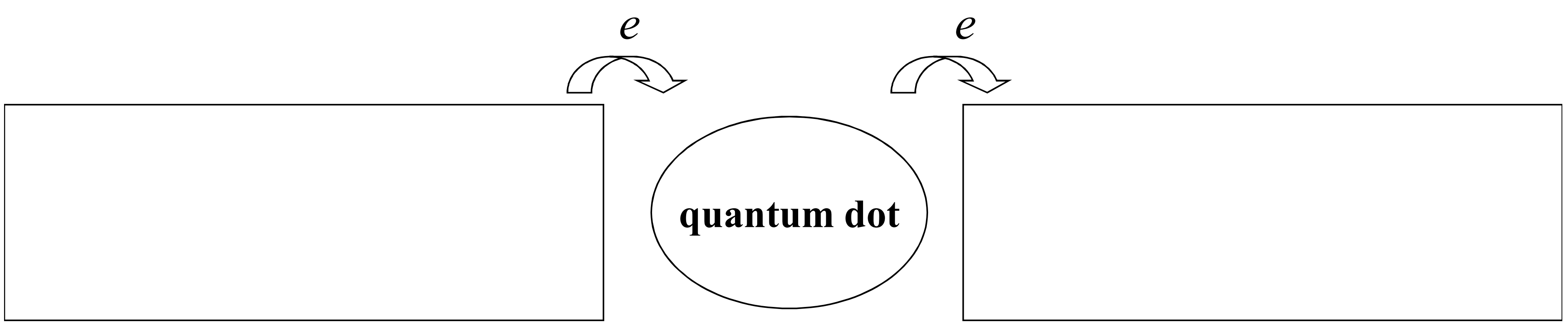
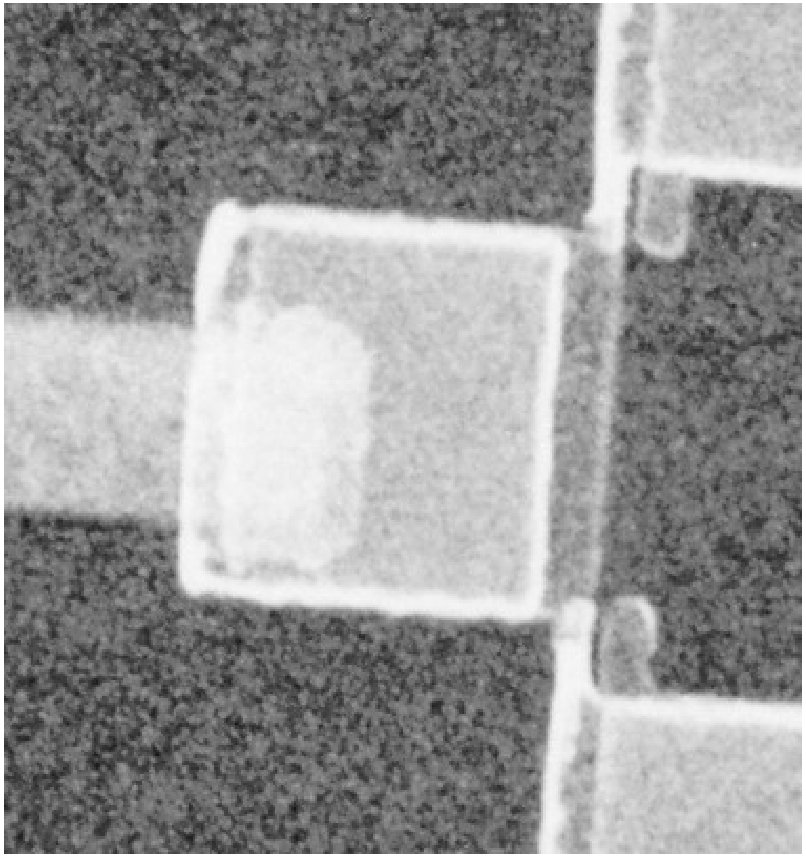
Charging effects

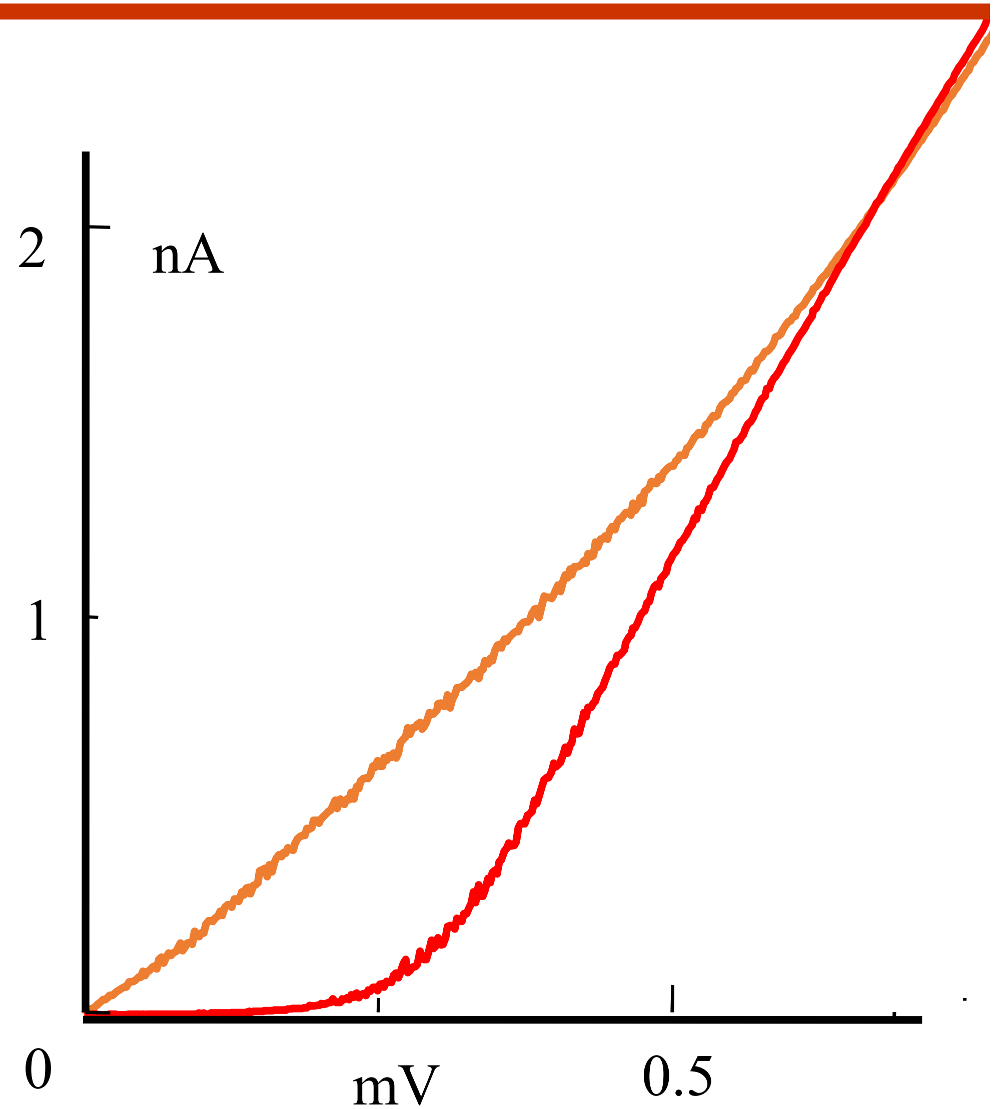
After screening, the next most simple approach to describing electron-electron interactions are charging effects.



