Review

Maxwell synthesis of electricity, magnetism and optics.

We determined the normal modes of the electromagnetic field. Each normal mode can be labeled by $\vec{k}$ and polarization.

We quantized the normal modes and found they have quantized energies

$$\hbar\omega(j+\tfrac{1}{2}) \qquad j = 0,1,2,3\cdots$$

We calculated the thermodynamic properties u, s, f, c_v, ...

Review II

The solutions to the wave equation in a periodic medium either have Bloch form

$$e^{i\vec{k}\cdot\vec{r}}u_{\vec{k}}(\vec{r}) \qquad \text{(band)}$$

or the solutions are exponentially growing or decaying (bandgap).

Bloch theorem

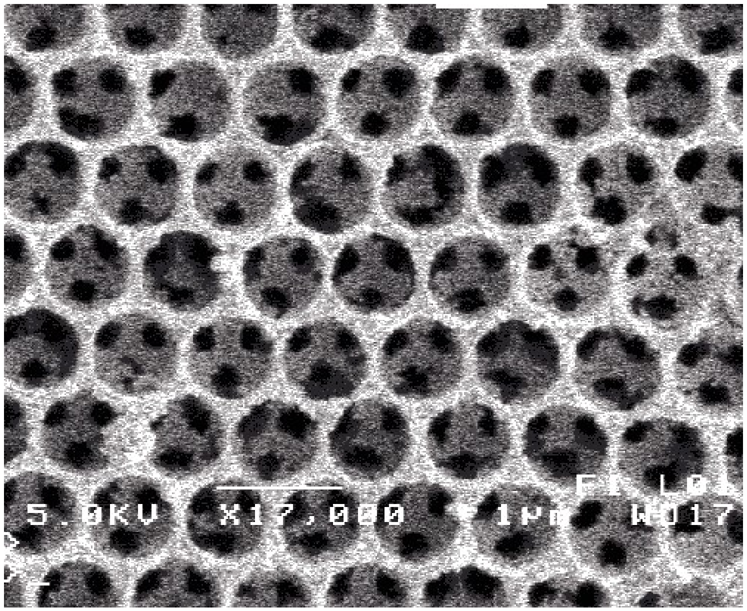
Fourier series for a periodic function.

$$C(\vec{r}) = \sum_{\vec{G}} C_{\vec{G}} e^{i\vec{G}\cdot\vec{r}}$$

$$f_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \sum_{\vec{G}} C_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} = e^{i\vec{k}\cdot\vec{r}} \underbrace{e^{i\vec{G}_0\cdot\vec{r}} e^{-i\vec{G}_0\cdot\vec{r}}}_{1} \sum_{\vec{G}} C_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} = e^{i\left(\vec{k}+\vec{G}_0\right)\cdot\vec{r}} \sum_{\vec{G}} C_{\vec{G}} e^{i\left(\vec{G}-\vec{G}_0\right)\cdot\vec{r}}$$

It is always possible to use a *k* in the first Brillouin zone.

Inverse opal photonic crystal



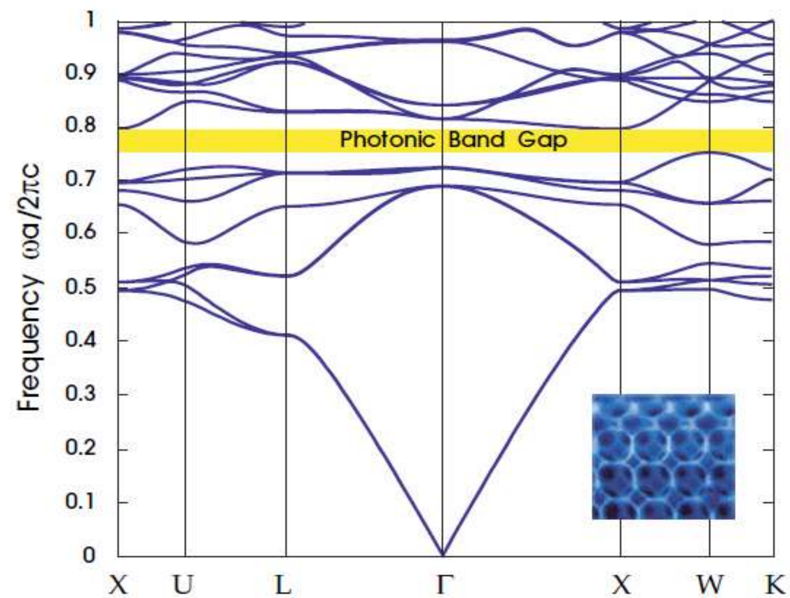
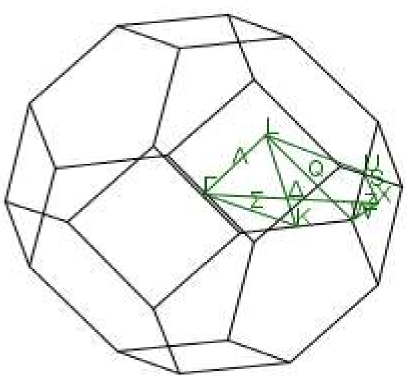


Figure 8: The photonic band structure for the lowest bands of an "inverse opal" structure: a face-centered cubic (fcc) lattice of close-packed air spheres in dielectric ($\varepsilon = 13$). (Inset shows fabricated structure from figure 9.) There is a complete photonic band gap (yellow) between the eighth and ninth bands. The wave vector varies across the irreducible Brillouin zone between the labelled high-symmetry points; see appendix B for a discussion of the Brillouin zone of an fcc lattice.



