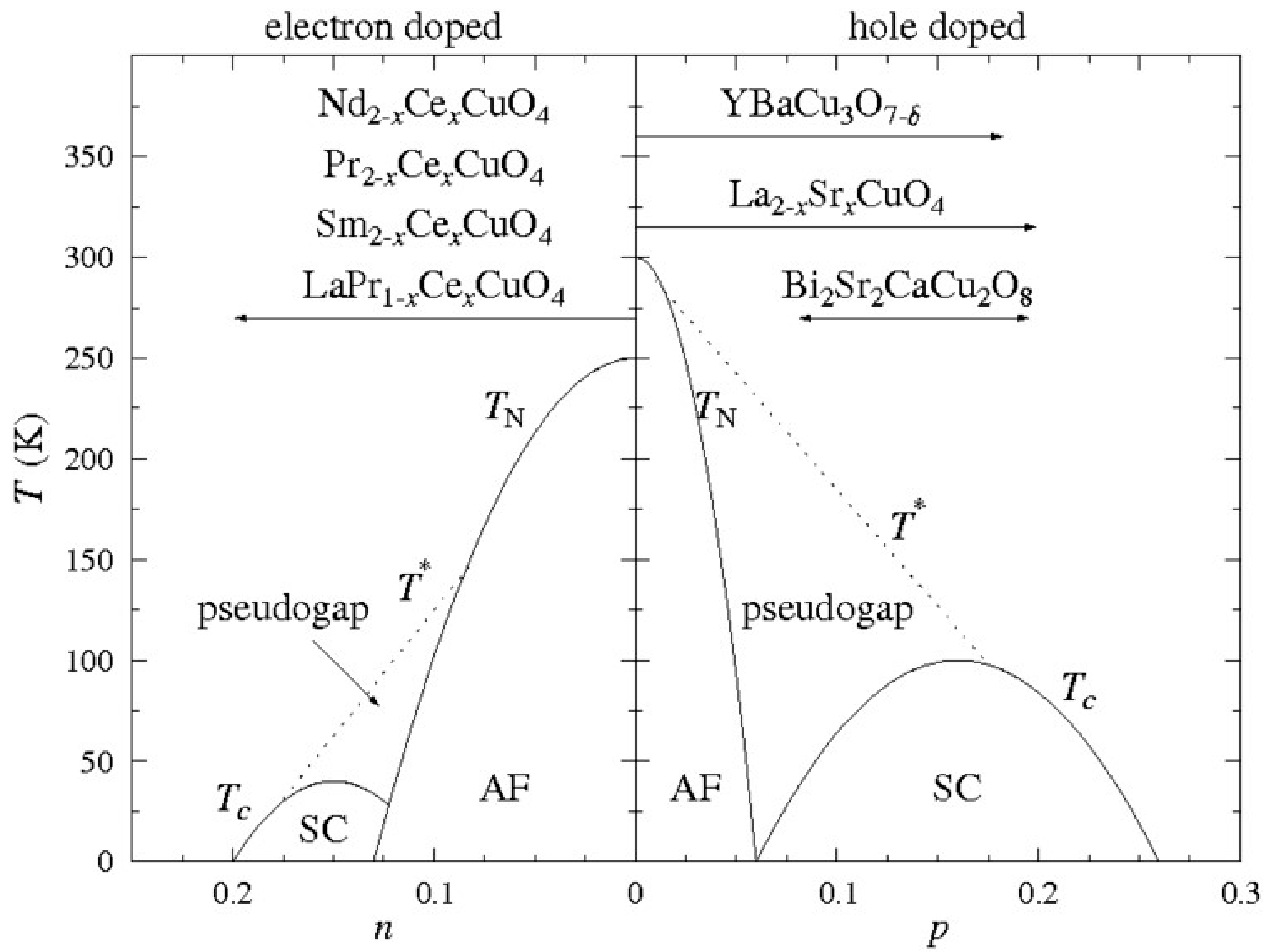
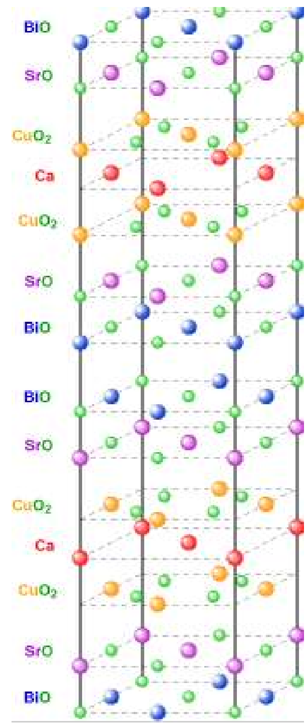


