Dielectric properties of insulators

Dielectric response of insulators

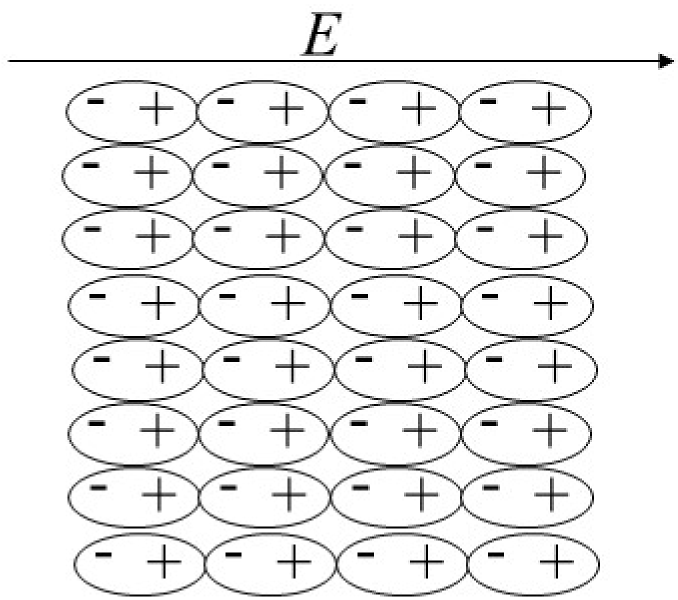
The electric polarization is related to the electric field

$$P_i = \varepsilon_0 \chi_{ij} E_j$$

The electric displacement vector *D* is also related to the electric field

$$D_i = P_i + \varepsilon_0 E_i = \varepsilon_0 (1 + \chi_{ij}) E_j = \varepsilon_0 \varepsilon_{ij} E_j$$

$$\varepsilon_{ij} = (1 + \chi_{ij})$$



E is decreased by a factor of the dielectric constant

Dielectric response of insulators

In an insulator, charge is bound. The response to an electric field can be modeled as a collection of damped harmonic oscillators

$$P = nex$$

Macroscopic polarization → P

density → n

ex = dipole moment

The core electrons of a metal respond to an electric field like this too.

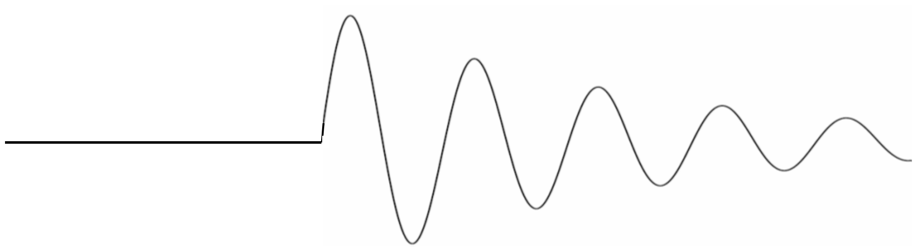
Dielectric response of insulators

The differential equation that describes how the position of the charge changes in time is:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = -eE(t)$$

The impulse response function is:

$$g(t) = -\frac{1}{b}\exp\left(\frac{-bt}{2m}\right)\sin\left(\frac{\sqrt{4mk-b^2}}{2m}t\right) \qquad t > 0$$



