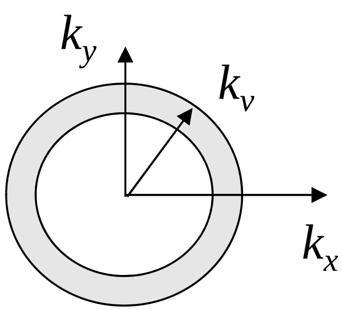
Electrons in a magnetic field

Density of states 2D

$$E_\nu = \hbar\omega_c\left(\nu + \tfrac{1}{2}\right)$$

The number of states between ring ν-1 and ring ν is



$$\frac{\pi\left(k_\nu^2 - k_{\nu-1}^2\right)}{\left(\dfrac{2\pi}{L}\right)^2}$$

$$\frac{\hbar^2 k_\nu^2}{2m} = \hbar\omega_c\left(\nu + \tfrac{1}{2}\right)$$

$$k_\nu^2 - k_{\nu-1}^2 = \frac{2m\omega_c}{\hbar}\left[\left(\nu + \tfrac{1}{2}\right) - \left(\nu - 1 + \tfrac{1}{2}\right)\right] = \frac{2m\omega_c}{\hbar}$$

The number of states between ring ν-1 and ring ν is $\dfrac{m\omega_c}{2\pi\hbar}L^2$

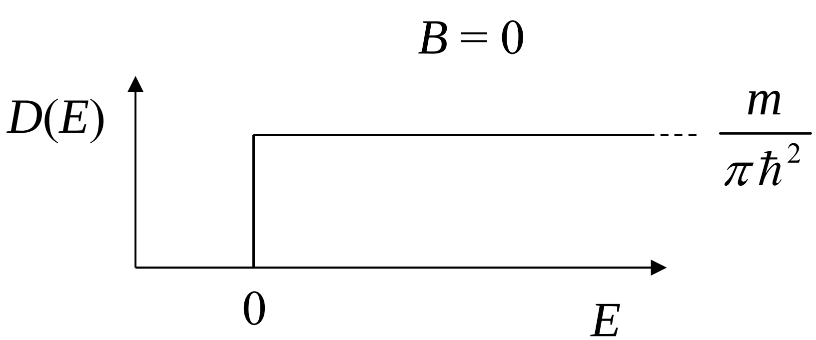
The density of states per spin is $\dfrac{m\omega_c}{2\pi\hbar}$

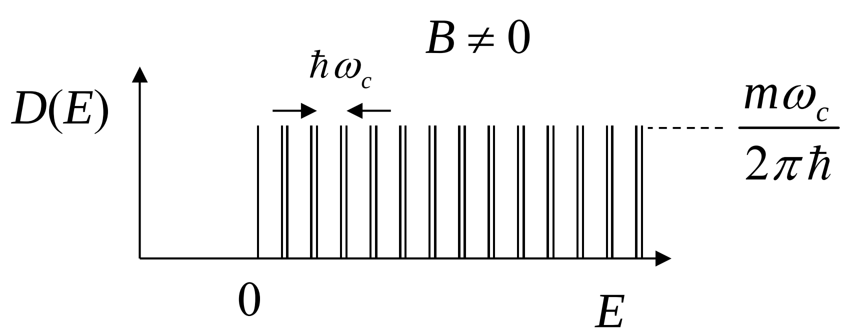
Spin

In a magnetic field, there is a shift of the energy of the electrons because of their spin.

$$E = -\vec{\mu} \cdot \vec{B} = \pm \frac{g}{2} \mu_B B$$

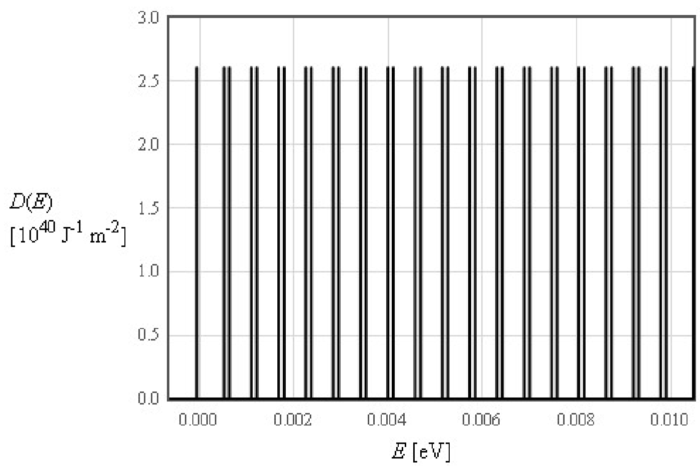
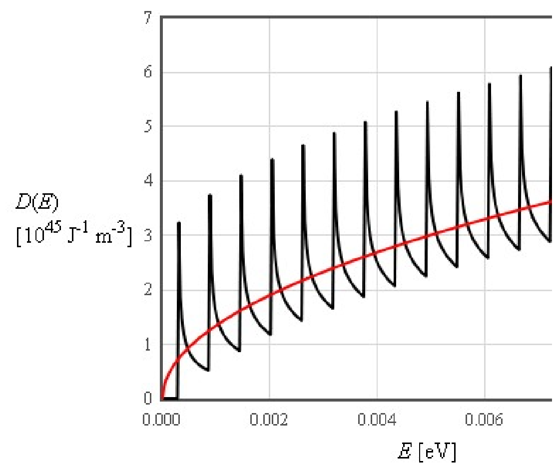
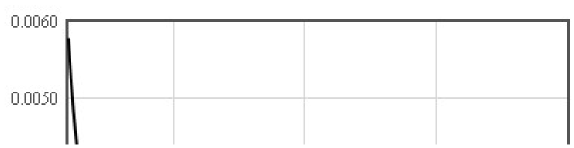
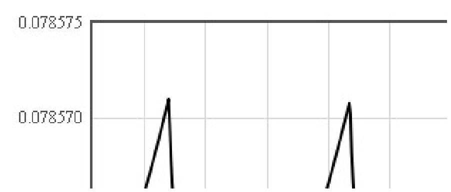
Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$ g-factor $g \approx 2$ $\hbar\omega_c = \frac{\hbar eB}{m} = 2\mu_B B$