http://www.ipap.jp/jpsj/announcement/announce2007May.htm

Cuprate superconductors

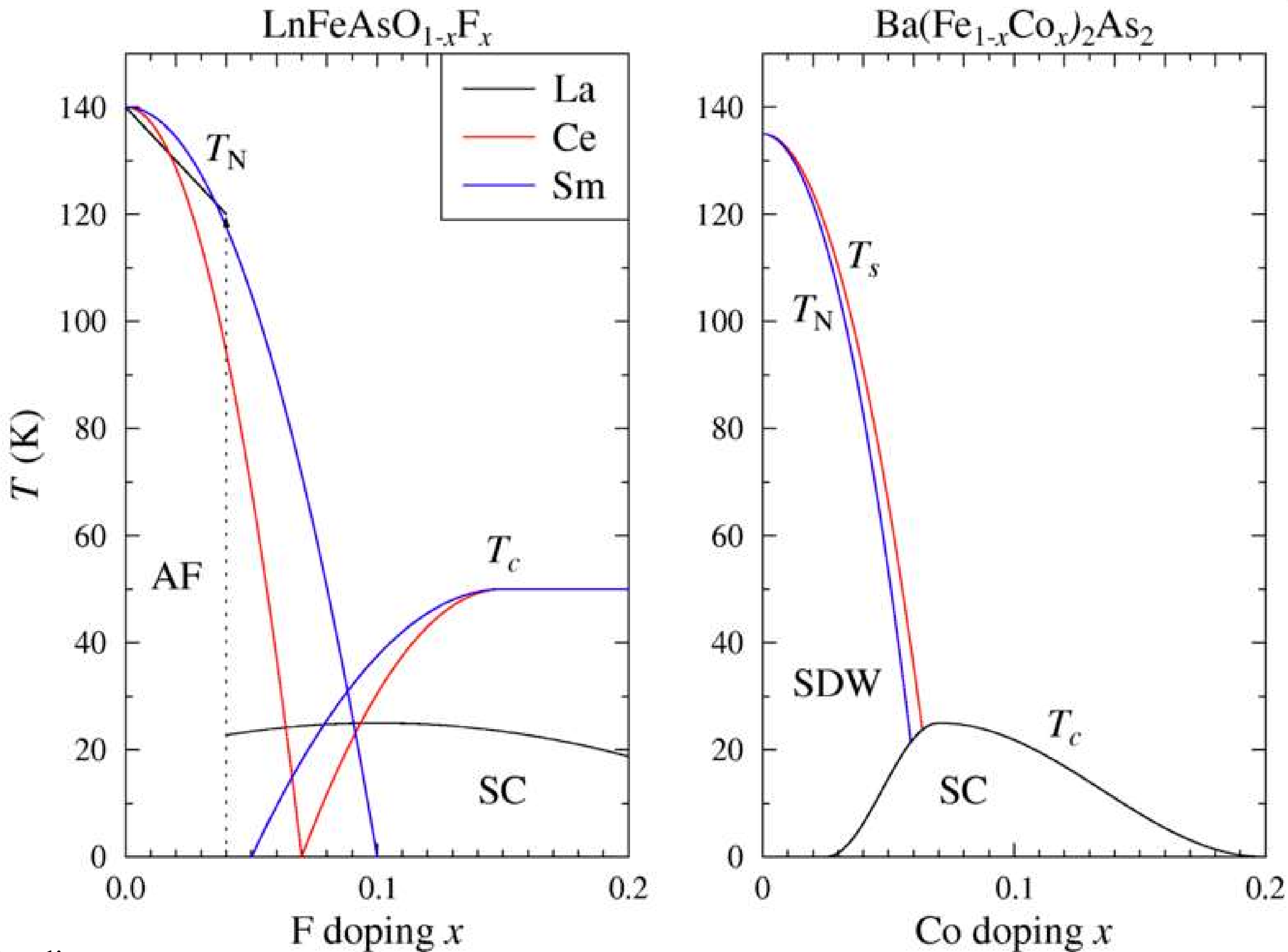


from Wikipedia



The unit cell of high-temperature cuprate superconductor BSCCO-2212

Iron based superconductors



from Wikipedia

Phonons

As usual, we start with the total Hamiltonian for a solid.