Electric susceptibility

$$\vec{P} = \varepsilon_0 \chi_E \vec{E} \qquad \vec{P} = nq\vec{x}$$

$$\chi_E = \frac{P}{\varepsilon_0 E} = \frac{nqx}{\varepsilon_0 E}$$

Assume a solution of the form $x(\omega)e^{i\omega t}$, $E(\omega)e^{i\omega t}$

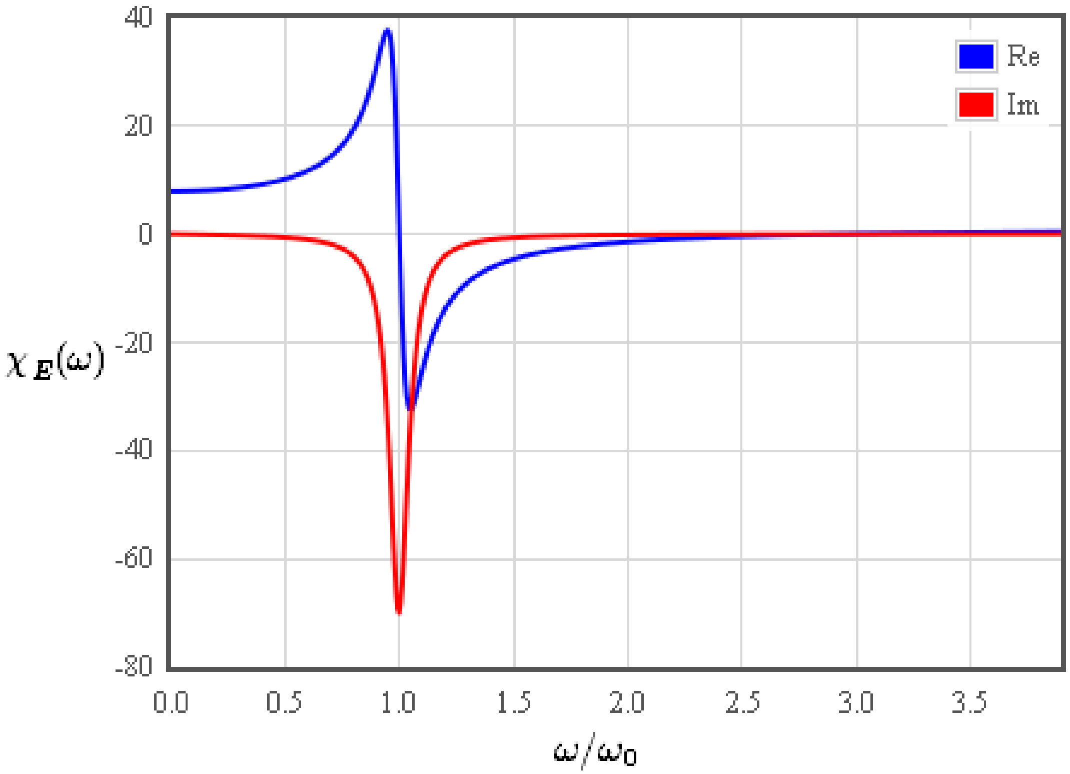
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = qE(t)$$

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = -\frac{qE}{m}$$

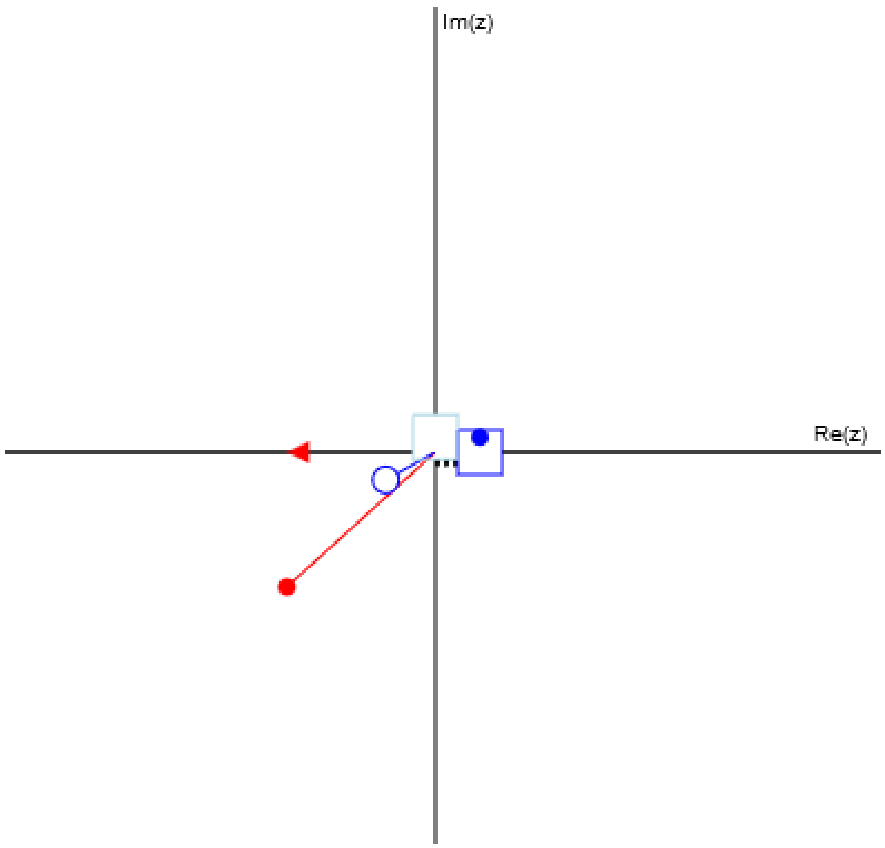
$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \gamma = \frac{b}{m}$$

Electric susceptibility

$$\chi_E(\omega) = \frac{n_{\omega_0} q^2}{\varepsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 + i\gamma\omega}$$