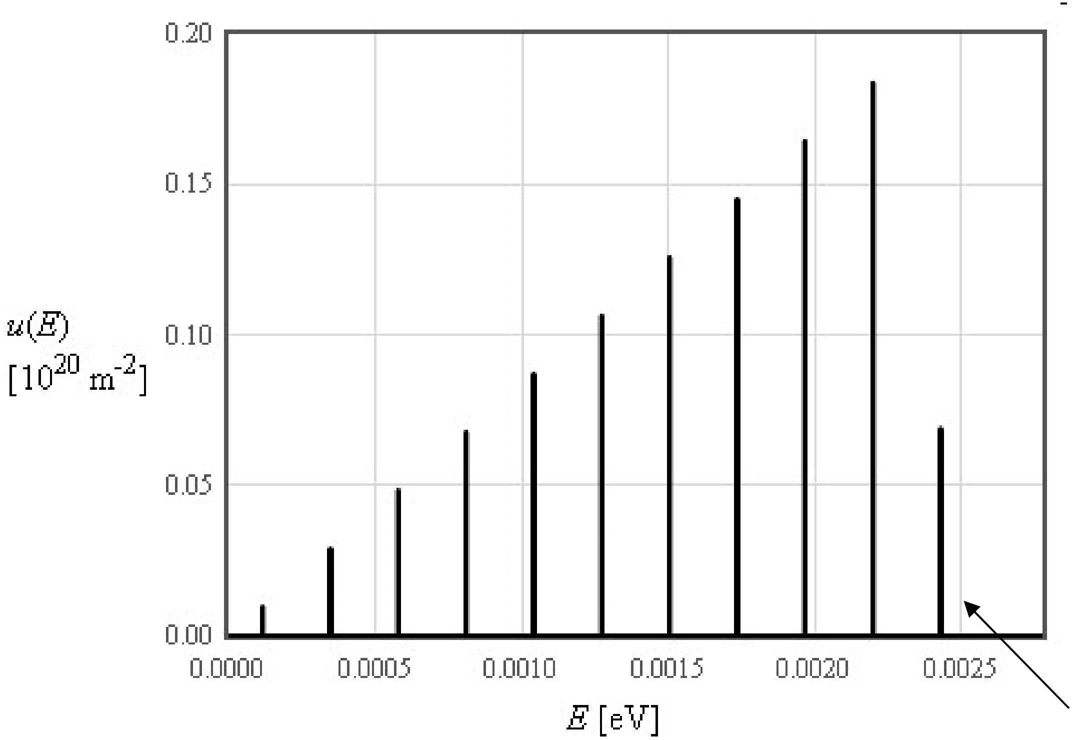
$$D(E) = \frac{m\omega_c}{2\pi\hbar} \sum_{\nu=0}^{\infty} \delta\left(E - \hbar\omega_c\left(\nu + \tfrac{1}{2} - \tfrac{g}{4}\right)\right) + \delta\left(E - \hbar\omega_c\left(\nu + \tfrac{1}{2} + \tfrac{g}{4}\right)\right)$$

ization of the Schrödinger equation for free electrons a magnetic field in 2 and 3 dimensions.

	2-D Schrödinger equation $i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$	3-D Schrödinger equation $i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$
Eigenfunction solutions	$\psi=g_v(x)\exp\left(ik_y y\right)$ $g_v(x)$ is a harmonic oscillator wavefunction	$\psi=g_v(x)\exp\left(ik_y y\right)\exp\left(ik_z z\right)$ $g_v(x)$ is a harmonic oscillator wavefun
Energy eigenvalues	$E=\hbar\omega_c\left(v+\frac{1}{2}\right)$ J $v=0,1,2,\cdots \qquad \omega_c=\frac{\lvert eB_z\rvert}{m}$	$E=\frac{\hbar^2k_z^2}{2m}+\hbar\omega_c\left(v+\frac{1}{2}\right)$ J $v=0,1,2,\cdots \qquad \omega_c=\frac{\lvert eB_z\rvert}{m}$
Density of states	$D(E)=\frac{m\omega_c}{2\pi\hbar}\sum_{v=0}^{\infty}\delta\left(E-\hbar\omega_c\left(v+\frac{1}{2}\right)-\frac{g\mu_B}{2}B\right)+\delta\left(E-\hbar\omega_c\left(v+\frac{1}{2}\right)+\frac{g\mu_B}{2}B\right)$ $\mathrm{J^{-1}m^{-2}}$  Calculate DoS	$D(E)=\frac{(2m)^{3/2}\omega_c}{4\pi^2\hbar^2}\sum_{v=0}^{\infty}\frac{H\left(E-\hbar\omega_c\left(v+\frac{1}{2}\right)\right)}{\sqrt{E-\hbar\omega_c\left(v+\frac{1}{2}\right)}}$  Calculate DoS
	$E_F=\hbar\omega_c\left(\mathrm{Int}\left(\frac{\pi\hbar n}{m\omega_c}\right)+\frac{1}{2}\right)$  	