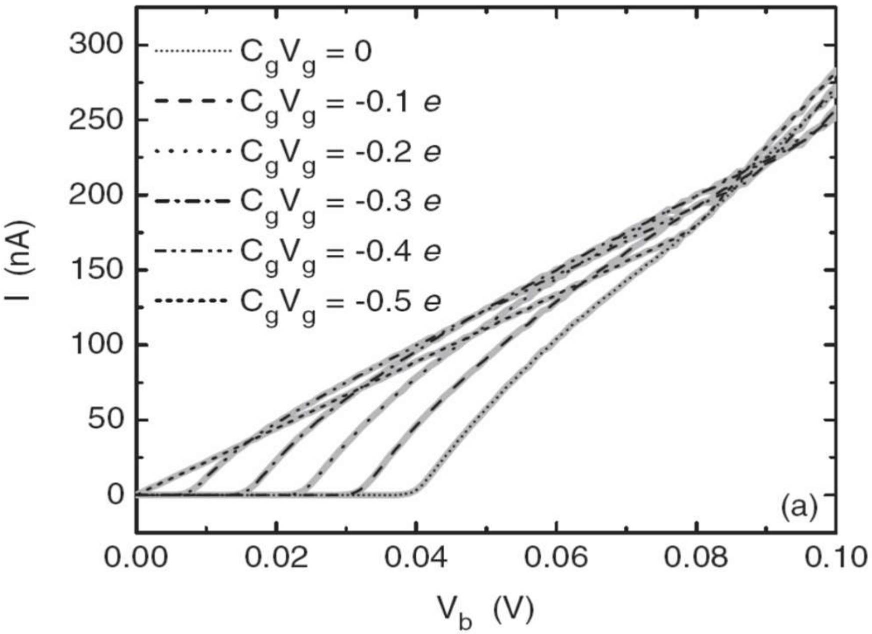
The motion of electrons through a single quantum dot is correlated.

