

Band structure

Tight Binding

Plane wave method bcc

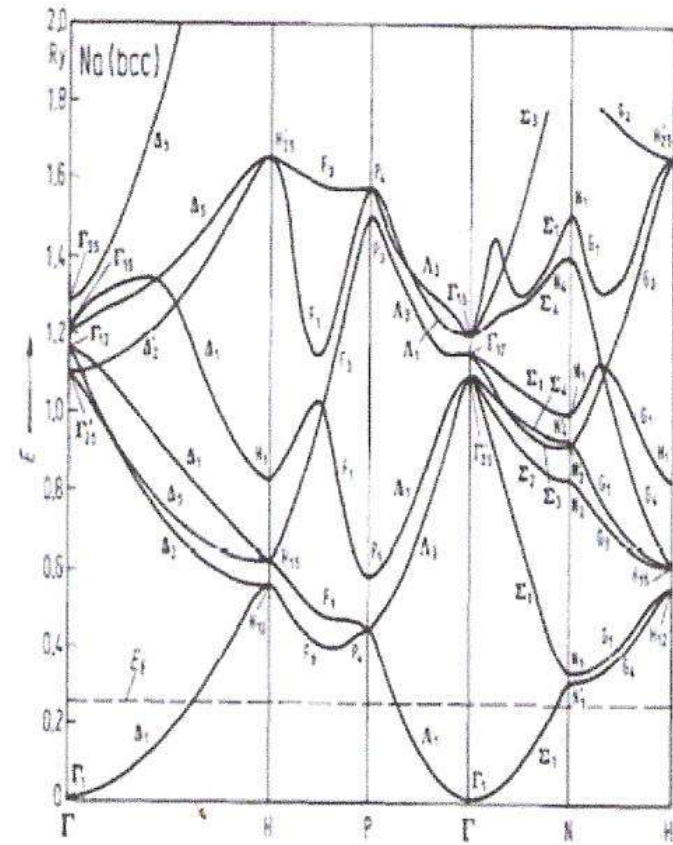
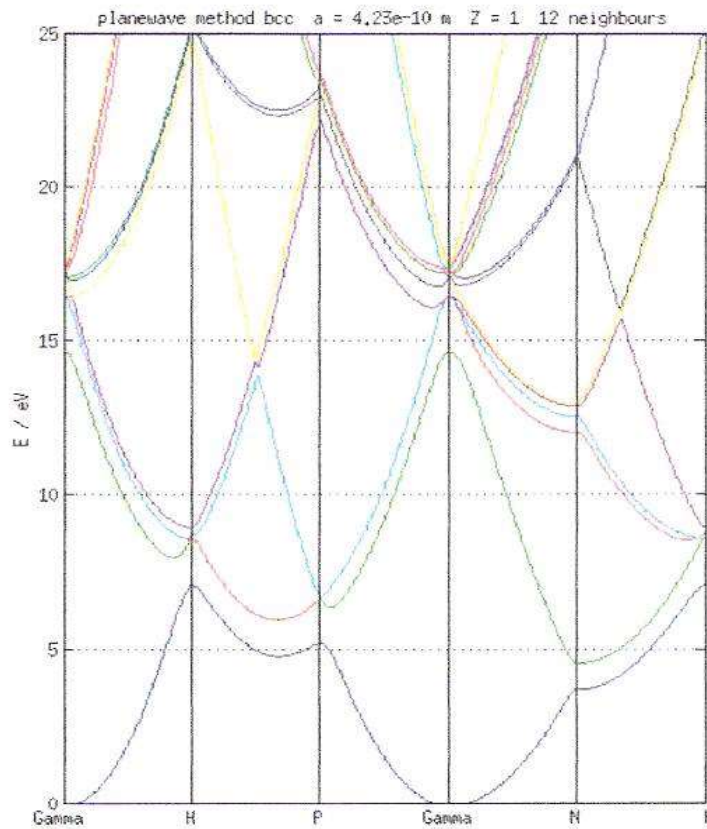


Abbildung 4: Planewave-Methode angewandt an Natrium vs. Literaturberechnung [1]

Muffin tin potentials, pseudopotentials



$$U(\vec{r}) = -\frac{Ze^2}{4\pi\epsilon_0 r} \quad \text{inside a radius } R \text{ and is constant outside}$$

$$U(\vec{r}) = \frac{Ze^2}{V\epsilon_0} \sum_{\vec{G}} \left(\frac{\cos(|G|R) - 1}{|G|^2} + \frac{\sin(|G|R) - |G|R \cos(|G|R)}{|G|^3} \right) \exp(i\vec{G} \cdot \vec{r}).$$



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PSEUDOPOTENTIALS

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[More about pseudopotentials](#)

[Naming convention for the pseudopotential](#)

[Unified Pseudopotential Format](#)

PSEUDOPOTENTIALS

Ready-to-use pseudopotentials are available from the periodic table below. Choose the options you desire from the menus (pseudopotentials from PSLibrary are recommended), then press "Filter". Elements for which at least a pseudopotential is available will appear in red. Click on the element entry and follow the link to access the pseudopotentials and a minimal description of their characteristics.