Energy spectral density 2d

At $T = 0$

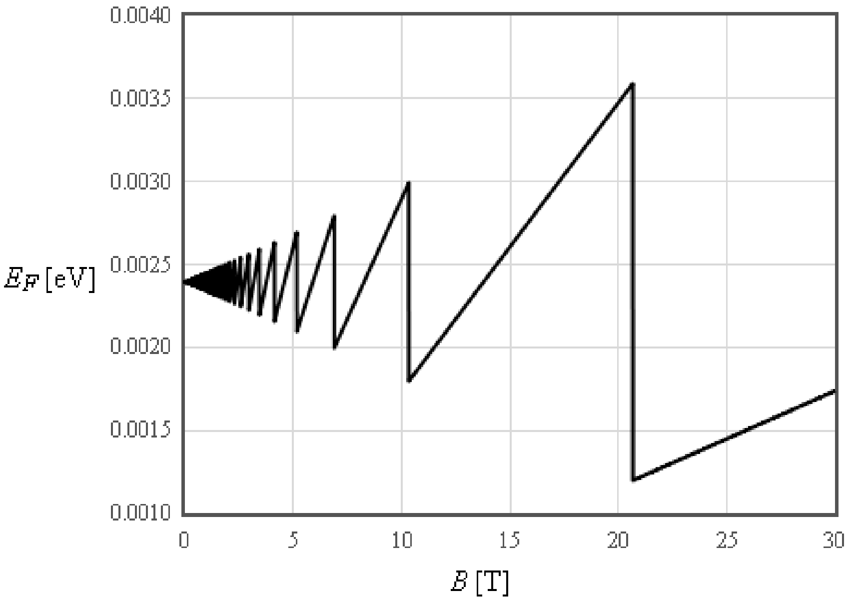


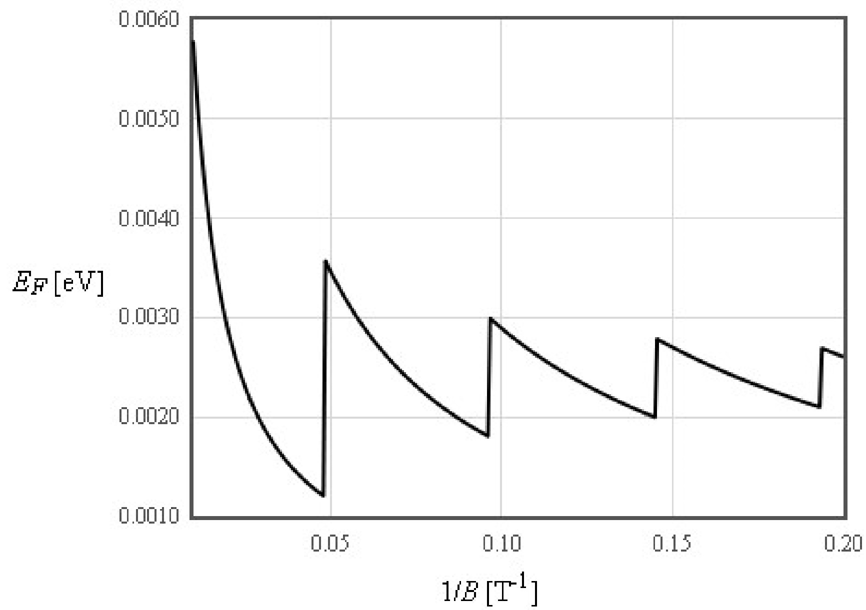
unfilled Landau level

analog to the Planck radiation law

Fermi energy 2d

$$n = \int_{-\infty}^{E_F} D(E)dE$$





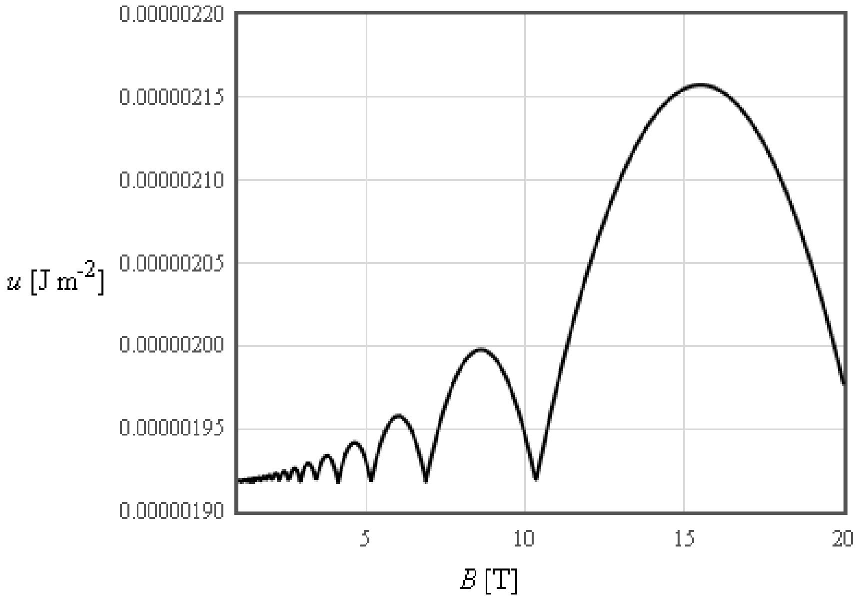
When there is only one Landau level, the Fermi energy rises linearly with field.

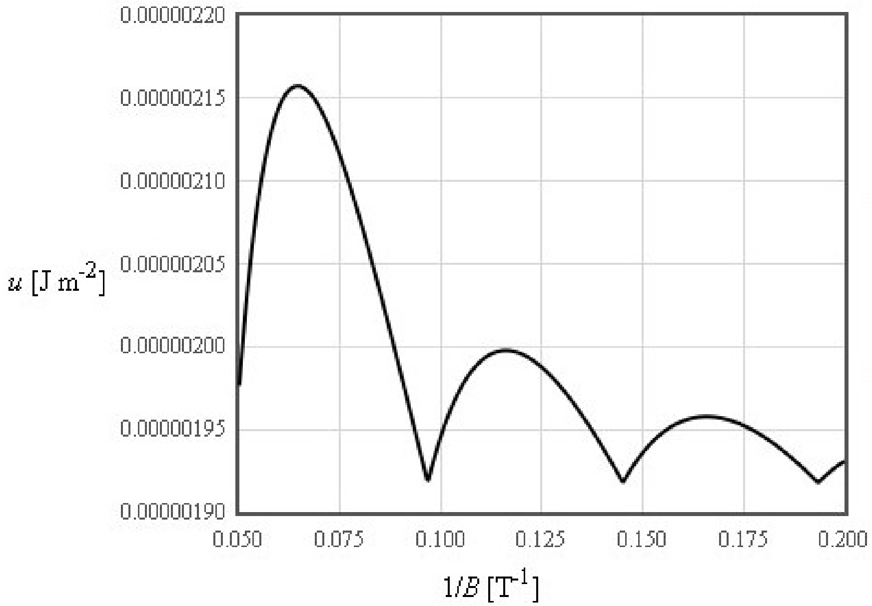
Periodic in $1/B$

Large field limit $\longrightarrow$ $E_F = \dfrac{\hbar\omega_c}{2} = \dfrac{\hbar e B}{2m}$

