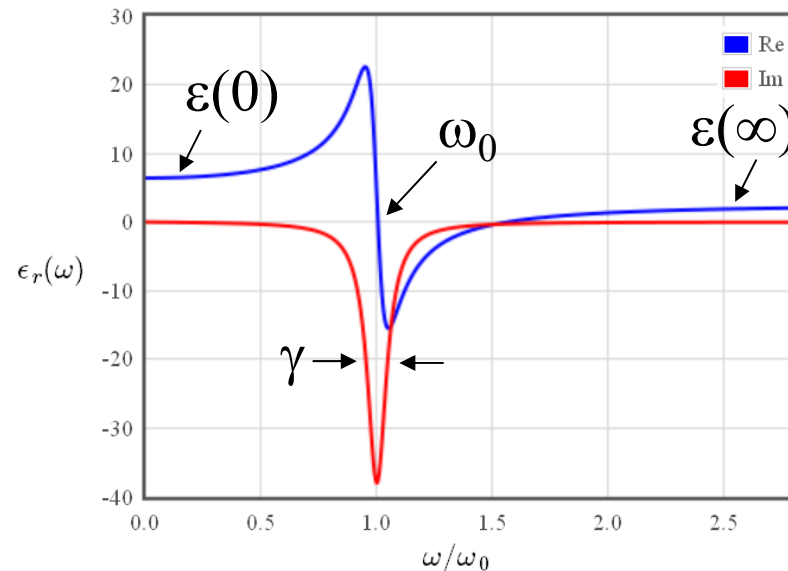# Dielectric properties of insulators

Dispersion relation

$$\varepsilon(\omega)\mu_0\varepsilon_0\omega^2 = k^2$$

If ε is real and positive: propagating electromagnetic waves $\quad \exp\left(i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right)$

If $\varepsilon_r < 0$: decaying solutions $\qquad \exp(-\vec{k}\cdot\vec{r} - i\omega t)$

If ε is complex, $\varepsilon_r > 0$: decaying electromagnetic waves $\quad \exp\left(i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right)\exp(-\kappa r)$
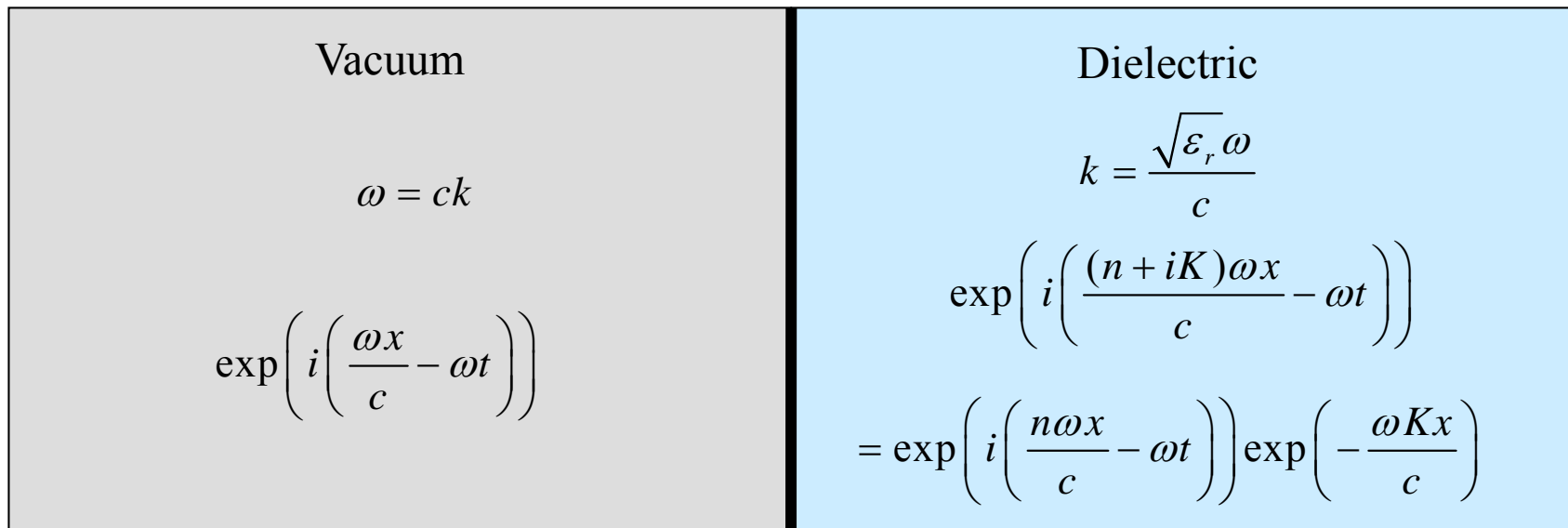
Dielectric function

Dispersion relation: $\varepsilon_r \mu_0 \varepsilon_0 \omega^2 = k^2$

$$k = \sqrt{\varepsilon_r \mu_0 \varepsilon_0}\,\omega = \frac{\sqrt{\varepsilon_r}\,\omega}{c}$$

