Ferromagnetism

Ferromagnetism

Below a critical temperature (called the Curie temperature) a magnetization spontaneously appears in a ferromagnet even in the absence of a magnetic field.

Iron, nickel, and cobalt are ferromagnetic.

Ferromagnetism overcomes the magnetic dipole-dipole interactions. Is arises from the Coulomb interactions of the electrons. The energy that is gained when the spins align is called the exchange energy.

Mean field theory (Molekularfeldtheorie)

Heisenberg Hamiltonian $H = -\sum_{i,j} J_{i,j}\vec{S}_i \cdot \vec{S}_j - g\mu_B B \sum_i \vec{S}_i$

Exchange energy

Mean field approximation

$$H_{MF} = \sum_i \vec{S}_i \cdot \left(\sum_\delta J_{i,\delta} \left\langle \vec{S} \right\rangle + g\mu_B \vec{B} \right)$$

δ sums over the neighbors of spin *i*

Looks like a magnetic field B_{MF}

$$\vec{B}_{MF} = \frac{1}{g\mu_B} \sum_\delta J_{i,\delta} \left\langle \vec{S} \right\rangle$$

magnetization → $\vec{M} = g\mu_B \frac{N}{V} \left\langle \vec{S} \right\rangle$

eliminate <*S*>

Mean field theory

$$\vec{B}_{MF} = \frac{V}{Ng^2\mu_B^2} zJ\vec{M}$$

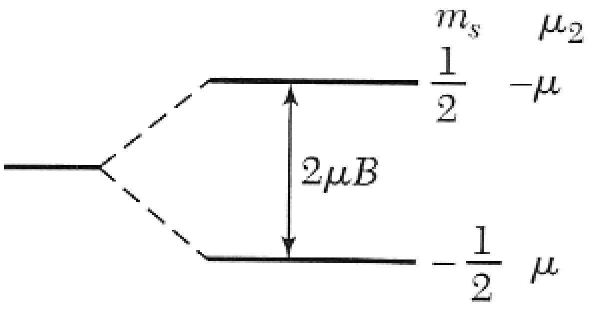
z is the number of nearest neighbors

In mean field, the energy of the spins is

$$E = \pm\frac{1}{2} g\mu_B (B_{MF} + B_a)$$

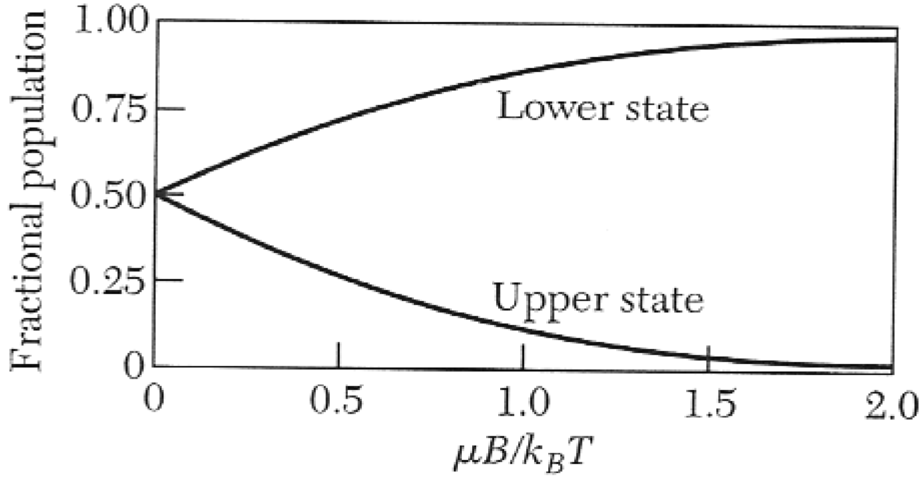
We calculated the populations of the spins in the paramagnetism section

Spin populations



$$\frac{N_1}{N} = \frac{\exp(\mu B/k_B T)}{\exp(\mu B/k_B T) + \exp(-\mu B/k_B T)}$$

$$\frac{N_2}{N} = \frac{\exp(-\mu B/k_B T)}{\exp(\mu B/k_B T) + \exp(-\mu B/k_B T)}$$



$$
\begin{aligned}
M &= (N_1 - N_2)\mu \\
&= N\mu \frac{\exp(\mu B/k_B T) - \exp(-\mu B/k_B T)}{\exp(\mu B/k_B T) + \exp(-\mu B/k_B T)} \\
&= N\mu \tanh\left(\frac{\mu B}{k_B T}\right)
\end{aligned}
$$

Mean field theory

$$M = \frac{1}{2} g \mu_B \frac{N}{V} \tanh\left(\frac{g \mu_B \left(B_{MF} + B_a\right)}{2 k_B T} \right)$$