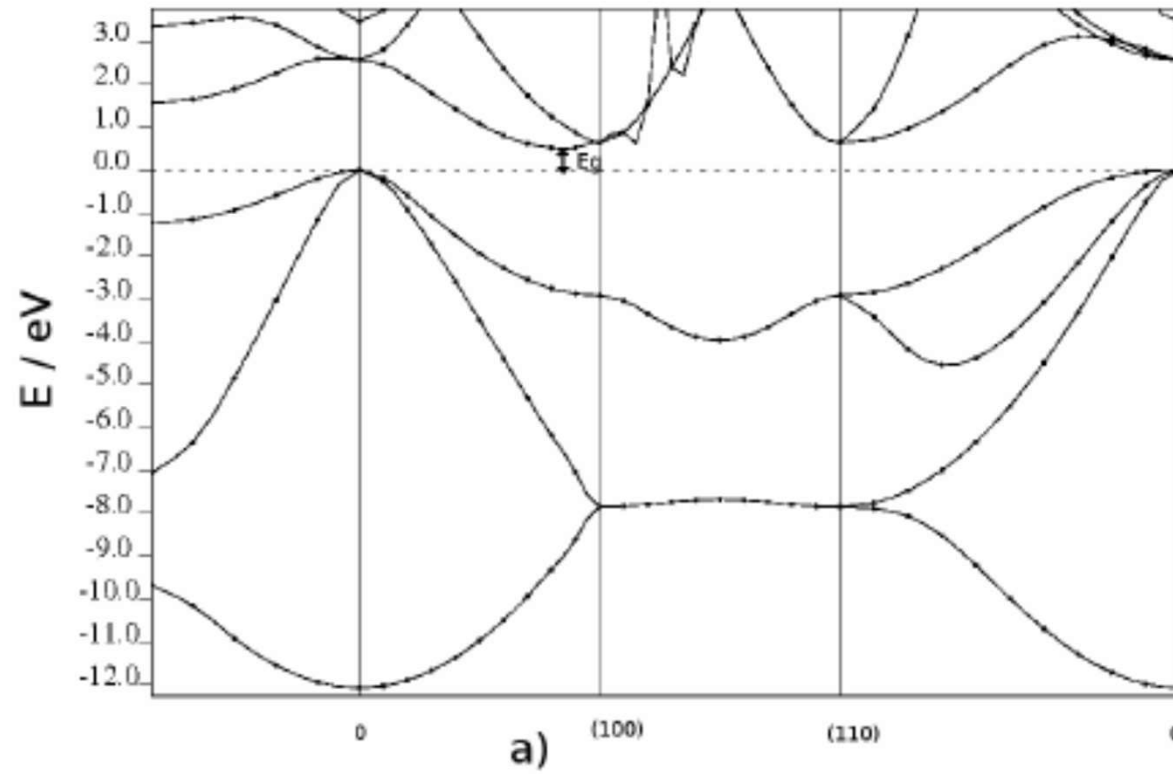
More information about [pseudopotentials in general](#), the [naming convention](#) adopted for pseudopotential files, the [Unified Pseudopotential Format](#), and on other pseudopotential databases, can be found via the links of the menu at the left.

Important Note: although most of these pseudopotentials were published or used with satisfactory results in published work, we cannot give any warranty whatsoever that they fit your actual needs.

1 H																	2 He	
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
55 Cs	56 Ba	57-70 *	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn

Si

Bachelor thesis Benedikt Tschofenig



QUANTUMESPRESSO

<http://www.quantum-espresso.org/>

Tight binding

Tight binding does not include electron-electron interactions

$$H_{MO} = \frac{-\hbar^2}{2m_e} \nabla^2 + V(\vec{r}) = \frac{-\hbar^2}{2m_e} \nabla^2 - \sum_A \frac{Z_A e^2}{4\pi\epsilon_0 |\vec{r} - \vec{r}_A|}$$

Assume a solution of the form.

$$\psi_k = \sum_{l,m,n} \exp\left(i\left(l\vec{k} \cdot \vec{a}_1 + m\vec{k} \cdot \vec{a}_2 + n\vec{k} \cdot \vec{a}_3\right)\right) \sum_a c_a \phi_a(\vec{r} - l\vec{a}_1 - m\vec{a}_2 - n\vec{a}_3)$$

↑
atomic orbitals:
choose the relevant
valence orbitals

Tight binding

$$\psi_k = \sum_{l,m,n} \exp\left(i\left(l\vec{k} \cdot \vec{a}_1 + m\vec{k} \cdot \vec{a}_2 + n\vec{k} \cdot \vec{a}_3\right)\right) \sum_a c_a \phi_a(\vec{r} - l\vec{a}_1 - m\vec{a}_2 - n\vec{a}_3)$$

$$H_{MO}\psi_k = E_k\psi_k$$

$$\langle \phi_a | H_{MO} | \psi_k \rangle = E_k \langle \psi_a | \psi_k \rangle$$

$$\begin{aligned} & c_a \langle \phi_a | H_{MO} | \phi_a \rangle + \sum_{\text{nearest neighbors } m} c_m \langle \phi_a | H_{MO} | \phi_m \rangle \exp(i(h\vec{k} \cdot \vec{a}_1 + j\vec{k} \cdot \vec{a}_2 + l\vec{k} \cdot \vec{a}_3)) + \text{small terms} \\ & = E_k c_a \langle \phi_a | \phi_a \rangle + \text{small terms} \end{aligned}$$

