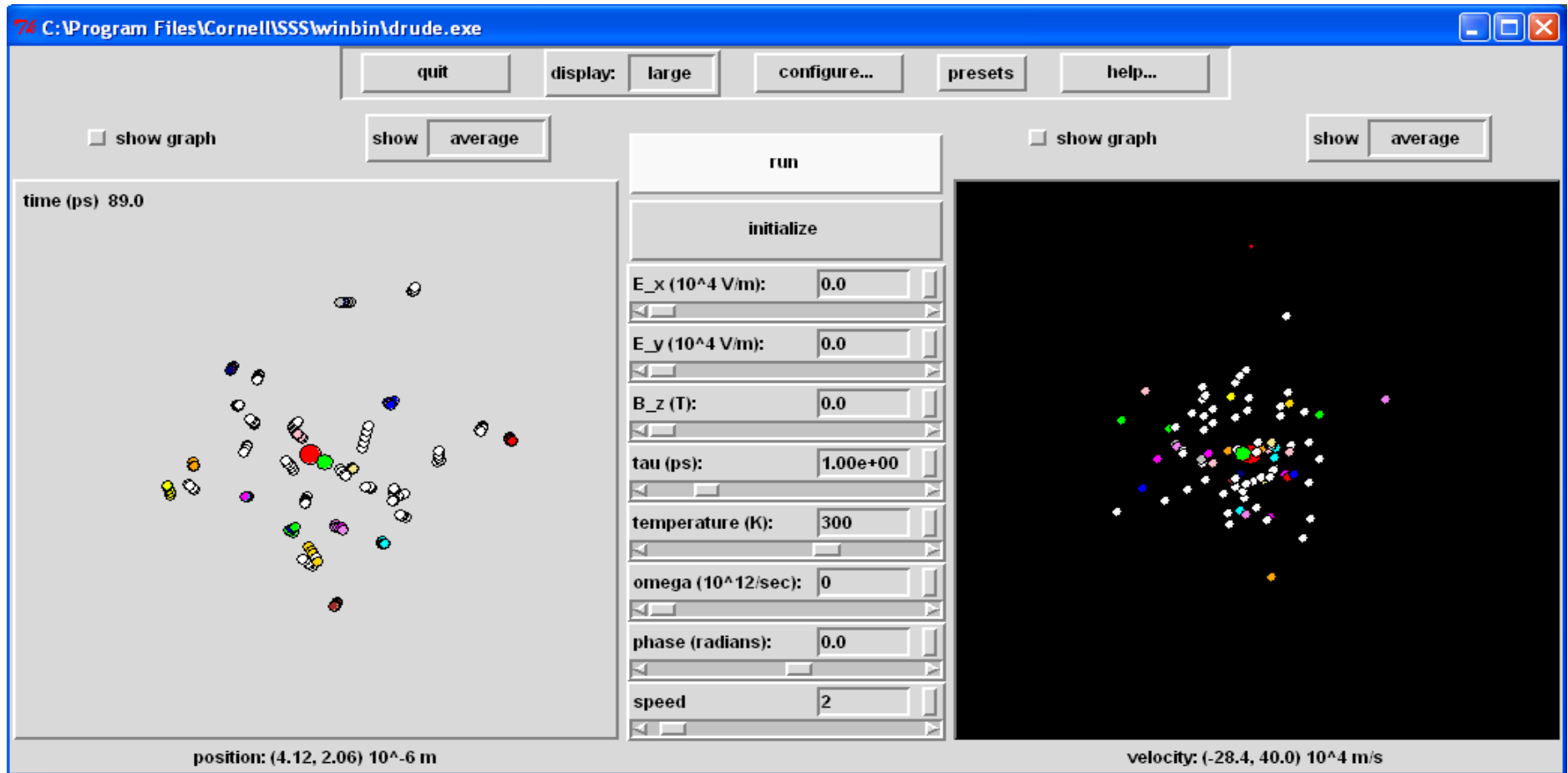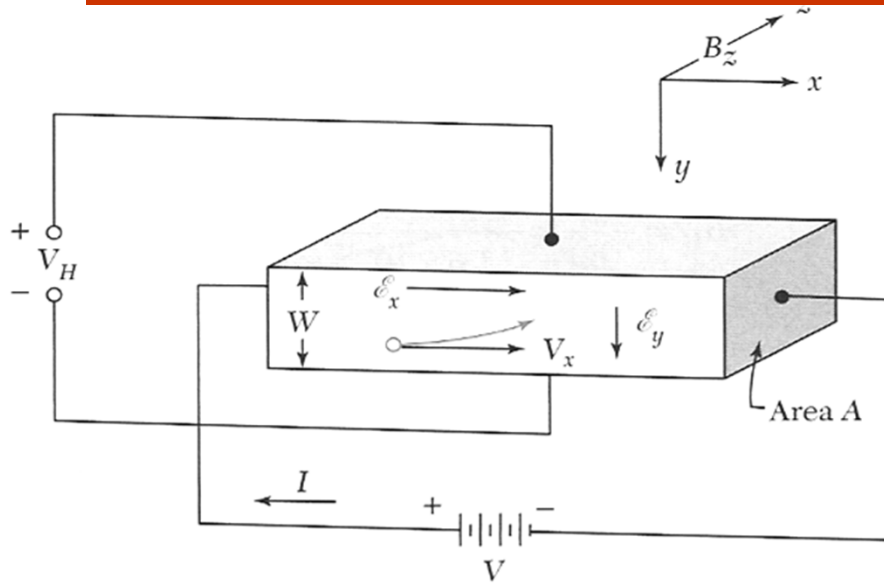# Quantum Hall Effect