$$H = -\sum_i \frac{\hbar^2}{2m_e}\nabla_i^2 - \sum_A \frac{\hbar^2}{2m_A}\nabla_A^2 - \sum_{i,A}\frac{Z_A e^2}{4\pi\varepsilon_0 r_{iA}} + \sum_{i<j}\frac{e^2}{4\pi\varepsilon_0 r_{ij}} + \sum_{A<B}\frac{Z_A Z_B e^2}{4\pi\varepsilon_0 r_{AB}}$$

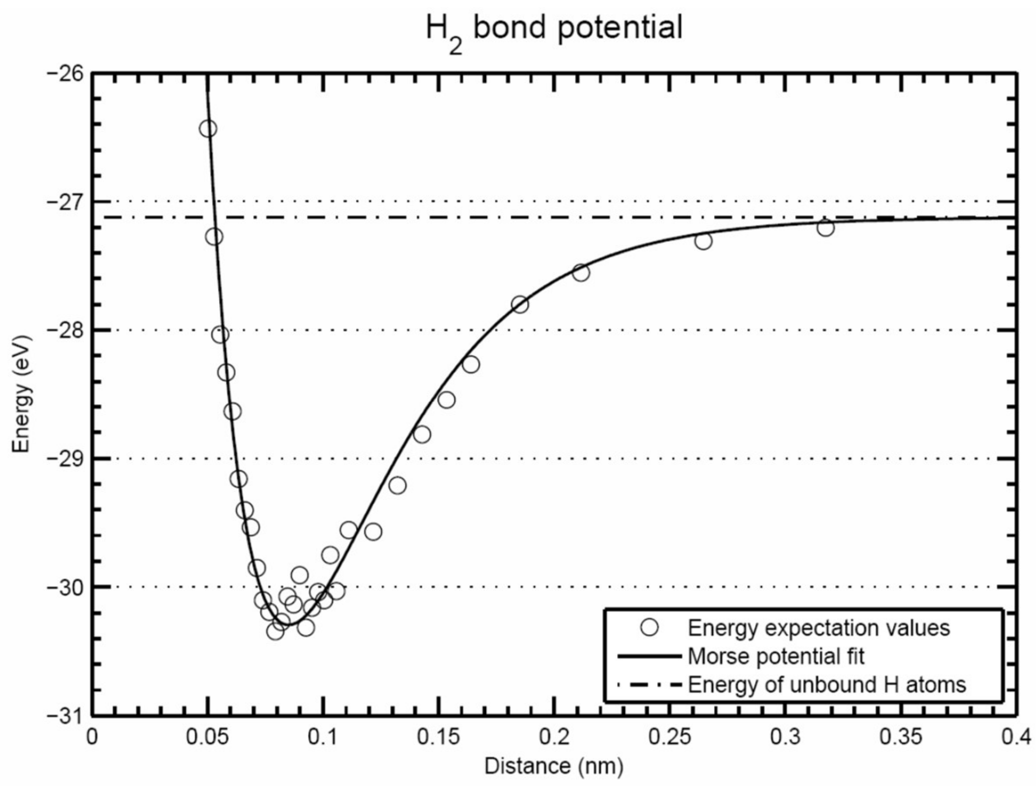
Fix the positions of the nuclei (Born - Oppenheimer approximation) and calculate the energy of the electrons (tight binding, DFT, etc).

Move the nuclei and recalculate until you find nuclear positions that minimize the energy. Check with x-ray diffraction data.

Calculate how the energy increases as nuclei are pushed a small distance from the minimum energy position. This is similar to determining a bond potential like a Morse potential.

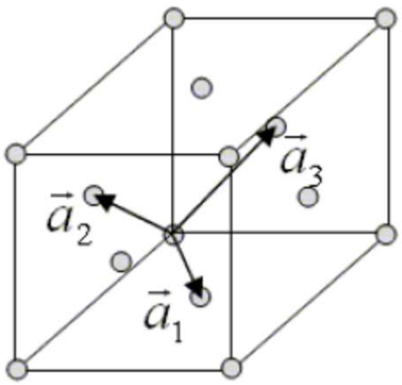
Phonons

In a crystal, the atoms are connected by nonlinear springs.