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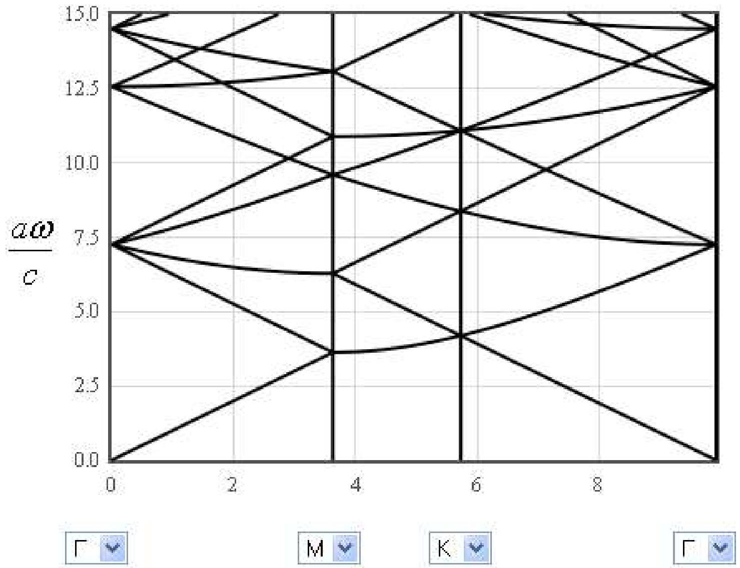
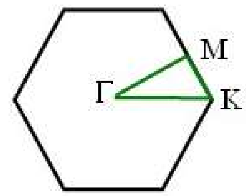
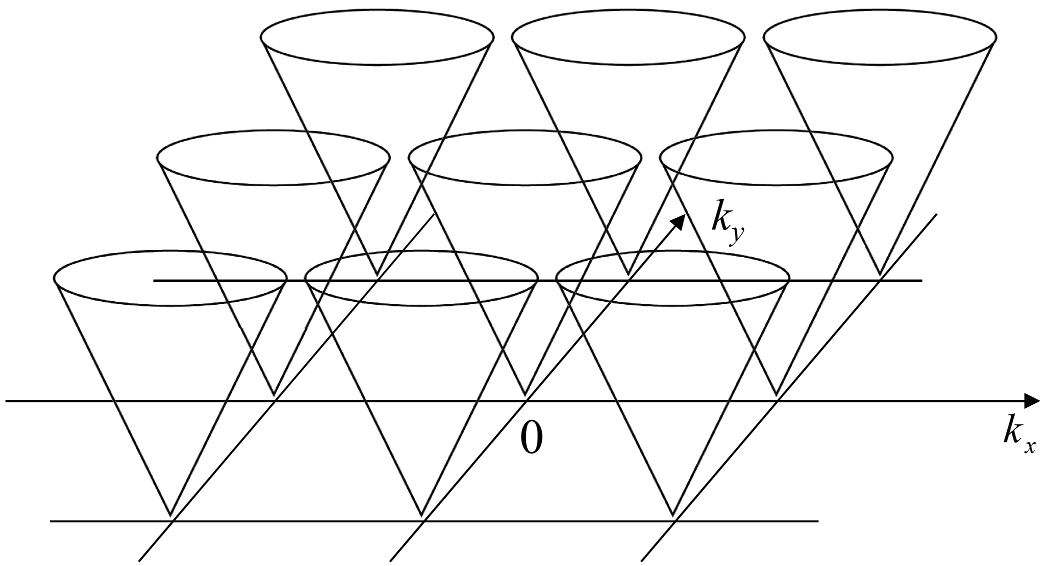
Empty lattice approximation



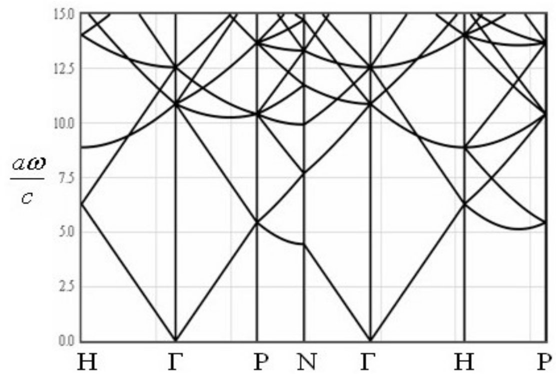




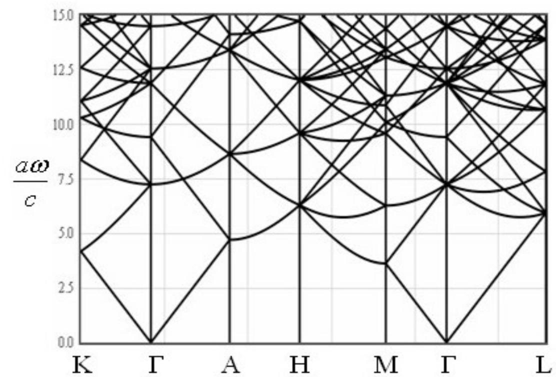
$$\omega = c\left|\vec{k}\right|$$

Empty lattice approximation

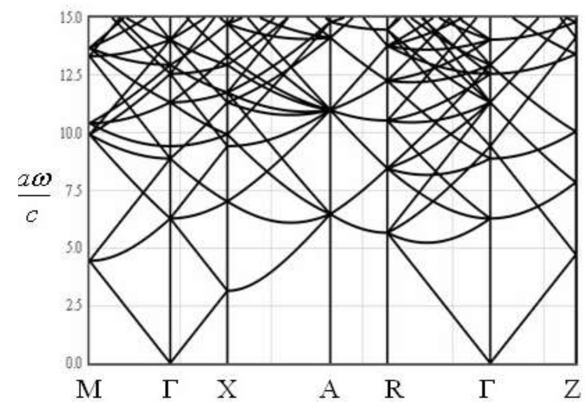
Body centered cubic

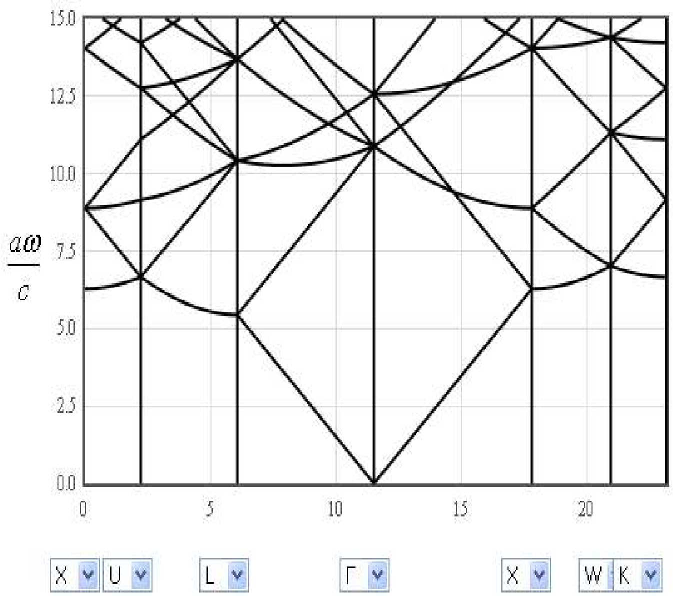


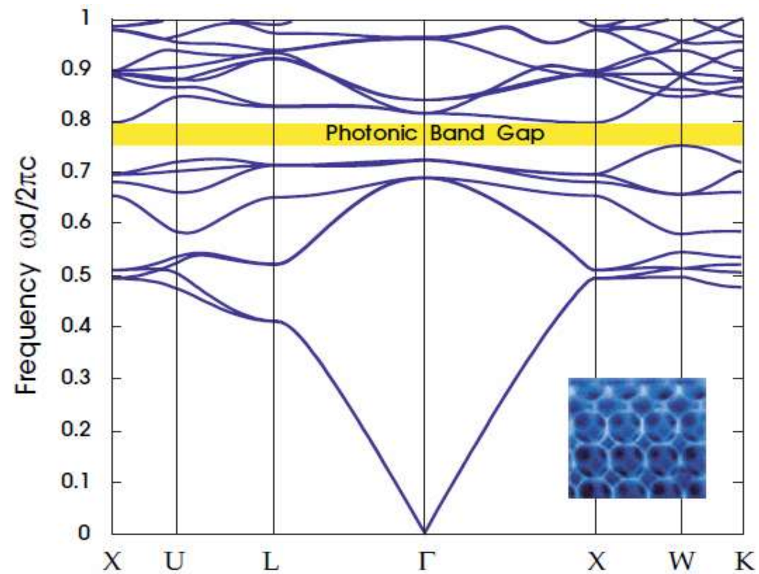
Hexagonal



Tetragonal







Plane wave method

$$c(\vec{r})^2 \nabla^2 A_j = \frac{d^2 A_j}{dt^2}$$

$$c(\vec{r})^2 = \sum_{\vec{G}} b_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} \qquad A_j = \sum_{\vec{k}} A_{\vec{k}} e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)}$$