Measurable: $\sqrt{\varepsilon} = n + iK$
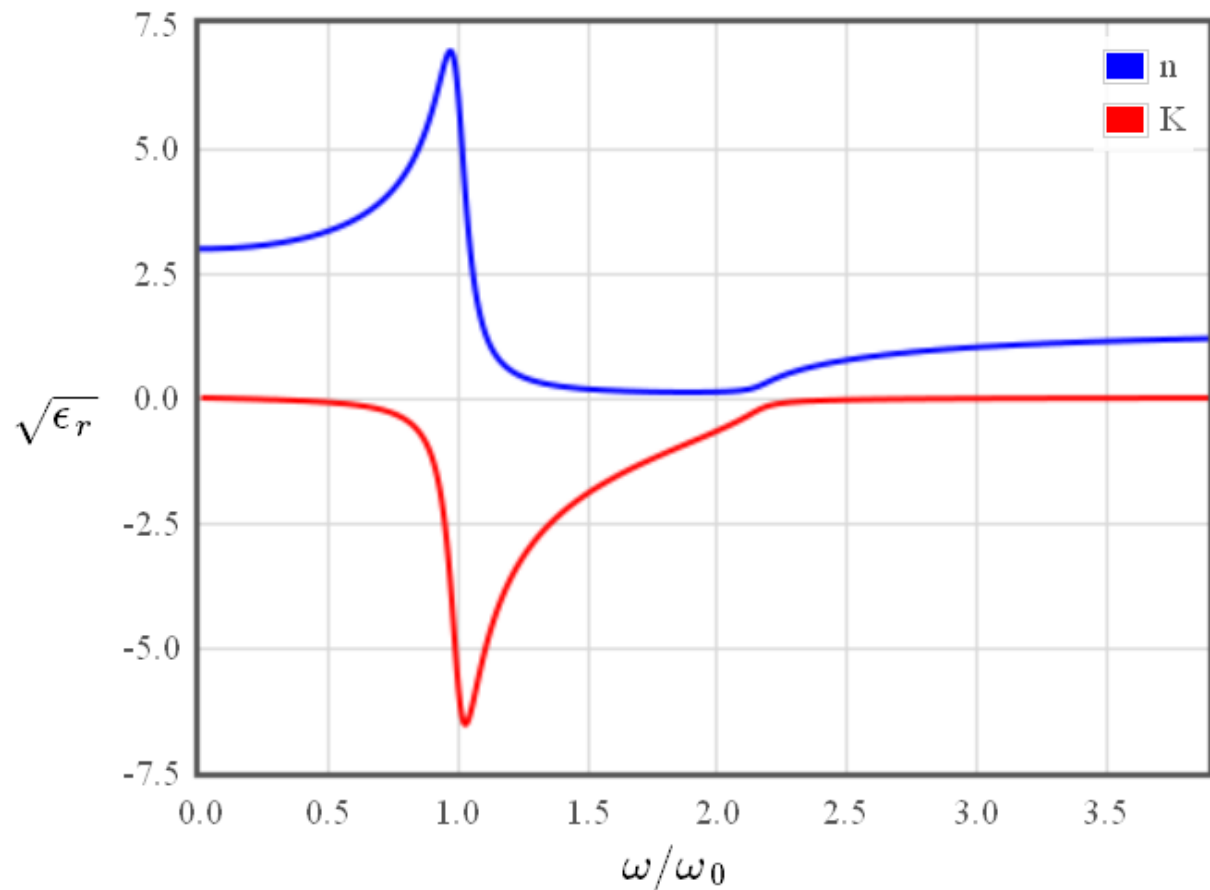
refractive index   extinction coefficient

Vacuum	Dielectric
$\omega = ck$	$k = \dfrac{\sqrt{\varepsilon_r}\,\omega}{c}$
$\exp\!\left(i\!\left(\dfrac{\omega x}{c} - \omega t \right) \right)$	$\exp\!\left(i\!\left(\dfrac{(n + iK)\omega x}{c} - \omega t \right) \right)$ $= \exp\!\left(i\!\left(\dfrac{n\omega x}{c} - \omega t \right) \right) \exp\!\left(-\dfrac{\omega K x}{c} \right)$

Intensity $\quad I(x) = I(0)\exp(-\alpha x) \; \mathrm{J\,m^{-2}\,s^{-1}} \quad$ Beer-Lambert