For zero applied field

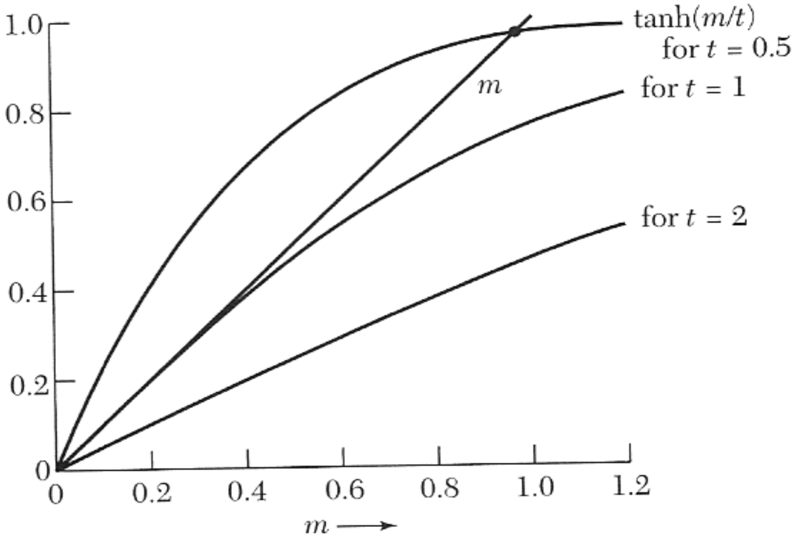
$$M = M_S \tanh\left(\frac{T_c}{T} \frac{M}{M_s} \right)$$

$$M_S = \frac{N}{2V} g \mu_B \quad \text{and} \quad T_c = \frac{z}{4 k_B} J$$

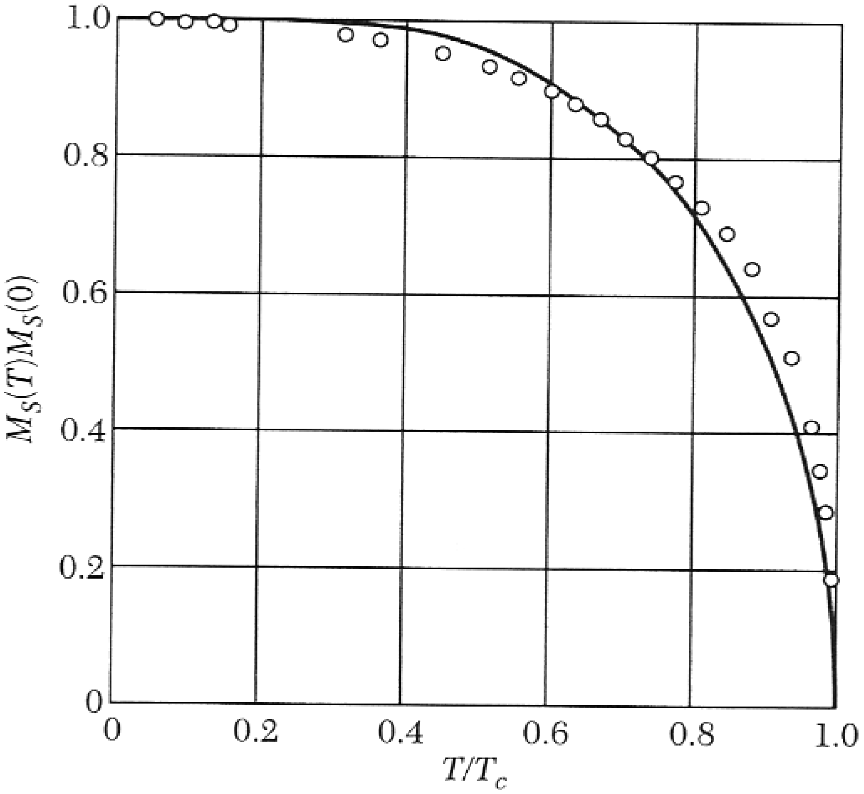
M_s = saturation magnetization T_c = Curie temperature

Mean field theory

$$M = M_S \tanh\left(\frac{T_c}{T}\frac{M}{M_s}\right)$$



$$m = \tanh\left(\frac{m}{t}\right)$$



Experimental points for Ni.

Ferromagnetism

Material	**Curie temp. (K)**	
Co	1388	
Fe	1043	
$FeOFe_2O_3$	858	
$NiOFe_2O_3$	858	
$CuOFe_2O_3$	728	
$MgOFe_2O_3$	713	
MnBi	630	
Ni	627	
MnSb	587	
$MnOFe_2O_3$	573	
$Y_3Fe_5O_{12}$	560	
CrO_2	386	
MnAs	318	
Gd	292	
Dy	88	
EuO	69	Electrical insulator
$Nd_2Fe_{14}B$	353	$M_s = 10\ M_s(\mathrm{Fe})$
Sm_2Co_{17}	700	rare earth magnets