$$\sum_{\vec{G}} b_{\vec{G}} e^{i\vec{G}\cdot\vec{r}} \sum_{\vec{\kappa}} \left(-\kappa^2\right) A_{\vec{\kappa}} e^{i\left(\vec{\kappa}\cdot\vec{r}-\omega t\right)} = -\omega^2 \sum_{\vec{k}} A_{\vec{k}} e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)}$$

$$\sum_{\vec{\kappa}} \sum_{\vec{G}} \left(-\kappa^2\right) b_{\vec{G}} A_{\vec{\kappa}} e^{i\left(\vec{G}\cdot\vec{r}+\vec{\kappa}\cdot\vec{r}-\omega t\right)} = -\omega^2 \sum_{\vec{k}} A_{\vec{k}} e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)}$$

collect like terms: $\vec{G}+\vec{\kappa}=\vec{k} \quad \Rightarrow \quad \vec{\kappa}=\vec{k}-\vec{G}$

Central equations: $$\sum_{\vec{G}} \left(\vec{k}-\vec{G}\right)^2 b_{\vec{G}} A_{\vec{k}-\vec{G}} = \omega^2 A_{\vec{k}}$$

Plane wave method

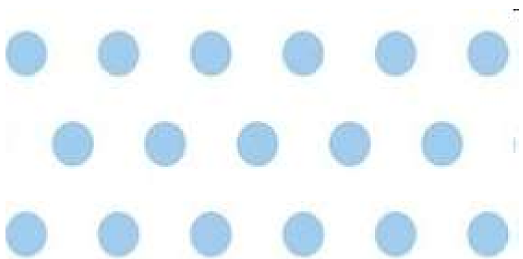
Central equations: $$\sum_{\vec{G}} \left(\vec{k}-\vec{G}\right)^2 b_{\vec{G}} A_{\vec{k}-\vec{G}} = \omega^2 A_{\vec{k}}$$

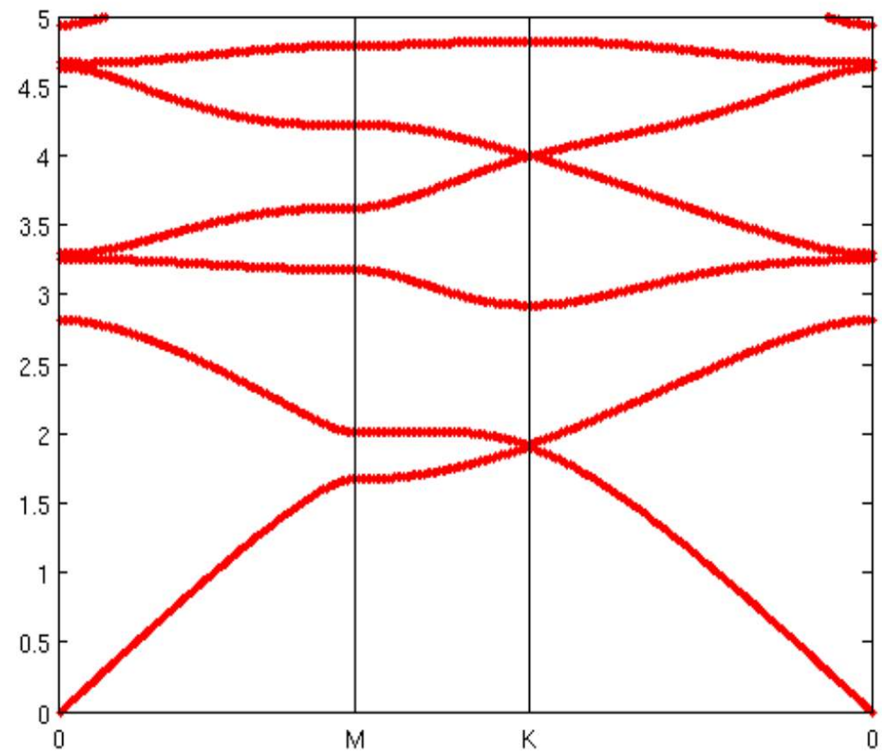
Choose a k value inside the 1st Brillouin zone. The coefficient A_k is coupled by the central equations to coefficients A_k outside the 1st Brillouin zone. Write these coupled equations in matrix form.

$$\begin{bmatrix} \left(\vec{k}+\vec{G}_2\right)^2 b_0 & \left(\vec{k}+\vec{G}_2-\vec{G}_1\right)^2 b_{\vec{G}_1} & k^2 b_{\vec{G}_2} & \left(\vec{k}+\vec{G}_2-\vec{G}_3\right)^2 b_{\vec{G}_3} & \left(\vec{k}+\vec{G}_2-\vec{G}_4\right)^2 b_{\vec{G}_4} \\ \left(\vec{k}+2\vec{G}_1\right)^2 b_{-\vec{G}_1} & \left(\vec{k}+\vec{G}_1\right)^2 b_0 & k^2 b_{\vec{G}_1} & \left(\vec{k}+\vec{G}_1-\vec{G}_2\right)^2 b_{\vec{G}_2} & \left(\vec{k}+\vec{G}_1-\vec{G}_3\right)^2 b_{\vec{G}_3} \\ \left(\vec{k}+\vec{G}_2\right)^2 b_{-\vec{G}_2} & \left(\vec{k}+\vec{G}_1\right)^2 b_{-\vec{G}_1} & k^2 b_0 & \left(\vec{k}-\vec{G}_1\right)^2 b_{\vec{G}_1} & \left(\vec{k}-\vec{G}_2\right)^2 b_{\vec{G}_2} \\ \left(\vec{k}-\vec{G}_1+\vec{G}_3\right)^2 b_{-\vec{G}_3} & \left(\vec{k}-\vec{G}_1+\vec{G}_2\right)^2 b_{-\vec{G}_2} & k^2 b_{-\vec{G}_1} & \left(\vec{k}-\vec{G}_1\right)^2 b_0 & \left(\vec{k}-2\vec{G}_1\right)^2 b_{\vec{G}_1} \\ \left(\vec{k}-\vec{G}_2+\vec{G}_4\right)^2 b_{-\vec{G}_4} & \left(\vec{k}-\vec{G}_2+\vec{G}_3\right)^2 b_{-\vec{G}_3} & k^2 b_{-\vec{G}_2} & \left(\vec{k}-\vec{G}_2+\vec{G}_1\right)^2 b_{-\vec{G}_1} & \left(\vec{k}-\vec{G}_2\right)^2 b_0 \end{bmatrix} \begin{bmatrix} A_{k+G_2} \\ A_{k+G_1} \\ A_k \\ A_{k-G_1} \\ A_{k-G_2} \end{bmatrix} = \omega^2 \begin{bmatrix} A_{k+G_2} \\ A_{k+G_1} \\ A_k \\ A_{k-G_1} \\ A_{k-G_2} \end{bmatrix}$$

There is a matrix like this for every k value in the 1st Brillouin zone.