There is one equation for each atomic orbital

Tight binding, one atomic orbital

$$c_a \langle \phi_a | H_{MO} | \phi_a \rangle + \sum_{\text{nearest neighbors } m} c_m \langle \phi_a | H_{MO} | \phi_m \rangle \exp(i(h\vec{k} \cdot \vec{a}_1 + j\vec{k} \cdot \vec{a}_2 + l\vec{k} \cdot \vec{a}_3)) + \text{small terms}$$
$$= E_k c_a \langle \phi_a | \phi_a \rangle + \text{small terms}$$

For only one atomic orbital in the sum over valence orbitals

$$E_k c_a \langle \phi_a | \phi_a \rangle = c_a \langle \phi_a | H_{MO} | \phi_a \rangle + \sum_{\text{nearest neighbors } m} c_a \langle \phi_a | H_{MO} | \phi_m \rangle \exp(i(h\vec{k} \cdot \vec{a}_1 + j\vec{k} \cdot \vec{a}_2 + l\vec{k} \cdot \vec{a}_3))$$

one atomic orbital

$$E_k = \varepsilon - t \sum_m e^{i\vec{k} \cdot \vec{\rho}_m}$$

$$\varepsilon = \langle \phi_a(\vec{r}) | H_{MO} | \phi_a(\vec{r}) \rangle$$

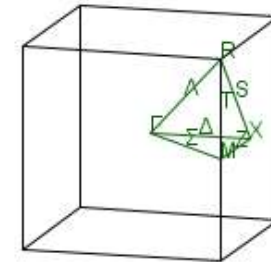
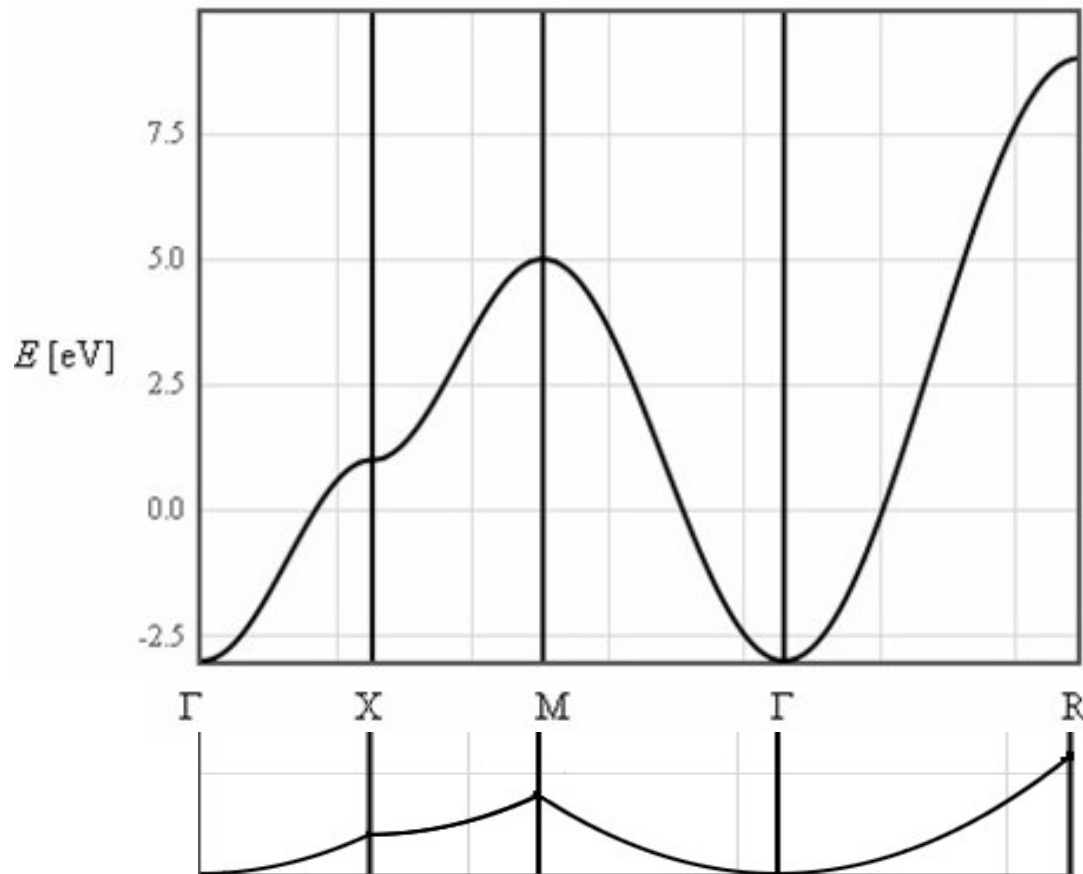
$$t = -\langle \phi_a(\vec{r}) | H_{MO} | \phi_a(\vec{r} - \vec{\rho}_m) \rangle$$

Tight binding, simple cubic

$$E = \varepsilon - t \sum_m e^{i\vec{k} \cdot \vec{\rho}_m}$$

$$E = \varepsilon - t \left(e^{ik_x a} + e^{-ik_x a} + e^{ik_y a} + e^{-ik_y a} + e^{ik_z a} + e^{-ik_z a} \right)$$

