Landau Theory of First Order Phase Transitions

Superconductivity

First order transitions

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \beta < 0 \qquad \gamma > 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 = 0$$

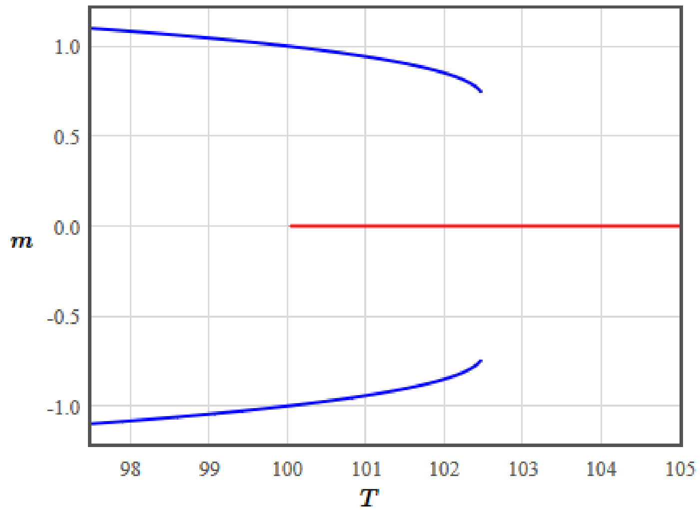
One solution for $m = 0$.

$$\alpha_0 (T - T_c) + \beta m^2 + \gamma m^4 = 0$$

$$m^2 = 0, \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}$$

There will be a minimum at finite m as long as m^2 is real

$$T_1 = \frac{\beta^2}{4\alpha_0 \gamma} + T_c$$



Jump in the order parameter

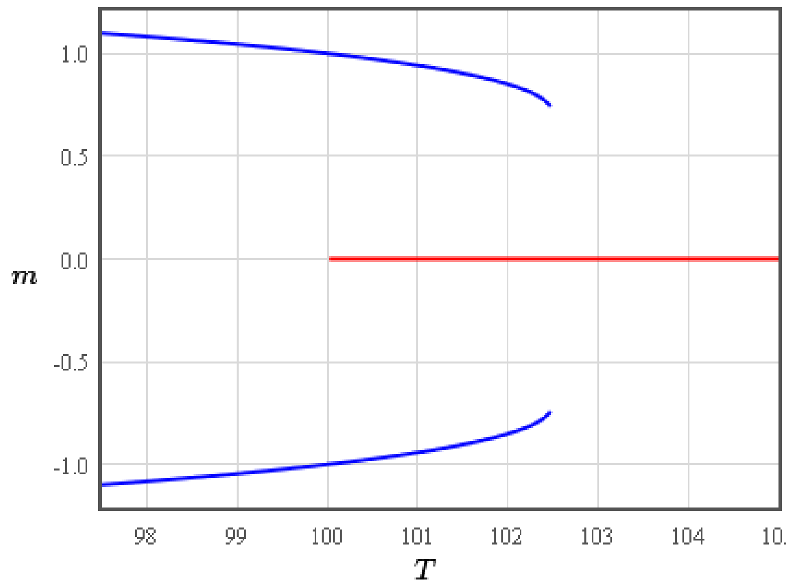
$$m^2 = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}$$

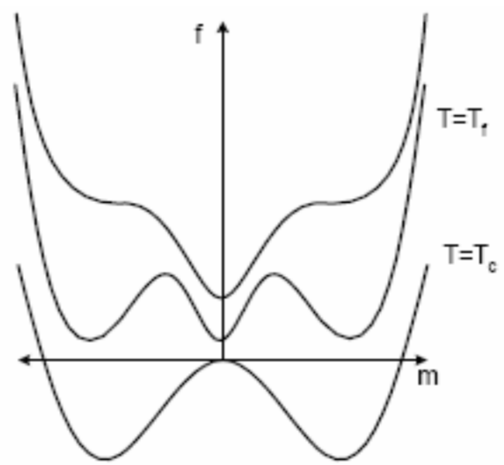
At T_c

$$\Delta m = \sqrt{\frac{-\beta}{\gamma}}$$

At T_1

$$\Delta m = \sqrt{\frac{-\beta}{2\gamma}}$$





First order transitions, entropy, c_v

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \beta < 0$$

$$m = 0, \pm\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}}$$

$$s = -\frac{\partial f}{\partial T} = \frac{\alpha_0}{2\gamma}\left(\beta - \sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}\right)$$