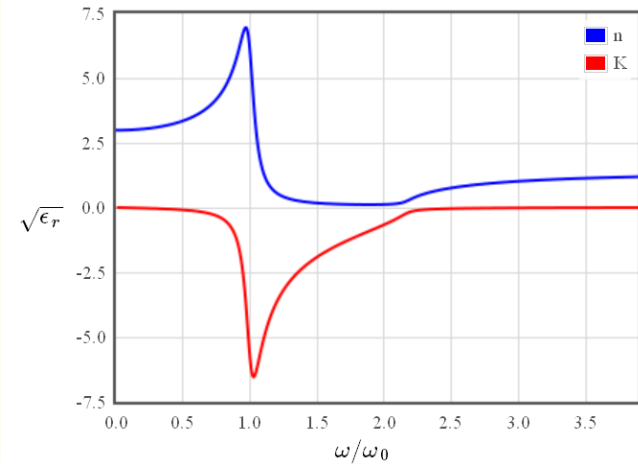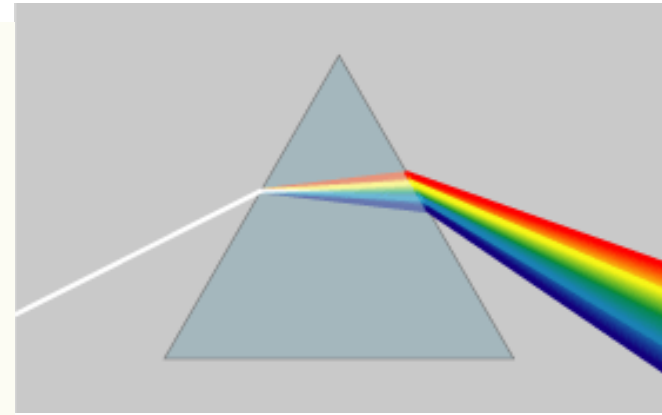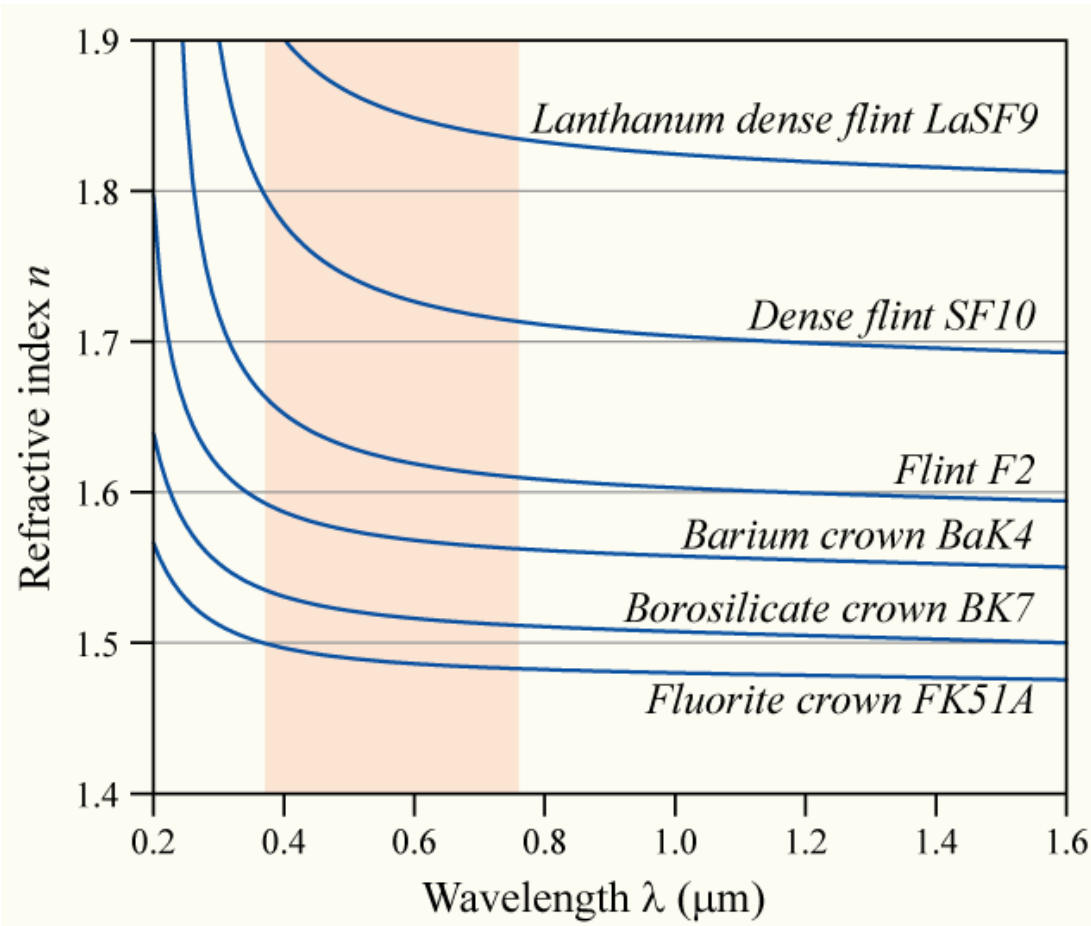absorption coefficient $\quad \longrightarrow \quad \alpha = \dfrac{2\omega K}{c}$