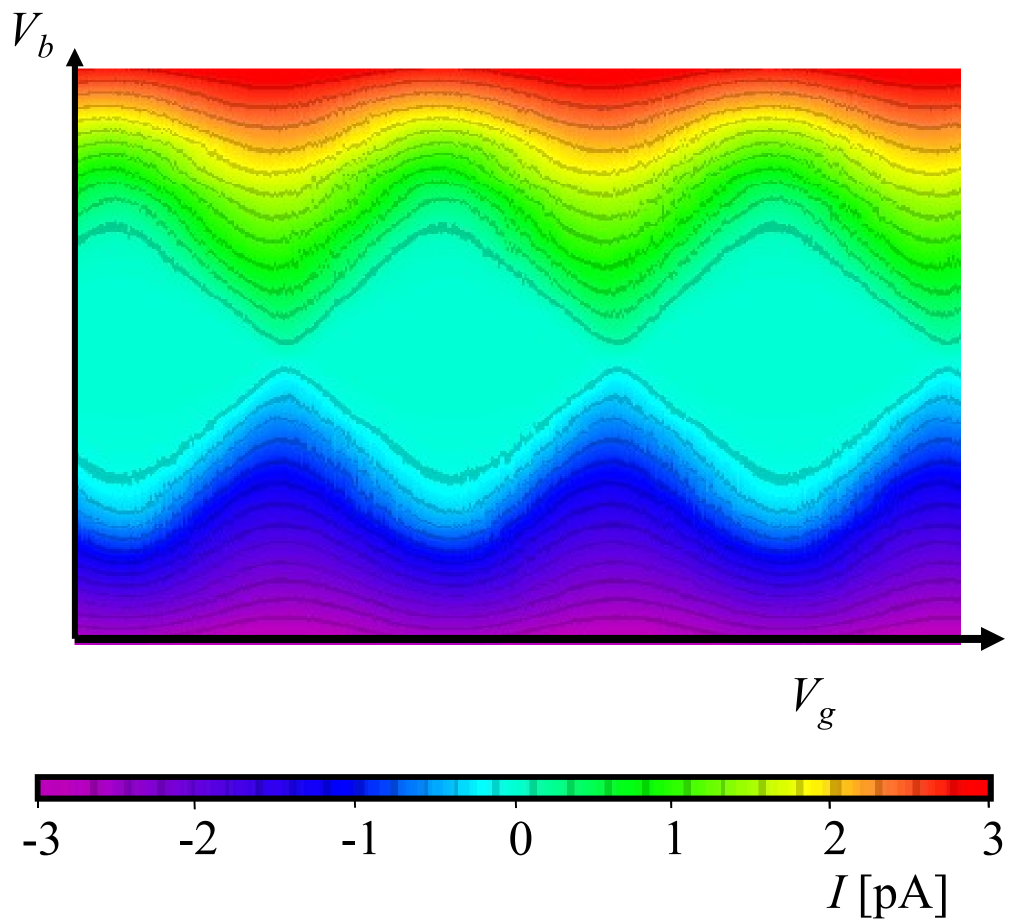
Single electron transistor

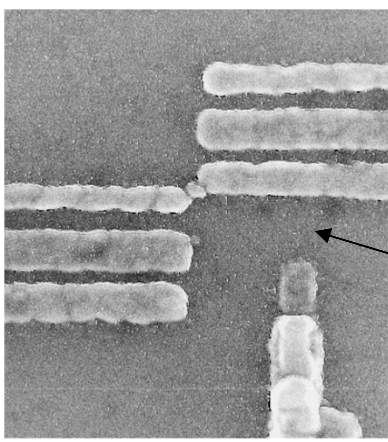




Coulomb blockade

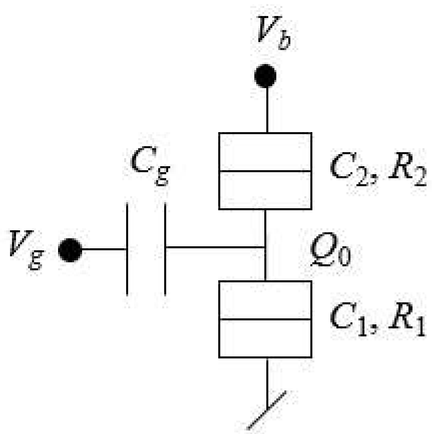


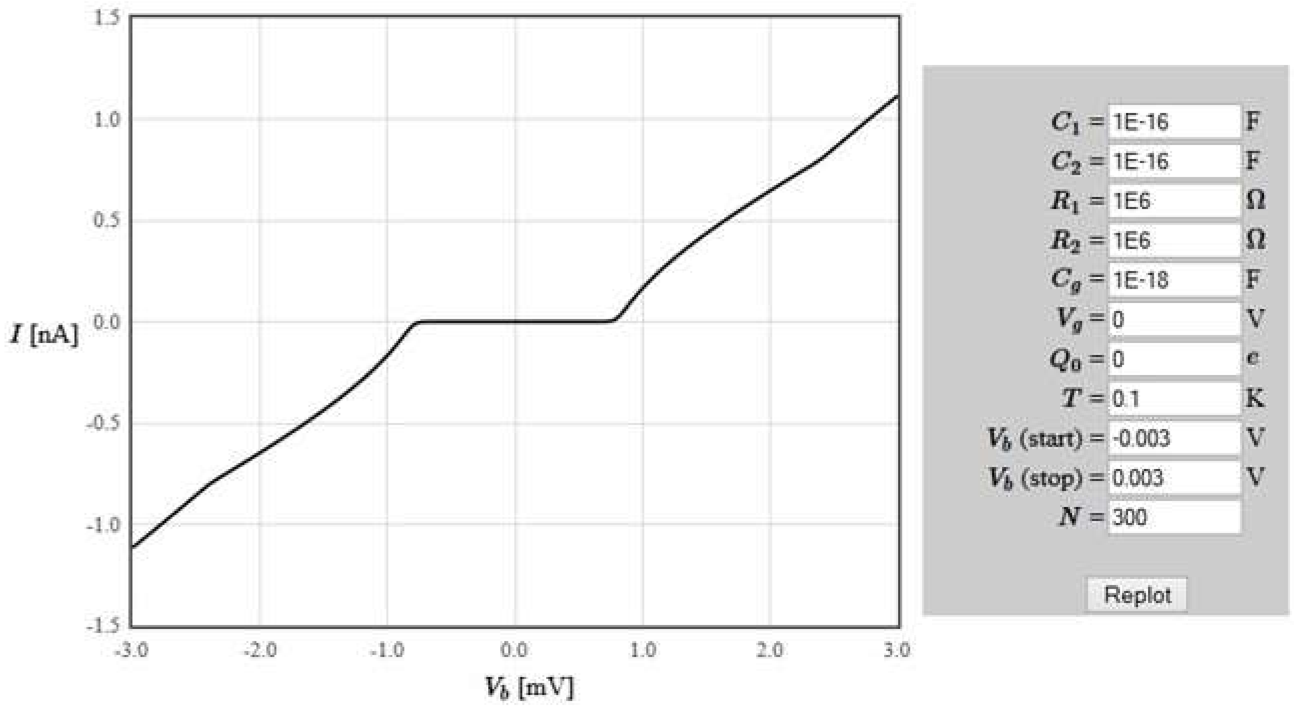




2 nm room temperature SET Pashkin/Tsai NEC

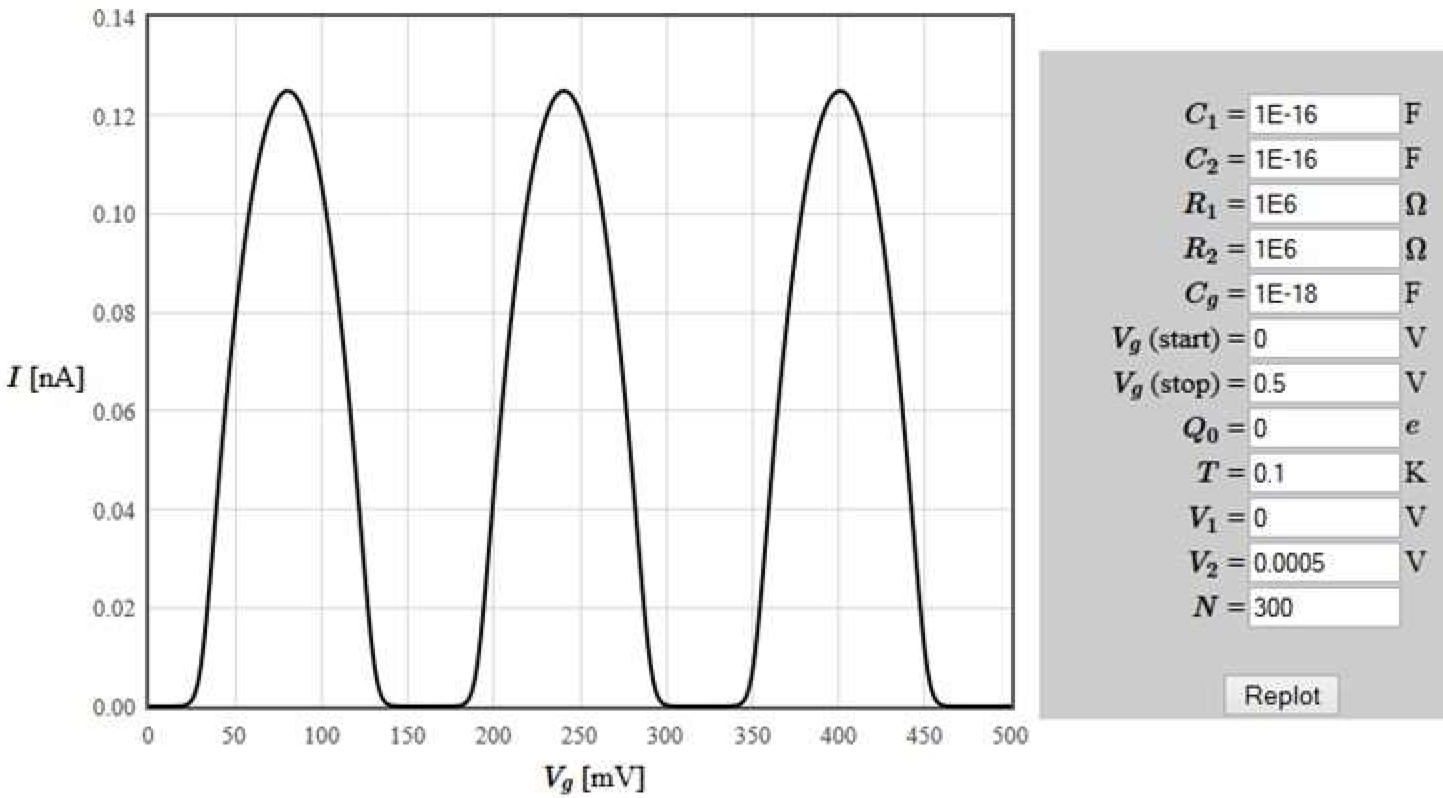
Single electron transistors





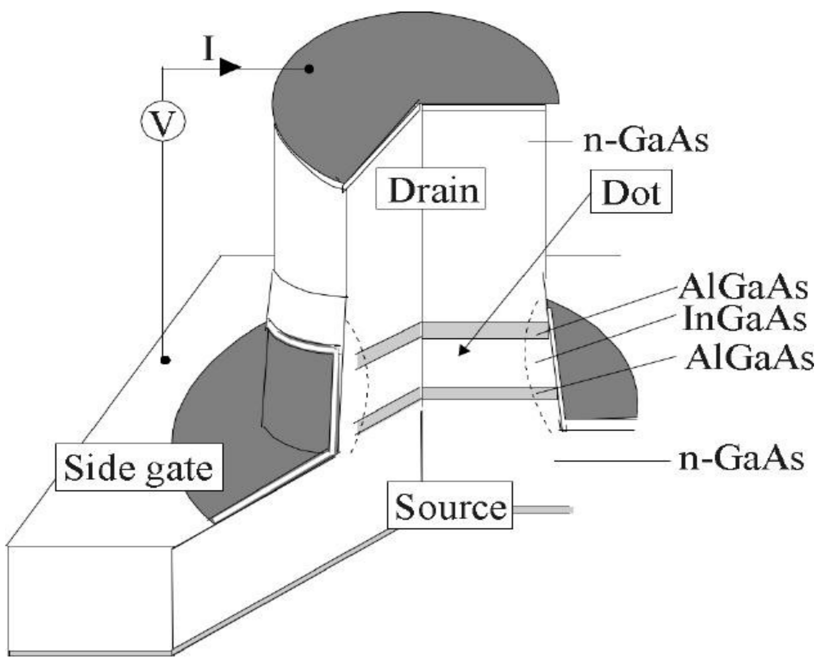
http://lamp.tu-graz.ac.at/~hadley/set/asymIV/SETIV.html
http://lamp.tu-graz.ac.at/~hadley/set/symIV/SETIV.html