$$c_v = T\frac{\partial s}{\partial T} = \frac{\alpha_0^2 T}{\sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}}$$

branch where the order parameter is nonzero

First order transitions, susceptibility

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 - mB \qquad \beta < 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 - B = 0$$

At the minima $$B = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5$$

For small m,

$$\chi = \left.\frac{dm}{dB}\right|_{m=0} = \frac{1}{2\alpha_0 (T - T_c)}$$ Curie - Weiss

$$\chi = \left.\frac{dm}{dB}\right|_{m=\sqrt{\frac{-\beta}{2\gamma}}} = \frac{1}{2\alpha_0 (T - T_1)}$$

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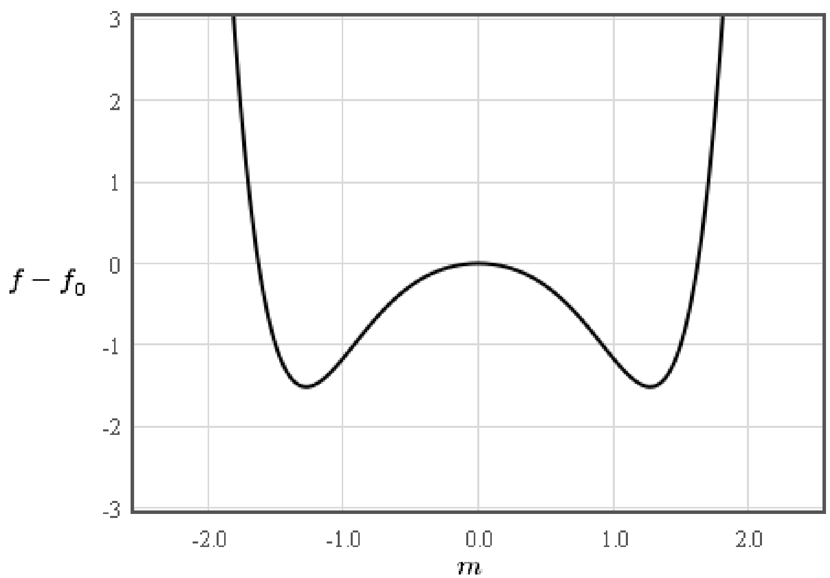
Landau theory of a first order phase transition

The free energy for a first order transition in Landau theory is,

$$f(T) = f_0(T) + \alpha_0(T - T_c)m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 \qquad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0.$$

Here $f_0(T)$ describes the temperature dependence of the high temperature phase near the phase transition.