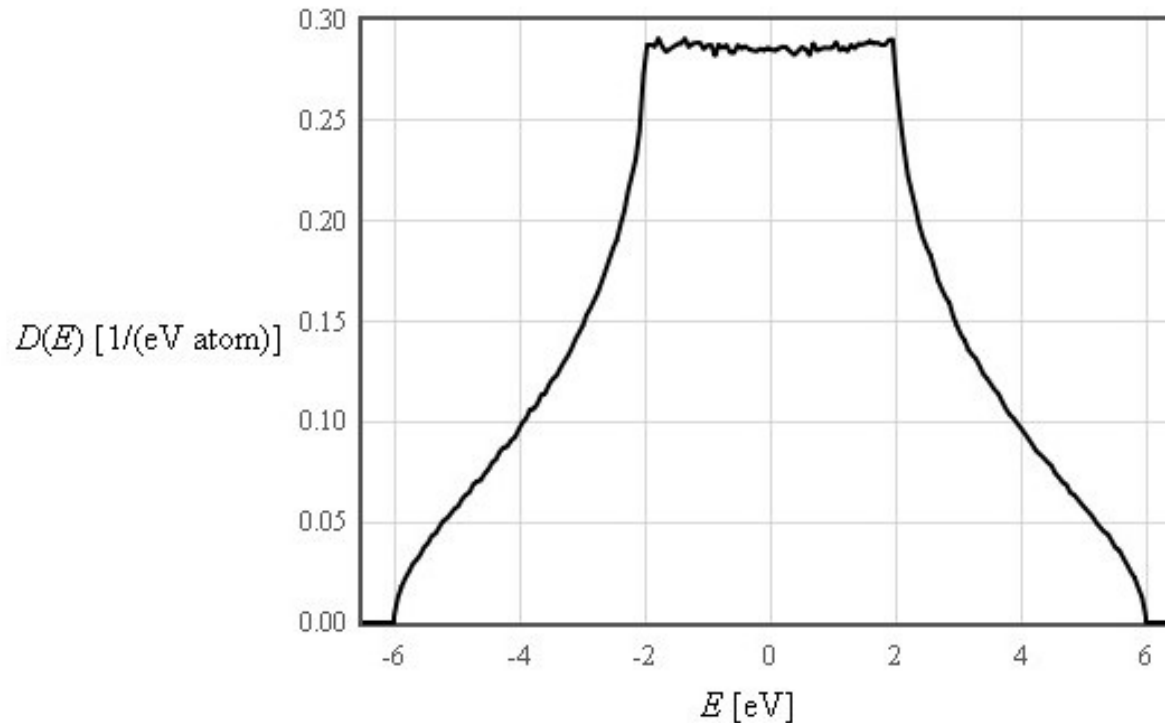
$$= \varepsilon - 2t \left(\cos(k_x a) + \cos(k_y a) + \cos(k_z a) \right)$$



Effective mass $m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}} = \frac{\hbar^2}{2ta^2}$

Narrow bands \rightarrow high effective mass

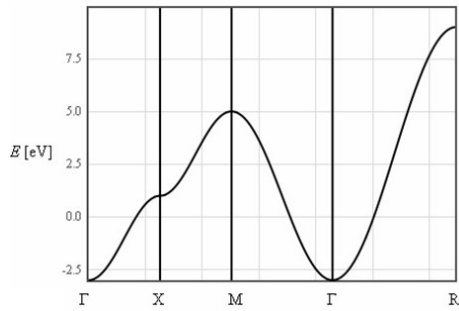
Density of states (simple cubic)