Close packed circles in 2-D

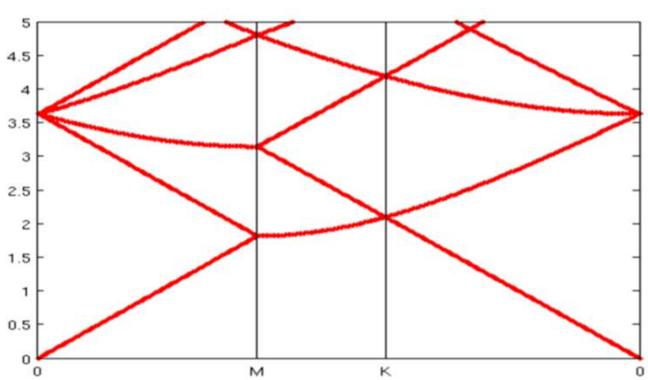


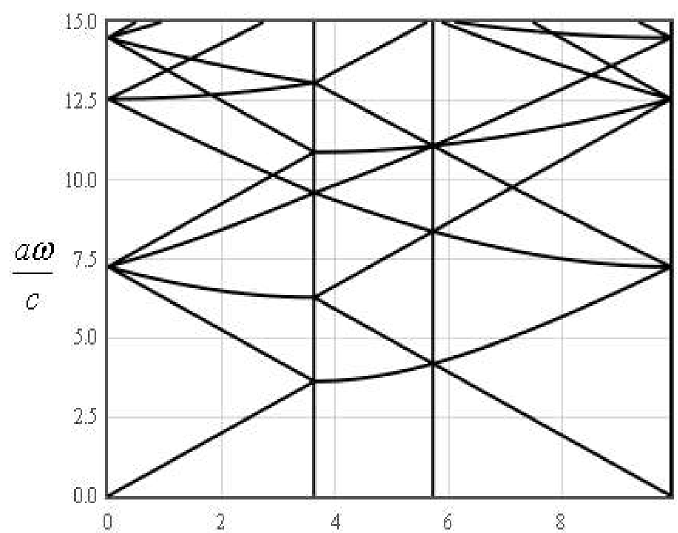


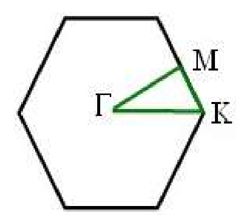
Solved by a student with the plane wave method

Uniform speed of light

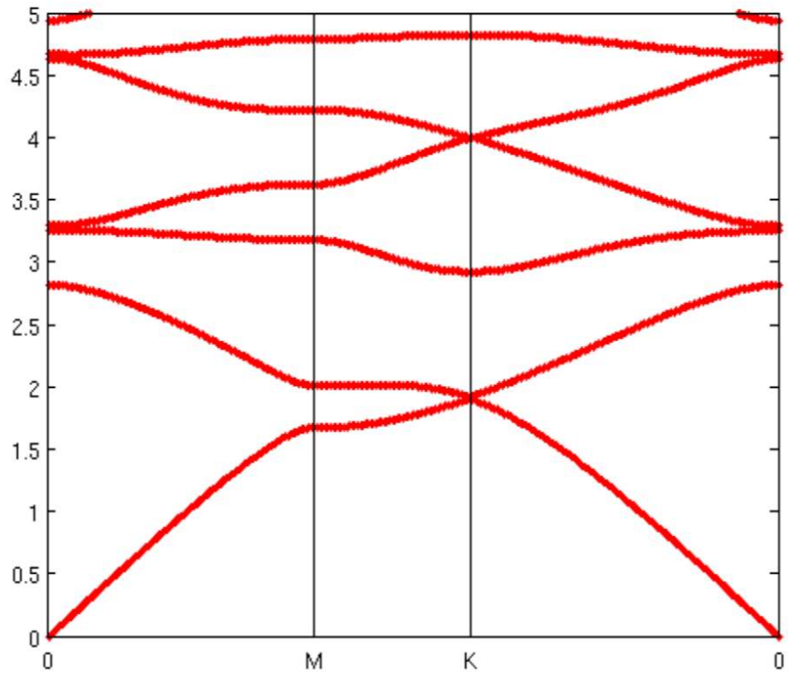
$$
\begin{bmatrix}
\left(\vec{k}+\vec{G}_2\right)^2 b_0-\omega^2 & 0 & 0 & 0 & 0 \\
0 & \left(\vec{k}+\vec{G}_1\right)^2 b_0-\omega^2 & 0 & 0 & 0 \\
0 & 0 & k^2 b_0-\omega^2 & 0 & 0 \\
0 & 0 & 0 & \left(\vec{k}-\vec{G}_1\right)^2 b_0-\omega^2 & 0 \\
0 & 0 & 0 & 0 & \left(\vec{k}-\vec{G}_2\right)^2 b_0-\omega^2
\end{bmatrix}
\begin{bmatrix}
A_{k+G_2} \\ A_{k+G_1} \\ A_k \\ A_{k-G_1} \\ A_{k-G_2}
\end{bmatrix} = 0
$$