Single electron transistors I-V_g

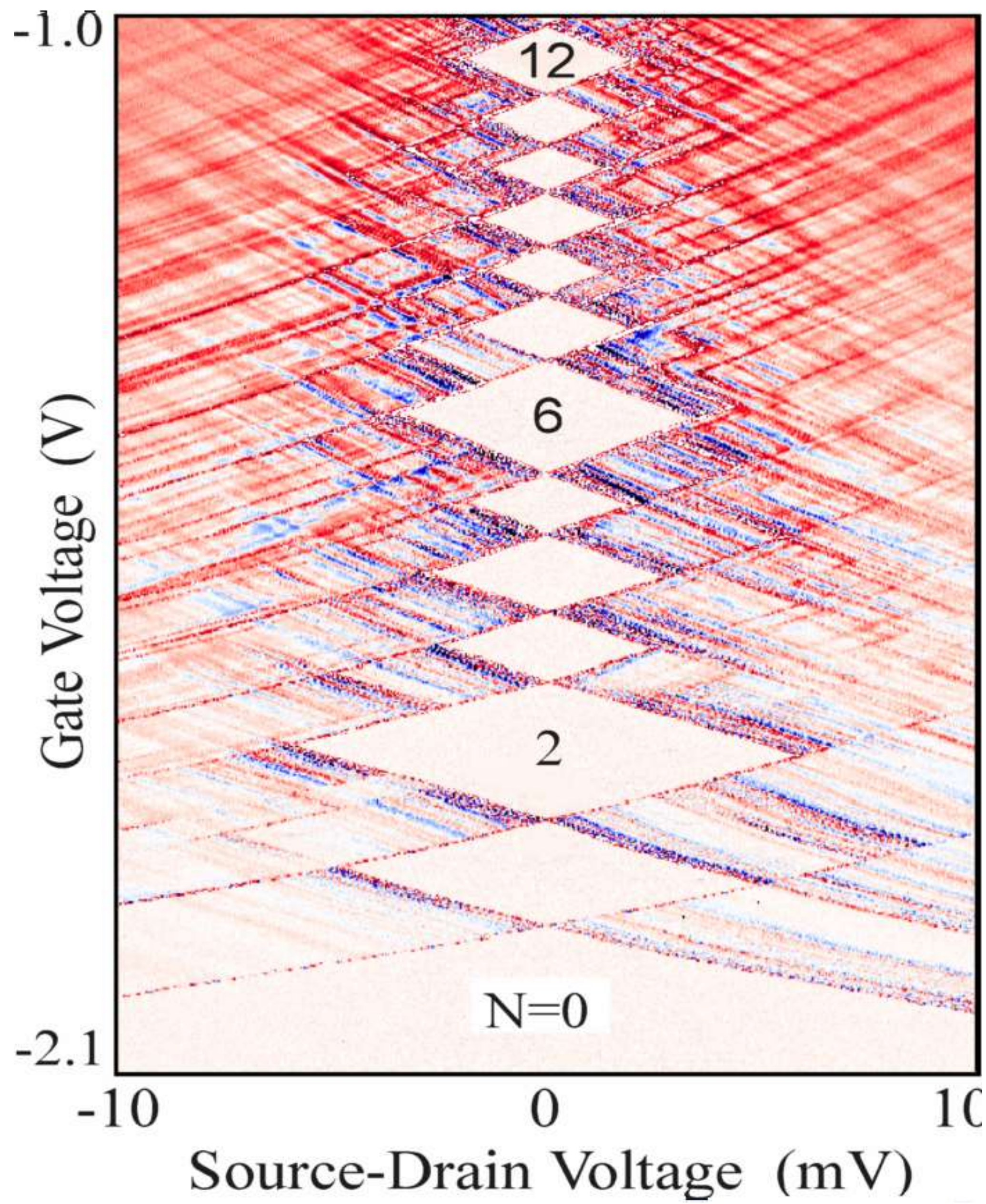


http://lamp.tu-graz.ac.at/~hadley/set/ivg/ivg.html

Quantum dot

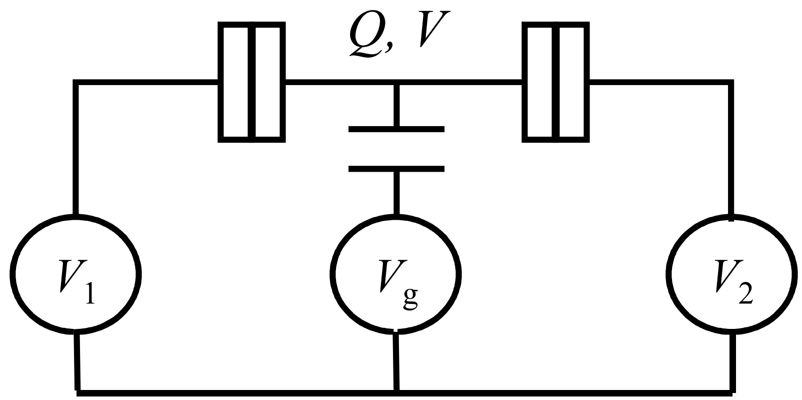


I
V
Drain
n-GaAs
Dot
AlGaAs
InGaAs
AlGaAs
Side gate
Source
n-GaAs



-1.0
-2.1
12
6
2
N=0
Gate Voltage (V)
-10
0
10
Source-Drain Voltage (mV)

Single electron transistor



$$Q = C_1(V - V_1) + C_2(V - V_2) + C_g(V - V_g)$$

$$Q = -ne$$

$$V(n) = \left(-ne + C_1V_1 + C_2V_2 + C_gV_g\right) / C_\Sigma$$

$$C_\Sigma = C_1 + C_2 + C_g$$

Single electron transistor

The potential of the island with *n* electrons on it:

$$V(n) = \left(-ne + C_1V_1 + C_2V_2 + C_g\right) / C_\Sigma$$

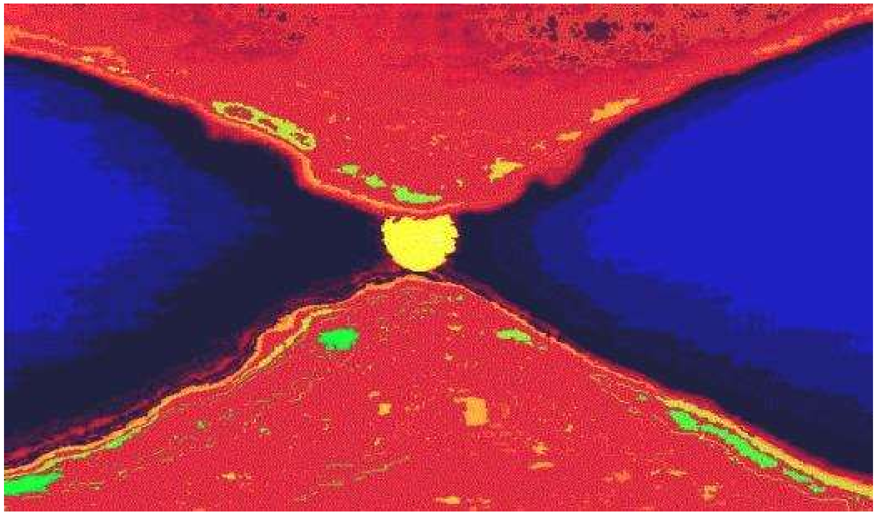
The energy needed to add an infinitesimal charge *dq* to an island at voltage *V*(*n*) is *V*(*n*)*dq*. The energy needed to add a whole electron is:

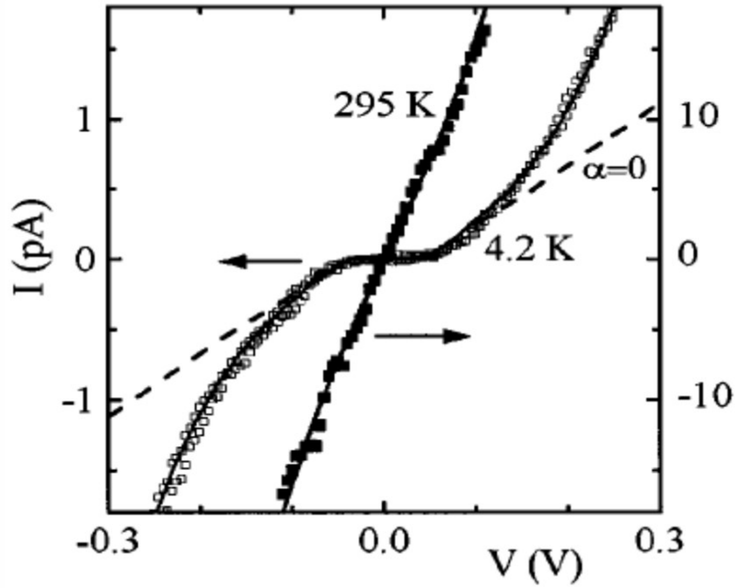
$$\Delta E = \int_0^{-e} V(n)dq = -eV(n) + \frac{e^2}{2C_\Sigma}$$

Charging energy

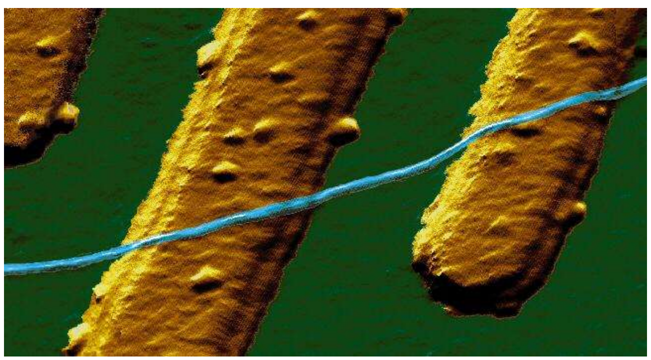
The energy needed to remove a whole electron is:

$$\Delta E = \int_0^{e} V(n)dq = eV(n) + \frac{e^2}{2C_\Sigma}$$

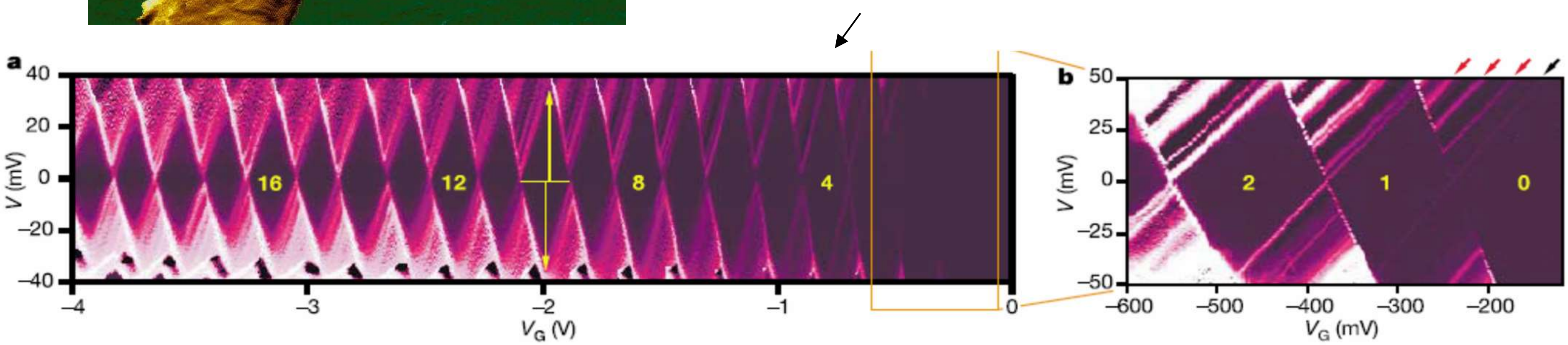




A. Bezryadin et al. Appl. Phys. Lett., 71, p. 1273.



Jarillo-Herrero, et al., Nature 429, 389 (2004).



Coulomb blockade suppressed by thermal and quantum fluctuations

Thermal fluctuations $\frac{e^2}{2C_\Sigma} >> k_B T$

Quantum fluctuations $\Delta E \Delta t > \hbar$

Duration of a quantum fluctuation: $\Delta t \sim \frac{\hbar 2 C_\Sigma}{e^2}$

RC charging time of the capacitance: RC_Σ

Charging faster than a quantum fluctuation $RC_\Sigma < \frac{\hbar 2 C_\Sigma}{e^2}$