If no forces are applied, the electrons diffuse.

The average velocity moves against an electric field.

In just a magnetic field, the average velocity is zero.

In an electric and magnetic field, the electrons move in a straight line at the Hall angle.

The drift velocity decreases as the B field increases.

The Hall Effect (diffusive regime)

$$v_{d,x} = -\frac{eE_x \tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y \tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$
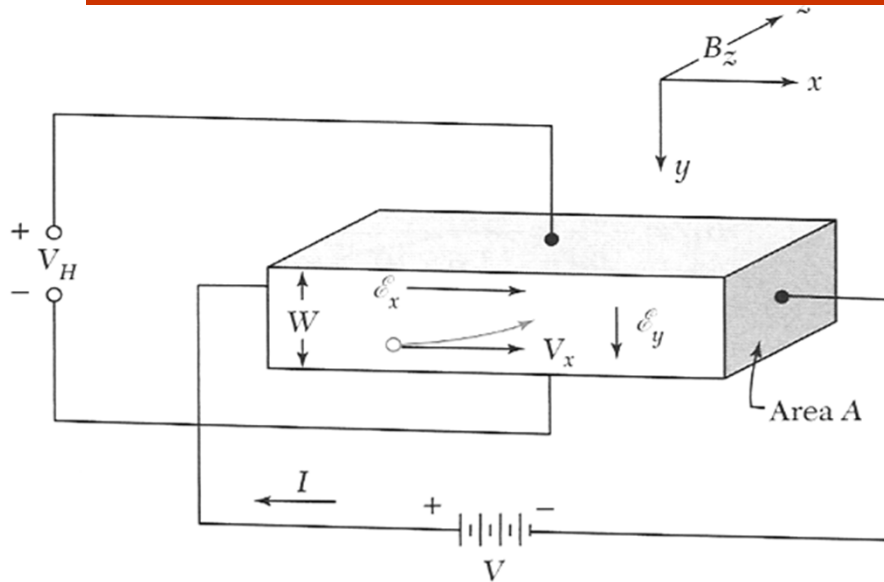
$$v_{d,z} = -\frac{eE_z \tau_{sc}}{m}$$

If $v_{d,y} = 0$,

$$E_y = v_{d,x}B_z = V_H/W = R_H j_x B_z \qquad V_H = \text{Hall voltage}, R_H = \text{Hall Constant}$$

$$v_{d,x} = -j_x/ne$$

$$\boxed{R_H = E_y/j_x B_z = -1/ne}$$

The Hall Effect (diffusive regime)