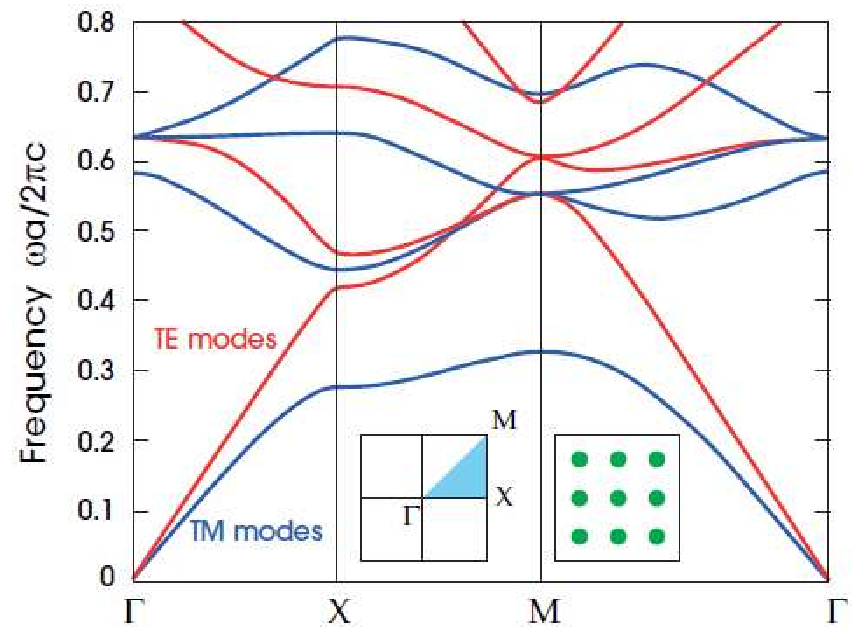






TM and TE modes





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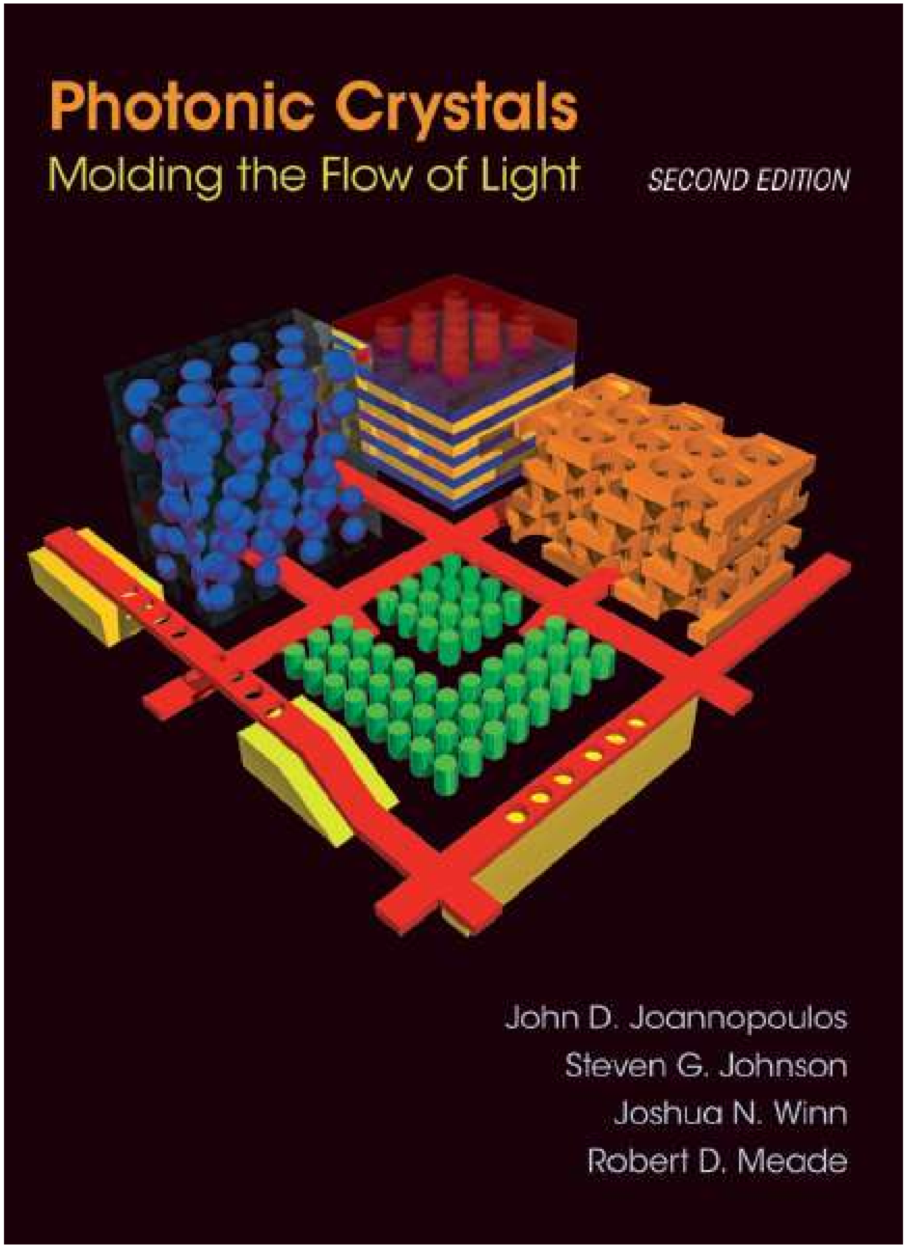
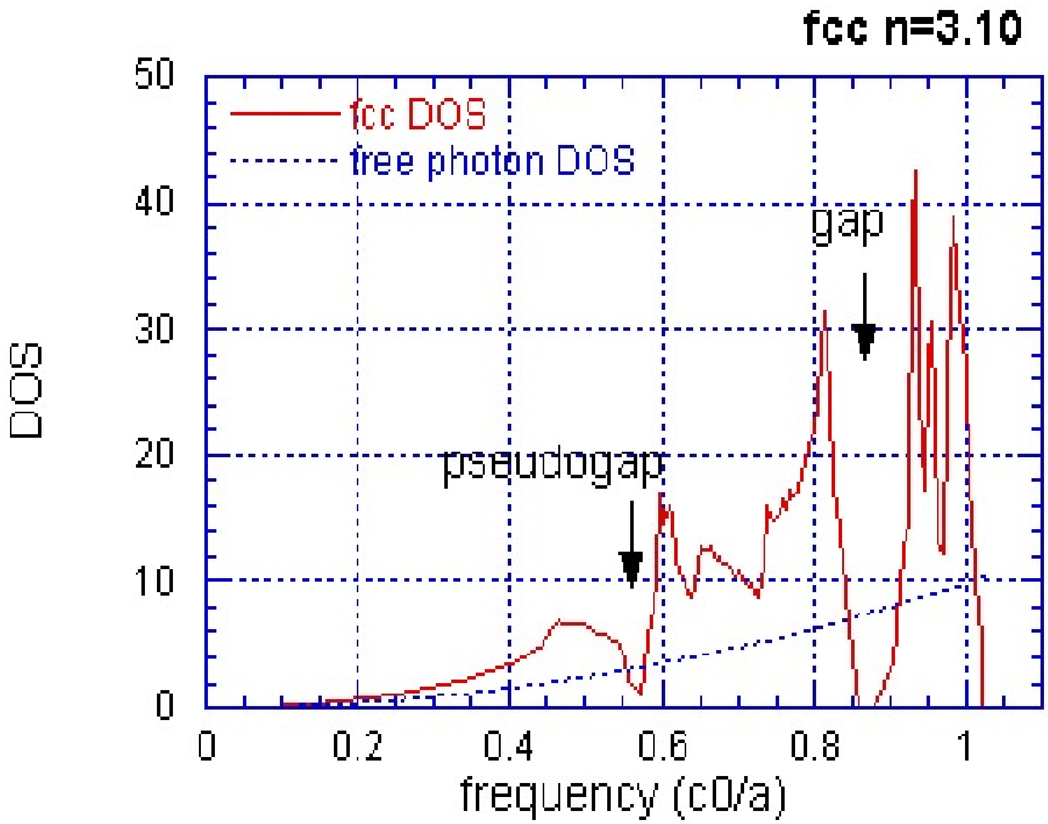
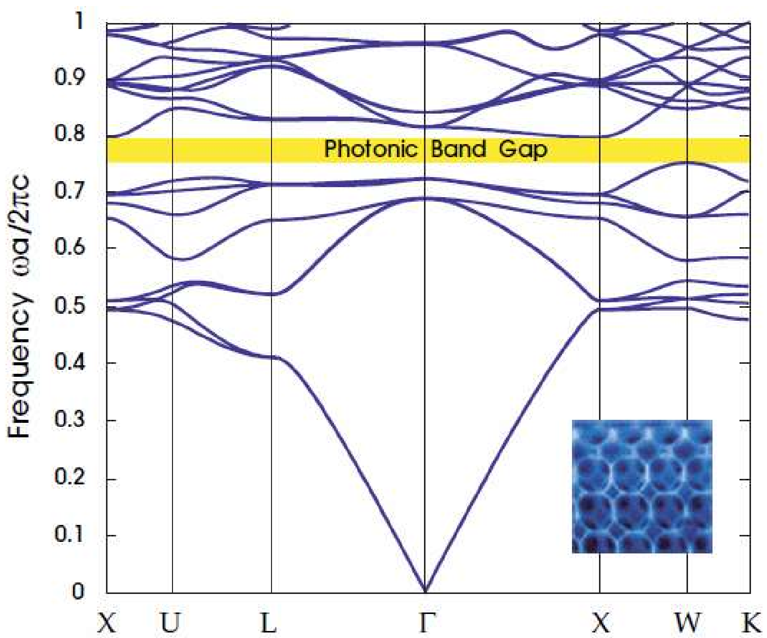