Internal energy 2d

At $T = 0$ $\quad u = \int_{-\infty}^{E_F} E D(E) dE$



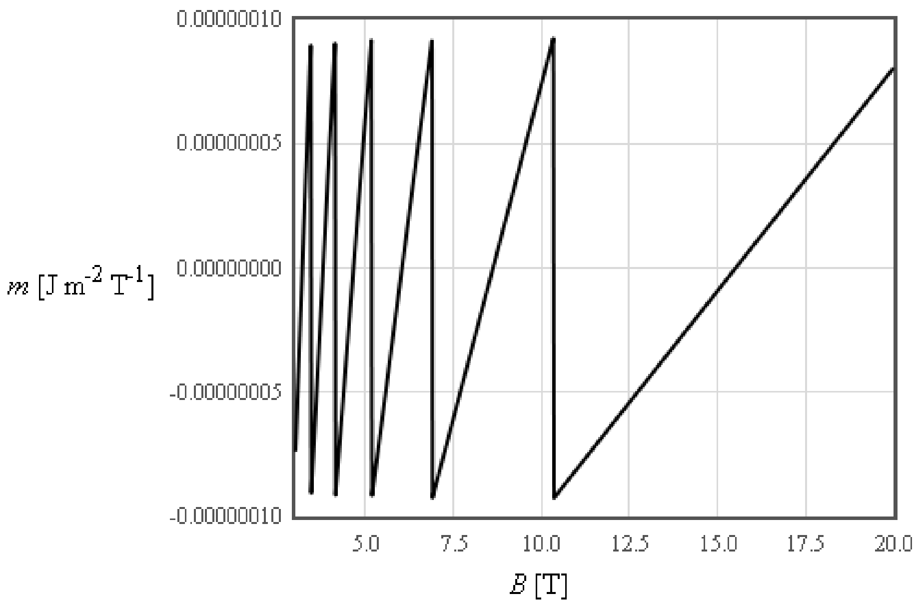


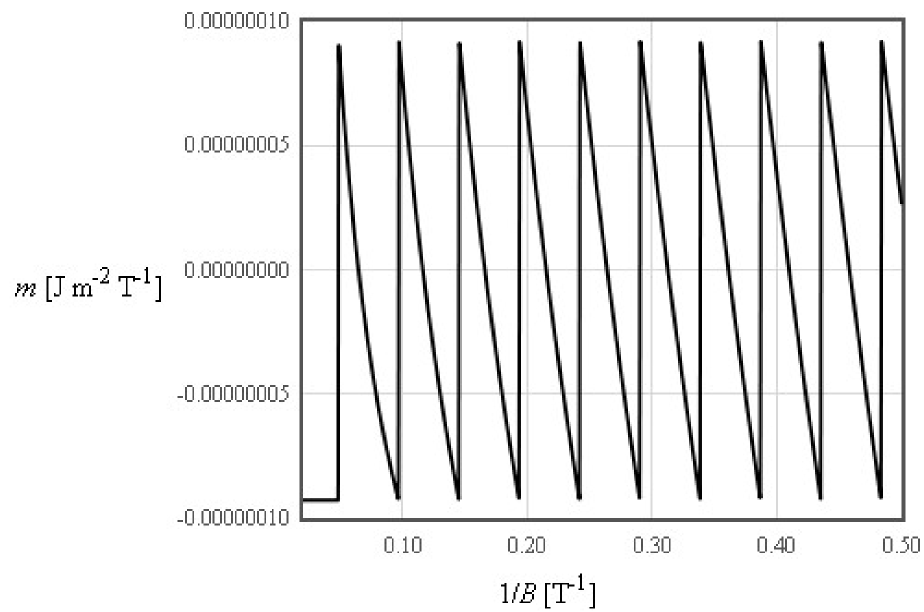
$$u = n\frac{\hbar\omega_c}{2} = n\frac{\hbar eB}{2m}$$

Large field limit

Magnetization 2d

At $T = 0$





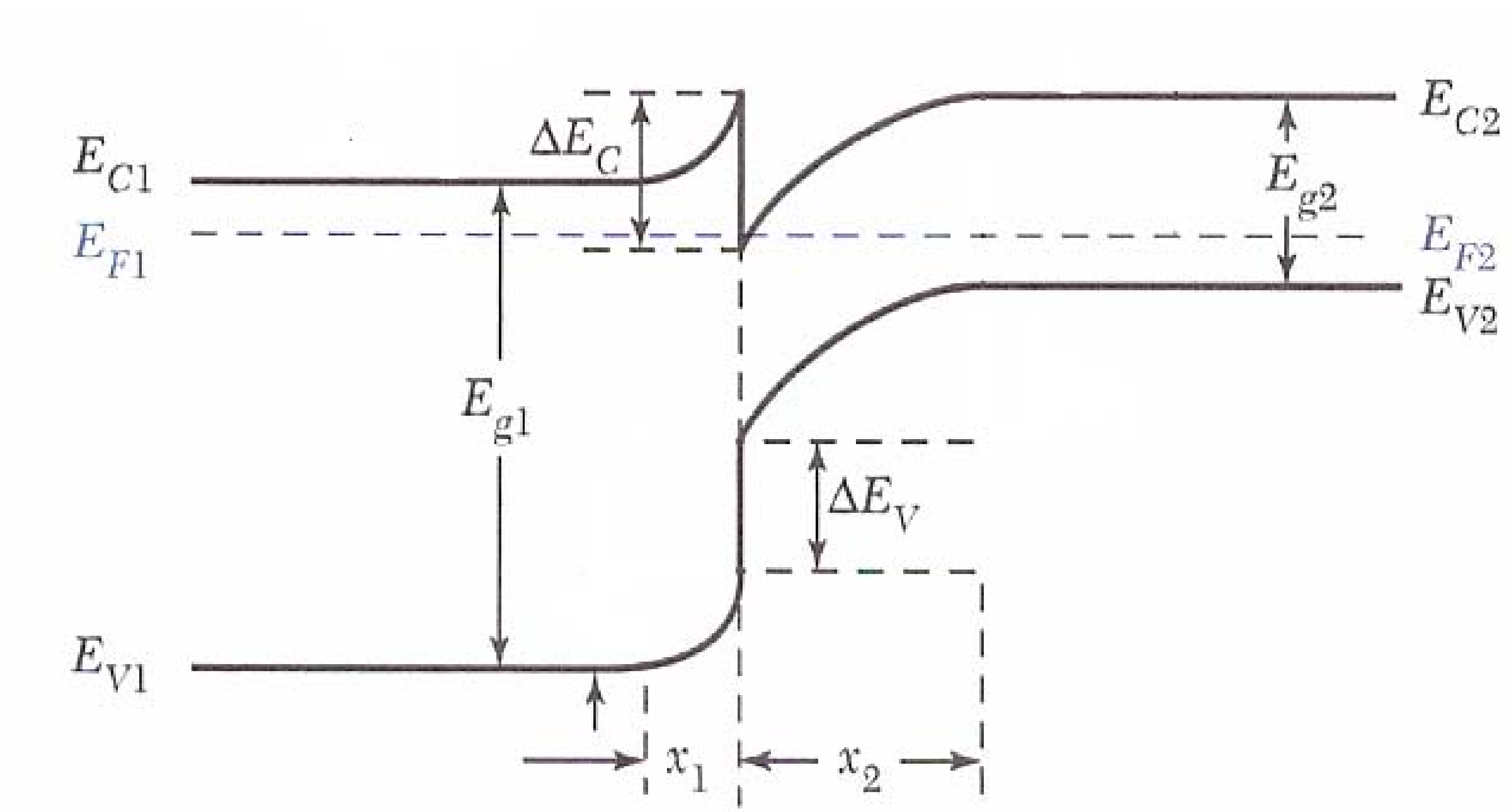
$$m = -\frac{du}{dB} = -n\frac{\hbar e}{2m}$$