$\alpha_0 =$ 0.1

$\beta =$ -1

$\gamma =$ 1

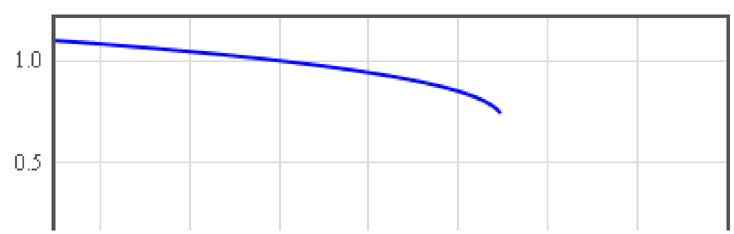
$T =$ 90

$T_c =$ 100

$f_0(T) =$ -0.01*T*T

submit

Order parameter



Superconductivity

Primary characteristic: zero resistance at dc

There is a critical temperature T_c above which superconductivity disappears

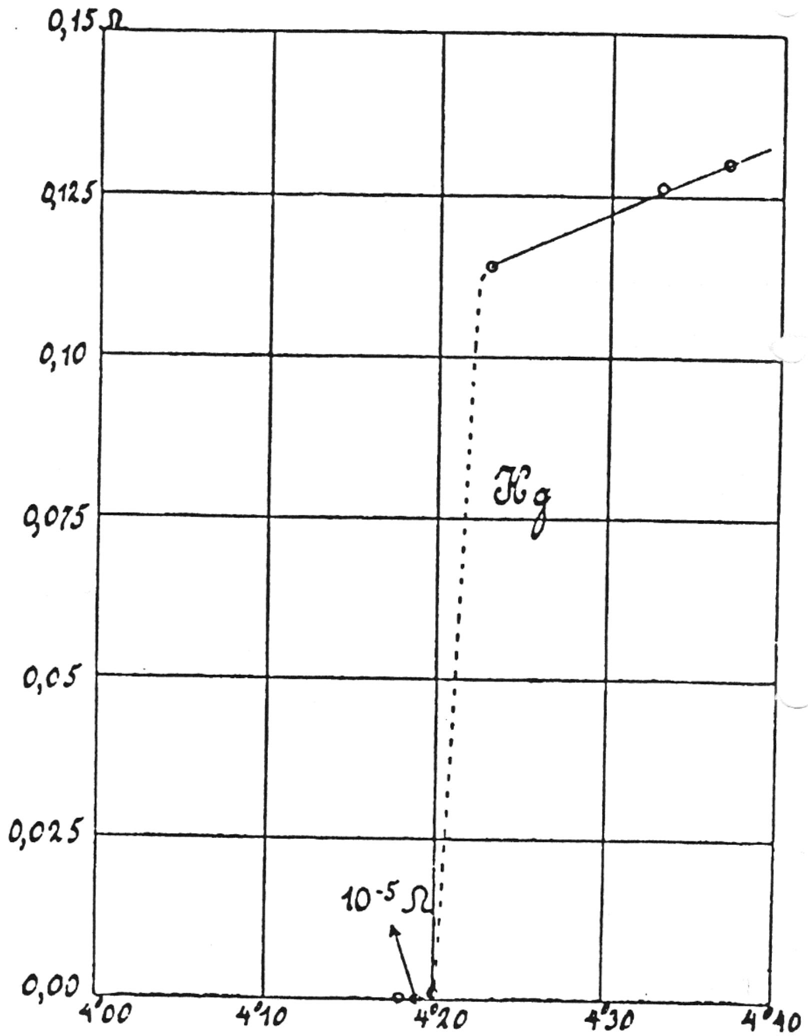
About 1/3 of all metals are superconductors