Calculate the energy for every allowed k in the Brillouin zone

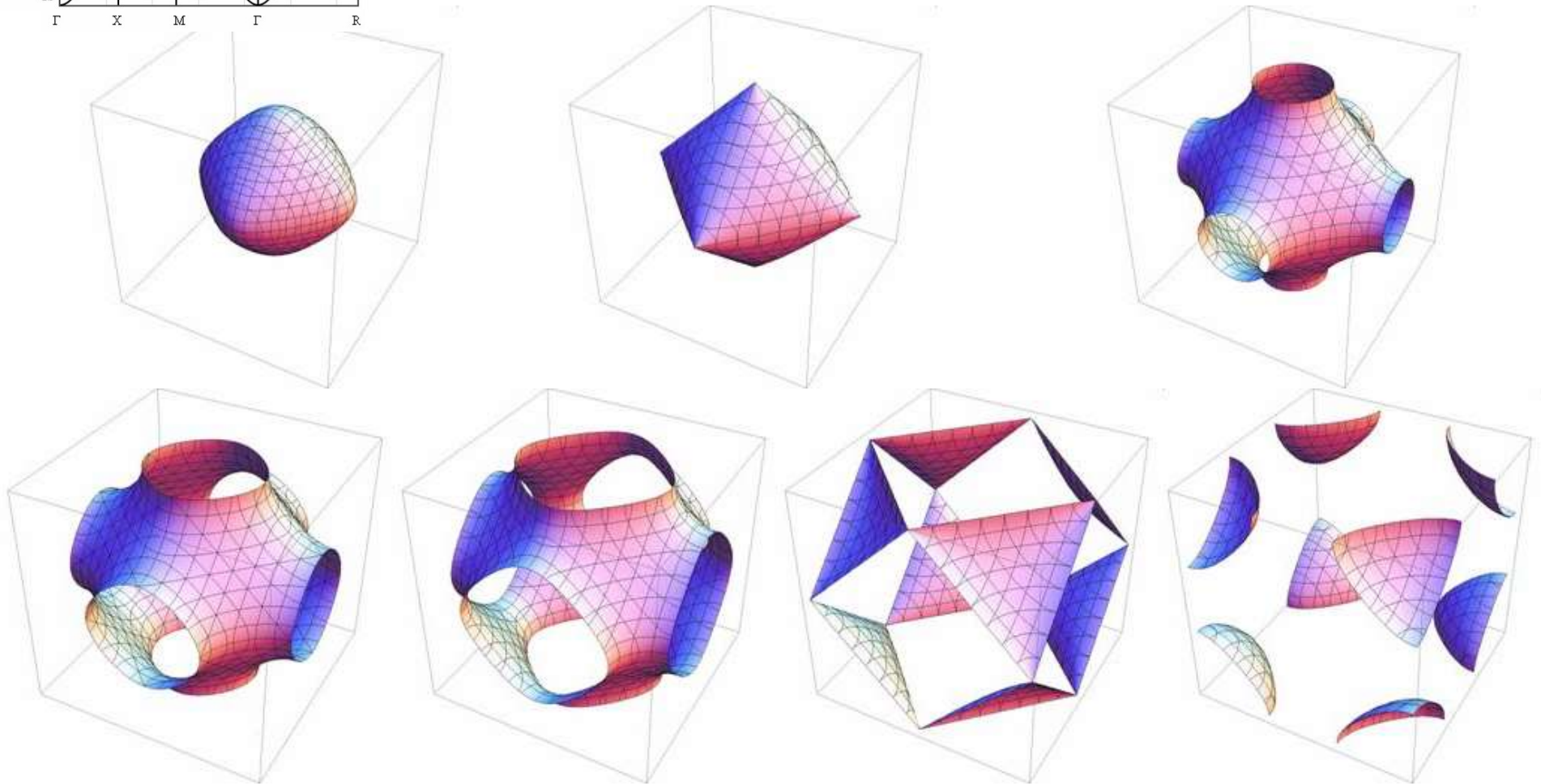
$$E = \varepsilon - 2t \left(\cos(k_x a) + \cos(k_y a) + \cos(k_z a) \right)$$

<http://lamp.tu-graz.ac.at/~hadley/ss1/bands/tbtable/tbtable.html>



Tight binding, simple cubic

$$E = \varepsilon - 2t(\cos(k_x a) + \cos(k_y a) + \cos(k_z a))$$

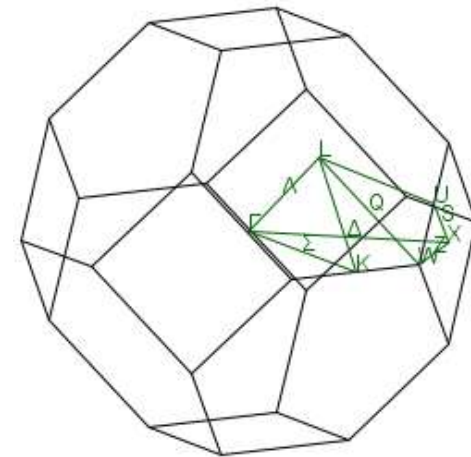
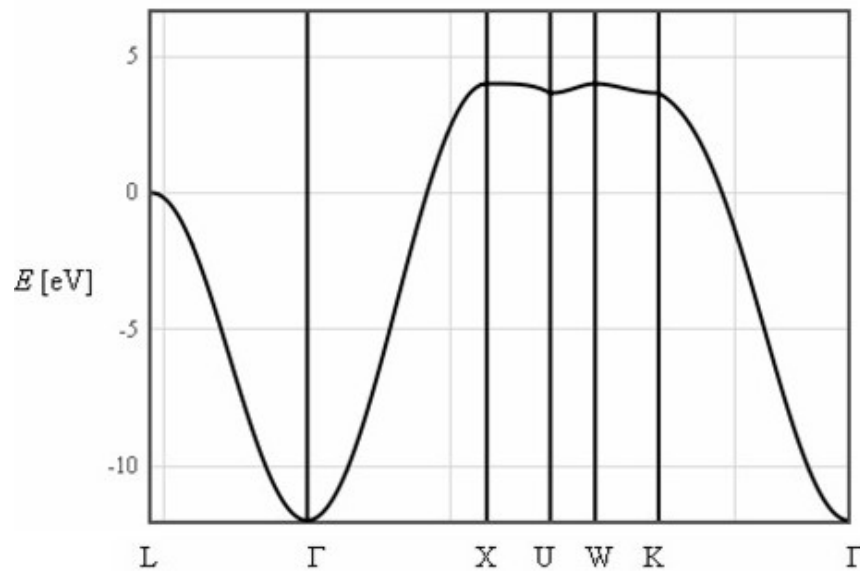