Curie - Weiss law

$$M = \frac{1}{2} g \mu_B \frac{N}{V} \tanh\left(\frac{g \mu_B \left(B_{MF} + B_a \right)}{2 k_B T} \right)$$

$$\vec{B}_{MF} = \frac{V}{N g^2 \mu_B^2} z J \vec{M}$$

Above T_c we can expand the hyperbolic tangent $\tanh(x) \approx x$ for $x << 1$

$$M \approx \frac{1}{4} g^2 \mu_B^2 \frac{N}{V k_B T} \left(\frac{V}{N g^2 \mu_B^2} z J M + B_a \right)$$

Solve for M

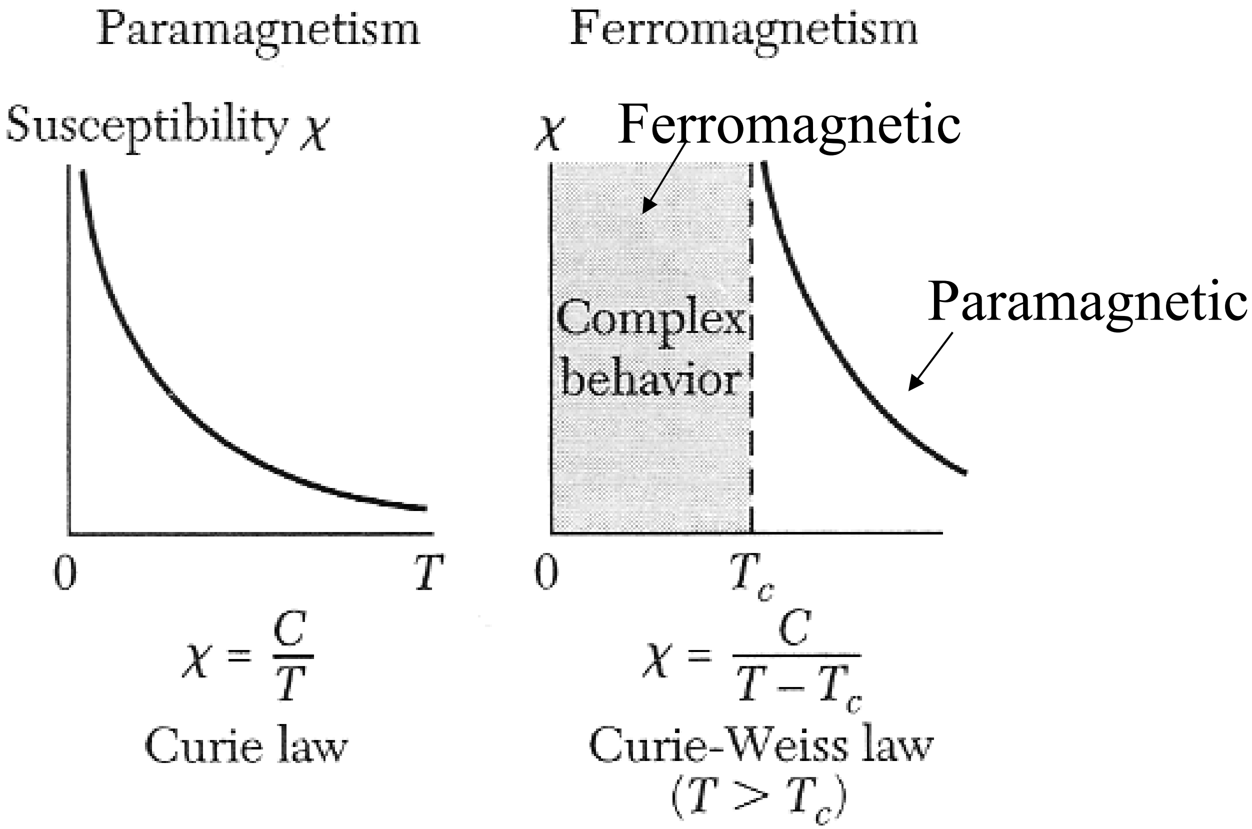
$$M \approx \frac{g^2 \mu_B^2 N}{4 V k_B} \frac{B_a}{T - T_c}$$

$$T_c = \frac{z}{4 k_B} J$$

Curie Weiss Law $\chi = \frac{dM}{dH} \approx \frac{C}{T - T_c}$

Critical fluctuations near T_c

Ferromagnets are paramagnetic above T_c



Critical fluctuations near T_c.

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Magnetization of a Magnetite Single Crystal Near the Curie Point*

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(Received January 20, 1956)