Metals are usually superconductors OR magnetic, not both

Good conductors are bad superconductors

Kittel chapter 10

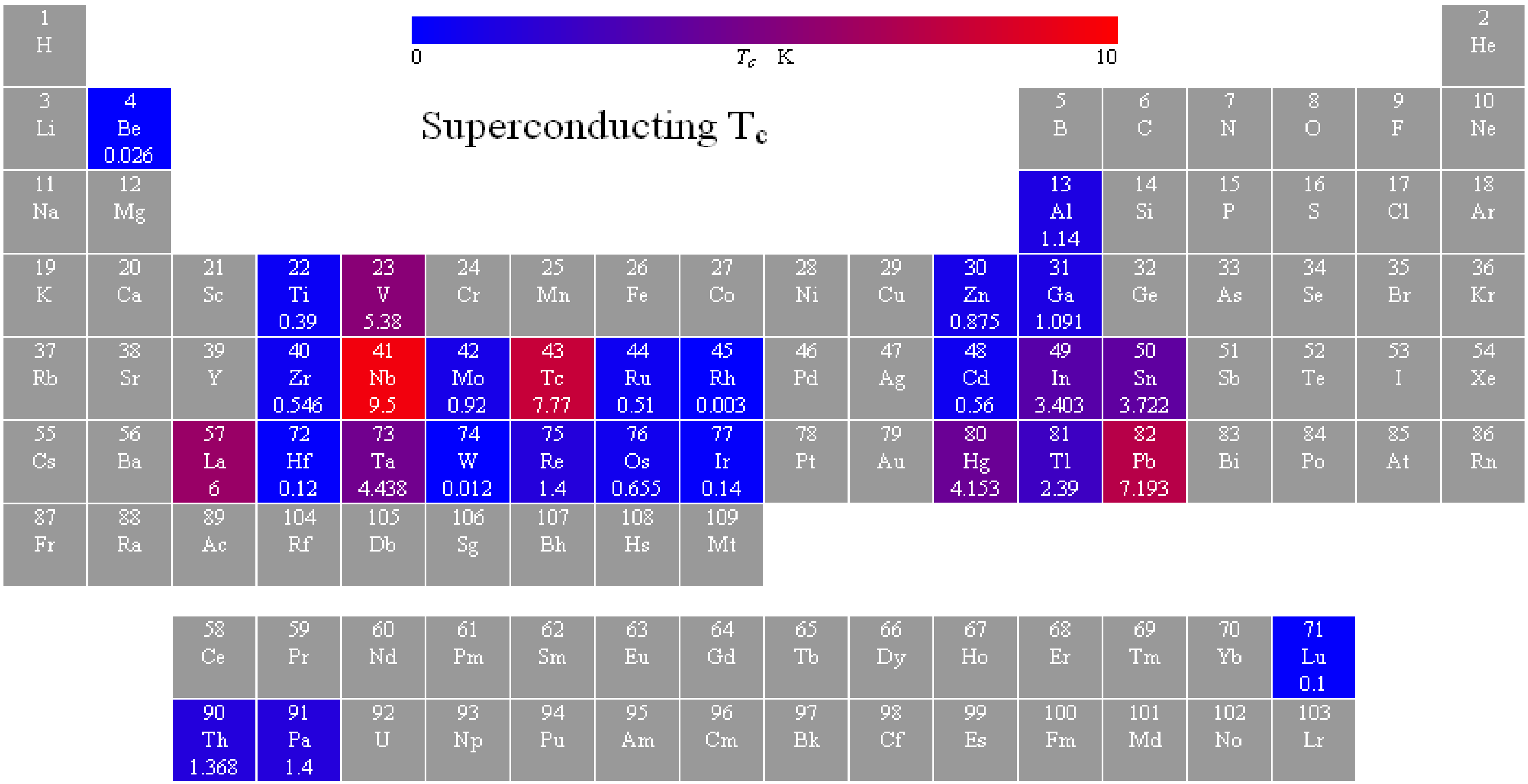
Superconductivity



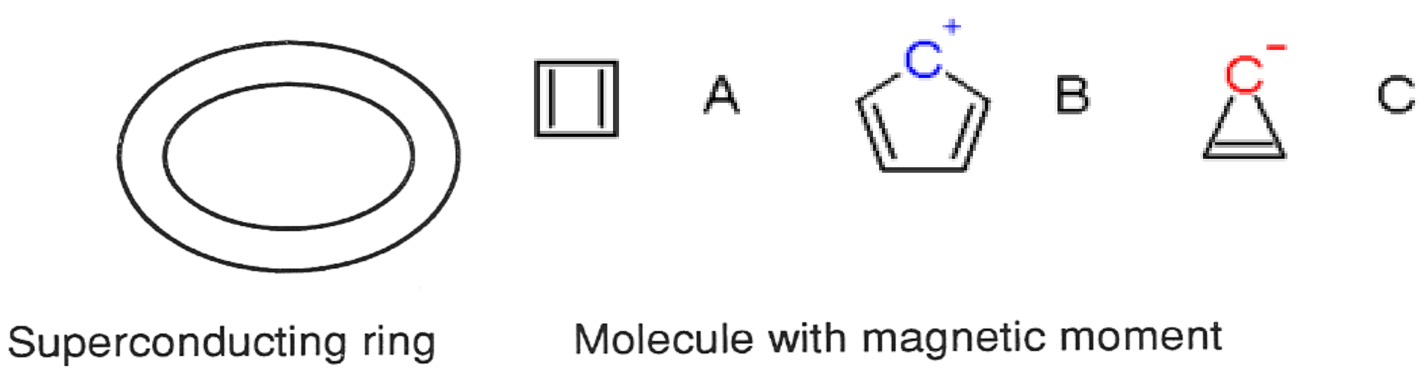
Heike Kammerling-Onnes

Superconductivity was discovered in 1911

Critical temperature



