

Dielectric properties of metals

- Outline
- Quantization
- Photons
- Electrons
- Magnetic effects and Fermi surfaces
- Linear response
- Transport
- Crystal Physics
- Electron-electron interactions
- Quasiparticles
- Structural phase transitions
- Landau theory of second order phase transitions
- Superconductivity
- Exam questions
- Appendices
- Lectures
- Books
- Course notes
- TUG students
- Making presentations

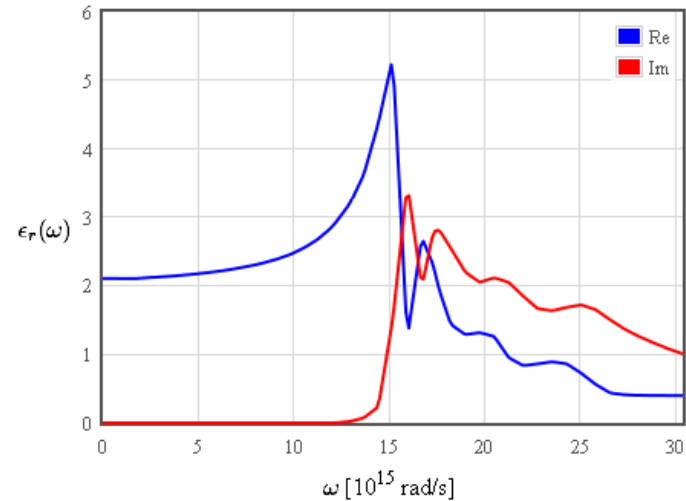
Cu
Si
SiO2
diamond

The optical properties of SiO₂ (glass)

nanophotonics.csic.es

Dielectric function

The relative dielectric constant describes the relationship between the electric displacement \vec{D} and the electric field \vec{E} , $\vec{D} = \epsilon_r \epsilon_0 \vec{E} = \vec{P} + \epsilon_0 \vec{E}$.



There are two conventions for dielectric function. Either it is assumed that the time dependence of \vec{D} , \vec{P} , and \vec{E} is $\exp(-i\omega t)$ and the plot of the dielectric function looks as it is shown above, or it is assumed that the time dependence of \vec{D} , \vec{P} , and \vec{E} is $\exp(i\omega t)$ and the imaginary part of the has the opposite sign as in the plot above. Here we will assume a time dependence of $\exp(-i\omega t)$.

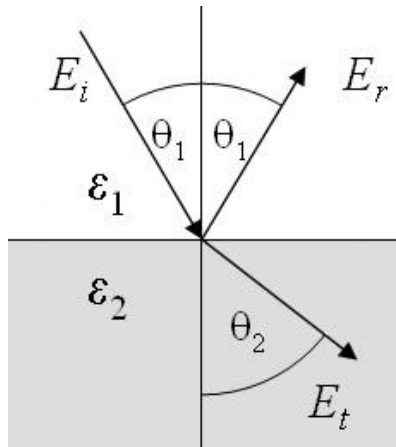
Electric susceptibility

The electric susceptibility χ_E describes the relationship between the polarization \vec{P} and the electric field \vec{E} , $\vec{P} = \epsilon_0 \chi_E \vec{E}$.

$$\chi_E = \epsilon_r - 1$$



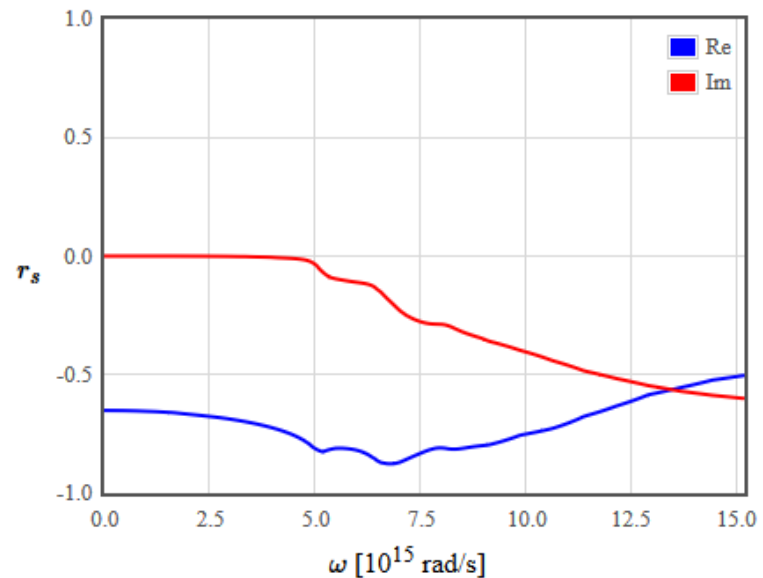
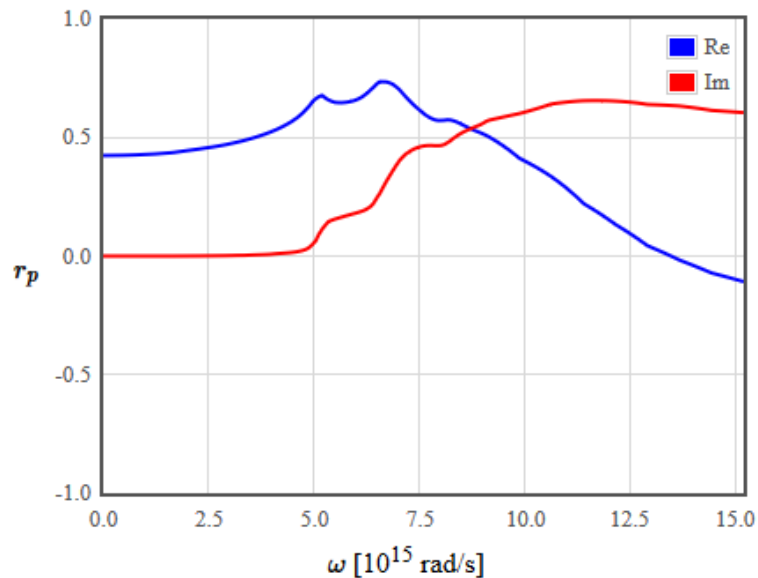
Ellipsometry