Use the plane wave method to calculate photon dispersion relations and densities of states.

Photon density of states

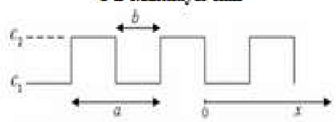
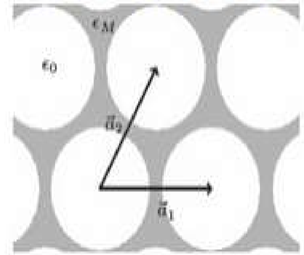
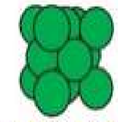
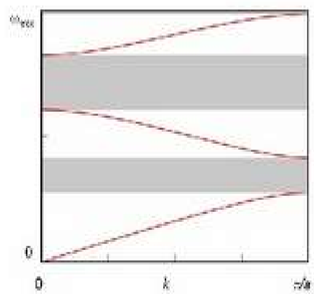
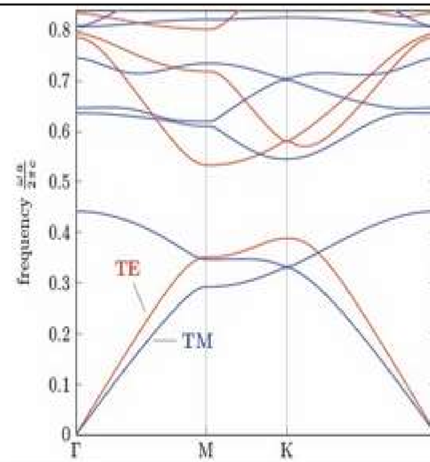
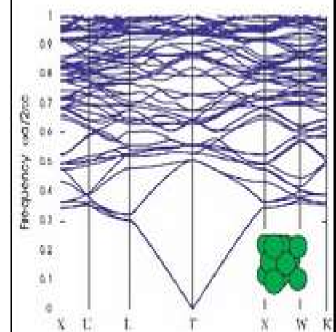
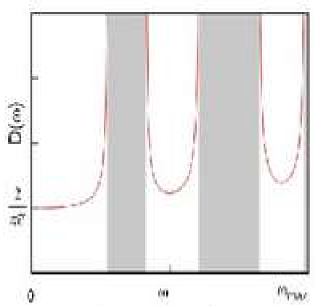
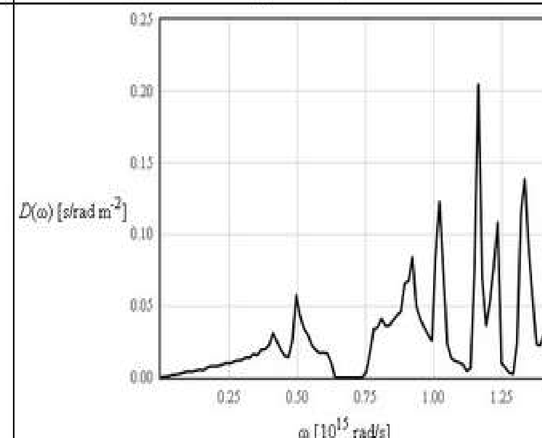
Diffraction causes gaps in the density of modes for *k* vectors near the planes in reciprocal space where diffraction occurs.





photon density of states for voids in an fcc lattice
http://www.public.iastate.edu/~cmpexp/groups/PBG/pres_mit_short/sld002.htm

Photonic crystals

	1-D Multilayer film  1-D Multilayer film	2-D triangular array of air holes 	fcc  http://ab-initio.mit.edu/book
Dispersion relation	 Calculate ω(k)		 http://ab-initio.mit.edu/book
Density of states	 Calculate DoS	 Show DoS	

