

# Magnons and Plasmons

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# Ferromagnetic magnons - simple cubic

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The dispersion relation in one dimension:

$$\hbar\omega = 4J|S|(1 - \cos(ka))$$

The dispersion relation for a cubic lattice in three dimensions:

$$\hbar\omega = 2J|S|\left(z - \sum_{\vec{\delta}} \cos(\vec{k} \cdot \vec{\delta})\right)$$

The magnon contribution to thermodynamic properties can be calculated similar to the phonon contribution to the thermodynamic properties.

# Magnons

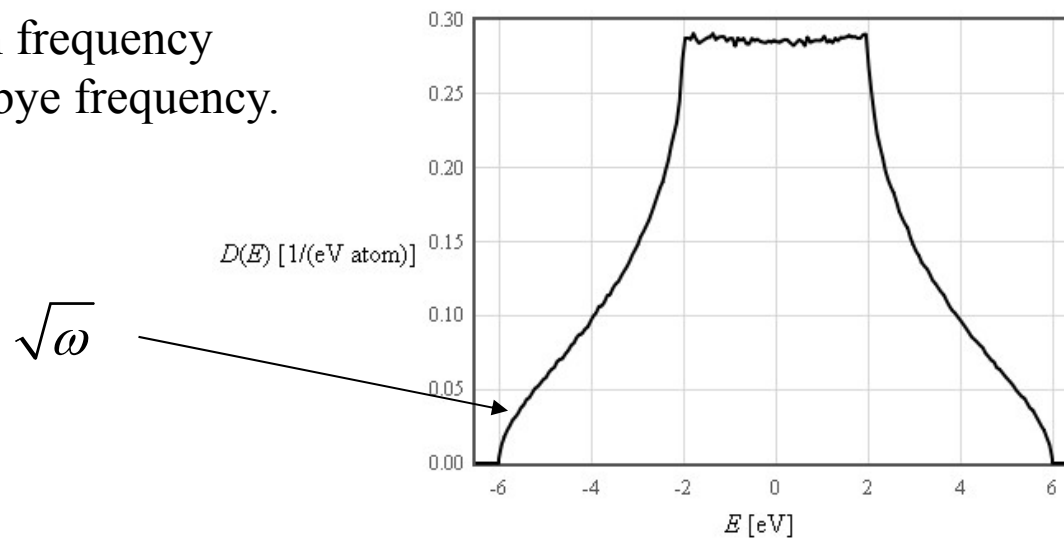
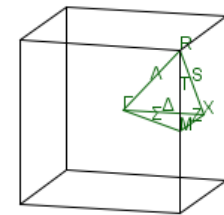
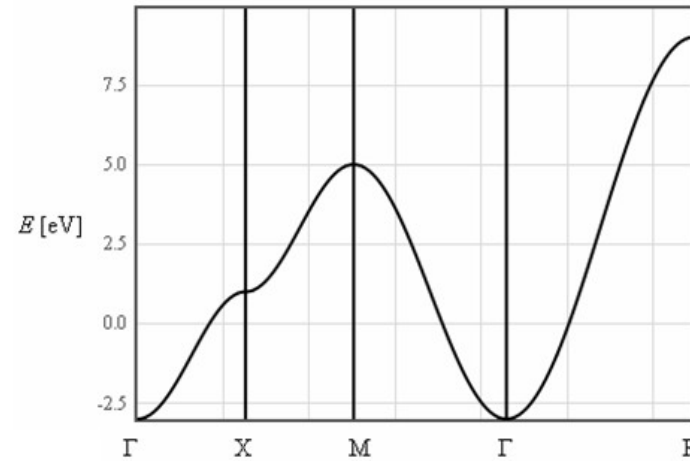
simple cubic 3-D

$$E = \varepsilon - 2t (\cos(k_x a) + \cos(k_y a) + \cos(k_z a))$$

$$\hbar\omega = 2J |S| \left( z - \sum_{\vec{\delta}} \cos(\vec{k} \cdot \vec{\delta}) \right)$$

Dispersion relation is mathematically equivalent to tight binding model for electrons.

There is a maximum frequency analogous to the Debye frequency.



# Long wavelength / low temperature limit

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Dispersion relation:  $\hbar\omega \approx 2JSk^2a^2$

The density of states:  $D(\omega) \propto \sqrt{\omega}$

Magnons are bosons:  $\langle n_k \rangle = \frac{1}{\exp\left(\frac{\hbar\omega_k}{k_B T}\right) - 1}$

$$u = \int_0^{\infty} \frac{\hbar\omega D(\omega) d\omega}{\exp\left(\frac{\hbar\omega_k}{k_B T}\right) - 1} \propto T^{5/2}$$

$$c_v \propto T^{3/2}$$

# Magnons

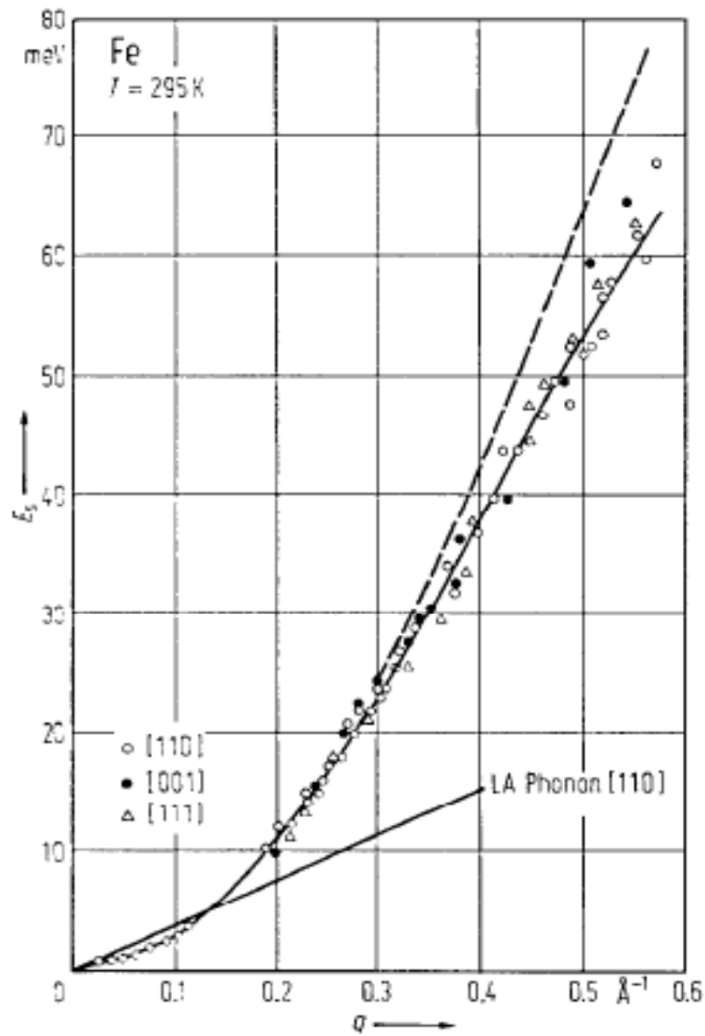


Fig. 1. Constant- $E$  scan TAS-measured spin wave dispersion relation for various directions in a single crystal of Fe at 295 K. The dashed line corresponds to the Heisenberg model with  $D=281 \text{ meV \AA}^2$  and  $\beta=1.0 \text{ \AA}^2$  [68 S 3], see also [73 M 1].