$$\rho_{xx} = \frac{E_x}{j_x}$$
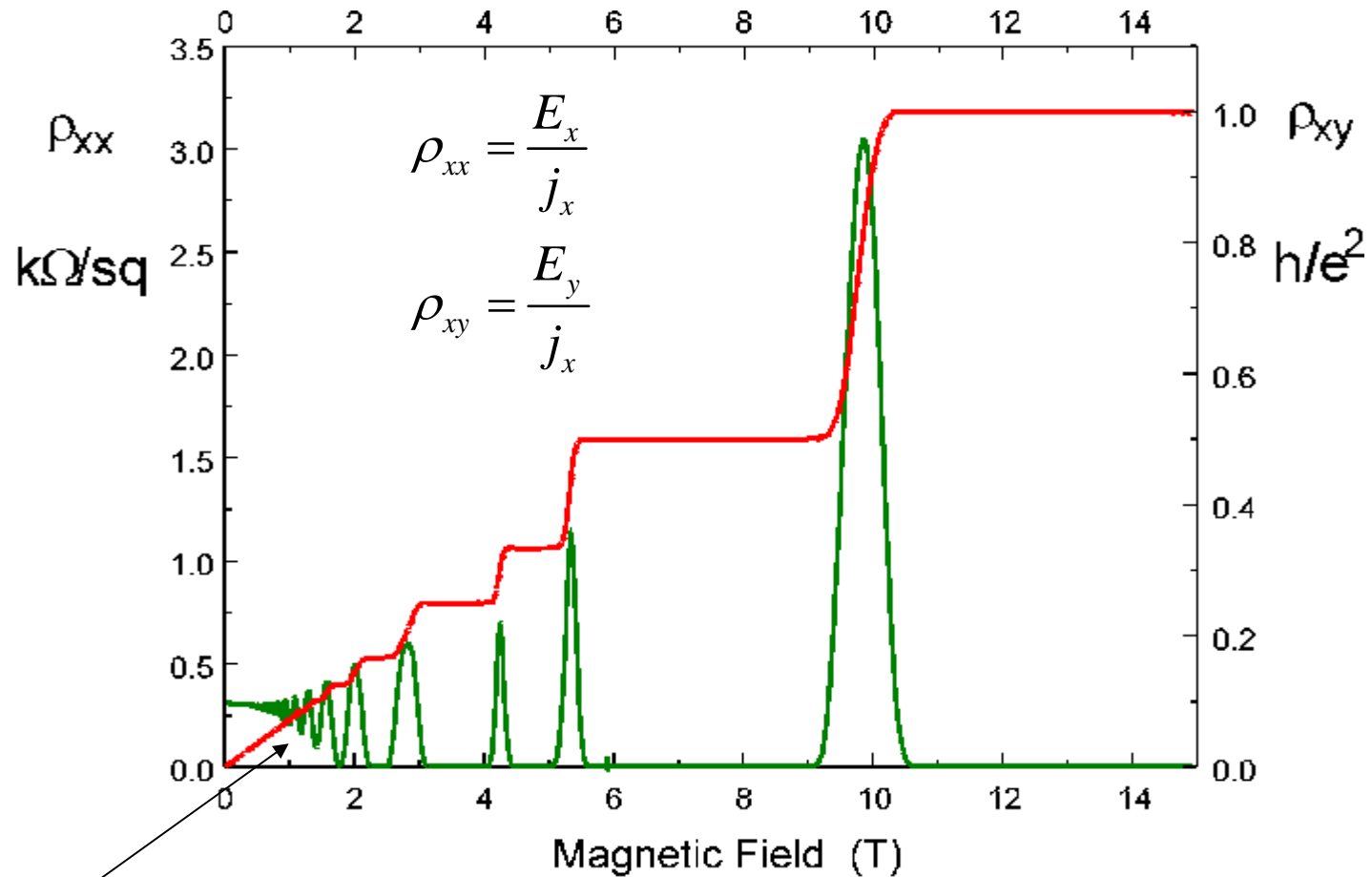
$$\rho_{xy} = \frac{E_y}{j_x}$$