$$R < \frac{2\hbar}{e^2} \approx 8\ \mathrm{k\Omega}$$

$$\frac{h}{e^2} \approx 25.5\ \mathrm{k\Omega}$$

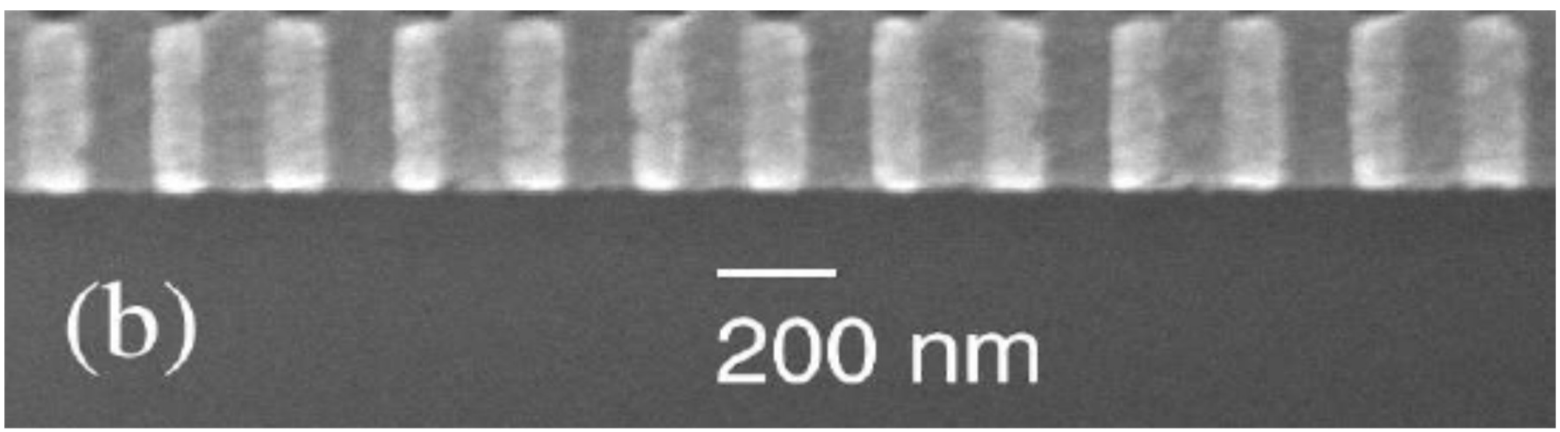
Resistance quantum

Metal - insulator transition in 1-d arrays

Charging energy $\Delta E = e^2/2C$

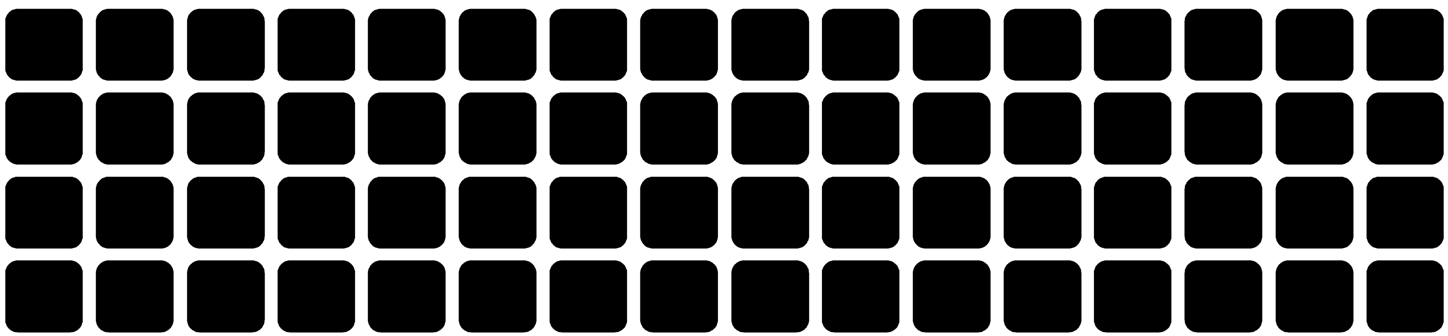
$$\Delta t = \frac{\hbar}{\Delta E} = \frac{2C\hbar}{e^2} > \frac{1}{\Gamma} = RC$$

$$R < \frac{2\hbar}{e^2} \qquad \text{extended state}$$

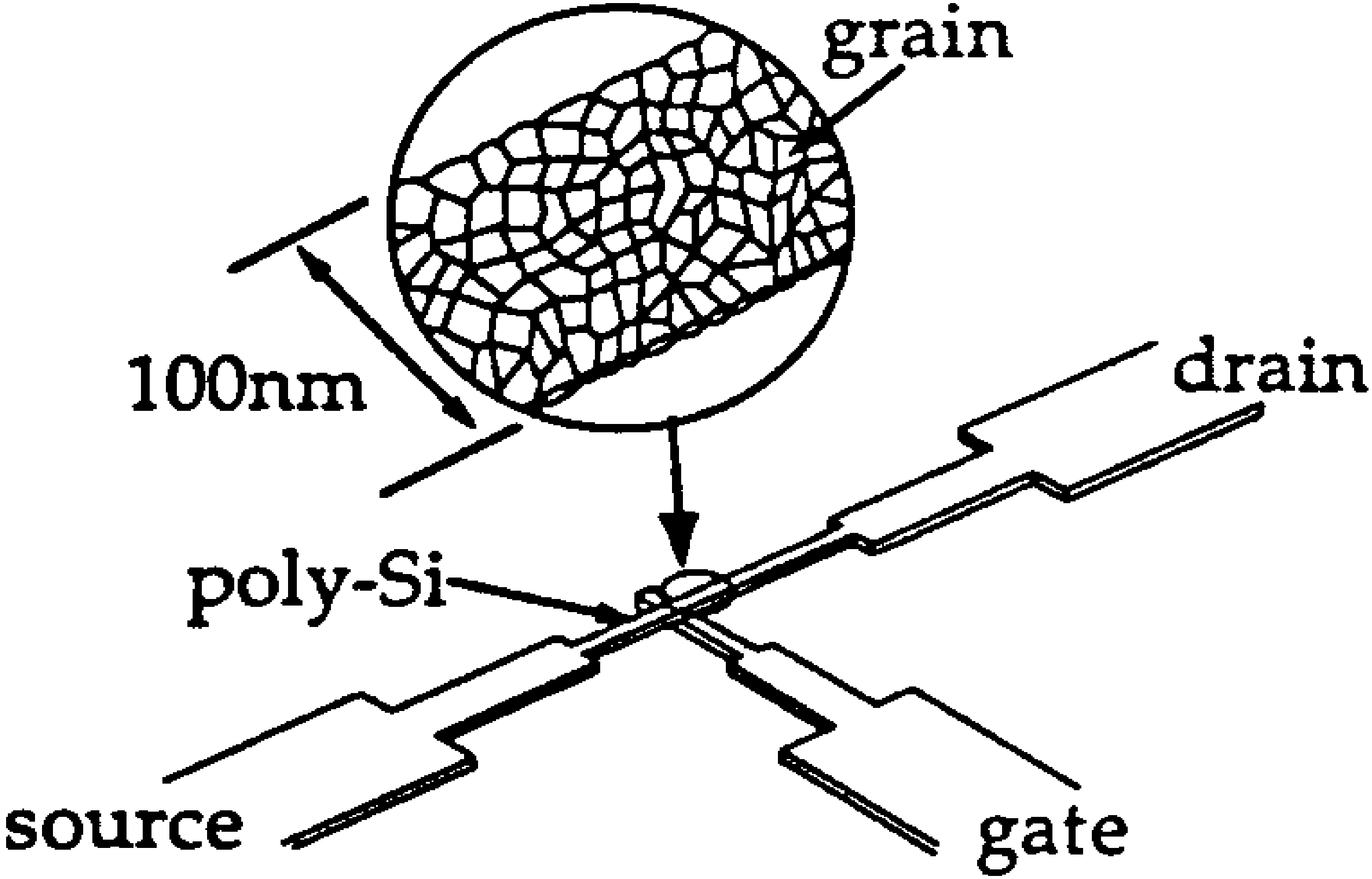


Metal insulator transition

If the tunnel resistances between the crystals is > 25 kΩ, the material will be an insulator at low temperature