fcc

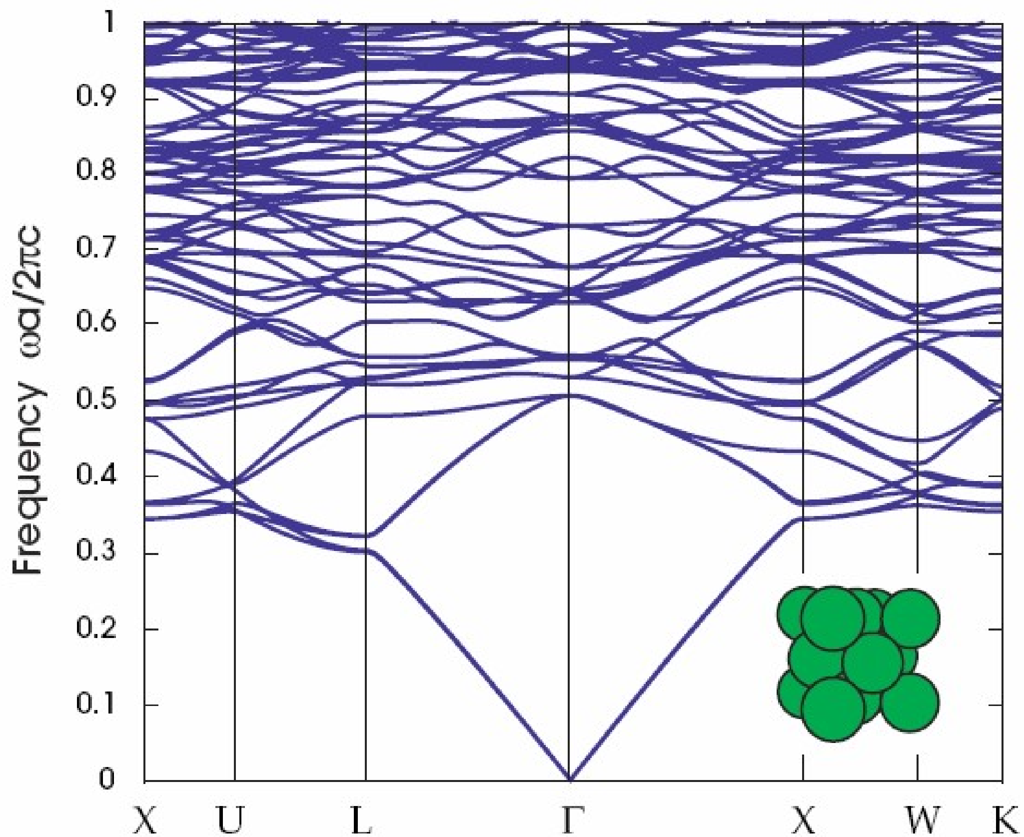


Figure 2: The photonic band structure for the lowest-frequency electromagnetic modes of a face-centered cubic (fcc) lattice of close-packed dielectric spheres ($\varepsilon = 13$) in air (inset). Note the *absence* of a complete photonic band gap. The wave vector varies across the irreducible Brillouin zone between the labelled high-symmetry points; see appendix B for a discussion of the Brillouin zone of an fcc lattice.

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diamond

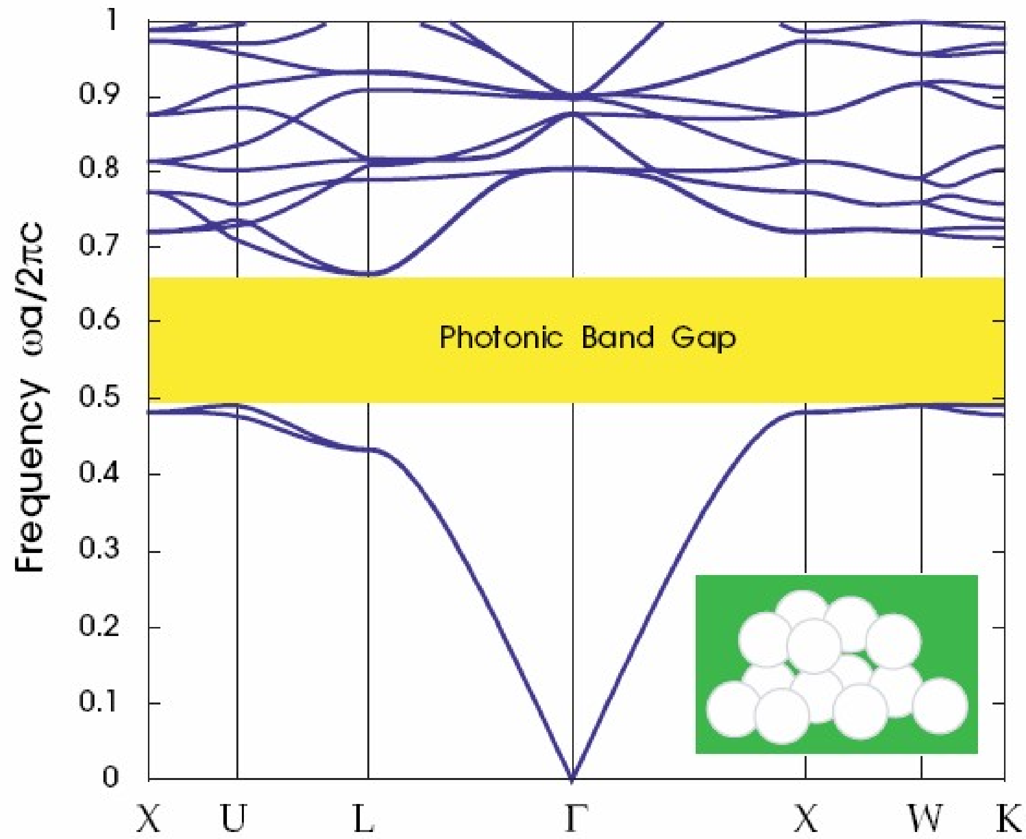

Figure 3: The photonic band structure for the lowest bands of a diamond lattice of air spheres in a high dielectric ($\varepsilon = 13$) material (inset). A complete photonic band gap is shown in yellow. The wave vector varies across the irreducible Brillouin zone between the labelled high-symmetry points; see appendix B for a discussion of the Brillouin zone of an fcc lattice.

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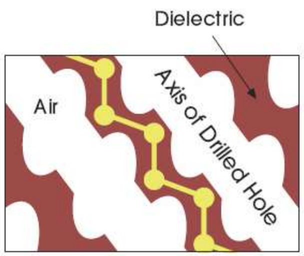
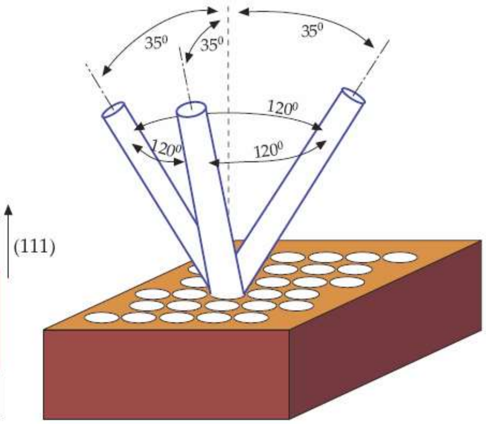
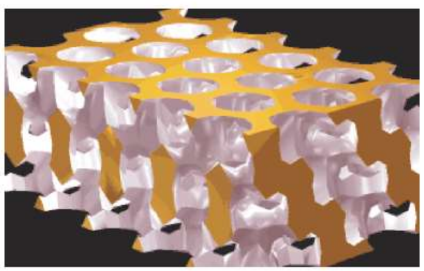
Yablonovite