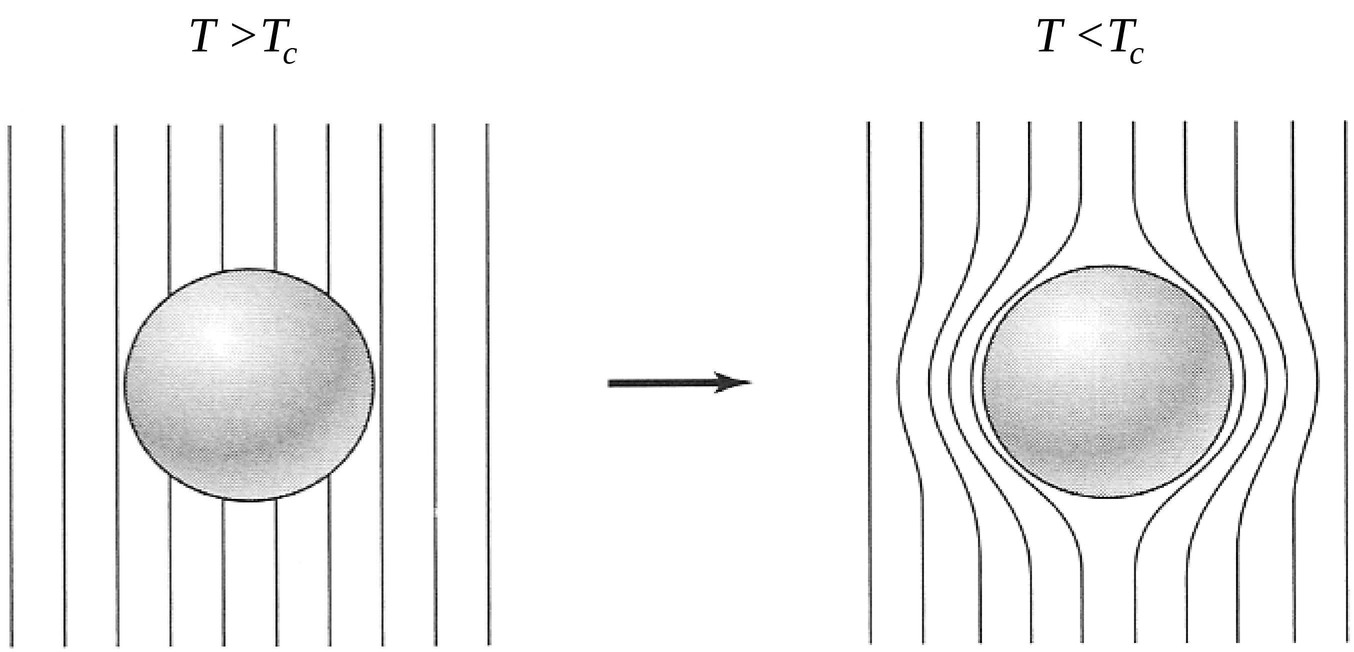
Superconductivity



Antiaromatic molecules are unstable and highly reactive

No measurable decay in current after 2.5 years. $\rho < 10^{-25}\ \Omega\text{m}$.

Meissner effect



Superconductors are perfect diamagnets at low fields.
$B = 0$ inside a bulk superconductor.