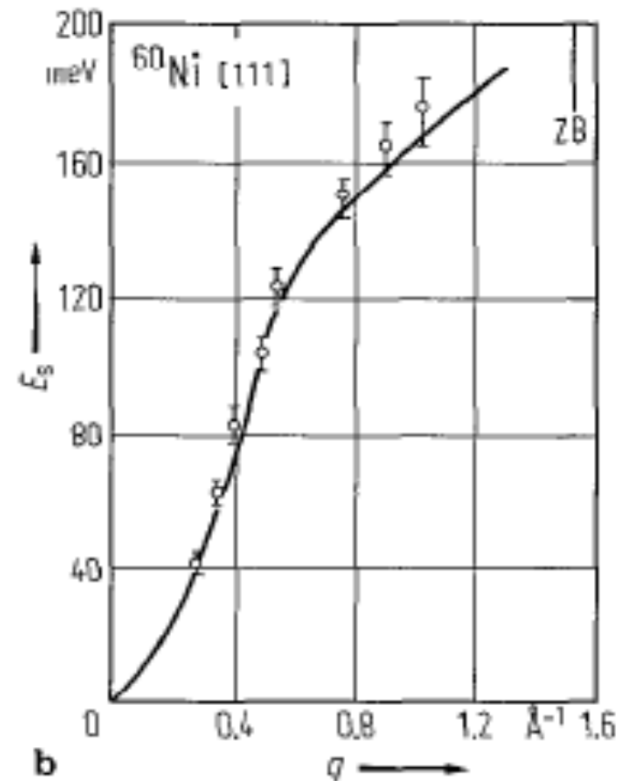
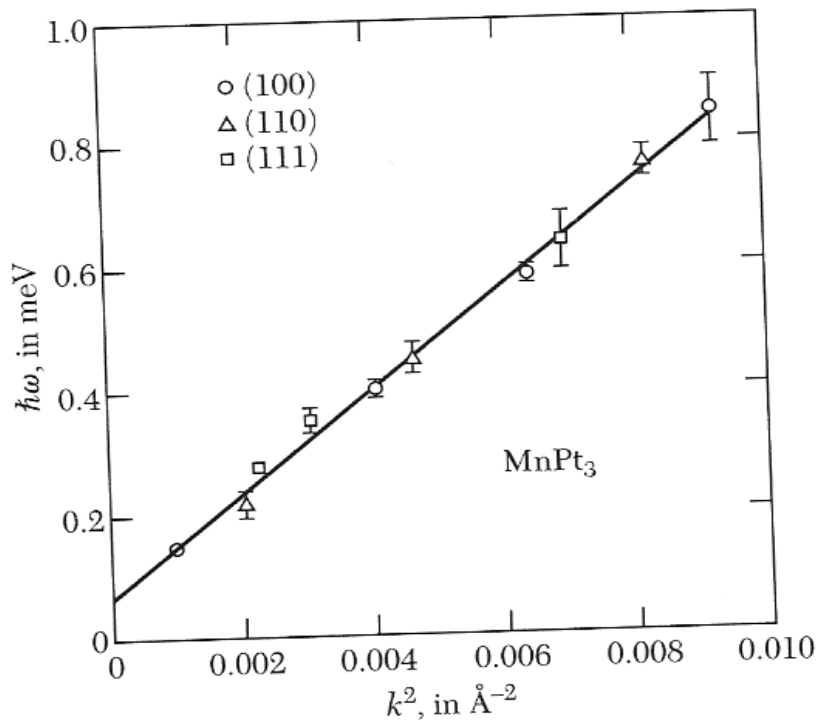
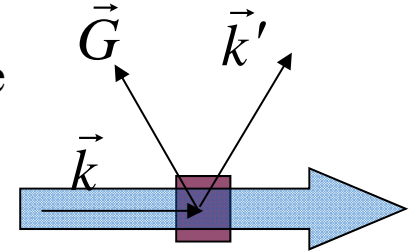


Fig. 6b. Room-temperature spin wave dispersion curve for the [111] direction of  $^{60}\text{Ni}$ . ZB shows the position of the zone boundary [85 M 1]. The solid curve is from calculations [85 C 1, 83 C 1].