Large field limit

de Haas - van Alphen oscillations

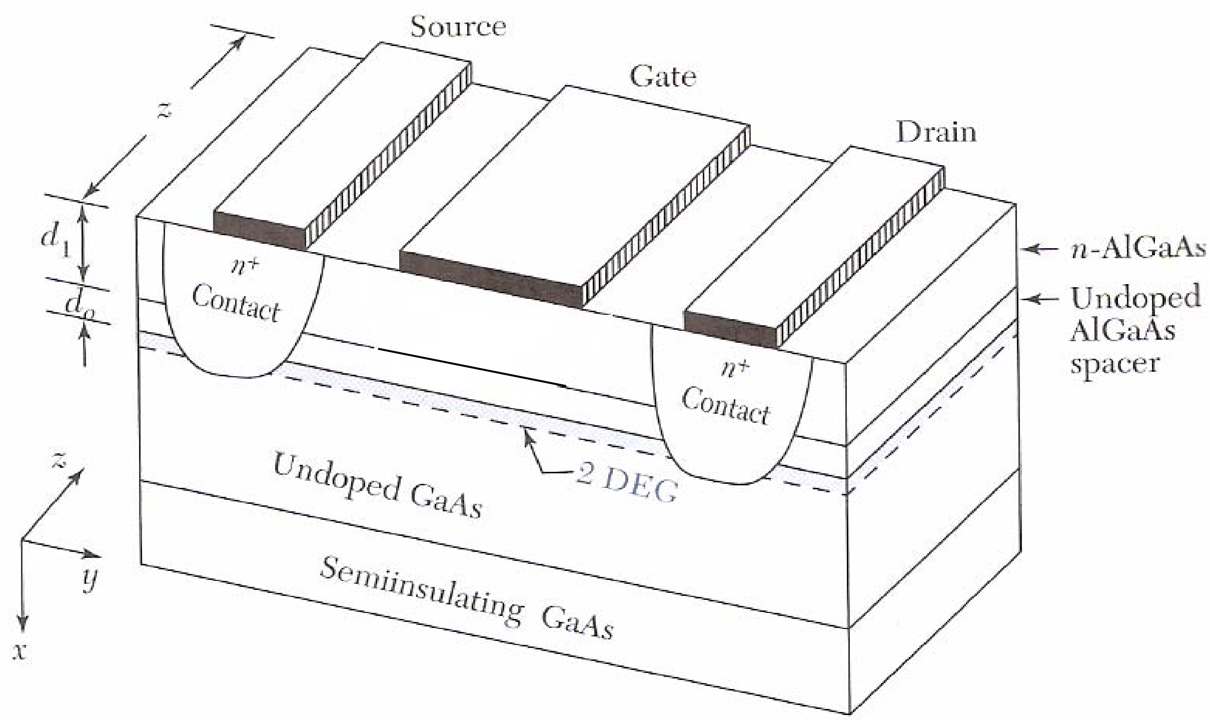
Heterostructure

pn junction formed from two semiconductors with different band gaps



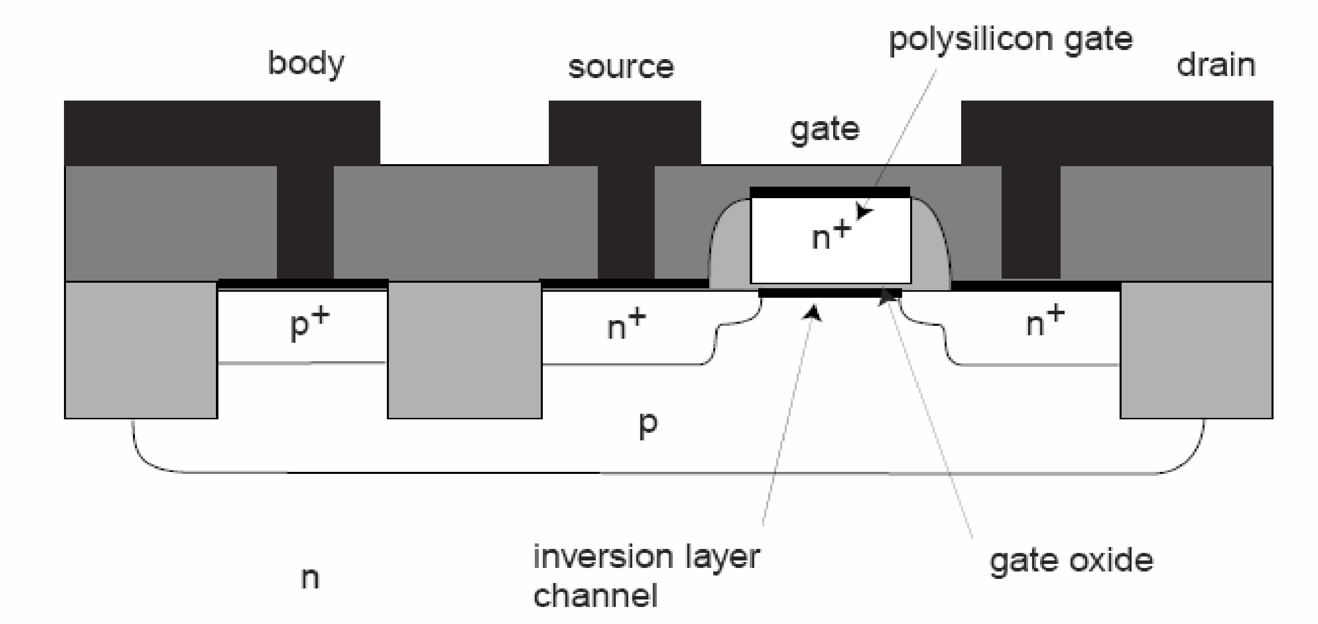
MODFET (HEMT)

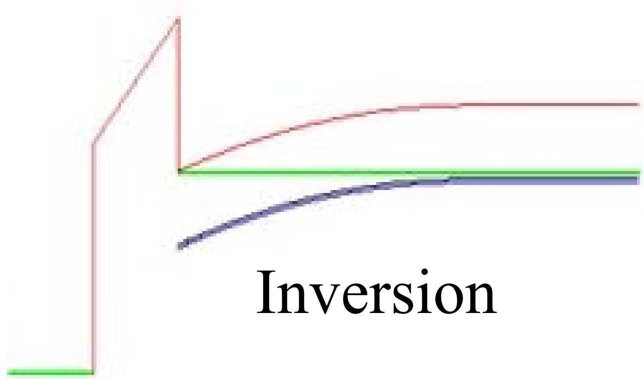
Modulation doped field effect transistor (MODFET)
High electron mobility transistor (HEMT)



The magnetic field can be at an angle to the 2DEG. The Landau splitting experiences the component perpendicular to the plane. The Zeeman splitting experiences the full field.