Phonons are the quasiparticles you get when you linearize this problem.

fcc phonons



3N degrees of freedom

$$
\begin{aligned}
F^x_{lmn} = \frac{C}{2}\Big[& \left(u^x_{l+1mn} - u^x_{lmn}\right) + \left(u^x_{l-1mn} - u^x_{lmn}\right) + \left(u^x_{lm+1n} - u^x_{lmn}\right) + \left(u^x_{lm-1n} - u^x_{lmn}\right) \\
& + \left(u^x_{l+1mn-1} - u^x_{lmn}\right) + \left(u^x_{l-1mn+1} - u^x_{lmn}\right) + \left(u^x_{lm+1n-1} - u^x_{lmn}\right) + \left(u^x_{lm-1n+1} - u^x_{lmn}\right) \\
& + \left(u^y_{l+1mn} - u^y_{lmn}\right) + \left(u^y_{l-1mn} - u^y_{lmn}\right) - \left(u^y_{lm+1n-1} - u^y_{lmn}\right) - \left(u^y_{lm-1n+1} - u^y_{lmn}\right) \\
& + \left(u^z_{lm+1n} - u^z_{lmn}\right) + \left(u^z_{lm-1n} - u^z_{lmn}\right) - \left(u^z_{l+1mn-1} - u^z_{lmn}\right) - \left(u^z_{l-1mn+1} - u^z_{lmn}\right)\Big]
\end{aligned}
$$

The ground state is all of the atoms at their equilibrium positions.

phonon normal mode solutions

Newton's laws are a set of $3N_{\text{atom}}$ coupled differential equations. In a normal mode solution, all of the atoms move with the same frequency. The translational symmetry of the crystal requires that the normal mode solutions are eigenfunctions of the translation operator. The normal mode solutions are therefore

$$\vec{u}_k \exp\left(i\left(\vec{k}\cdot l\vec{a}_1 + \vec{k}\cdot m\vec{a}_2 + \vec{k}\cdot n\vec{a}_3 - \omega t\right)\right)$$

The components of the vector u_k describe the displacements of the atoms of the basis away from their equilibrium positions. If there are p atoms in the basis, u_k will have $3p$ components $u_k = (u^{Ax}{}_k, u^{Ay}{}_k, u^{Az}{}_k, u^{Bx}{}_k, u^{By}{}_k, u^{Bz}{}_k, \ldots)$, where the superscripts label the atoms of the basis.

Phonons

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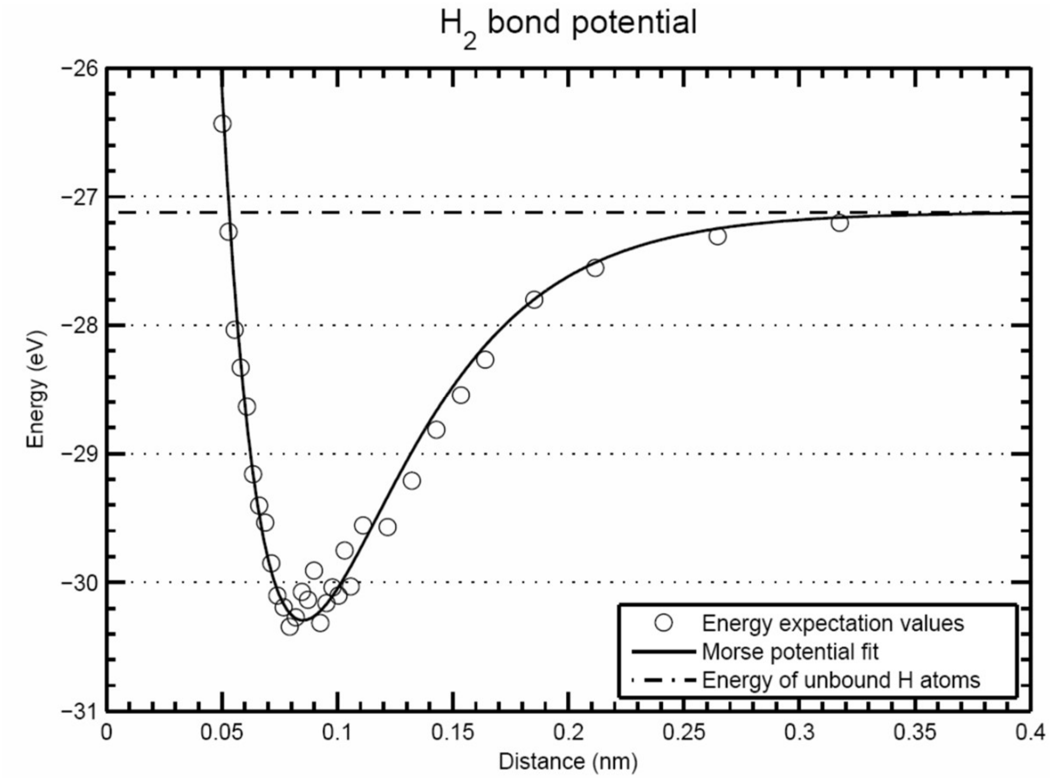