# Neutron magnetic scattering

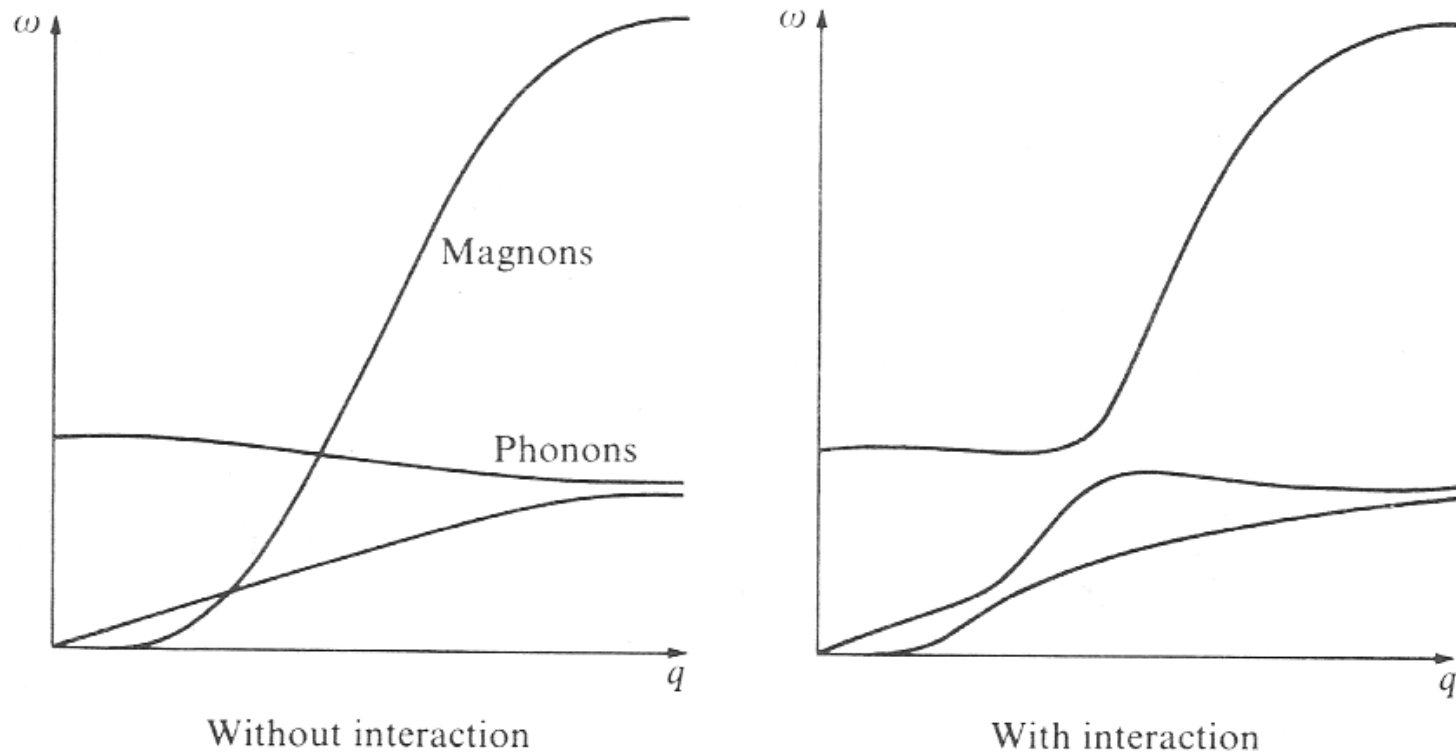
Neutrons can scatter inelastically from magnetic material and create or annihilate magnons

$$\vec{k}_n = \vec{k}'_n + \vec{k}_{magnon} + \vec{G}$$



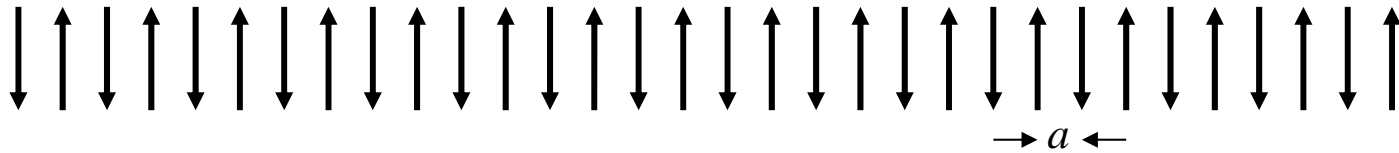
$$\frac{\hbar^2 k'^2}{2m_n} \pm \hbar\omega_{magnon} = \frac{\hbar^2 k^2}{2m_n} + \frac{\hbar^2 G^2}{2m_{crystal}}$$

**Fig. 5.7** Schematic magnon and phonon dispersion curves. The magnon curve has been compressed by a factor of order 10 for illustrative purposes.



From: *Solid State Theory*, Harrison

# Antiferromagnet magnons



$$\hbar \frac{dS_p^{Ax}}{dt} = 2J |S| (-S_p^{By} - 2S_p^{Ay} - S_{p-1}^{By})$$

$$\hbar \frac{dS_p^{Ay}}{dt} = -2J |S| (-S_p^{Bx} - 2S_p^{Ax} - S_{p-1}^{Bx})$$

$$\hbar \frac{dS_p^{Bx}}{dt} = 2J |S| (S_{p+1}^{Ay} + 2S_p^{By} + S_p^{Ay})$$

$$\hbar \frac{dS_p^{By}}{dt} = -2J |S| (S_{p+1}^{Ax} + 2S_p^{Bx} + S_p^{Ax})$$

$$\hbar \frac{dS_p^{Az}}{dt} = 0$$

$$\hbar \frac{dS_p^{Bz}}{dt} = 0$$

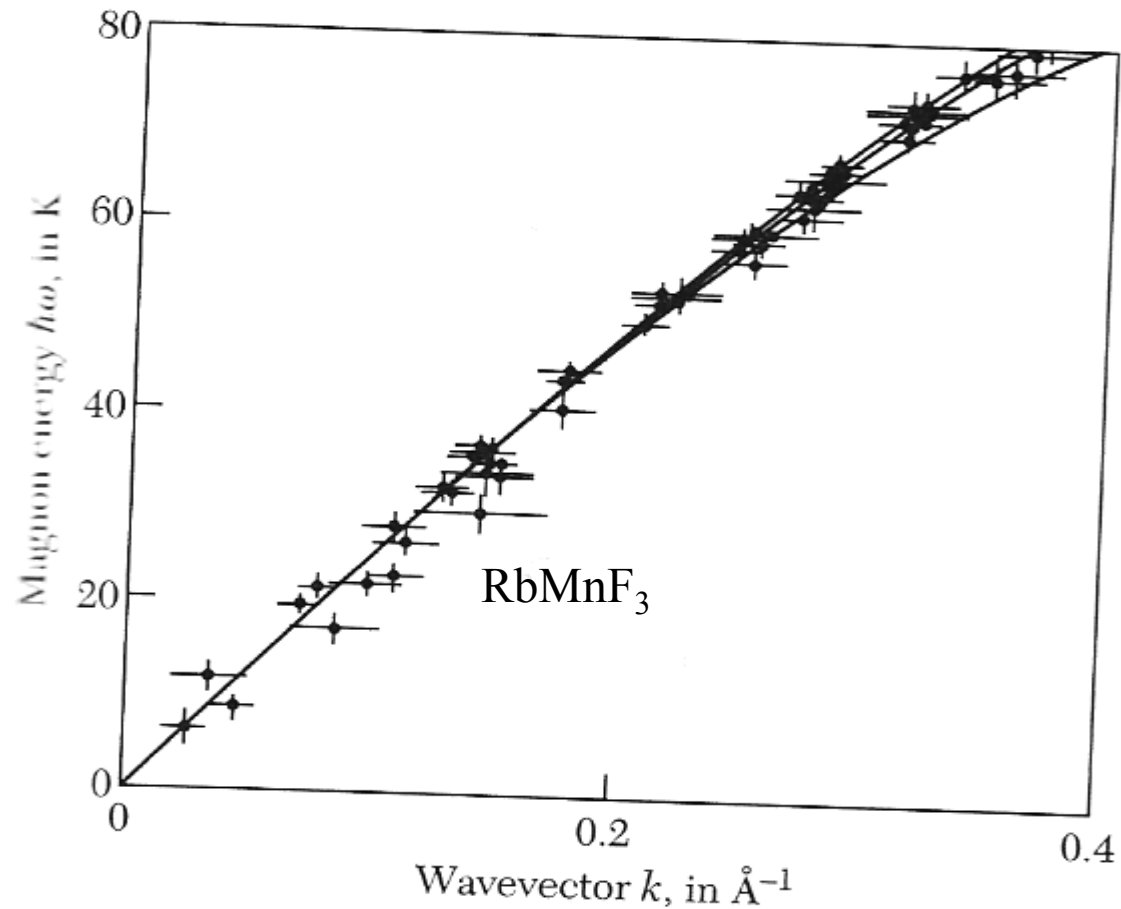
$$\begin{pmatrix} S_p^{Ax} \\ S_p^{Ay} \\ S_p^{Bx} \\ S_p^{By} \end{pmatrix} = \begin{pmatrix} u_k^{Ax} \\ u_k^{Ay} \\ u_k^{Bx} \\ u_k^{By} \end{pmatrix} \exp[i(2kpa - \omega t)]$$