MOSFETs





Inversion

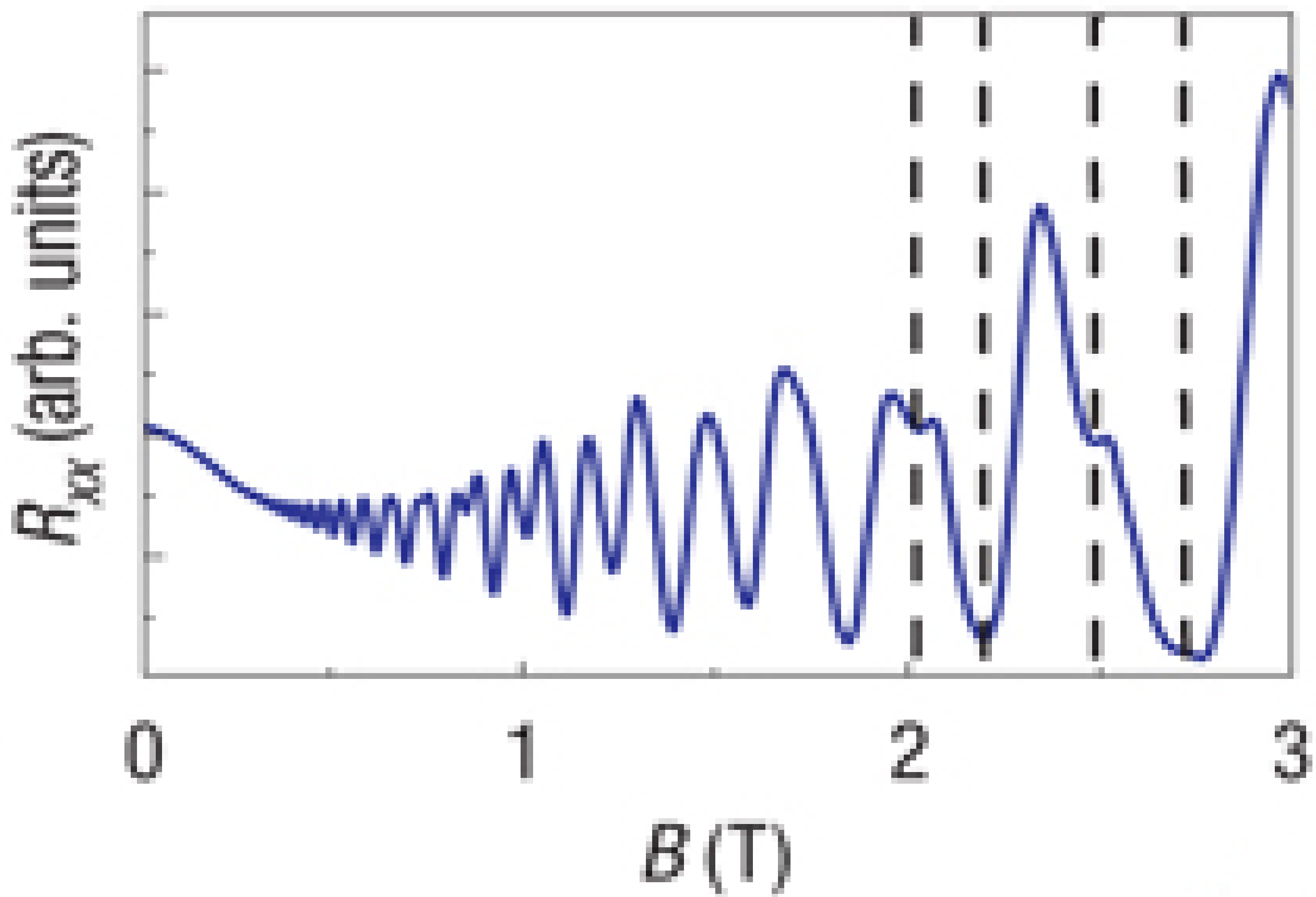
Scattering at the Fermi surface

At room temperature, phonon energies are much less than the Fermi energy. The energy of electrons hardly changes as they scatter from phonons. Electrons scatter from a point close to the Fermi surface to another point close to the Fermi surface.

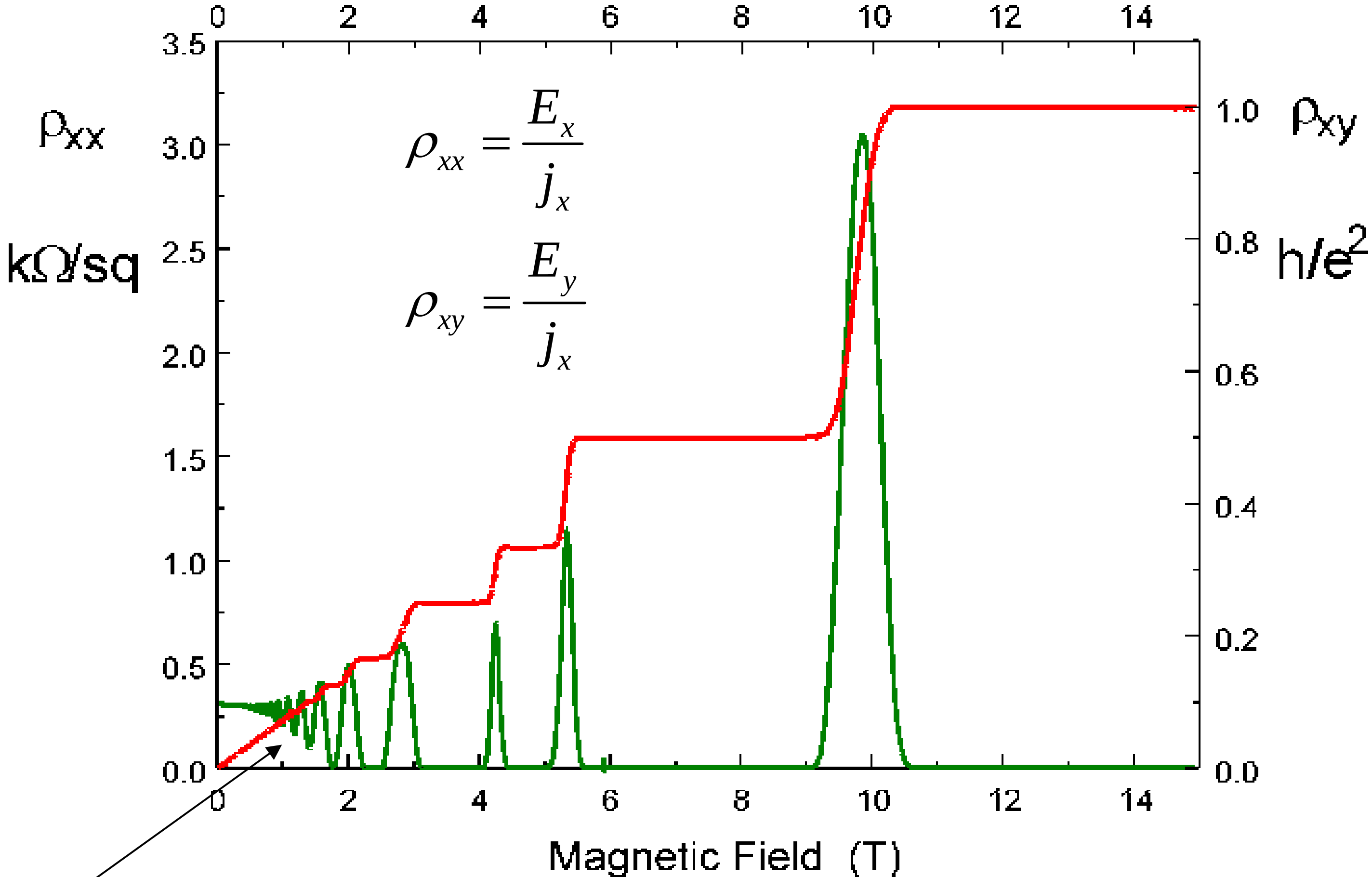
Changing the magnetic field changes the number of states at the Fermi energy.