The index of refraction *n* and the extinction coefficient *K*

$$\sqrt{\epsilon_r} = n + iK$$
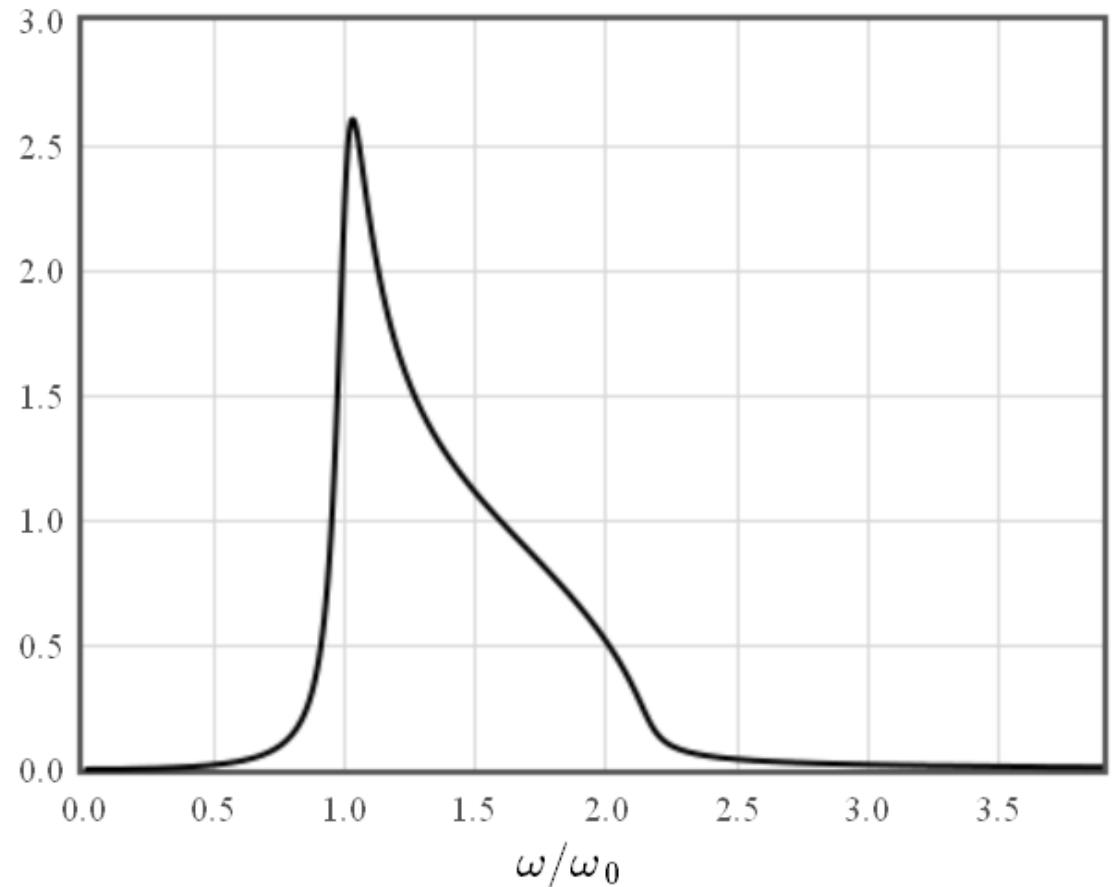
Dispersion

Cause of chromatic aberration in lenses.