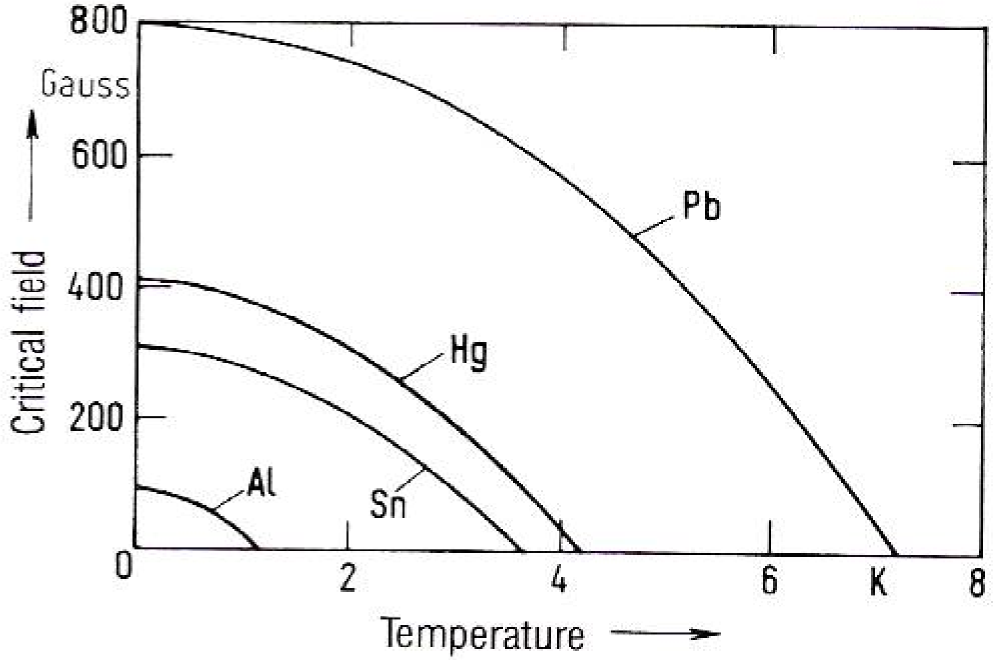
Superconductors are used for magnetic shielding.

Superconductivity

Critical temperature T_c

Critical current density J_c

Critical field H_c



$$n\Delta \approx nk_B T_c \approx \mu_0 H_c^2 \approx \tfrac{1}{2} nmv^2 = \frac{m}{2ne^2} J_c^2$$

Probability current

Schrödinger equation for a charged particle in an electric and magnetic field is

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}(-i\hbar\nabla - qA)^2 \psi + V\psi$$

write out the $(-i\hbar\nabla - qA)^2 \psi$ term

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}\left(-\hbar^2\nabla^2 + i\hbar qA\nabla + ihq\nabla A + q^2 A^2\right)\psi + V\psi$$

write the wave function in polar form

$$\psi = |\psi| e^{i\theta}$$

$$\nabla \psi = \nabla |\psi| e^{i\theta} + i\nabla \theta |\psi| e^{i\theta}$$

$$\nabla^2 \psi = \nabla^2 |\psi| e^{i\theta} + 2i\nabla \theta \nabla |\psi| e^{i\theta} + i\nabla^2 \theta |\psi| e^{i\theta} - \left(\nabla \theta\right)^2 |\psi| e^{i\theta}$$

Probability current

Schrödinger equation becomes:

$$i\hbar\frac{\partial|\psi|}{\partial t}-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(\nabla^2|\psi|+2i\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)$$

$$+i\hbar qA\left(\nabla|\psi|+i\nabla\theta|\psi|\right)+ihq\nabla A|\psi|+q^2A^2|\psi|\Big]+V|\psi|$$

Real part:

$$-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{-\hbar^2}{2m}\left(\nabla^2-\left(\nabla\theta-\frac{q}{\hbar}\vec{A}\right)^2\right)|\psi|+V|\psi|$$