# Antiferromagnet magnons

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$$\hbar\omega = 4|J|S|\sin(ka)|$$



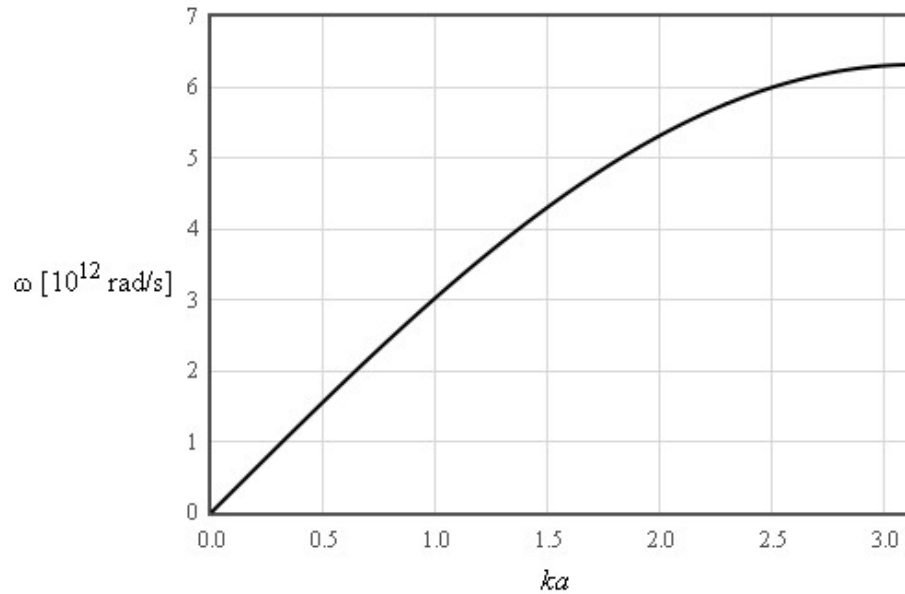
Brillouin zone boundary is at  $k = \pi/2a$

# Antiferromagnet magnons

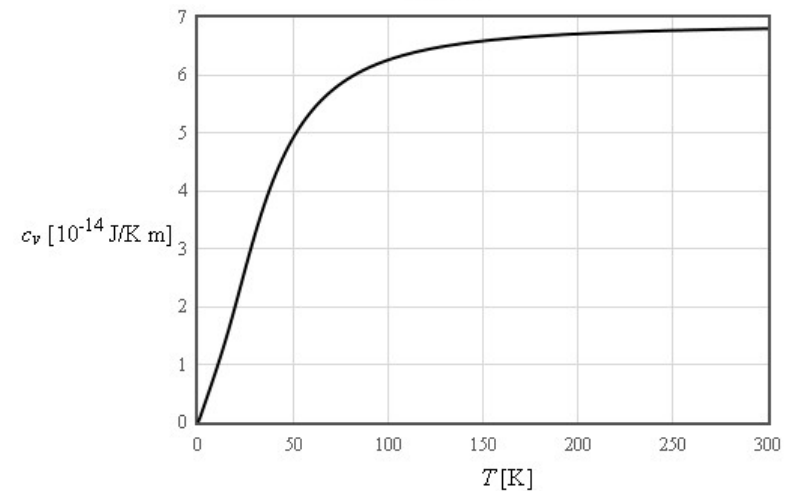
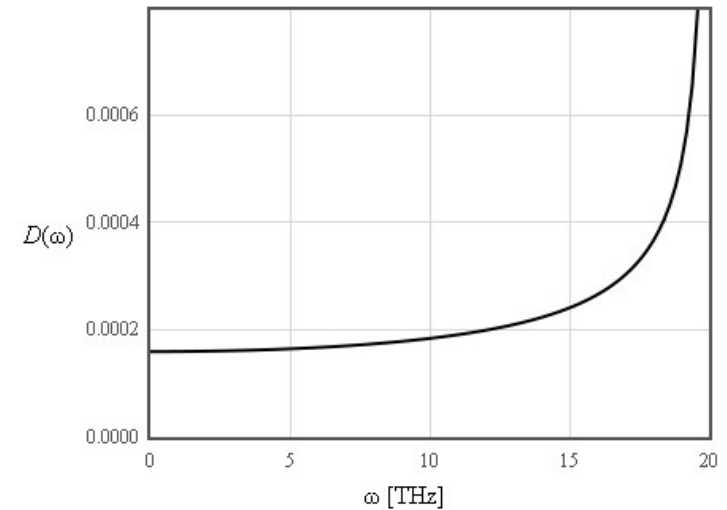
$$\hbar\omega = 4|J|S|\sin(ka)|$$