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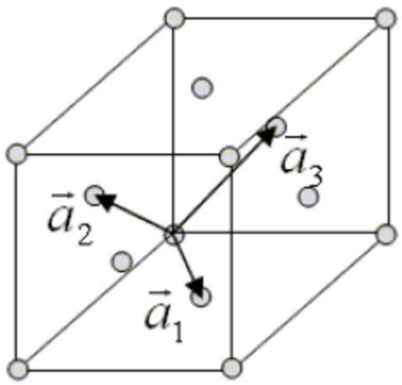
fcc phonons



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Phonons

N_{atom} atoms in crystal
$3N_{\mathrm{atom}}$ normal modes
p atoms in the basis
N_{atom}/p unit cells
N_{atom}/p translational symmetries
N_{atom}/p k-vectors
$3p$ modes for every k vector
3 acoustic branches and $3p$-3 optical branches

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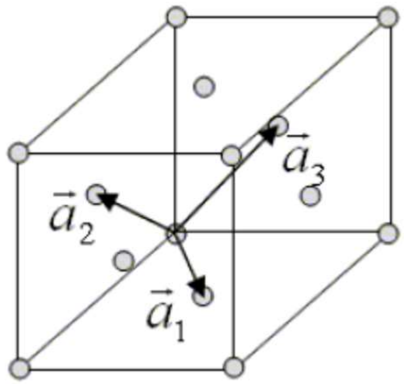
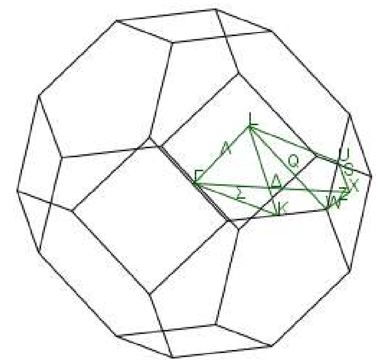
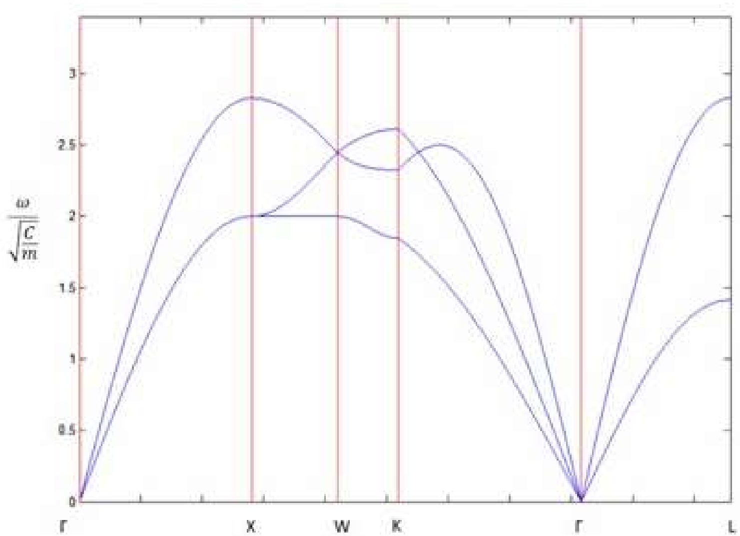
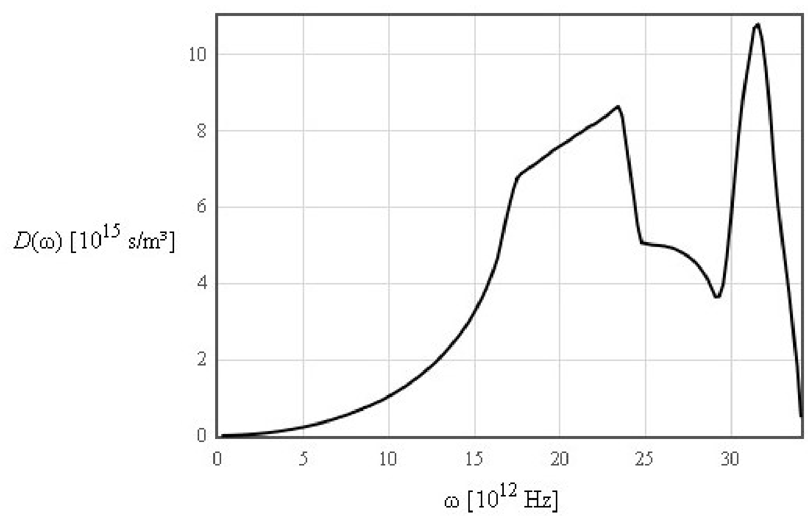