Resonance of a damped driven harmonic oscillator



m =4 [kg]

b =1 [N s/m]

k =6 [N/m]

F_0 =0.9 [N]

ω =0.8 [rad/s]

$\omega_0 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ = 1.22 [rad/s] = 0.194 [Hz]

$\theta = \text{atan}\left(\frac{\omega b}{k - m\omega^2}\right)$ = 0.228 [rad] = 13.1 [deg]

$|A| = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + \omega^2 b^2}}$ = 0.255 [m]

$Q = \frac{\sqrt{mk}}{b}$ = 4.90

Display $F_0 e^{i\omega t}$: ☑ Display $|A|e^{i(\omega t - \theta)}$: ☑

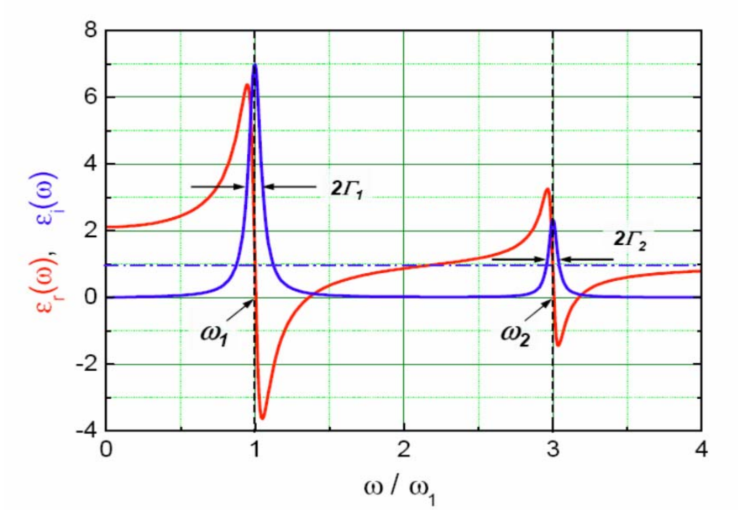
Display transients z: ☑ Display x_2: ☑

http://lamp.tu-graz.ac.at/~hadley/physikm/apps/resonance.en.php

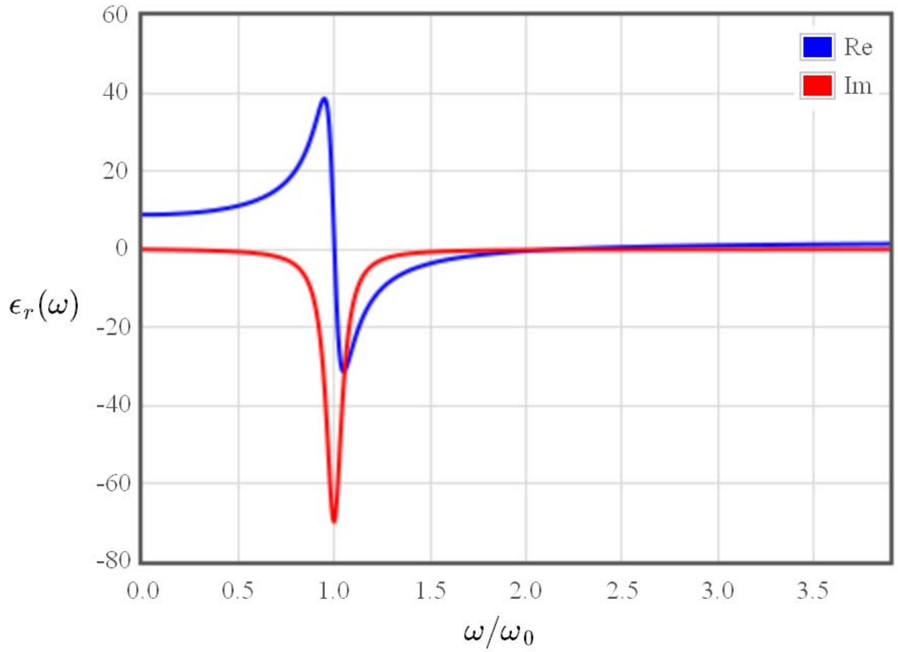
Dielectric function

$$\vec{D} = \epsilon_r \epsilon_0 \vec{E} = \vec{P} + \epsilon_0 \vec{E}$$

$$\varepsilon_r(\omega) = 1 + \chi_E(\omega) = 1 + \frac{n_{\omega_0} q^2}{\varepsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 + i\gamma\omega}$$