Imaginary part:

$$\hbar\frac{\partial|\psi|}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(2\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)+2\hbar qA\nabla|\psi|+\hbar q|\psi|\nabla A\Big]$$

Probability current

Imaginary part:

$$\hbar \frac{\partial |\psi|}{\partial t} = \frac{1}{2m}\left[-\hbar^2 \left(2\nabla\theta\nabla|\psi| + i\nabla^2\theta|\psi| - (\nabla\theta)^2|\psi| \right) + 2\hbar q A\nabla|\psi| + \hbar q|\psi|\nabla A \right]$$

Multiply by |ψ| and rearrange $\quad \frac{\partial}{\partial t}|\psi|^2 = 2|\psi|\frac{\partial}{\partial t}|\psi|$

$$\frac{\partial}{\partial t}|\psi|^2 + \nabla\cdot\left[\frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)\right] = 0$$

This is a continuity equation for probability

$$\frac{\partial P}{\partial t} + \nabla\cdot\vec{S} = 0$$

The probability current: $\quad \vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$

Probability current / supercurrent

The probability current: $$\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$$

The particles are Cooper pairs q = -2e, m = 2m_e, $|\psi|^2 = n_{cp}$.

All superconducting electrons are in the same state so

$$\vec{j} = -2e\vec{S}$$

$$\boxed{\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)}$$

London gauge $\nabla\theta = 0$

$$\vec{j} = \frac{-2n_{cp}e^2}{m_e}\vec{A} = \frac{-n_s e^2}{m_e}\vec{A}$$

$$n_s = 2n_{cp}$$

1st London equation

$$\vec{j} = \frac{-n_s e^2}{m_e}\vec{A}$$

$$\frac{d\vec{j}}{dt} = \frac{-n_s e^2}{m_e}\frac{d\vec{A}}{dt} = \frac{n_s e^2}{m_e}\vec{E}$$

$$\frac{d\vec{A}}{dt} = -\vec{E}$$

First London equation:

$$\boxed{\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e}\vec{E}}$$

Classical derivation:

$$-e\vec{E} = m\frac{d\vec{v}}{dt} = -\frac{m}{n_s e}\frac{d\vec{j}}{dt}$$

$$\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e}\vec{E}$$

2nd London equation

$$\vec{j} = \frac{-n_s e^2}{m_e} \vec{A}$$

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \nabla \times \vec{A}$$

Second London equation:

$$\boxed{\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B}}$$

Meissner effect

Combine second London equation with Ampere's law

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B} \qquad \nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla \times \nabla \times \vec{B} = \frac{-n_s e^2 \mu_0}{m_e} \vec{B}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \left(\nabla \cdot \vec{B} \right) - \nabla^2 \vec{B}$$

Helmholtz equation: $\lambda^2 \nabla^2 \vec{B} = \vec{B}$

London penetration depth: $\lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$

Meissner effect

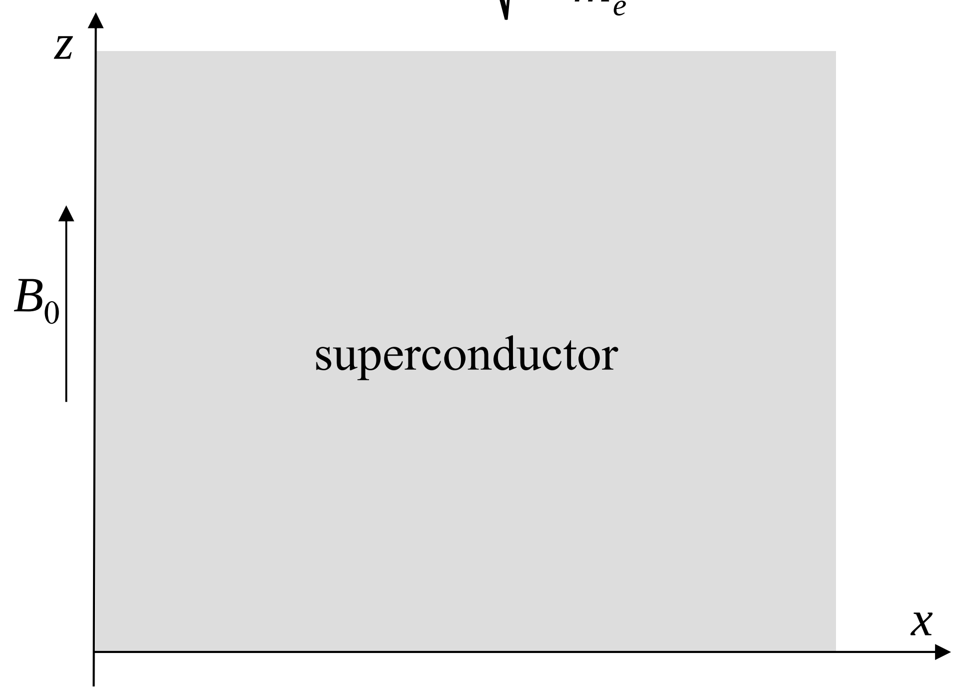
$$\lambda^2 \nabla^2 \vec{B} = \vec{B}$$