Christian Gruber, 2008

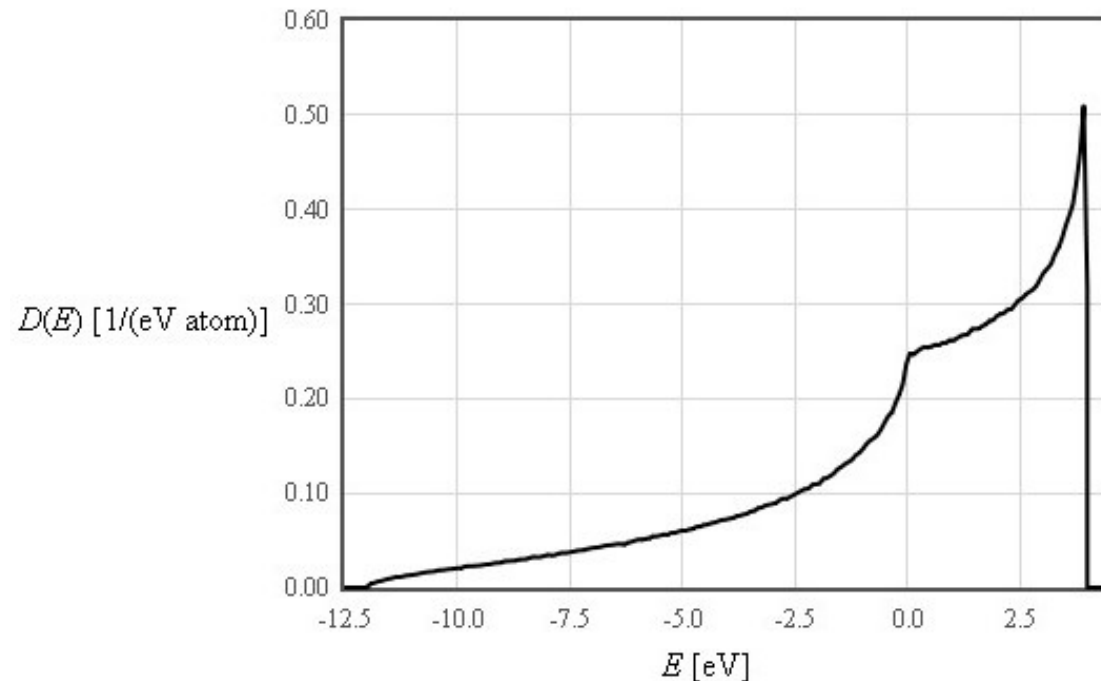
Tight binding, fcc

$$E = \varepsilon - t \sum_m e^{i\vec{k} \cdot \vec{\rho}_m}$$

$$E = \varepsilon - 4t \left(\cos\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_y a}{2}\right) + \cos\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_z a}{2}\right) + \cos\left(\frac{k_y a}{2}\right) \cos\left(\frac{k_z a}{2}\right) \right)$$



Density of states (fcc)

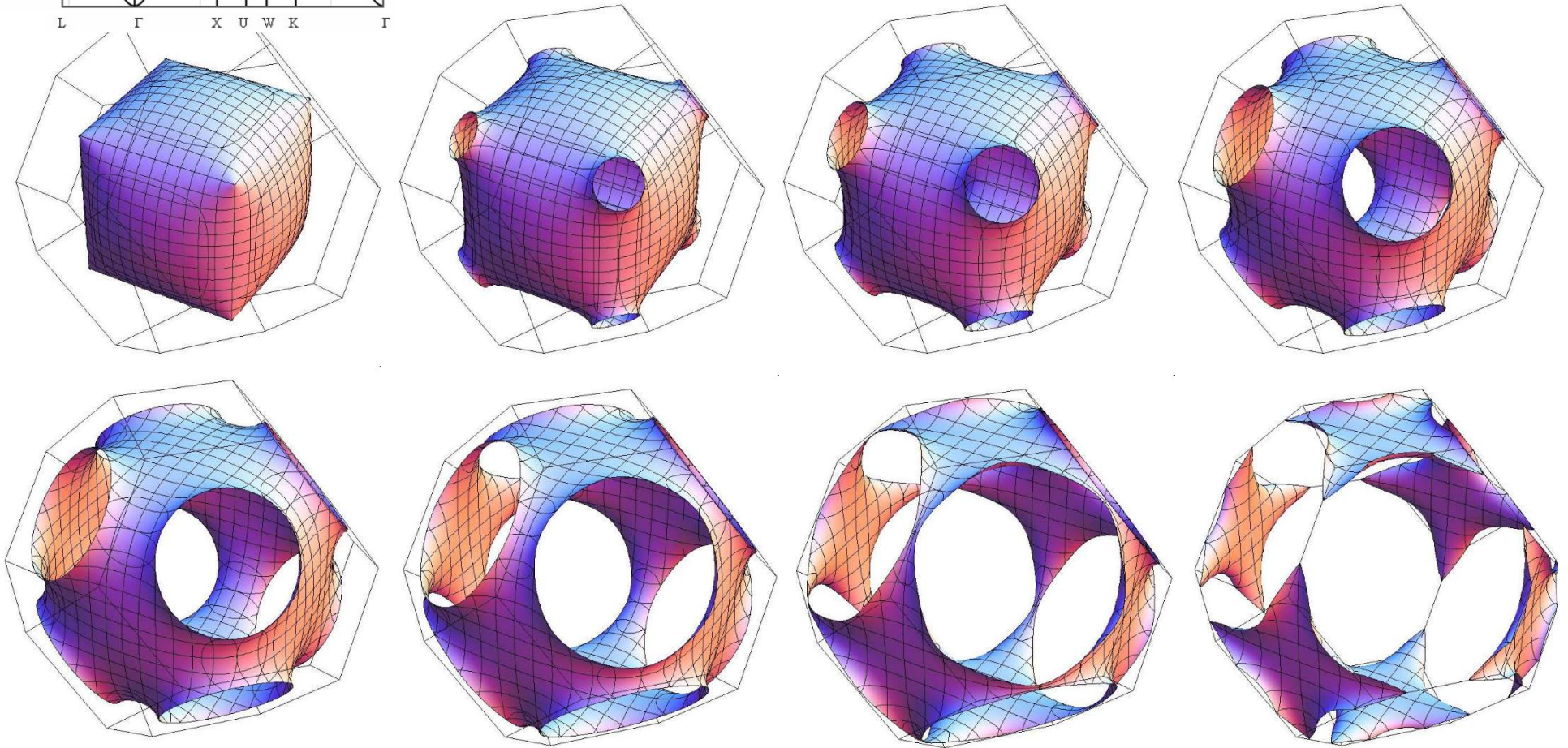
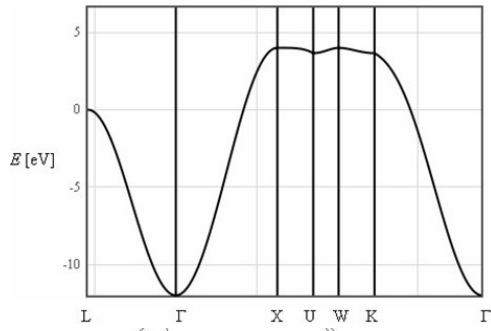