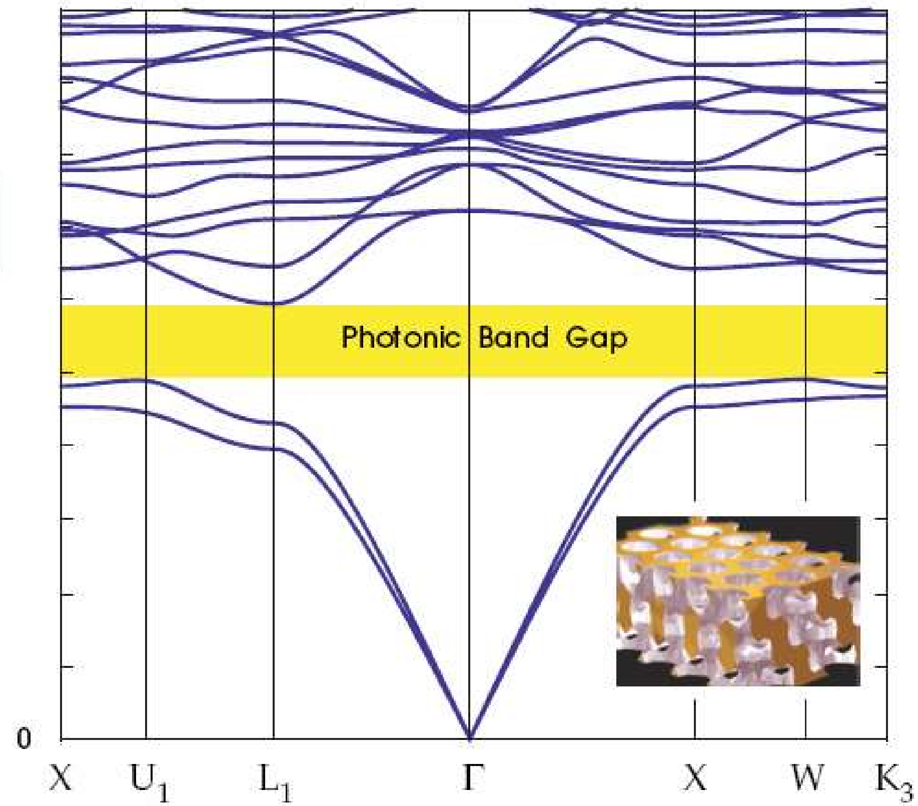


Figure 5: The photonic band structure for the lowest bands of Yablonovite (inset, from figure 4). Wave vectors are shown for a portion of the irreducible Brillouin zone that includes the edges of the complete gap (yellow). A detailed discussion of this band structure can be found in Yablonovitch et al. (1991*a*).

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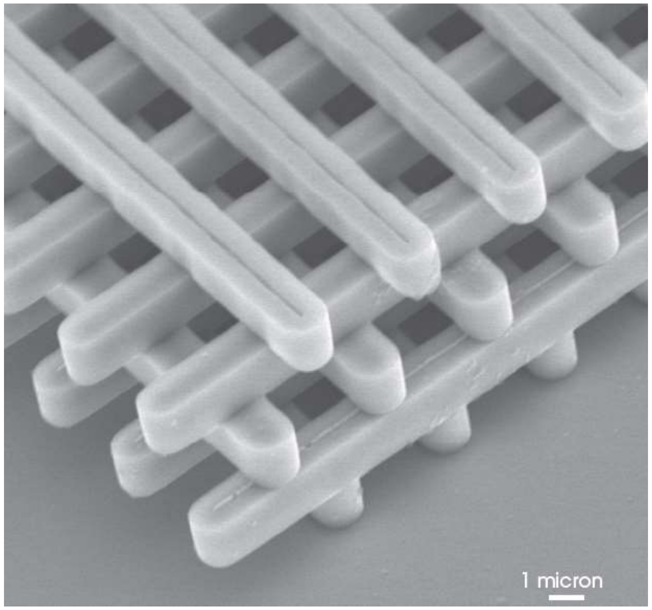
Woodpile



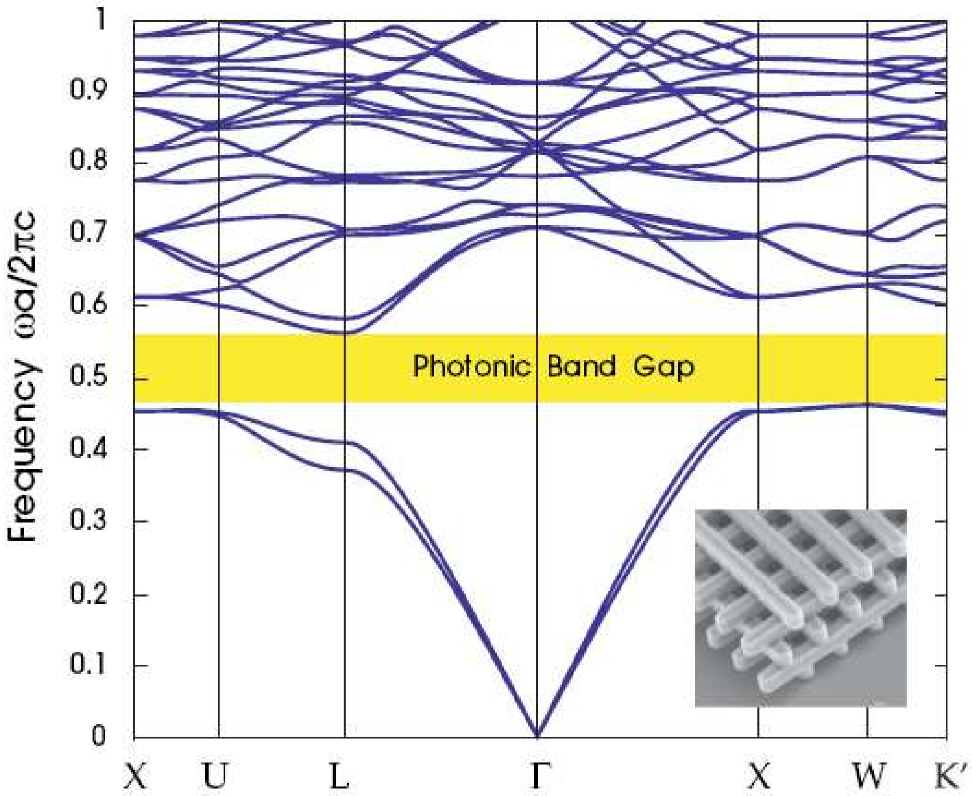


Figure 7: The photonic band structure for the lowest bands of the woodpile structure (inset, from figure 6) with $\varepsilon = 13$ logs in air. The irreducible Brillouin zone is larger than that of the fcc lattice described in appendix B, because of reduced symmetry—only a portion is shown, including the edges of the complete photonic band gap (yellow).

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Student projects

Describe the plane wave method.

Calculate the band structure and density of states for a photonic crystal.

Help complete the table of the empty lattice approximation

Plot the thermodynamic properties of some photonic crystal (you need the density of states)