$$R_H = E_y/j_x B_z = -1/ne$$

multiply both sides by B_z

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$

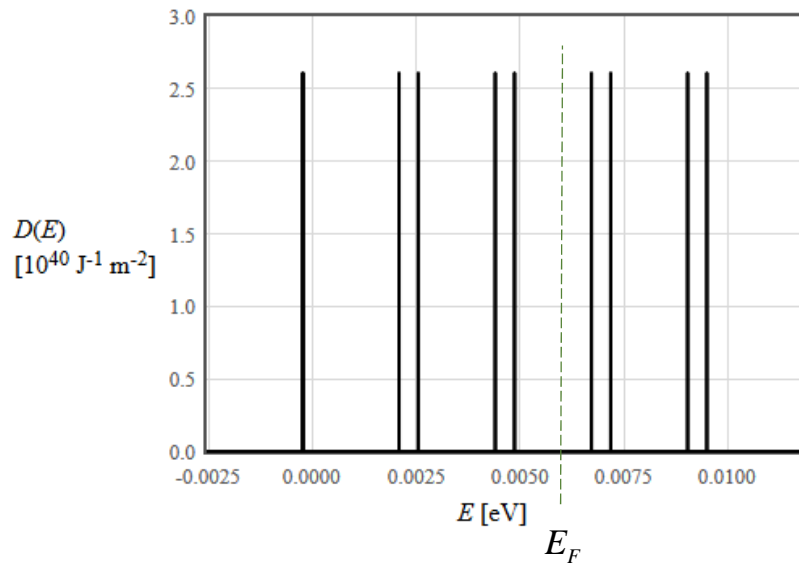The Hall resistivity is proportional to the magnetic field.

Quantum Hall Effect

$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{xy} = \frac{E_y}{j_x}$$

Shubnikov-De Haas oscillations

Resistance standard
25812.807557(18) Ω

Quantum hall effect

If the Fermi energy is between Landau levels, the electron density n is an integer v times the degeneracy of the Landau level $n = D_0 v$

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$
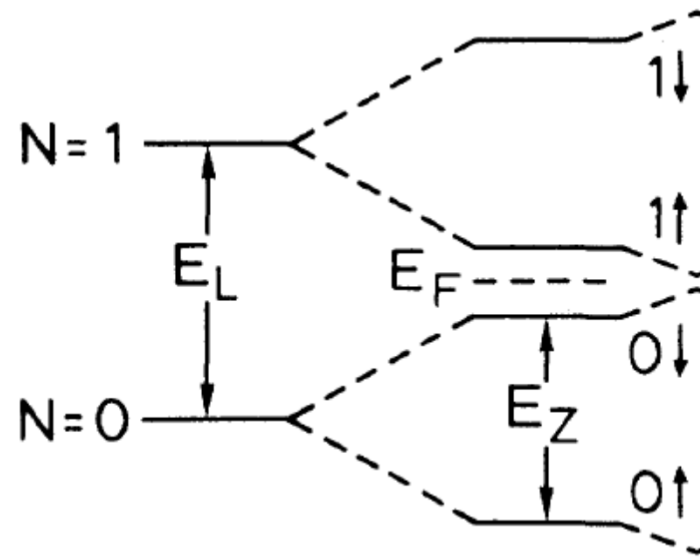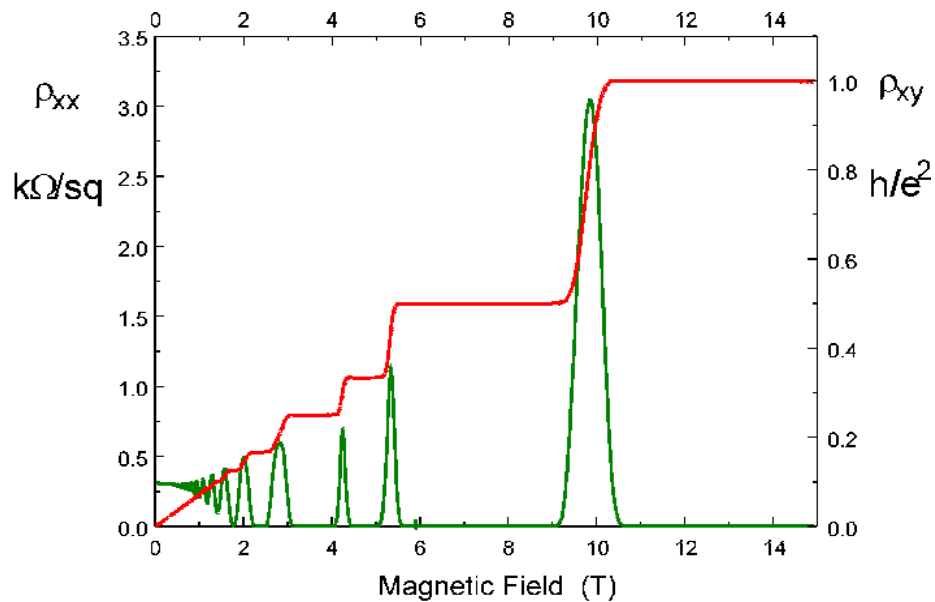
Each Landau level can hold the same number of electrons.