Calculate the energy for every allowed k in the Brillouin zone

$$E = \varepsilon - 4t \left(\cos\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_y a}{2}\right) + \cos\left(\frac{k_x a}{2}\right) \cos\left(\frac{k_z a}{2}\right) + \cos\left(\frac{k_y a}{2}\right) \cos\left(\frac{k_z a}{2}\right) \right)$$

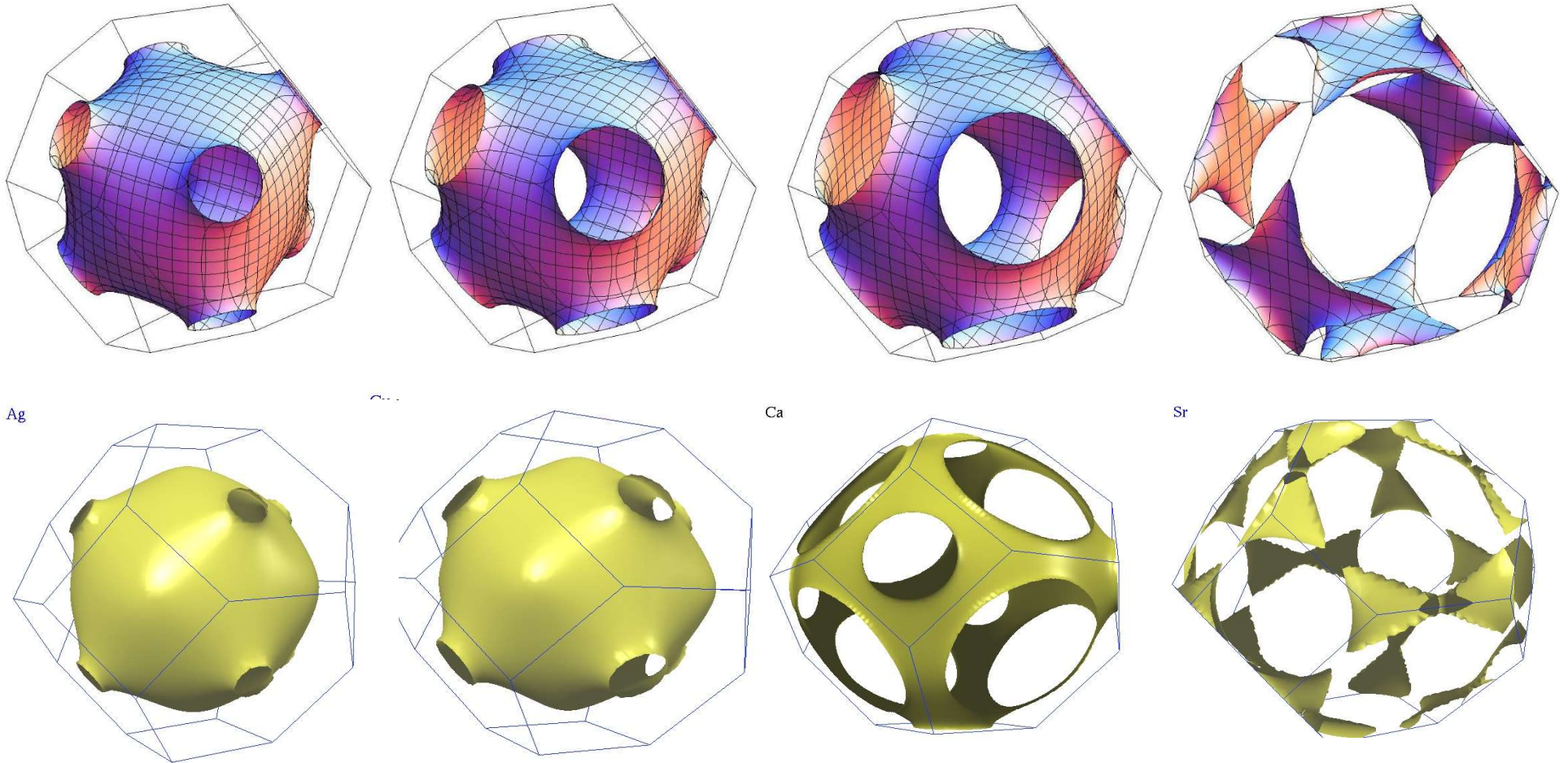
<http://lamp.tu-graz.ac.at/~hadley/ss1/bands/tbtable/tbtable.html>