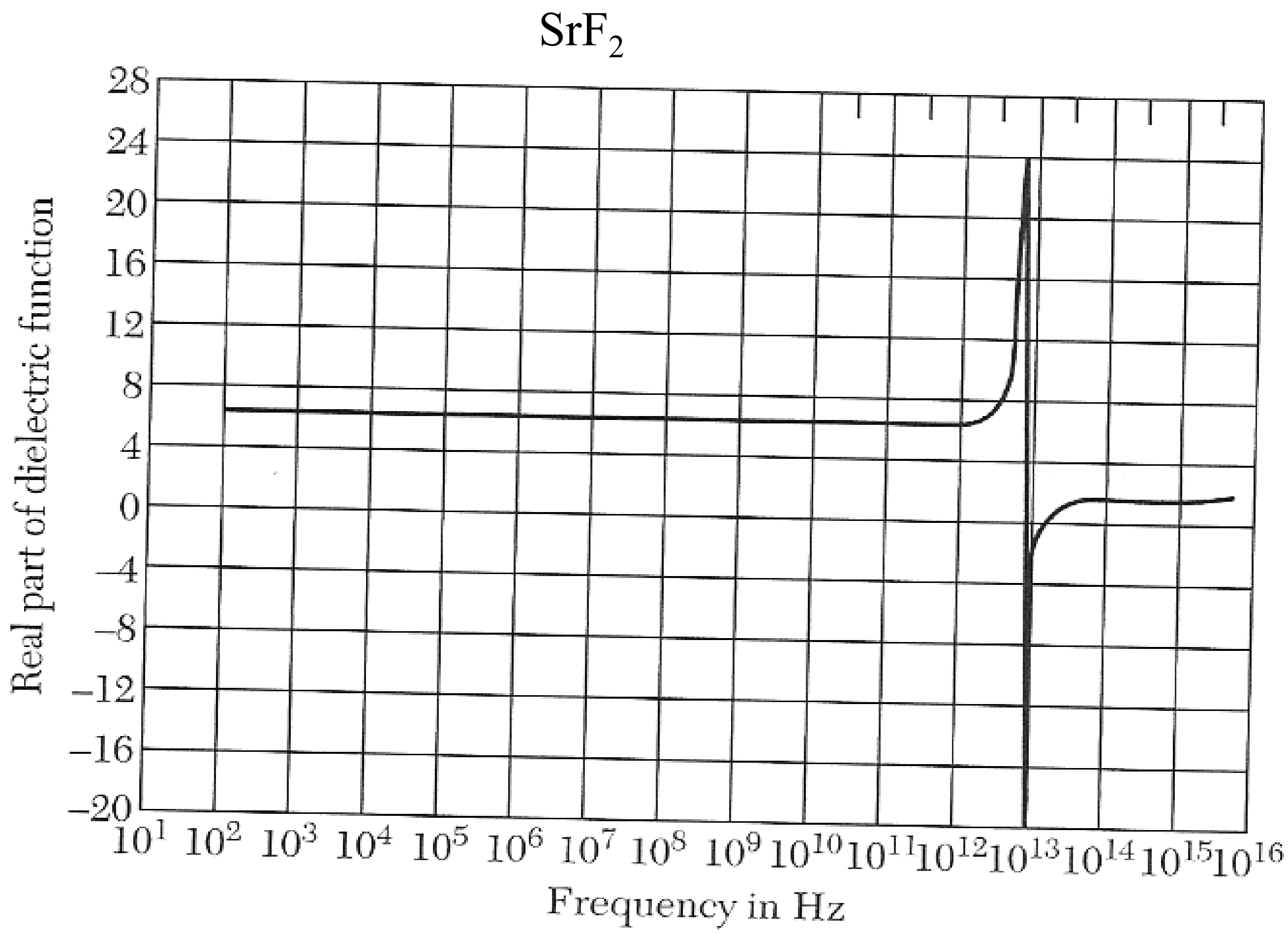


Gross and Marx



There can be more resonances.

Dielectric function of insulators



Insulators can often be modeled as a simple resonance.

Dispersion relation

In the section on photons, we derived the wave equation for light in vacuum. Here the wave equation for light in a dielectric material is derived.