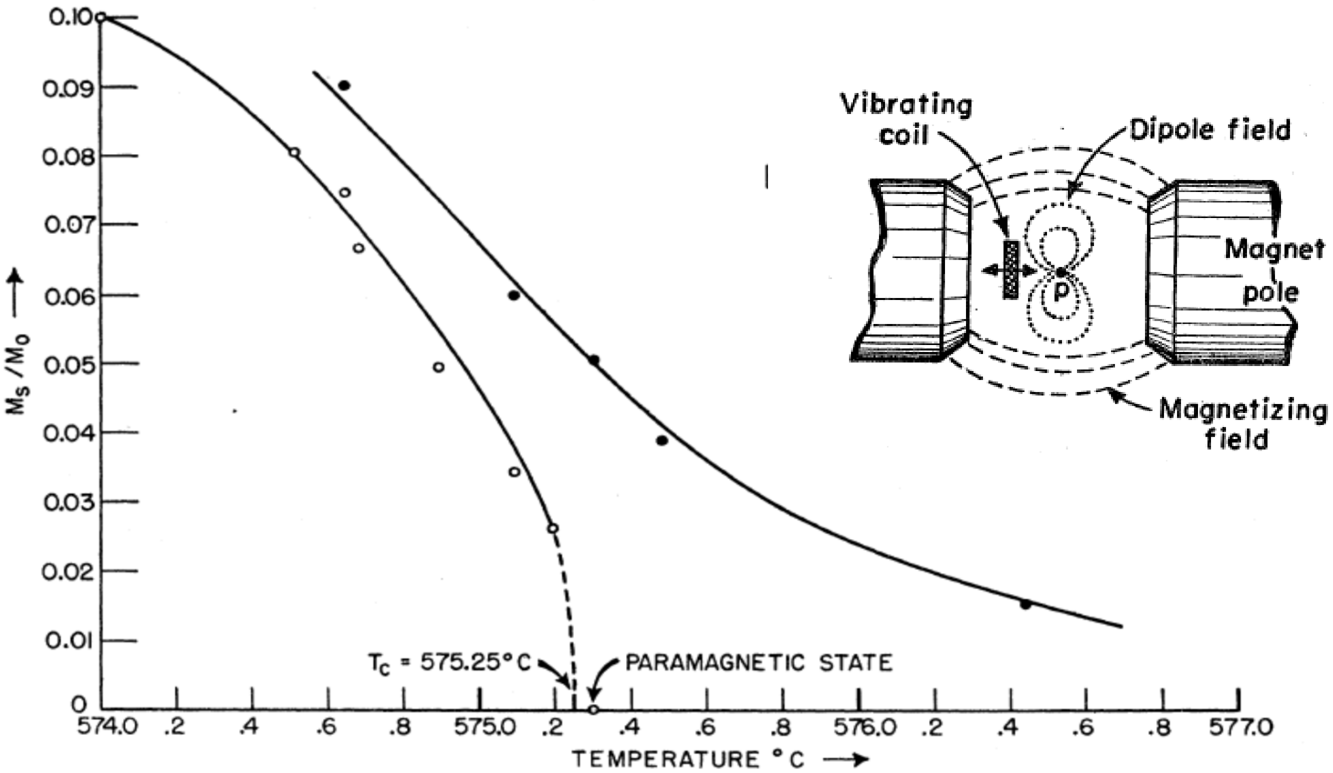


FIG. 9. M_s/M_0 *vs* T in the [111] direction near the Curie point for single-crystal magnetite.

FIG. 2. Principle of the vibrating-coil magnetometer.