$$\rho_{xy} = \frac{-B_z}{ne} = \frac{-hD_0}{ve^2 D_0} = \frac{-h}{ve^2}$$

$$D_0 = \frac{m\omega_c}{2\pi\hbar} = \frac{eB_z}{h}$$

$$\omega_c = \frac{eB_z}{m} \qquad\qquad B_z = \frac{hD_0}{e}$$

Quantum hall effect

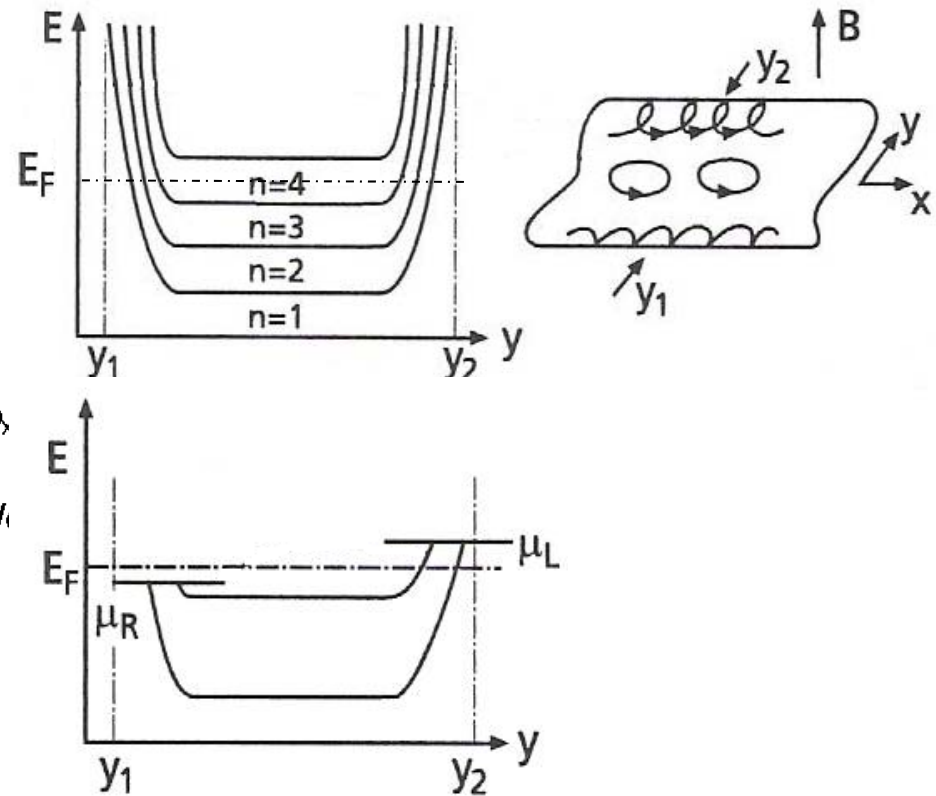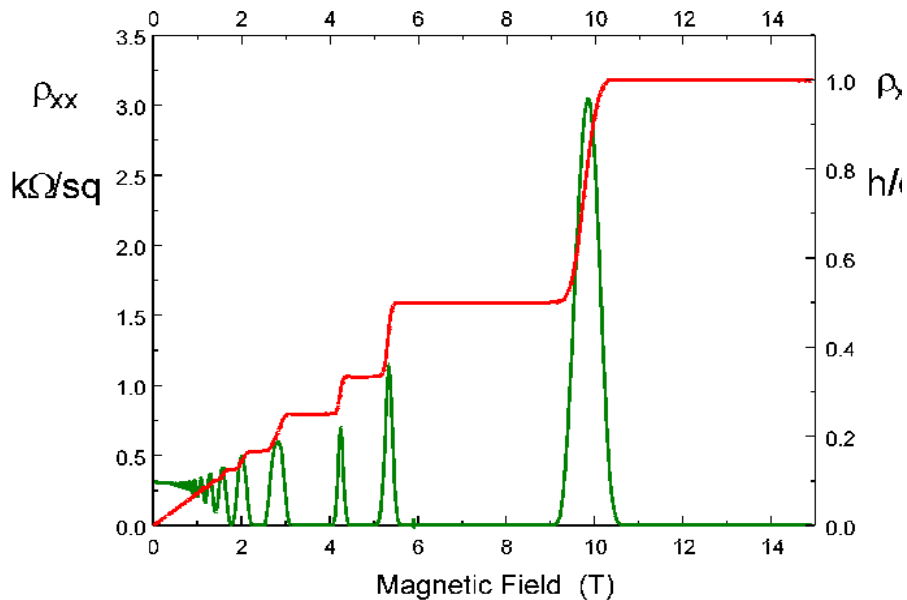$$\rho_{xy} = \frac{h}{\nu e^2}$$