Absorption coefficient α

$$I = I_0 \exp(-\alpha x)$$

$$\alpha = \frac{2\omega K}{c}$$
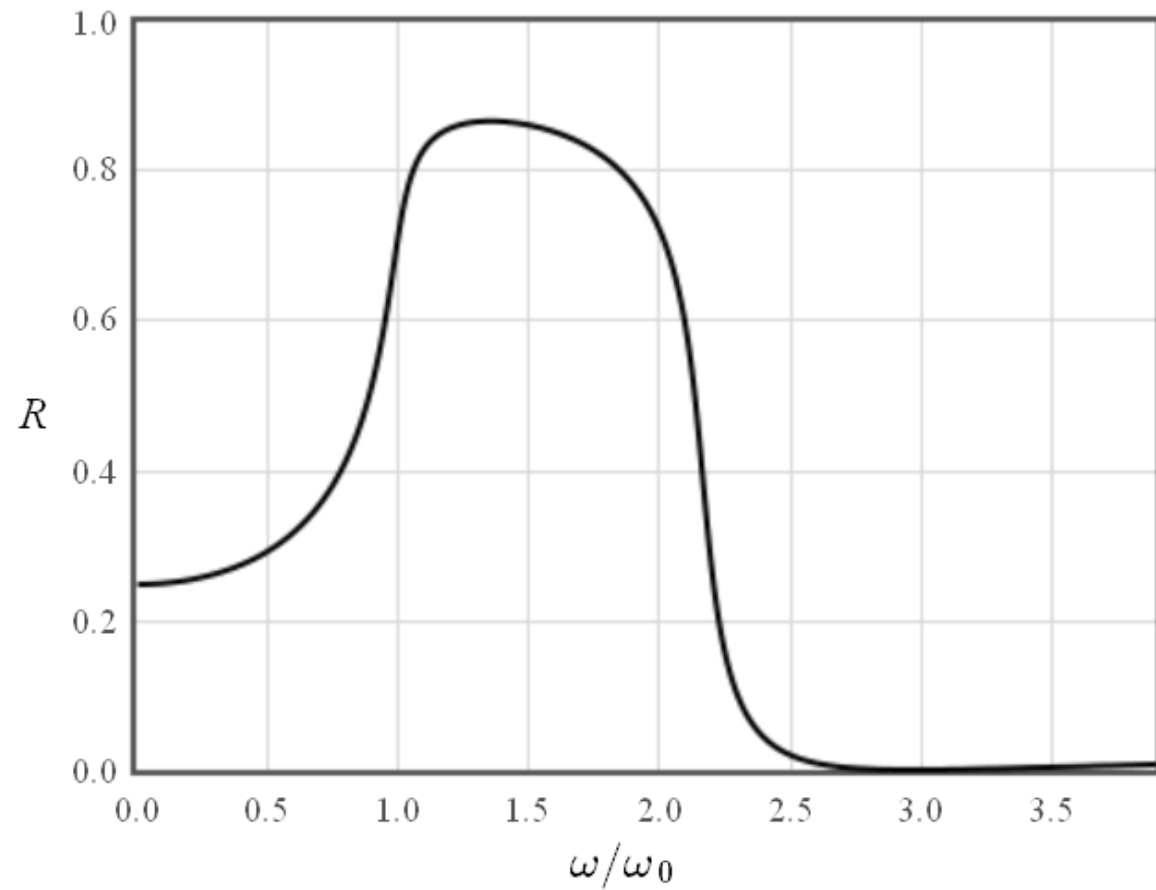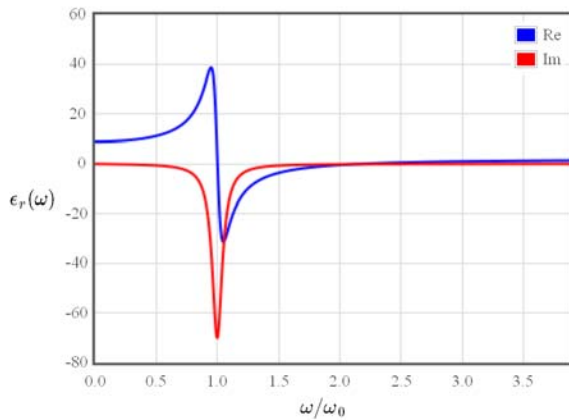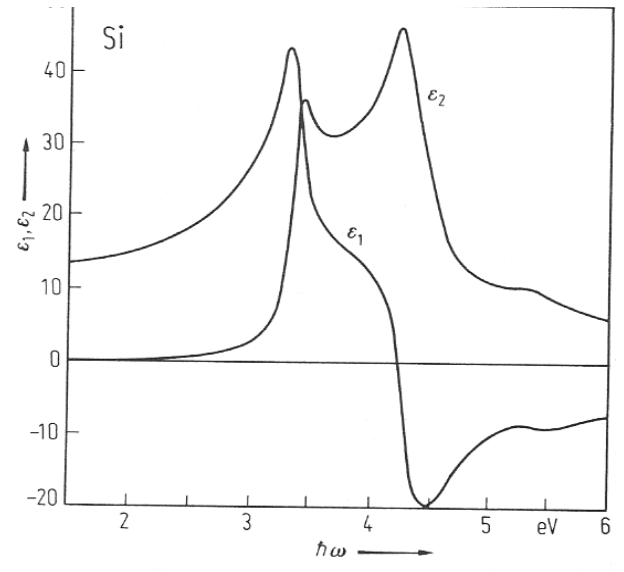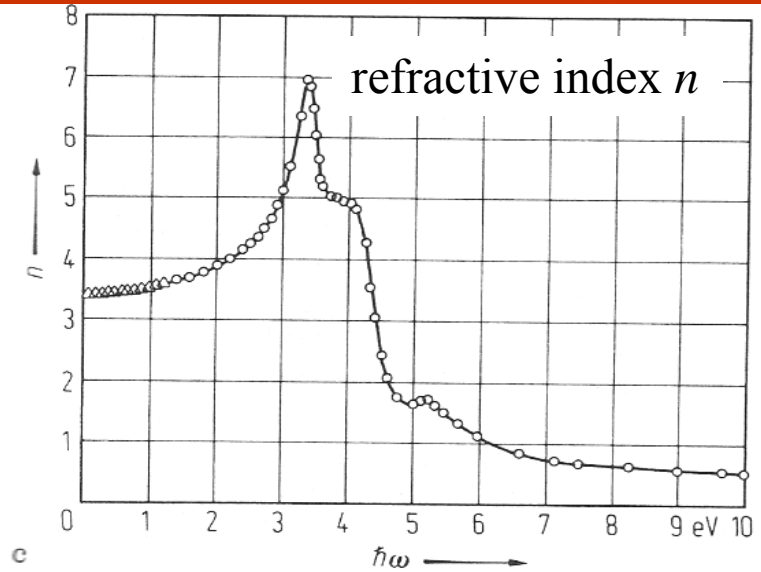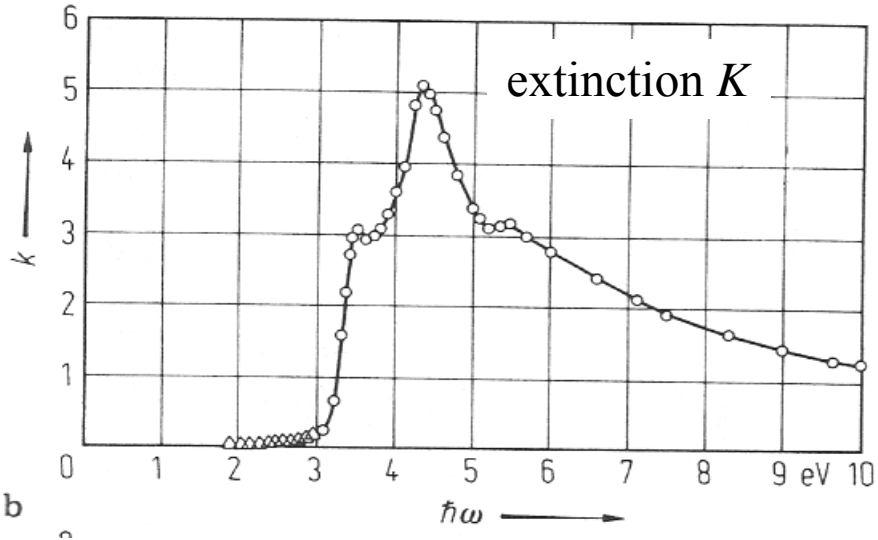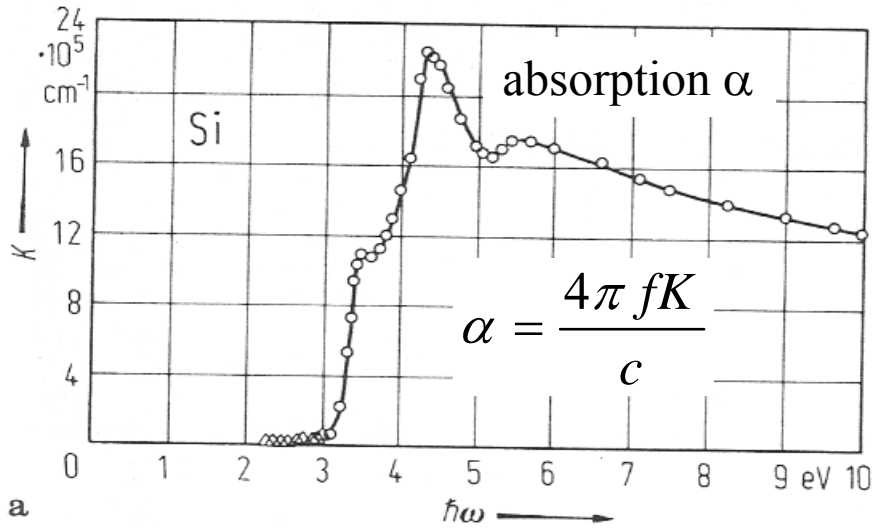
α
$[10^6 \text{ m}^{-1}]$