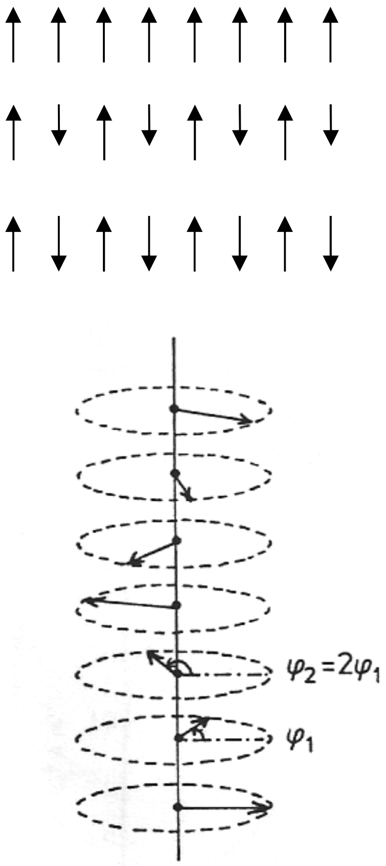
Magnetic ordering

Ferromagnetism

Ferrimagnetism

Antiferromagnetism

Helimagnetism



All ordered magnetic states have excitations called magnons

Ferrimagnets

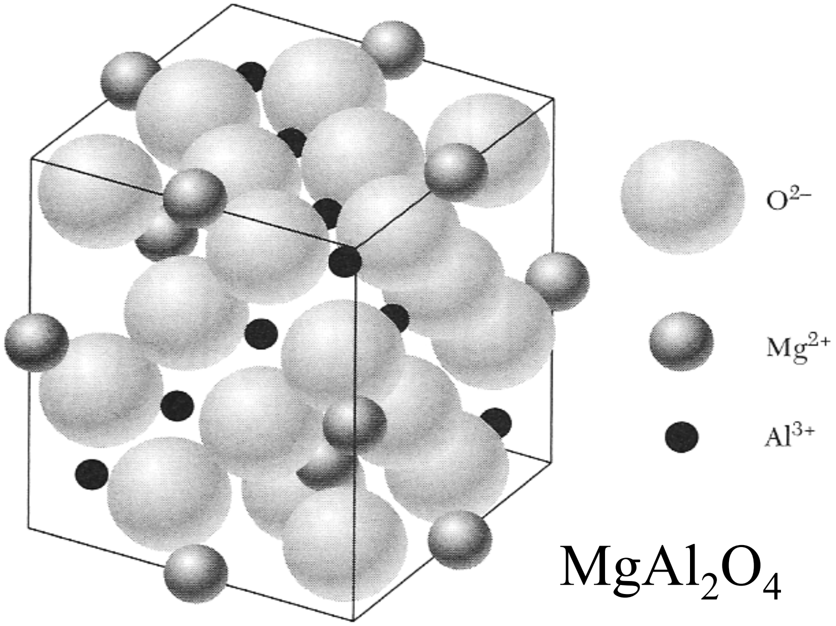
Magnetite Fe_3O_4
(Magneteisen)

Ferrites $MO \cdot Fe_2O_3$

M = Fe, Zn, Cd, Ni, Cu, Co, Mg

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Two sublattices A and B.



$MgAl_2O_4$

Spinel crystal structure XY_2O_4

8 tetrahedral sites A (surrounded by 4 O) $5\mu_B \uparrow$

16 octahedral sites B (surrounded by 6 O) $9\mu_B \downarrow$

per unit cell

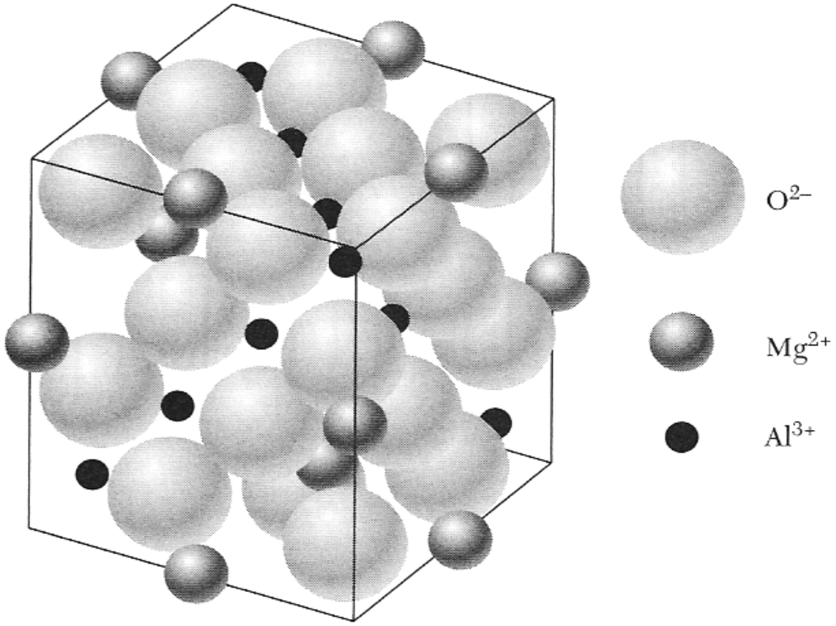
Ferrimagnets

Magnetite Fe_3O_4

Ferrites $MO \cdot Fe_2O_3$

M = Fe, Zn, Cd, Ni, Cu, Co, Mg

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓



Exchange integrals J_{AA}, J_{AB}, and J_{BB} are all negative (antiparallel preferred)

$|J_{AB}| > |J_{AA}|, |J_{BB}|$

Mean field theory

Heisenberg Hamiltonian $$H = -\sum_{i,j} J_{i,j}\vec{S}_i \cdot \vec{S}_j - g\mu_B B \sum_i \vec{S}_i$$

Exchange energy

Mean field approximation

$$\vec{B}_{MF,A} = \frac{1}{g\mu_B}\sum_{\delta} J_{i,AB}\left\langle \vec{S}_B \right\rangle + \frac{1}{g\mu_B}\sum_{\delta} J_{i,AA}\left\langle \vec{S}_A \right\rangle$$

$$\vec{B}_{MF,B} = \frac{1}{g\mu_B}\sum_{\delta} J_{i,AB}\left\langle \vec{S}_A \right\rangle + \frac{1}{g\mu_B}\sum_{\delta} J_{i,BB}\left\langle \vec{S}_B \right\rangle$$

$$\vec{M}_A = g\mu_B \frac{N}{V}\left\langle \vec{S}_A \right\rangle \qquad \vec{M}_B = g\mu_B \frac{N}{V}\left\langle \vec{S}_B \right\rangle$$

Mean field theory

The spins can take on two energies. These energies are different on the A sites and B because the A spins see a different environment as the B spins.

$$E_A = \pm\frac{1}{2} g \mu_B (B_{MF,A} + B_a) \qquad E_B = \pm\frac{1}{2} g \mu_B (B_{MF,B} + B_a)$$