3N degrees of freedom





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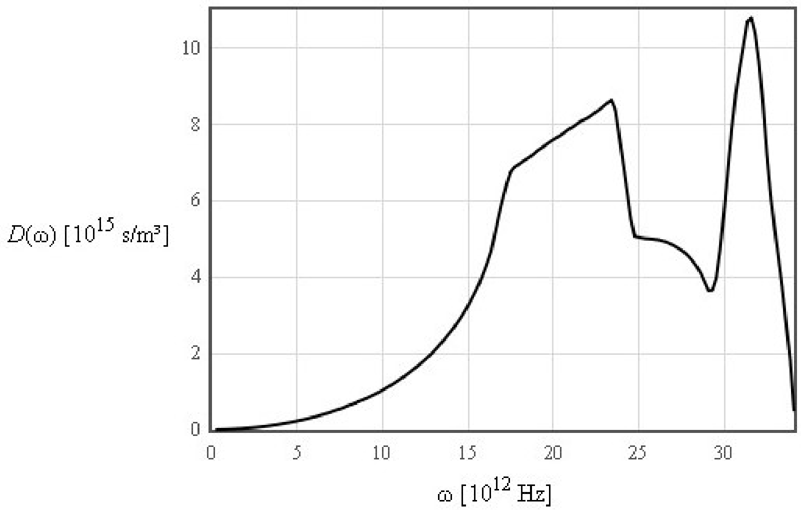
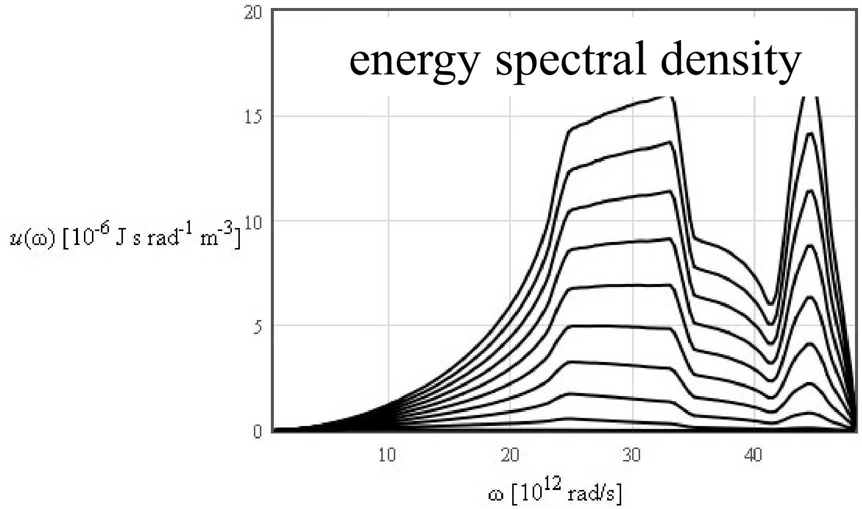
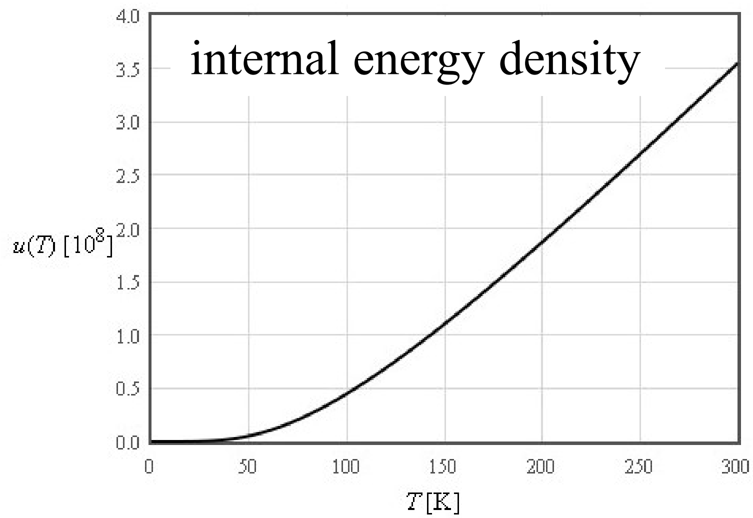