Reflectance

$$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$$

Dielectric function of silicon

$$\sqrt{\varepsilon(\omega)} = n(\omega) + iK(\omega)$$

absorption α

$$\alpha = \frac{4\pi fK}{c}$$

extinction K

refractive index n

Optical properties of insulators and semiconductors

In an insulator, all charges are bound. By applying an electric field, the electrons and ions can be pulled out of their equilibrium positions. When this electric field is turned off, the charges oscillate as they return to their equilibrium positions. A simple model for an insulator can be constructed by describing the motion of the charge as a damped mass-spring system. The differential equation that describes the motion of a charge is,

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = -qE.$$

Rewriting above equation using $\omega_0 = \sqrt{\frac{k}{m}}$ and the damping constant $\gamma = \frac{b}{m}$ yields,

$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = -\frac{qE}{m}.$$

If the electric field is pulsed on, the response of the charges is described by the **impulse response function** $g(t)$. The impulse response function satisfies the equation,

$$\frac{d^2 g}{dt^2} + \gamma \frac{dg}{dt} + \omega_0^2 g = -\frac{q}{m} \delta(t).$$

The solution to this equation is zero before the electric field is pulsed on and at the time of the pulse the charges suddenly start oscillating with the frequency $\omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$. The amplitude of the oscillation decays exponentially to zero in a characteristic time $\frac{2}{\gamma}$.

$$g(t) = -\frac{q}{m\omega_1} \exp\left(-\frac{\gamma}{2} t\right) \sin(\omega_1 t).$$
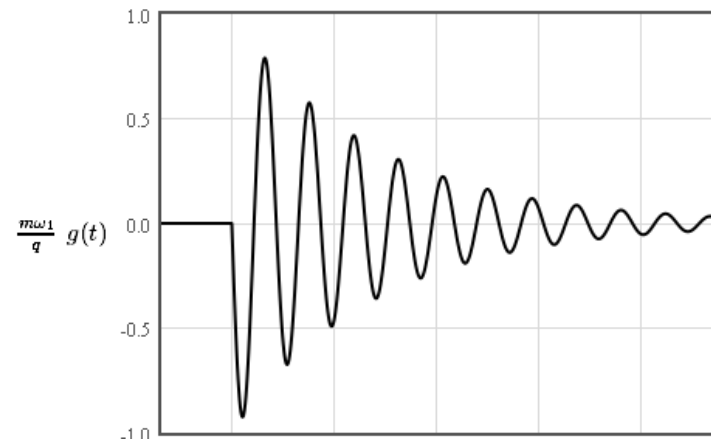
Dielectrics

Dielectrics used as electrical insulators should not conduct.