There are oscillations in the electrical conductivity as a function of magnetic field.

Shubnikov-De Haas oscillations



Quantum Hall Effect

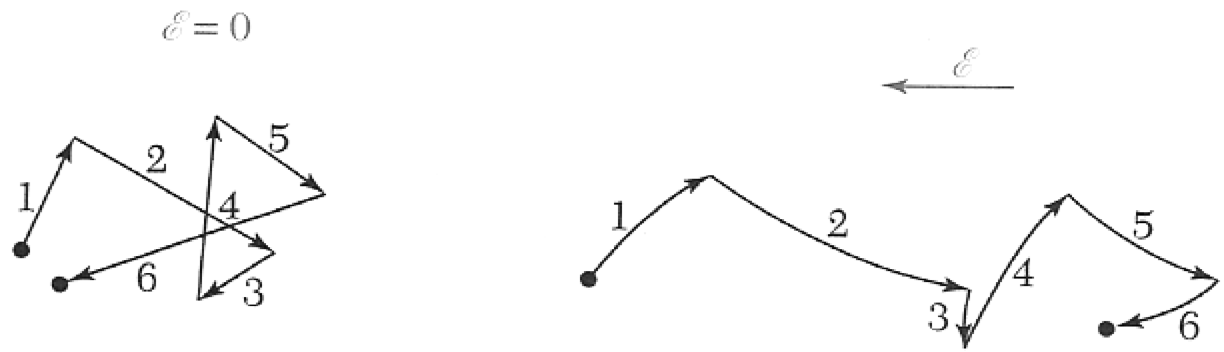


Klaus von Klitzing

Shubnikov-De Haas oscillations

Resistance standard
25812.807557(18) Ω

Review of the Hall effect: Diffusive transport



$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{\text{sc}}}$$

$$-\frac{e\tau_{\text{sc}}}{m}\vec{E} = \vec{v}_d$$

mobility: $$-\mu_e\vec{E} = \vec{v}_d$$

$$\vec{j} = -n|e|\vec{v}_d = n|e|\mu_e\vec{E} = \frac{ne^2\tau_{sc}}{m}\vec{E} = \sigma\vec{E}$$

Review of the Hall effect

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{\mathrm{sc}}} \longleftarrow \quad \text{diffusive regime}$$

$$\vec{F} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right) = m\frac{\vec{v}_d}{\tau_{\mathrm{sc}}}$$

If B is in the z-direction, the three components of the force are

$$-e\left(E_x + v_{dy}B_z\right) = m\frac{v_{dx}}{\tau_{\mathrm{sc}}}$$

$$-e\left(E_y - v_{dx}B_z\right) = m\frac{v_{dy}}{\tau_{\mathrm{sc}}}$$

$$-e\left(E_z\right) = m\frac{v_{dz}}{\tau_{\mathrm{sc}}}$$

Magnetic field (diffusive regime)