Mathematically equivalent to phonons in 1-d

$$\omega = \sqrt{\frac{4C}{m}} \left| \sin\left(\frac{ka}{2}\right) \right|$$




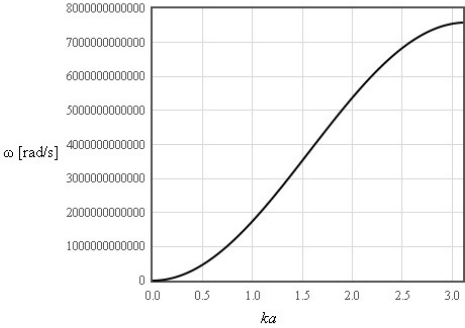
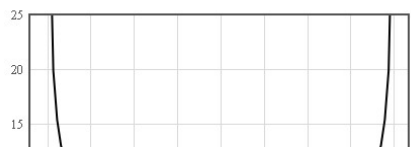
$$D(\omega) = \frac{1}{\pi a \sqrt{\frac{C}{m}} \sqrt{1 - \frac{\omega^2 m}{4C}}}$$



# Student project

Make a table of magnon properties like the table of phonon properties

Magnons

	1-D ferromagnetic magnons	1-D antiferromagnetic magnons	3-D low temperature limit
Equations of motion in mean field theory			
Eigenfunction solutions	$\begin{pmatrix} S_p^x \\ S_p^y \end{pmatrix} = \begin{pmatrix} u_k^x \\ u_k^y \end{pmatrix} \exp[i(kpa - \omega t)]$		
Dispersion relation	$\hbar\omega = 4JS(1 - \cos(ka))$  <p>Calculate ω(k)</p>		
			$D(E)$

Fe bcc  
Ni fcc  
Co hcp

1 student / column

# Longitudinal plasma waves

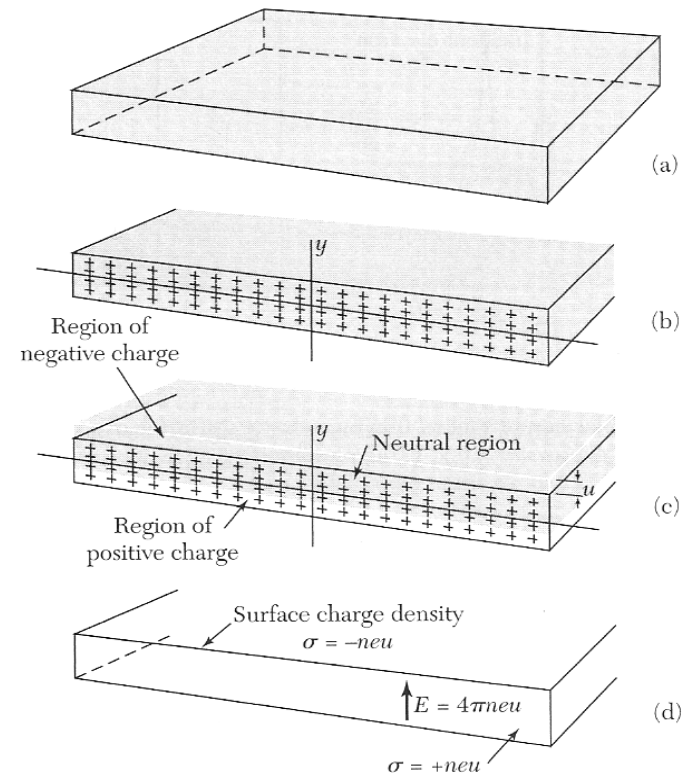
$$nm \frac{d^2 y}{dt^2} = -neE$$

$$E = \frac{ney}{\epsilon_0}$$

$$nm \frac{d^2 y}{dt^2} = -\frac{n^2 e^2 y}{\epsilon_0}$$

$$\frac{d^2 y}{dt^2} + \omega_p^2 y = 0$$

Plasma frequency  $\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}}$



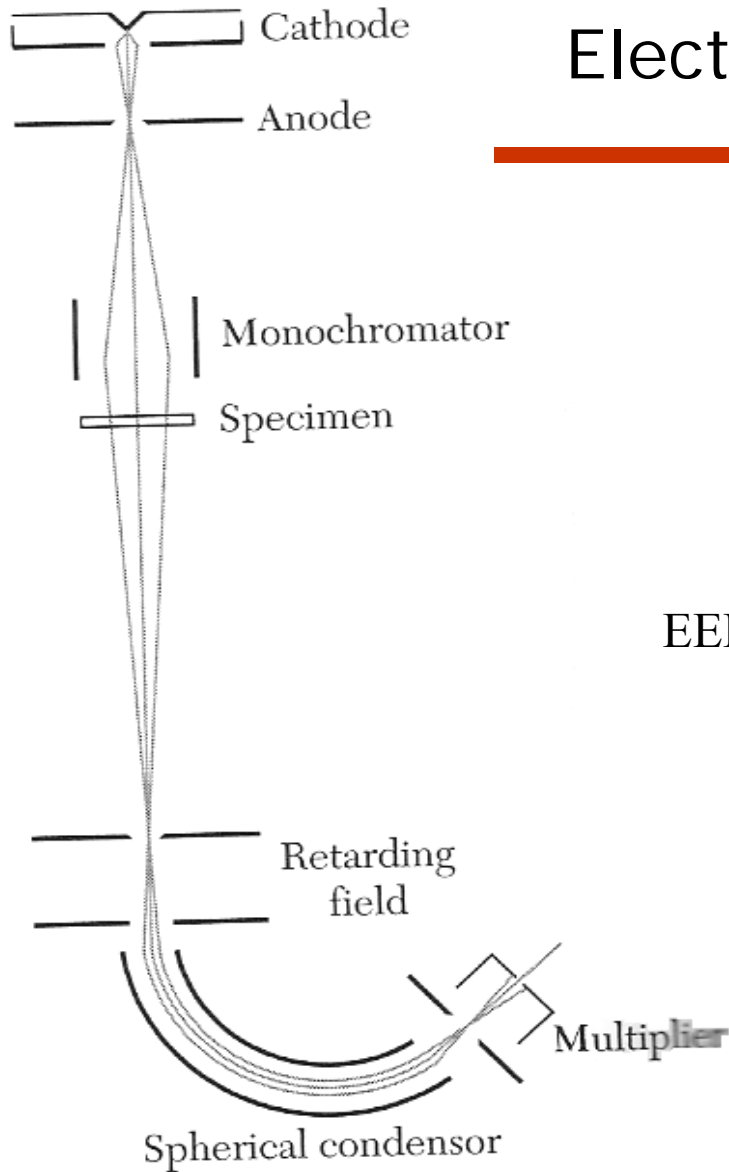
Kittel

There is no magnetic component of the wave.