Large breakdown field.

Low AC losses.

Sometimes a low dielectric constant is desired (CMOS interconnects)

Sometimes a high dielectric constant is desired (supercapacitors).

Breakdown field

Typically 10^5-10^6 V/cm
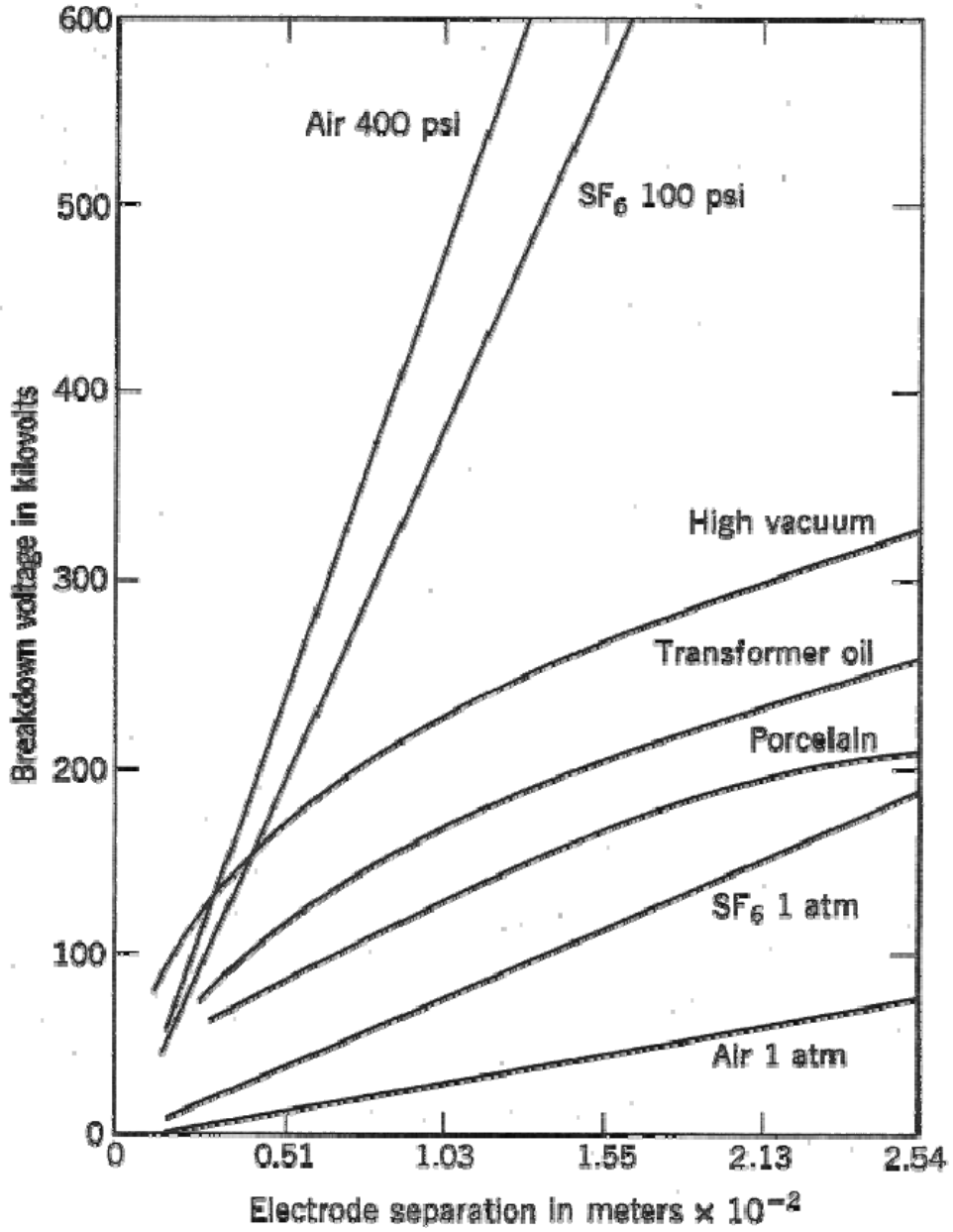
AC losses - loss tangent

In an ideal capacitor, current leads voltage by 90°.

Because the dielectric constant is complex, in real materials current leads voltage by 90° - δ.

$$\text{Power loss} = \frac{\omega \varepsilon_1 V_0^2}{2} \tan \delta$$

Becomes more of an issue at high frequencies (microwaves)