S. Koch, R. J. Haug, and K. v. Klitzing,
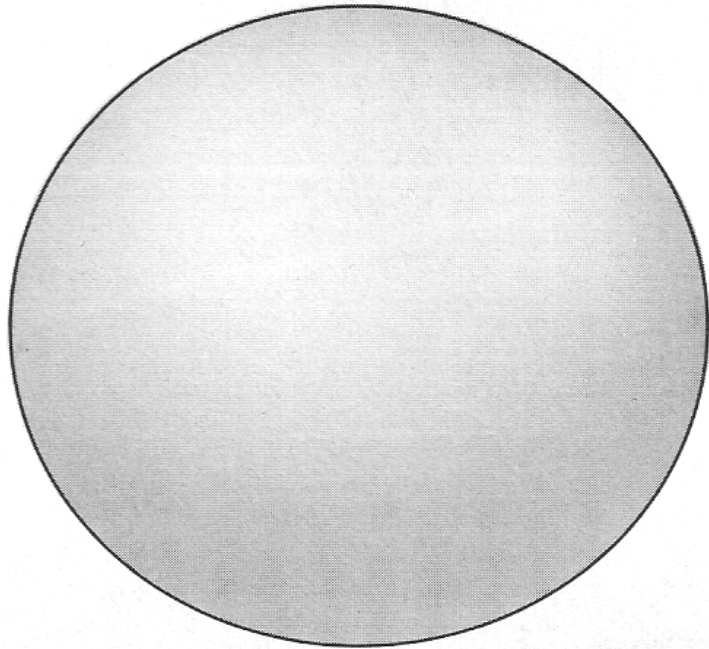Phys. Rev. B 47, 4048–4051 (1993)

Quantum Hall effect

Edge states are responsible for the zero resistance in ρ_{xx}

On the plateaus, resistance goes to zero because there are no states to scatter into.
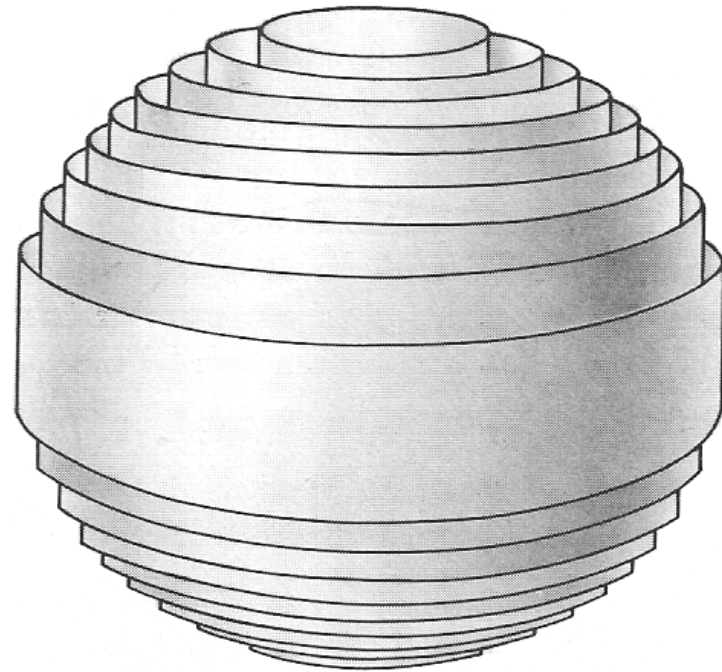

Ibach & Lueth (modified)

Fermi sphere in a magnetic field

$B = 0$

$B \neq 0$

Landau cylinders