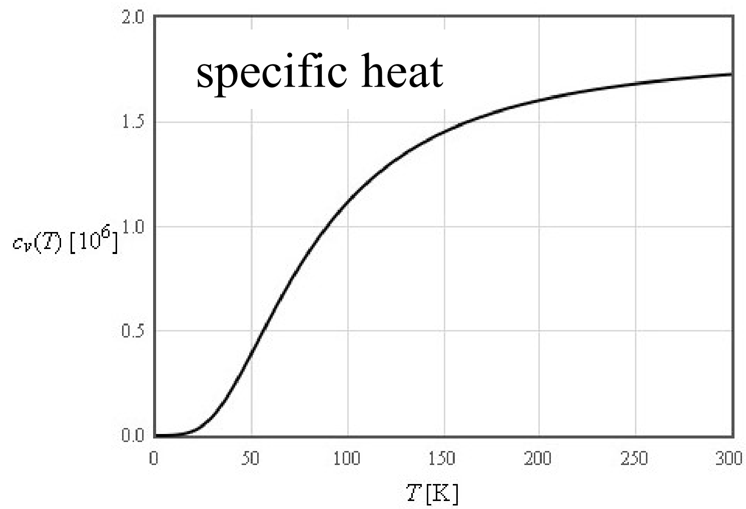
$D(\omega)$ [10^{15} s/m³]
ω [10^{12} Hz]



energy spectral density
$u(\omega)$ [10^{-6} J s rad^{-1} m^{-3}]
ω [10^{12} rad/s]

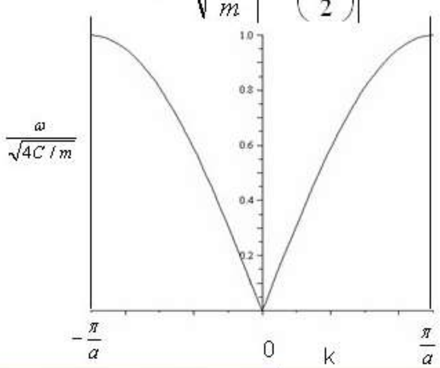


internal energy density
$u(T)$ [10^{8}]
T [K]



specific heat
$c_V(T)$ [10^{6}]
T [K]

http://lamp.tu-graz.ac.at/~hadley/ss1/phonons/phonontable.html

	Linear Chain $m\frac{d^2u_s}{dt^2}=C(u_{s+1}-2u_s+u_{s-1})$	Linear chain 2 masses $M_1\frac{d^2u_s}{dt^2}=C(v_{s-1}-2u_s+v_s)$ $M_2\frac{d^2v_s}{dt^2}=C(u_s-2v_s+u_{s+1})$	Linear chain 2 spring constants $M\frac{d^2u_s}{dt^2}=C_1(v_{s-1}-2u_s+v_s)$ $M\frac{d^2v_s}{dt^2}=C_2(u_s-2v_s+u_{s+1})$	n
Eigenfunction solutions	$u_s=A_k e^{i(ksa-\omega t)}$	$u_s=ue^{i(ksa-\omega t)}$ $v_s=ve^{i(ksa-\omega t)}$	$u_s=ue^{i(ksa-\omega t)}$ $v_s=ve^{i(ksa-\omega t)}$	
Dispersion relation	$\omega=\sqrt{\frac{4C}{m}}\left\lvert\sin\left(\frac{ka}{2}\right)\right\rvert$ 	$\omega^2=C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\pm C\sqrt{\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2-\frac{4\sin^2 ka}{M_1M_2}}$