Plasma waves can be quantized like any other wave

# Electron energy loss spectroscopy

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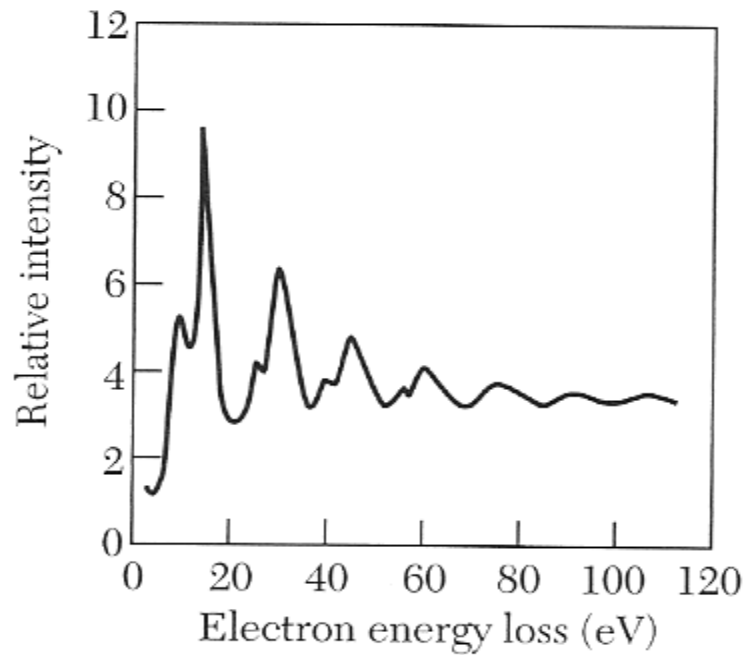


$$\Delta E = n\hbar\omega_p$$

EELS is often used to measure phonons

# Electron energy loss spectroscopy

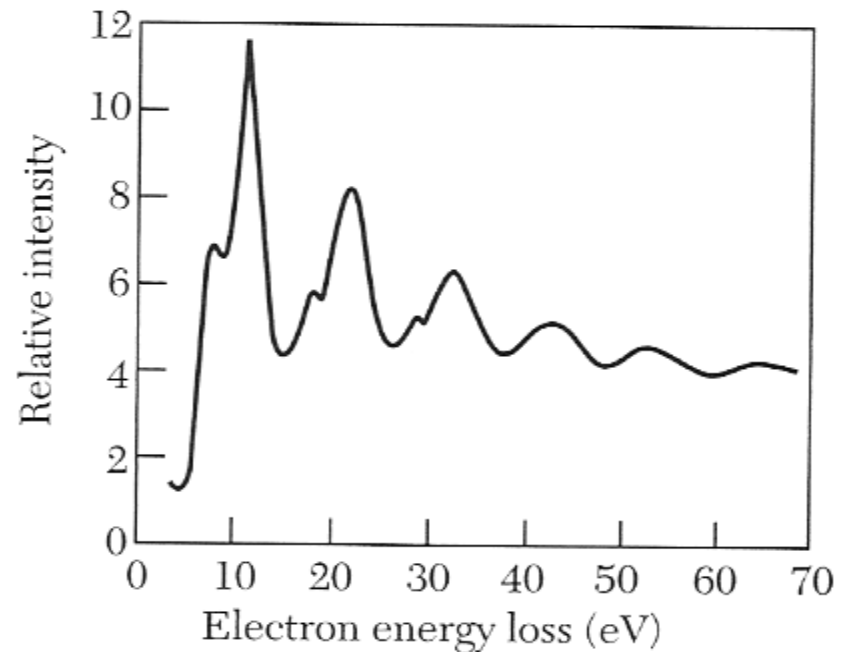
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Aluminum

Plasmons 15.3 eV

Surface plasmons 10.3 eV



Magnesium

Plasmons 10.6 eV

Surface plasmons 7.1 eV

# Transverse optical plasma waves

The dispersion relation for light

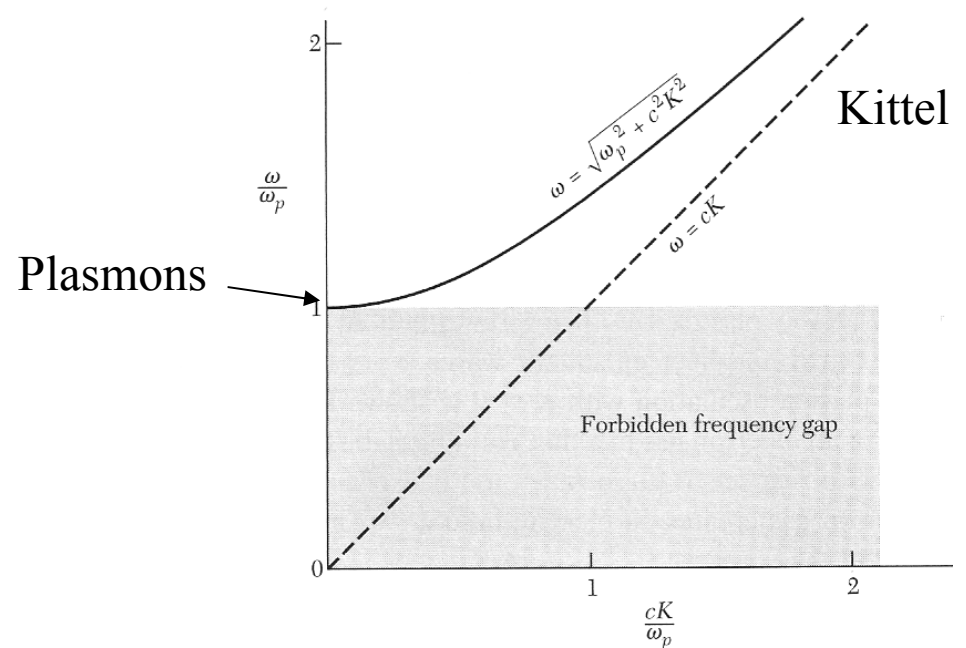
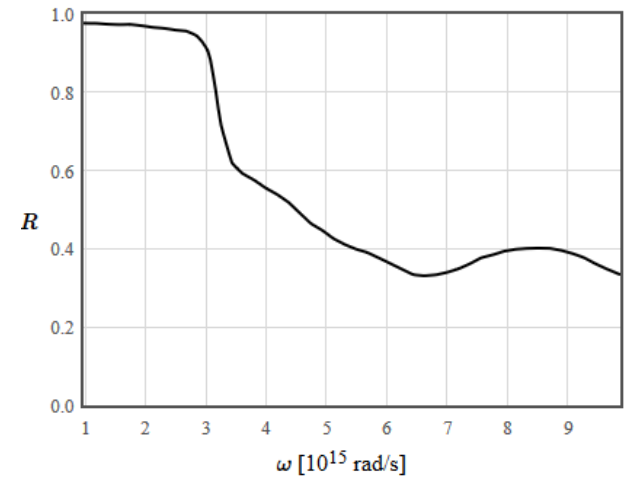
$$\epsilon\epsilon_0\mu_0\omega^2 = \frac{\epsilon\omega^2}{c^2} = k^2$$