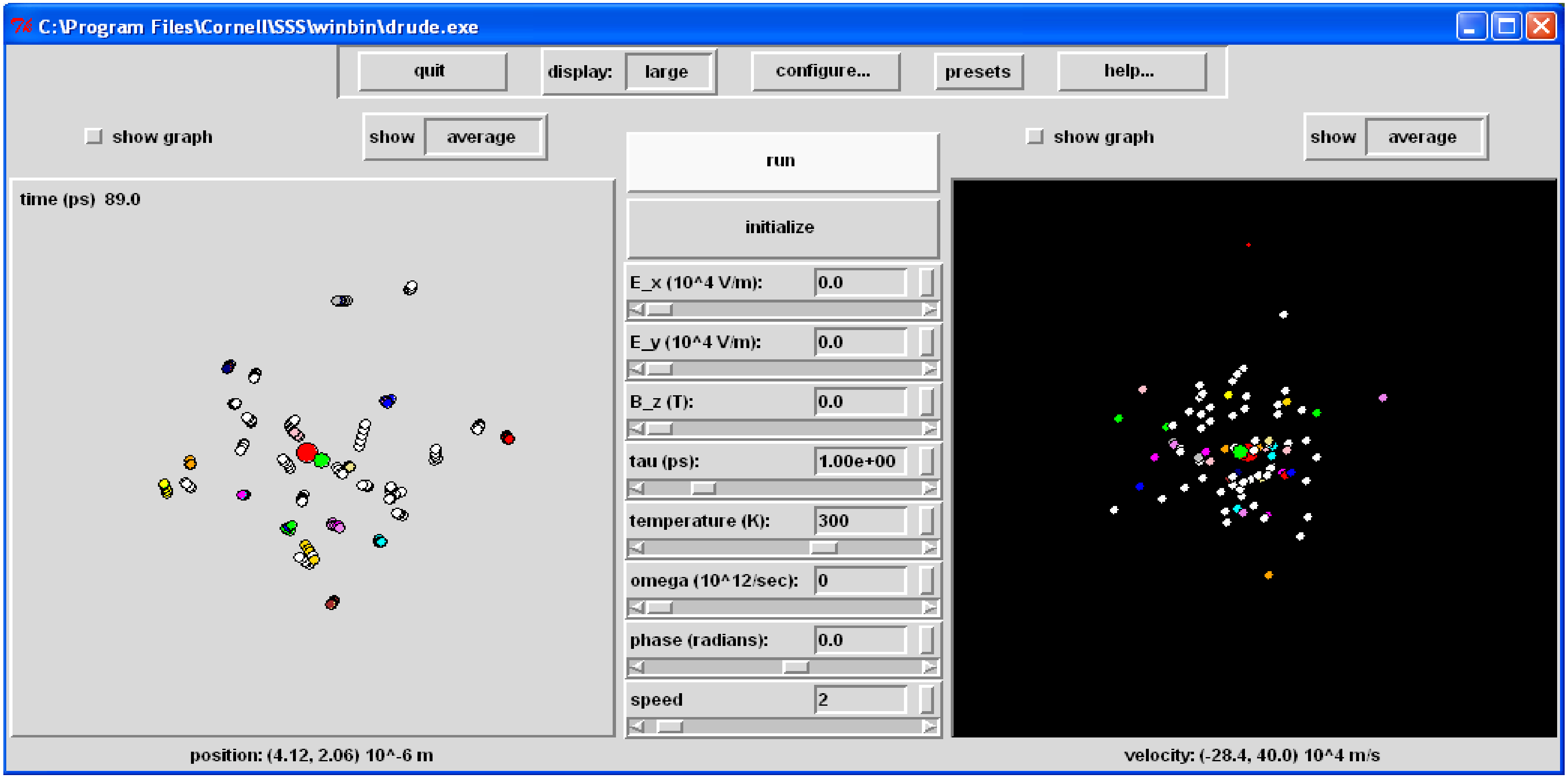
For a magnetic field in the z-direction

$$\begin{cases} v_{d,x} = -\dfrac{eE_x\tau_{sc}}{m} - \dfrac{eB_z}{m}\tau_{sc}v_{d,y} \\ v_{d,y} = -\dfrac{eE_y\tau_{sc}}{m} + \dfrac{eB_z}{m}\tau_{sc}v_{d,x} \\ v_{d,z} = -\dfrac{eE_z\tau_{sc}}{m} \end{cases}$$

If $E_y = 0$,

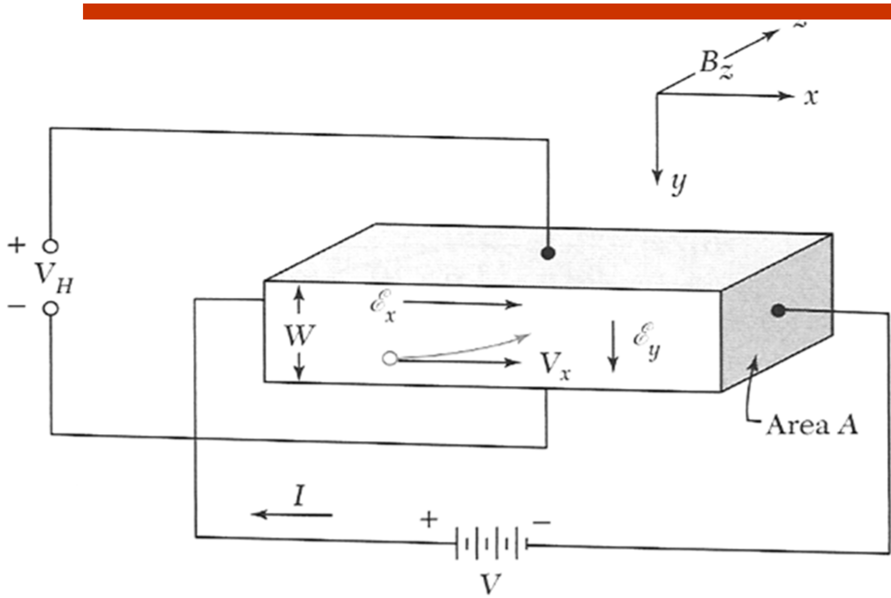
$$v_{d,y} = -\frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$\tan\theta_H = -\frac{eB_z}{m}\tau_{sc}$$



If no forces are applied, the electrons diffuse.
The average velocity moves against an electric field.
In just a magnetic field, the average velocity is zero.
In an electric and magnetic field, the electrons move in a straight line at the Hall angle.
The drift velocity decreases as the B field increases.

The Hall Effect (diffusive regime)



$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

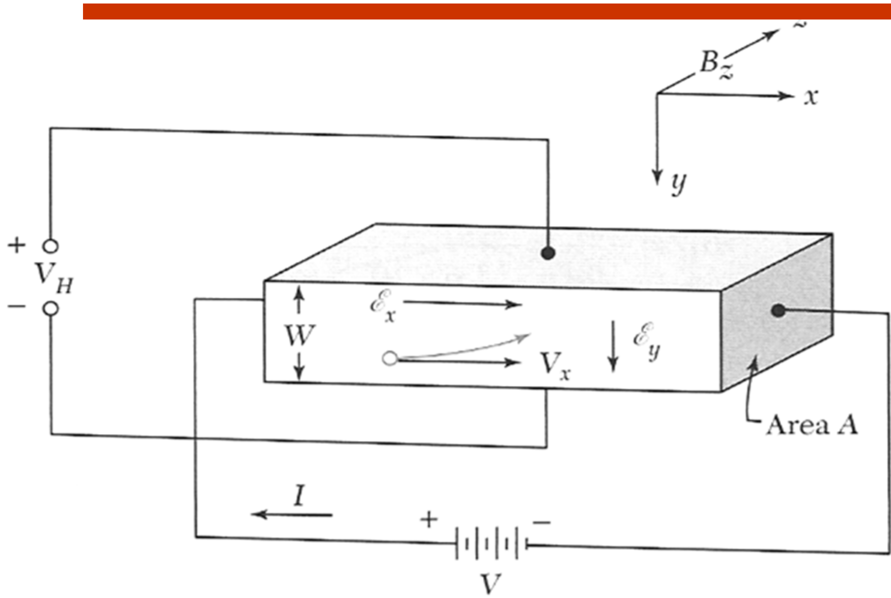
If $v_{d,y} = 0$,

$E_y = v_{d,x}B_z = V_H/W = R_H j_x B_z$ $\quad$ V_H = Hall voltage, R_H = Hall Constant

$v_{d,x} = - j_x/ne$

$$\boxed{R_H = E_y/j_xB_z = -1/ne}$$

The Hall Effect (diffusive regime)



$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{xy} = \frac{E_y}{j_x}$$

$R_H = E_y / j_x B_z$ = - $1/ne$

multiply both sides by B_z

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$

The Hall resistivity is proportional to the magnetic field.