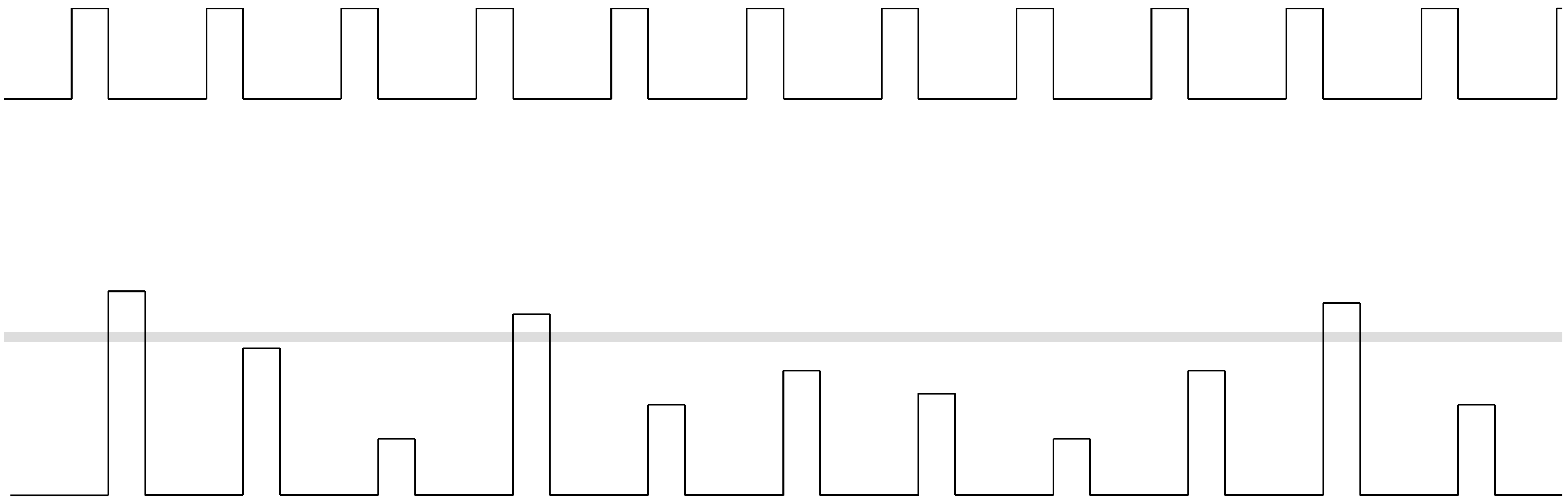
Strong coupling of metal particles results in a metal.
Weak coupling of metal particle results in an insulator.



Yano

Disorder => Favors insulating state

Uniform tunnel barriers



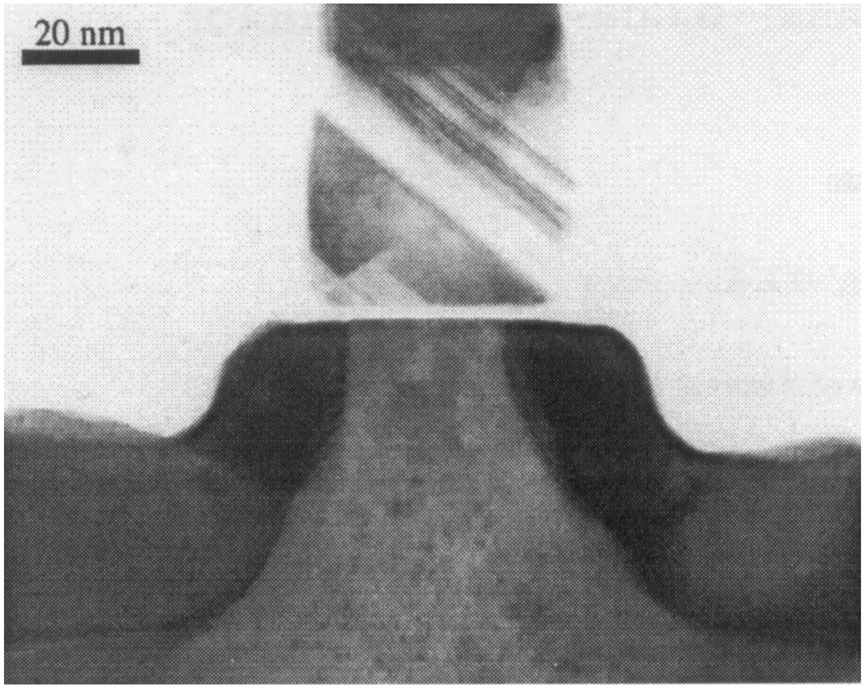
Random tunnel barriers, some with resistances above the resistance quantum

For bigger conducting regions, lower temperatures are needed to see insulating behavior.

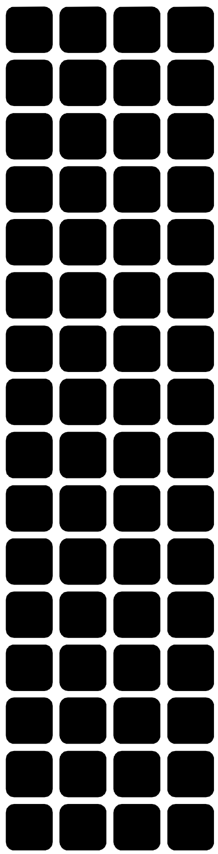
Single electron effects

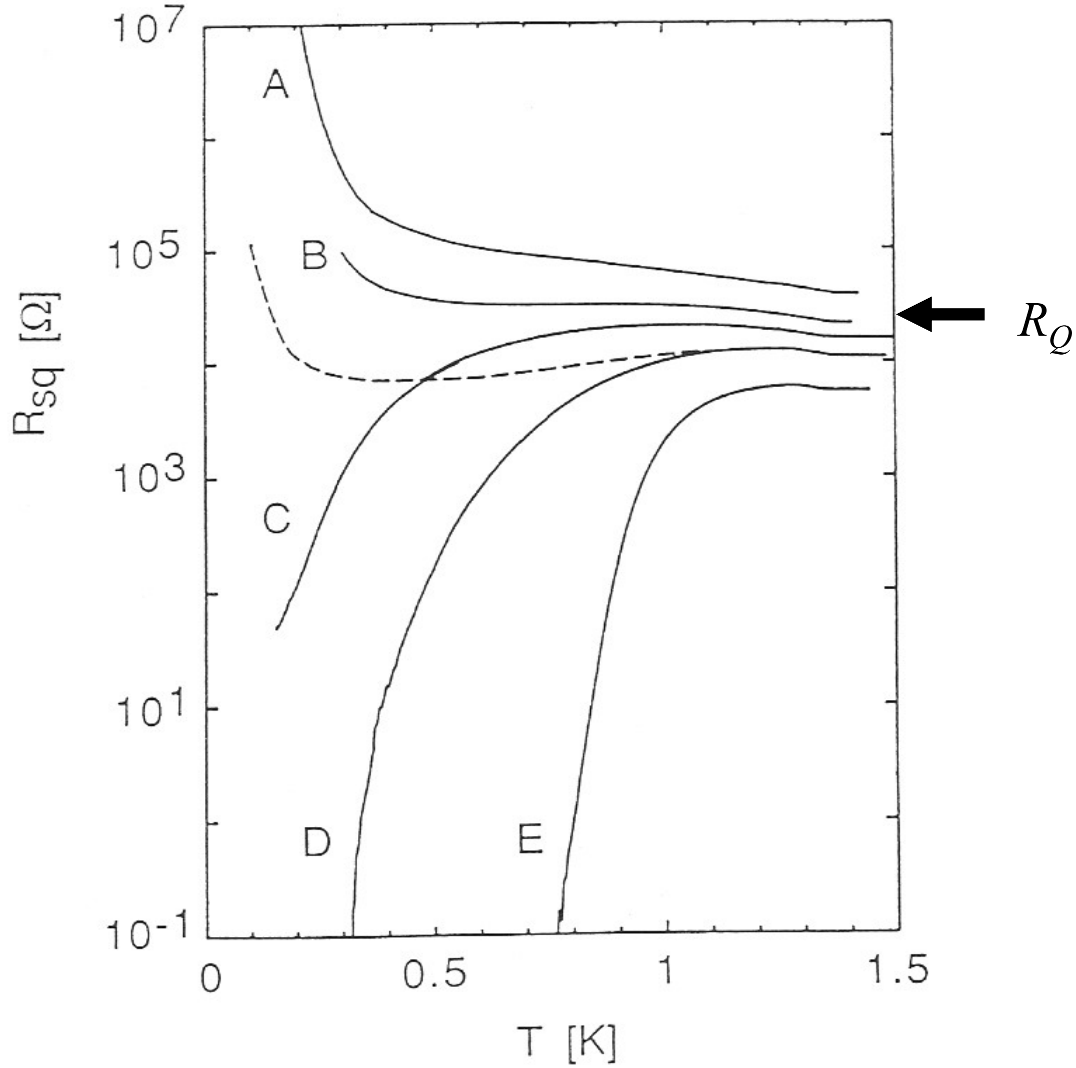
Single-electron effects will be present in any molecular scale circuit

Usually considered undesirable and are avoided by keeping the resistance below the resistance quantum.