Calculate the average magnetization with Boltzmann factors:

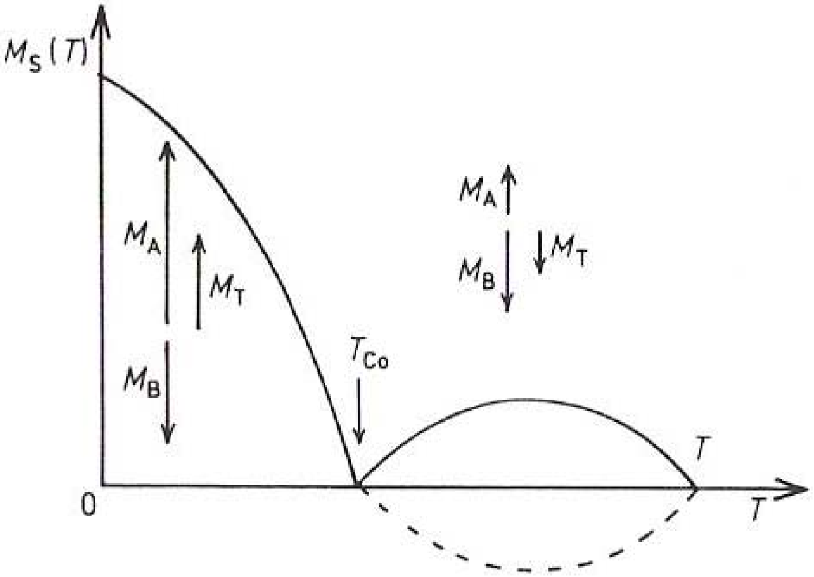
$$M_A = N\mu \tanh\left(\frac{\mu\left(B_{MF,A} + B_a\right)}{k_B T}\right) \qquad M_B = N\mu \tanh\left(\frac{\mu\left(B_{MF,B} + B_a\right)}{k_B T}\right)$$

$$M_A = M_{s,A} \tanh\left(\frac{\mu_0 \mu_{AB} M_B + \mu_0 \mu_{AA} M_A + \mu B_{ac}}{k_B T}\right)$$

$$M_B = M_{s,B} \tanh\left(\frac{\mu_0 \mu_{AB} M_A + \mu_0 \mu_{BB} M_B + \mu B_a}{k_B T}\right)$$

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓ Ferrimagnetism

gauss = 10^{-4} T
oersted = $10^{-4}/4\pi x 10^{-7}$ A/



D. Gignoux, magnetic properties of Metallic systems

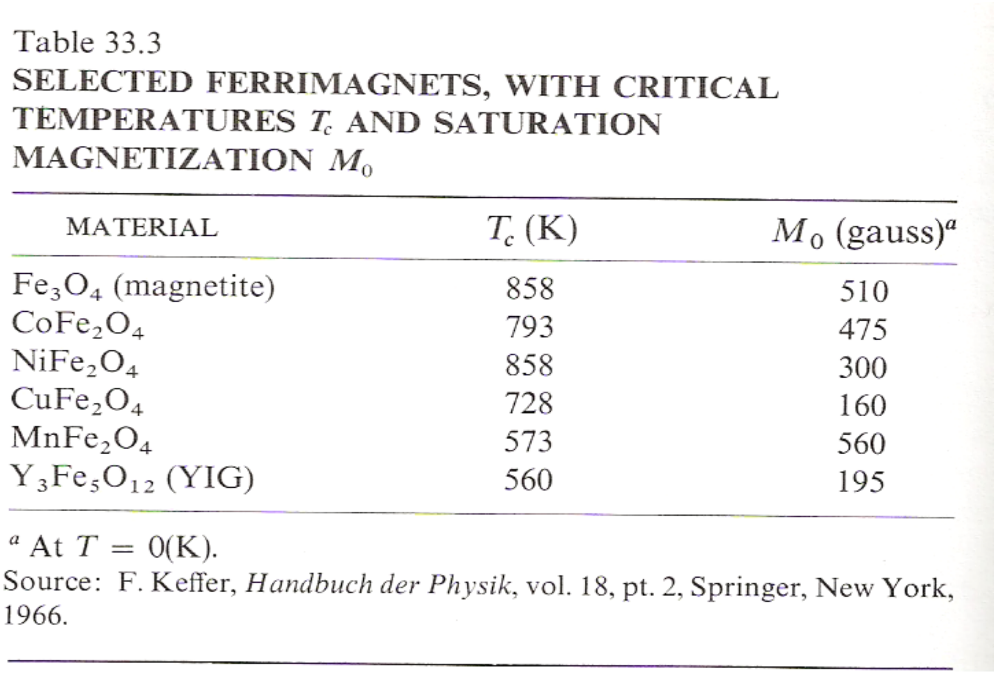


Table 33.3
SELECTED FERRIMAGNETS, WITH CRITICAL TEMPERATURES T_c AND SATURATION MAGNETIZATION M_0

MATERIAL	T_c (K)	M_0 (gauss)^a
Fe_3O_4 (magnetite)	858	510
$CoFe_2O_4$	793	475
$NiFe_2O_4$	858	300
$CuFe_2O_4$	728	160
$MnFe_2O_4$	573	560
$Y_3Fe_5O_{12}$ (YIG)	560	195

^{a} At $T = 0$(K).
Source: F. Keffer, *Handbuch der Physik*, vol. 18, pt. 2, Springer, New York, 1966.