Loss tangent

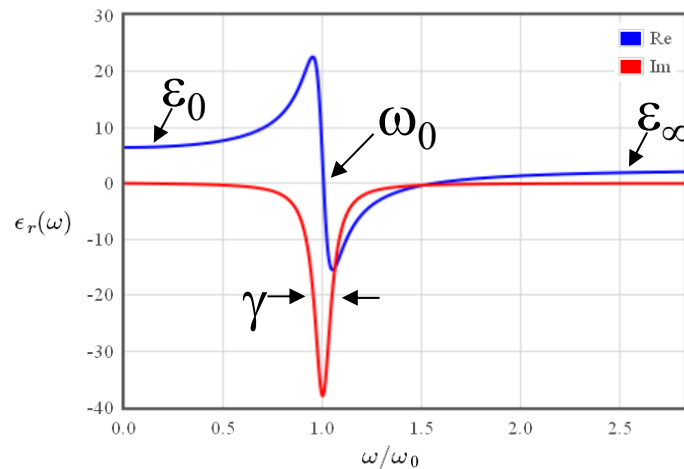| Substance	Dielectric Constant (relative to air)	Dielectric Strength (V/mil)	Loss Tangent	Max Temp (°F)
ABS (plastic), Molded	2.0 - 3.5	400 - 1350	0.00500 - 0.0190	171 - 228
Air	1.00054	30 - 70		
Alumina - 96% - 99.5%	10.0 9.6		0.0002 @ 1 GHz 0.0002 @ 100 MHz 0.0003 @ 10 GHz	
Aluminum Silicate	5.3 - 5.5			
Bakelite	3.7			
Bakelite (mica filled)	4.7	325 - 375		
Balsa Wood	1.37 @ 1 MHz 1.22 @ 3 GHz		0.012 @ 1 MHz 0.100 @ 3 GHz	
Beeswax (yellow)	2.53 @ 1 MHz 2.39 @ 3 GHz		0.0092 @ 1 MHz 0.0075 @ 3 GHz	
Beryllium oxide	6.7		0.006 @ 10 GHz	
Butyl Rubber	2.35 @ 1 MHz 2.35 @ 3 GHz		0.001 @ 1 MHz 0.0009 @ 3 GHz	
Carbon Tetrachloride	2.17 @ 1 MHz 2.17 @ 3 GHz		<0.0004 @ 1 MHz 0.0004 @ 3 GHz	
Diamond	5.5 - 10			
Delrin (acetyl resin)	3.7	500		180
Douglas Fir	1.9 @ 1 MHz		0.023 @ 1 MHz	
Douglas Fir Plywood	1.93 @ 1 MHz 1.82 @ 3 GHz		0.026 @ 1 MHz 0.027 @ 3 GHz	
Enamel	5.1	450		
Epoxy glass PCB	5.2	700		
Ethyl Alcohol (absolute)	24.5 @ 1 MHz 6.5 @ 3 GHz		0.09 @ 1 MHz 0.25 @ 3 GHz	
Ethylene Glycol	41 @ 1 MHz 12 @ 3 GHz		-0.03 @ 1 MHz 1 @ 3 GHz	
Formica XX	4.00			
FR-4 (G-10) - low resin - high resin	4.9 4.2		0.008 @ 100 MHz 0.008 @ 3 GHz	
Fused quartz	3.8		0.0002 @ 100 MHz 0.00006 @ 3 GHz	
Fused silica (glass)	3.8			
Gallium Arsenide (GaAs)	13.1		0.0016 @ 10 GHz	
Germanium	16			
Glass	4 - 10			
Glass (Corning 7059)	5.75		0.0036 @ 10 GHz	
Gutta-percha	2.6			
Halowax oil	4.8			
High Density Polyethylene (HDPE), Molded	1.0 - 5.0	475 - 3810	0.0000400 - 0.00100	158 - 248
Ice (pure distilled water)	4.15 @ 1 MHz 3.2 @ 3 GHz		0.12 @ 1 MHz 0.0009 @ 3 GHz	
Kapton® Type 100 Type 150	3.9 2.9	7400 4400		500

Polarizability

Overdamped modes
- Orientation polarizability
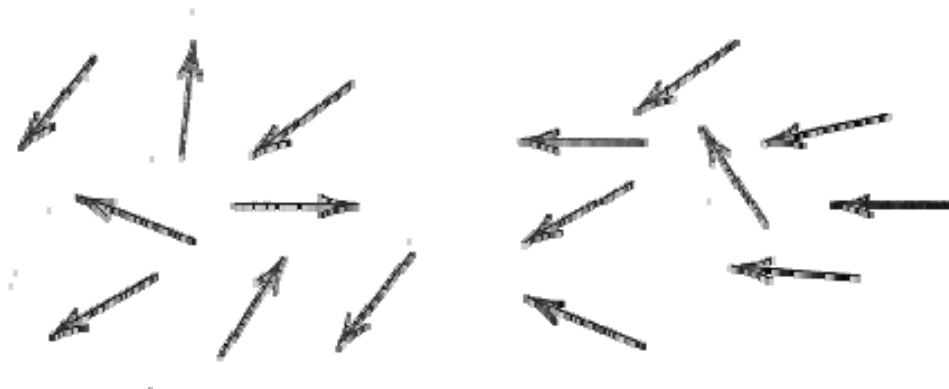- Space charge polarizability

Underdamped modes
- Ionic polarizability
- Electronic polarizability

Orientation (dipolar) Polarizability

For materials (gases, liquids, solids) with a permanent dipole moment.

The theory is very similar to paramagnetism.

$$\chi \propto \frac{1}{T} \qquad \text{Curie law}$$