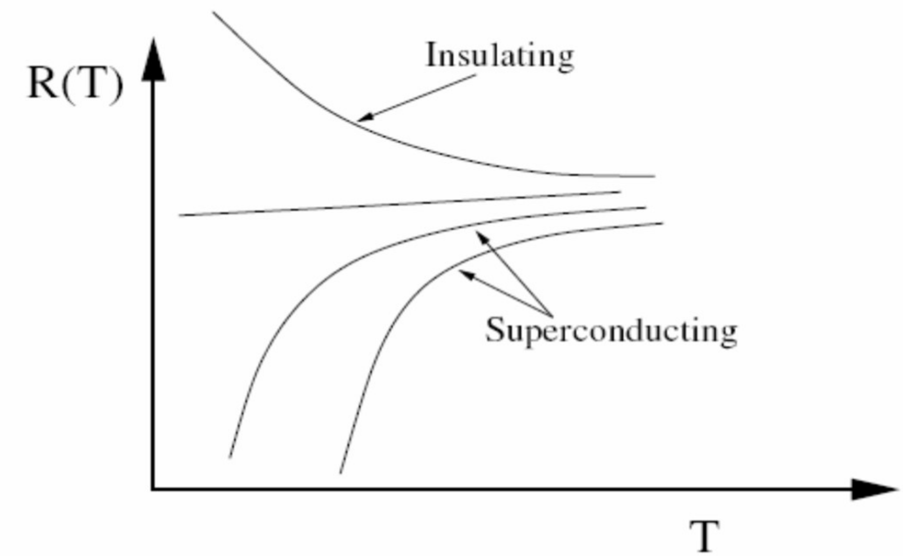


Josephson junction array





Geerligs PRL 63, p. 326 (1989).

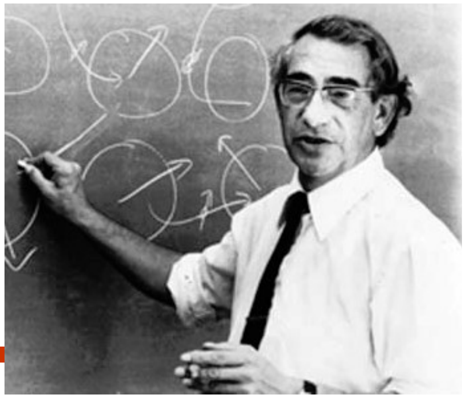


The Bose-Hubbard Model: From Josephson Junction Arrays to Optical Lattices

C. Bruder*[1], **Rosario Fazio****[2], and **Gerd Schön*****[3]

[1] Department of Physics and Astronomy, University of Basel, Klingelbergstr. 82, 4056 Basel, Switzerland
[2] NEST-INFM & Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy
[3] Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany

The Hubbard model



John Hubbard

The Hubbard model is an approximate model used, especially in solid state physics, to describe the transition between conducting and insulating systems. -Wikipedia

$$H = -t \sum_{\langle i,j \rangle,\sigma} c_{i,\sigma}^{\dagger} c_{j,\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

It is widely believed to be a good model for correlated electron systems including high temperature superconductors. The Hubbard model is solvable for a few electrons and a few sites but is extremely difficult to solve for many electrons on many sites.

http://nerdwisdom.com/tutorials/the-hubbard-model/

The Hubbard model

$$H = -t \sum_{\langle i,j \rangle,\sigma} c_{i,\sigma}^{\dagger} c_{j,\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Consider 2 electrons and two sites. If the electrons have the same spin:

$$\uparrow,\uparrow \quad \text{or} \quad \downarrow,\downarrow$$

They can't hop and the energy is zero.

If the electrons have opposite spin

$$\uparrow,\downarrow \quad \text{or} \quad \uparrow,\downarrow \quad \text{or} \quad \uparrow\downarrow,0 \quad \text{or} \quad 0,\uparrow\downarrow$$

the states couple together.

The Hubbard model

$$|\psi\rangle = a|\uparrow\downarrow,0\rangle + b|\uparrow,\downarrow\rangle + c|\downarrow,\uparrow\rangle + d|0,\uparrow\downarrow\rangle$$

$$H|\psi\rangle = E|\psi\rangle$$

$$H = -t\sum_{\langle i,j\rangle,\sigma} c_{i,\sigma}^{\dagger} c_{j,\sigma} + U\sum_i n_{i\uparrow} n_{i\downarrow}$$

$$\langle\uparrow\downarrow,0|H|\psi\rangle = Ua - tb - tc$$

$$\begin{bmatrix} U & -t & -t & 0 \\ -t & 0 & 0 & -t \\ -t & 0 & 0 & -t \\ 0 & -t & -t & U \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = E\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

States where electrons have opposite spin have lower energy (antiferromagnetic).

The Hubbard model

$with(LinearAlgebra):$
$U := 1:$
$t := 1:$
$Hubbard := Matrix([[U,-t,-t,0],[-t,0,0,-t],[-t,0,0,-t],[0,-t,-t,U]]);$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix}$$

$Eigenvectors(Hubbard);$

$$\begin{bmatrix} 0 \\ \frac{1}{2}+\frac{1}{2}\sqrt{17} \\ \frac{1}{2}-\frac{1}{2}\sqrt{17} \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & \frac{4}{\left(\frac{1}{2}+\frac{1}{2}\sqrt{17}\right)\left(-\frac{1}{2}+\frac{1}{2}\sqrt{17}\right)} & \frac{4}{\left(\frac{1}{2}-\frac{1}{2}\sqrt{17}\right)\left(-\frac{1}{2}-\frac{1}{2}\sqrt{17}\right)} & -1 \\ -1 & -\frac{2}{\frac{1}{2}+\frac{1}{2}\sqrt{17}} & -\frac{2}{\frac{1}{2}-\frac{1}{2}\sqrt{17}} & 0 \\ 1 & -\frac{2}{\frac{1}{2}+\frac{1}{2}\sqrt{17}} & -\frac{2}{\frac{1}{2}-\frac{1}{2}\sqrt{17}} & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Eigenvectors

$E = 0$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$E = 2.56$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -0.780776466 \\ -0.780776466 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.5615529319999997 \\ -2 \\ -2 \\ 2.5615529319999997 \end{bmatrix}$$

$E = -1.56$

One eigenvalue is less than zero

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1.2807764064 \\ 1.2807764064 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5615528128 \\ -2 \\ -2 \\ -1.5615528128 \end{bmatrix}$$

$E = 1$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The ground state of a half-filled band is antiferromagnetic.
The Hubbard model rapidly becomes intractable for more sites.