Tight binding, fcc



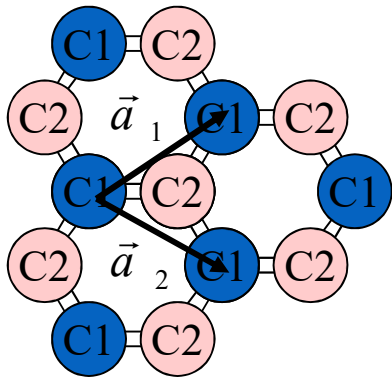
Christian Gruber, 2008

Tight binding, fcc

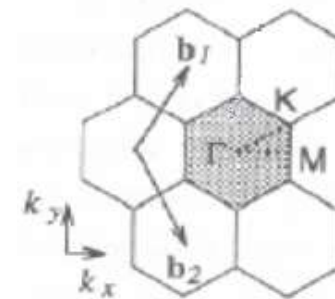


<http://www.phys.ufl.edu/fermisurface/>

Graphene



$$\vec{a}_1 = \frac{\sqrt{3}}{2} a \hat{x} + \frac{1}{2} a \hat{y}$$
$$\vec{a}_2 = \frac{\sqrt{3}}{2} a \hat{x} - \frac{1}{2} a \hat{y}$$



Two atoms per unit cell

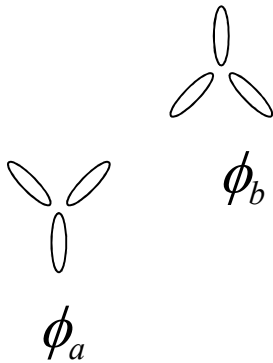
Graphene has an unusual dispersion relation in the vicinity of the Fermi energy.

2 carbon atoms / unit cell

The standard guess for the wave function in the tight binding model is

$$\psi_k = \sum_{j,l} \exp\left(i\left(j\vec{k} \cdot \vec{a}_1 + l\vec{k} \cdot \vec{a}_2\right)\right) \left(c_a \phi_{p_z a}(\vec{r} - j\vec{a}_1 - l\vec{a}_2) + c_b \phi_{p_z b}(\vec{r} - j\vec{a}_1 - l\vec{a}_2)\right)$$

For graphene, the valence orbitals are p_z orbitals



Substitute this wave function into the Schrödinger equation

$$H\psi_k = E\psi_k$$

2 carbon atoms / unit cell

$$\psi_k = \sum_{j,l} \exp(i(j\vec{k} \cdot \vec{a}_1 + l\vec{k} \cdot \vec{a}_2)) (c_a \phi_{p_z a}(\vec{r} - j\vec{a}_1 - l\vec{a}_2) + c_b \phi_{p_z b}(\vec{r} - j\vec{a}_1 - l\vec{a}_2))$$

$$H\psi_k = E\psi_k$$

Multiply by $\phi_{p_z a}^*(\vec{r})$ and integrate

the orbital for the atom at $j = 0, l = 0$.

$$c_a \langle \phi_a | H | \phi_a \rangle + c_b \langle \phi_a | H | \phi_b \rangle \sum_m e^{i\vec{k} \cdot \vec{\rho}_m} + \text{small terms}$$

$$= E \left(c_a \langle \phi_a | \phi_a \rangle + c_b \langle \phi_a | \phi_b \rangle \sum_m e^{i\vec{k} \cdot \vec{\rho}_m} + \text{small terms} \right)$$

1

0

m sums over the nearest neighbors

2 carbon atoms / unit cell

To get a second equation for c_a and c_b

Multiply $H\psi_k = E\psi_k$ by $\phi_{p_z b}^*(\vec{r})$ and integrate
 the orbital for the atom at $j=0, l=0$.

$$c_a \langle \phi_b | H | \phi_a \rangle \sum_m e^{-i\vec{k} \cdot \vec{\rho}_m} + c_b \langle \phi_b | H | \phi_b \rangle + \text{small terms}$$

$$= E \left(c_a \langle \phi_b | \phi_a \rangle \sum_m e^{i\vec{k} \cdot \vec{\rho}_m} + c_b \langle \phi_b | \phi_b \rangle + \text{small terms} \right)$$

↘ 0
↖ 1

Write as a matrix equation

Tight binding graphene

$$\begin{bmatrix} \langle \phi_a | H | \phi_a \rangle - E & \langle \phi_a | H | \phi_b \rangle \sum_m e^{i\vec{k} \cdot \vec{\rho}_m} \\ \langle \phi_b | H | \phi_a \rangle \sum_m e^{-i\vec{k} \cdot \vec{\rho}_m} & \langle \phi_b | H | \phi_b \rangle - E \end{bmatrix} \begin{bmatrix} c_a \\ c_b \end{bmatrix} = 0$$

m sums over the nearest neighbors.

There will be two eigen energies for every k .

N orbitals / unit cell results in N bands

$$\varepsilon = \langle \phi_a | H | \phi_a \rangle$$

$$t = - \langle \phi_a | H | \phi_b \rangle$$