$$\lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$$

solution to Helmholtz equation:

$$\vec{B} = \vec{B}_0 \exp\left(\frac{-x}{\lambda}\right)\hat{z}$$

Al	λ = 50 nm
In	λ = 65 nm
Sn	λ = 50 nm
Pb	λ = 40 nm
Nb	λ = 85 nm



$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

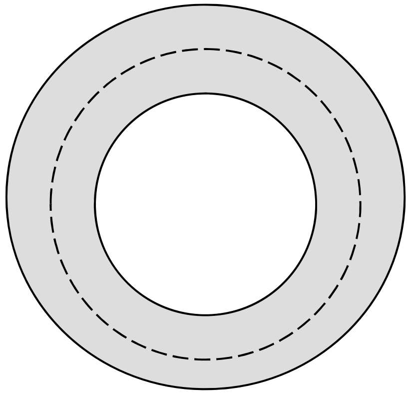
$$\vec{j} = \frac{\vec{B}_0}{\mu_0 \lambda} \exp\left(\frac{-x}{\lambda}\right)\hat{y}$$

Flux quantization

$$\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$

For a ring much thicker than the penetration depth, $j = 0$ along the dotted path.

$$0 = \left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$



Integrate once along the dotted path.

$$\oint \nabla\theta \cdot d\vec{l} = -\frac{2e}{\hbar}\oint \vec{A}\cdot d\vec{l} = -\frac{2e}{\hbar}\int_S \nabla\times\vec{A}\cdot d\vec{s} = -\frac{2e}{\hbar}\int_S \vec{B}\cdot d\vec{s} = -\frac{2e}{\hbar}\Phi$$

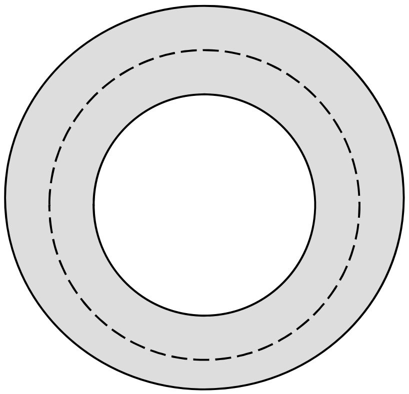
Stokes' theorem

magnetic flux

Flux quantization

$$\oint \nabla\theta \cdot d\vec{l} = 2\pi n = -\frac{2e}{\hbar}\Phi$$

n = ... -2, -1, 0, 1, 2 ...



$$2\pi n = \frac{2e}{\hbar}\Phi = \frac{\Phi}{\Phi_0}$$

Flux quantization: $\boxed{\Phi = n\Phi_0}$

$$\Phi_0 = \frac{h}{2e} = 2.0679\times 10^{-15}\ [\mathrm{W} = \mathrm{T\ m^2}]$$

Superconducting flux quantum