For a free electron gas

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\left(1 - \frac{\omega_p^2}{\omega^2}\right)\omega^2 = c^2k^2$$

$$\omega^2 = \omega_p^2 + c^2k^2$$

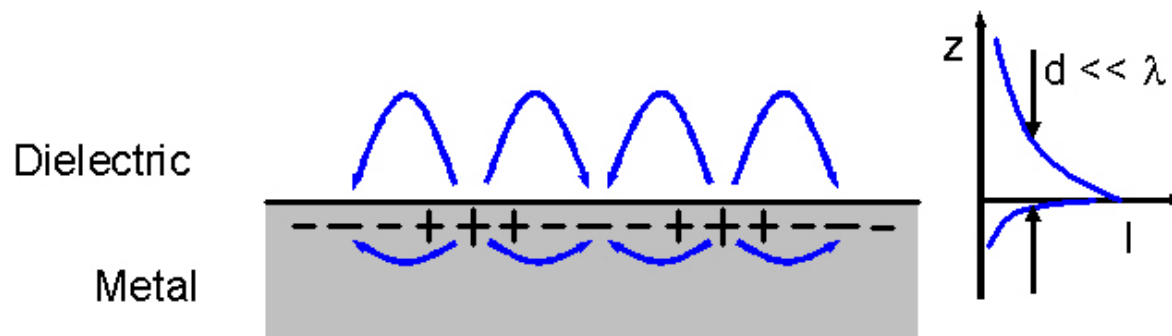


# Surface Plasmons

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Waves in the electron density at the boundary of two materials.

Surface plasmons have a lower frequency than bulk plasmons. This confines them to the interface.





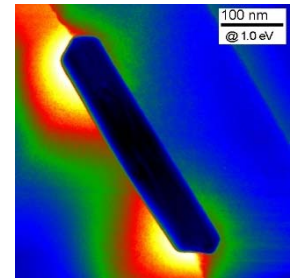
# Surface Plasmons



Green and blue require different sized particles.

High-resolution surface plasmon imaging of gold nanoparticles by energy-filtered transmission electron microscopy

PHYSICAL REVIEW B 79, 041401 R 2009



Surface plasmons on nanoparticles are efficient at scattering light.



# Organic plasmon-emitting diode

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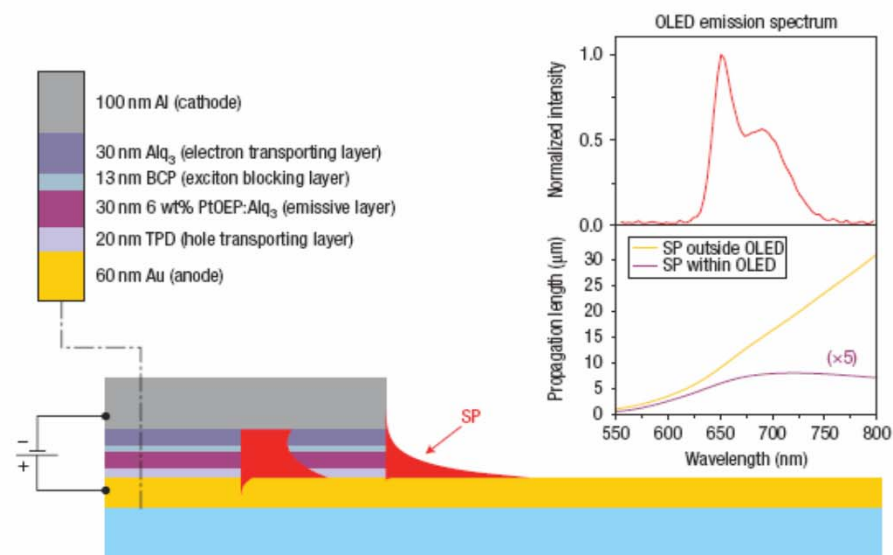
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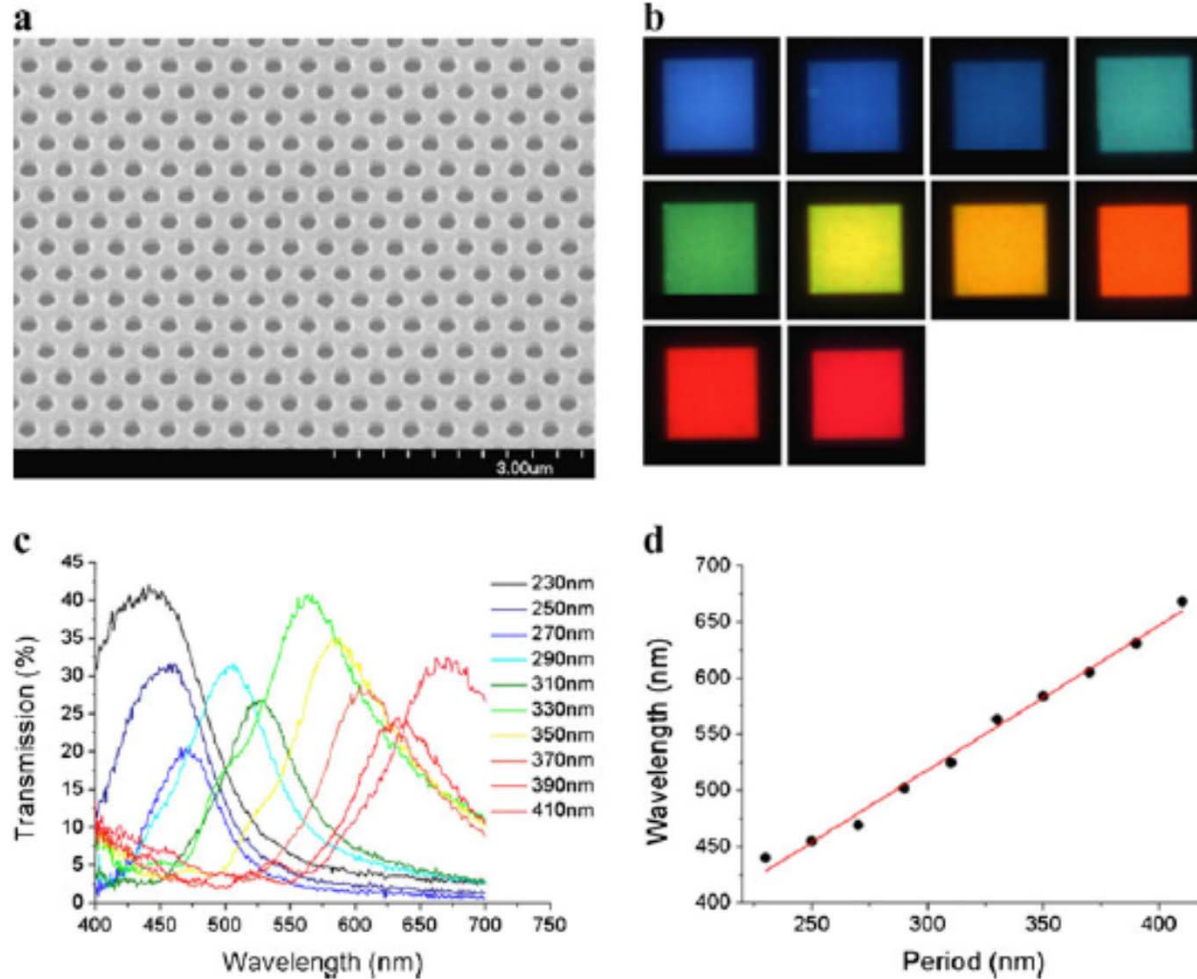
Published online: 28 September 2008; doi:10.1038/nphoton.2008.200