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

$$\vec{D} = \varepsilon\vec{E} \qquad \vec{B} = \mu\vec{H}$$

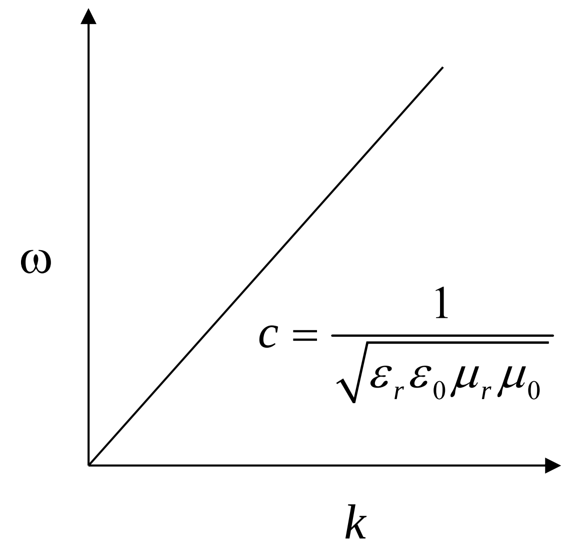
$$\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

Take the curl

$$\nabla\times\nabla\times\vec{E} = -\mu\frac{\partial\nabla\times\vec{H}}{\partial t}$$

$$\nabla\left(\cancelto{0}{\nabla\cdot\vec{E}}\right) - \nabla^2\vec{E} = -\mu\frac{\partial^2\vec{D}}{\partial t^2}$$

$$\varepsilon_r\varepsilon_0\mu_r\mu_0\frac{\partial^2\vec{D}}{\partial t^2} = \nabla^2\vec{D}$$



The normal mode solutions are plane waves: $\vec{D} = \vec{D}_0\exp(\vec{k}\cdot\vec{r} - \omega t)$

$$\boxed{\varepsilon(\omega,k)\mu_0\varepsilon_0\omega^2 = k^2}$$

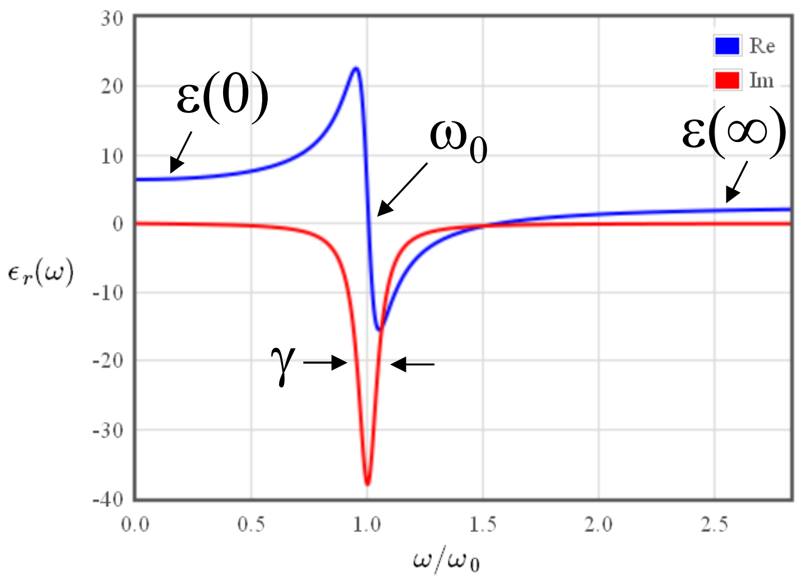
Dispersion relation

$$\varepsilon(\omega)\mu_0\varepsilon_0\omega^2 = k^2$$

If ε is real and positive: propagating electromagnetic waves $\exp\left(i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right)$

If $\varepsilon_r < 0$: decaying solutions $\exp(-\vec{k}\cdot\vec{r}-i\omega t)$

If ε is complex, $\varepsilon_r > 0$: decaying electromagnetic waves $\exp\left(i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right)\exp(-\kappa r)$



Dielectric function

Dispersion relation: $\varepsilon_r \mu_0 \varepsilon_0 \omega^2 = k^2$

$$k = \sqrt{\varepsilon_r \mu_0 \varepsilon_0}\,\omega = \frac{\sqrt{\varepsilon_r}\,\omega}{c}$$

Measurable: $\sqrt{\varepsilon} = n + iK$

refractive index (n) extinction coefficient (K)

Vacuum	Dielectric
$\omega = ck$	$k = \frac{\sqrt{\varepsilon_r}\,\omega}{c}$
$\exp\left(i\left(\frac{\omega x}{c} - \omega t\right)\right)$	$\exp\left(i\left(\frac{(n+iK)\omega x}{c} - \omega t\right)\right)$
	$= \exp\left(i\left(\frac{n\omega x}{c} - \omega t\right)\right)\exp\left(-\frac{\omega K x}{c}\right)$

Intensity $I(x) = I(0)\exp(-\alpha x)$ J m^{-2} s^{-1} Beer-Lambert

absorption coefficient $\longrightarrow$ $\alpha = \frac{2\omega K}{c}$