Kittel

Antiferromagnetism

Negative exchange energy $J_{AB} < 0$.

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

At low temperatures, below the Neel temperature T_N, the spins are aligned antiparallel and the macroscopic magnetization is zero.

Spin ordering can be observed by neutron scattering.

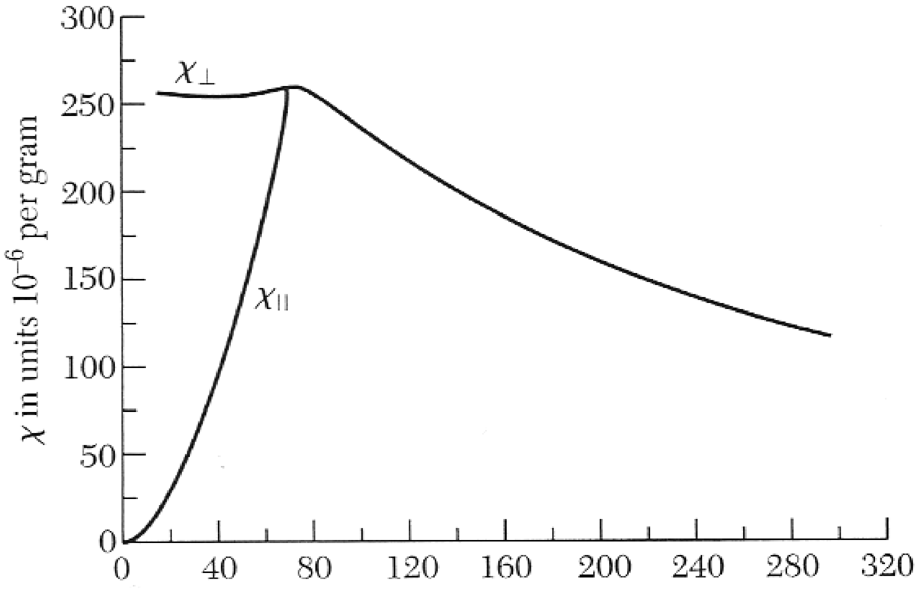
At high temperature antiferromagnets become paramagnetic. The macroscopic magnetization is zero and the spins are disordered in zero field.

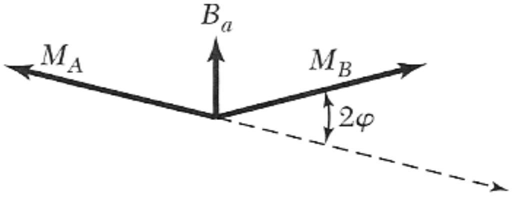
$$\chi = \mu_0 \frac{\vec{M}_A + \vec{M}_B}{\vec{B}_a} = \frac{C}{T + \Theta}$$

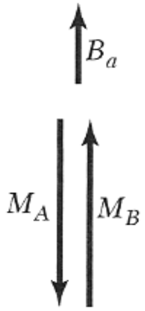
Curie-Weiss temperature

Antiferromagnetism ↑↓↑↓↑↓↑↓

Average spontaneous magnetization is zero at all temperatures.