Surface plasmons are hybrid modes of longitudinal electron oscillations and light fields at the interface of a metal and a dielectric<sup>1,2</sup>. Driven by advances in nanofabrication, imaging and numerical methods<sup>3,4</sup>, a wide range of plasmonic elements such as waveguides<sup>5,6</sup>, Bragg mirrors<sup>7</sup>, beamsplitters<sup>8</sup>, optical modulators<sup>9</sup> and surface plasmon detectors<sup>10</sup> have recently been reported. For introducing dynamic functionality to plasmonics, the rapidly growing field of organic optoelectronics<sup>11</sup> holds strong promise due to its ease of fabrication and integration opportunities. Here, we introduce an electrically switchable



Surface plasmons are used for biosensors.

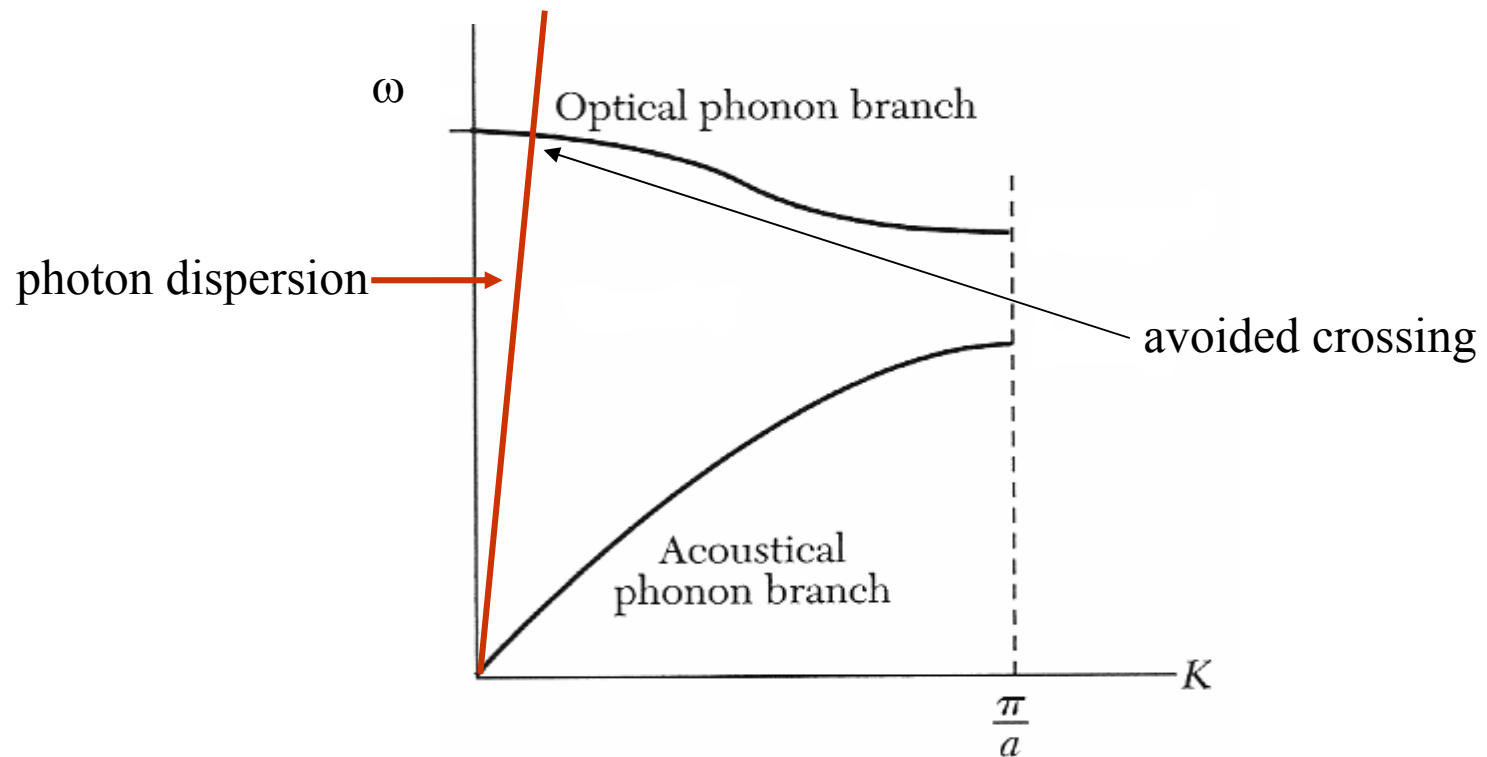
# Plasmon filter



Plasmon modes on the other side of the metal films are excited.

# Polaritons

Transverse optical phonons will couple to photons with the same  $\omega$  and  $k$ .



Light Bragg reflects off the sound wave; sound Bragg reflects off the light wave.