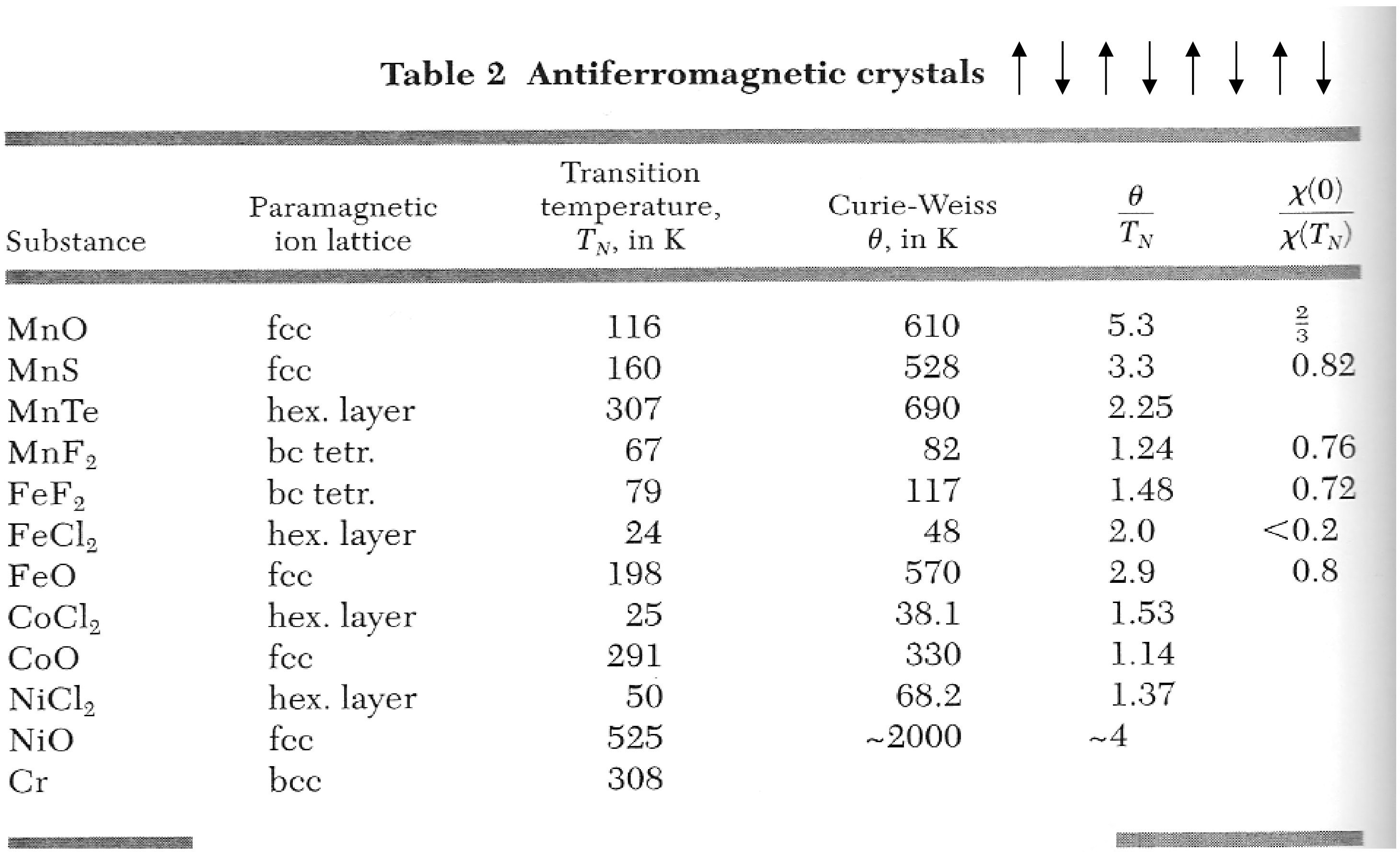
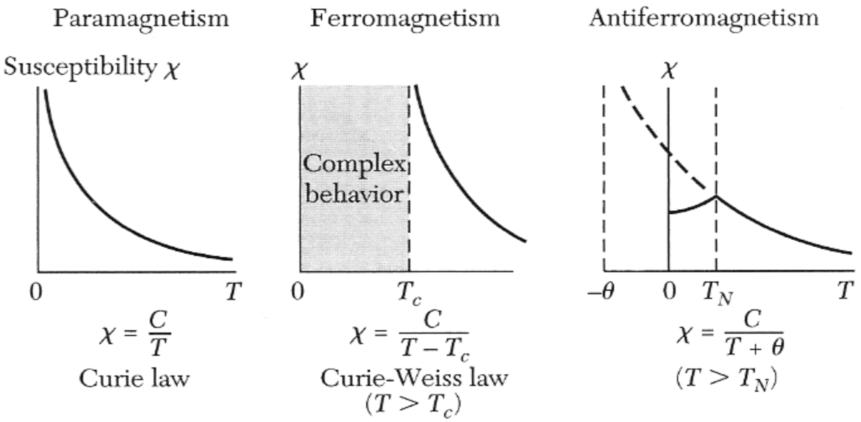


Table 2 Antiferromagnetic crystals ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Substance	Paramagnetic ion lattice	Transition temperature, T_N, in K	Curie-Weiss θ, in K	$\frac{\theta}{T_N}$	$\frac{\chi(0)}{\chi(T_N)}$
MnO	fcc	116	610	5.3	$\frac{2}{3}$
MnS	fcc	160	528	3.3	0.82
MnTe	hex. layer	307	690	2.25	
MnF_2	bc tetr.	67	82	1.24	0.76
FeF_2	bc tetr.	79	117	1.48	0.72
$FeCl_2$	hex. layer	24	48	2.0	<0.2
FeO	fcc	198	570	2.9	0.8
$CoCl_2$	hex. layer	25	38.1	1.53	
CoO	fcc	291	330	1.14	
$NiCl_2$	hex. layer	50	68.2	1.37	
NiO	fcc	525	~2000	~4	
Cr	bcc	308			



$\chi = \frac{C}{T}$
Curie law

$\chi = \frac{C}{T - T_c}$
Curie-Weiss law
$(T > T_c)$

$\chi = \frac{C}{T + \theta}$
$(T > T_N)$

from Kittel