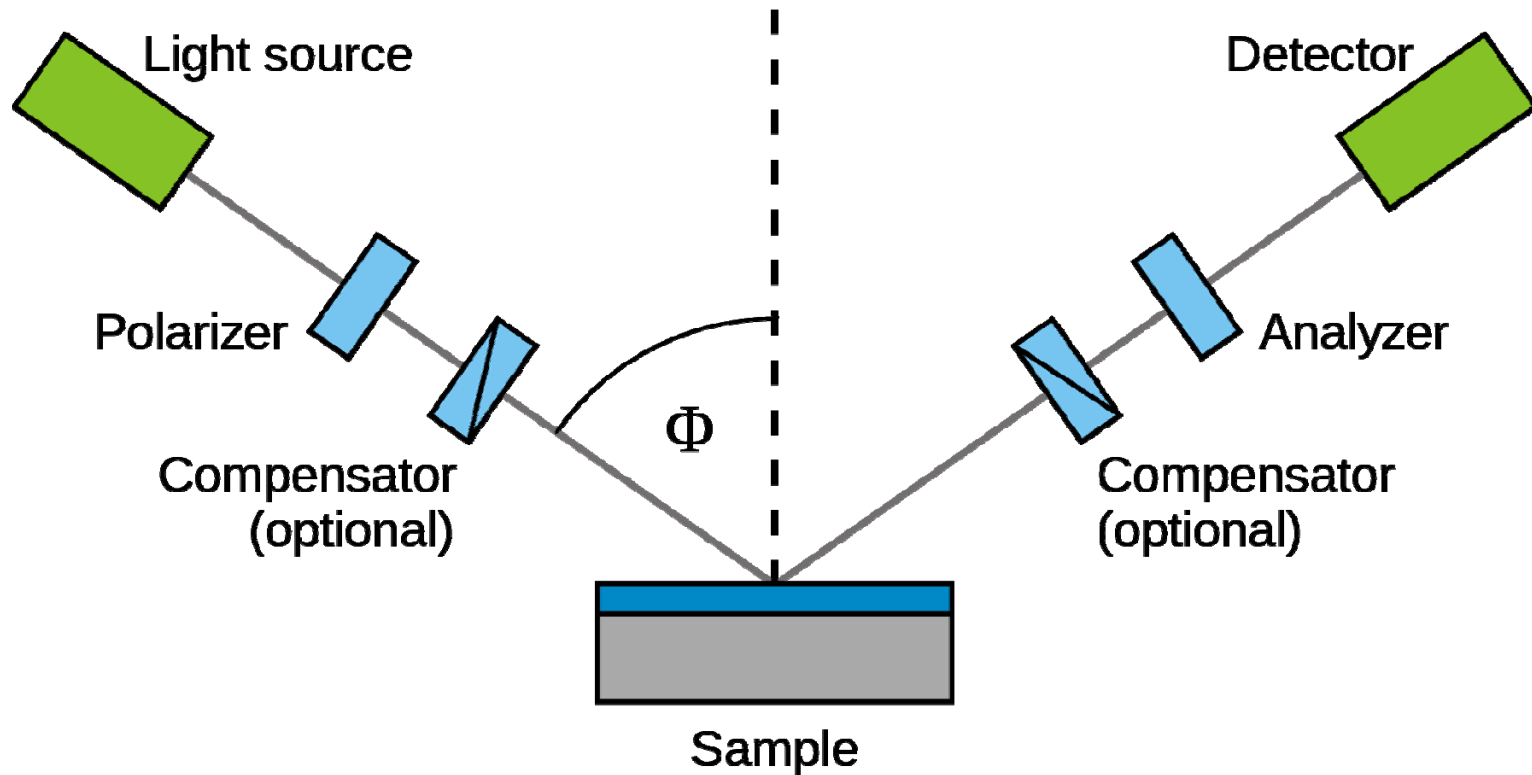
$$r_p = \frac{E_{rp}}{E_{ip}} = \frac{\sqrt{\epsilon_2} \cos \theta_1 - \sqrt{\epsilon_1} \cos \theta_2}{\sqrt{\epsilon_2} \cos \theta_1 + \sqrt{\epsilon_1} \cos \theta_2}$$

$$r_s = \frac{E_{sr}}{E_{si}} = \frac{\sqrt{\epsilon_2} \cos \theta_2 - \sqrt{\epsilon_1} \cos \theta_1}{\sqrt{\epsilon_1} \cos \theta_1 + \sqrt{\epsilon_2} \cos \theta_2}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



Ellipsometry

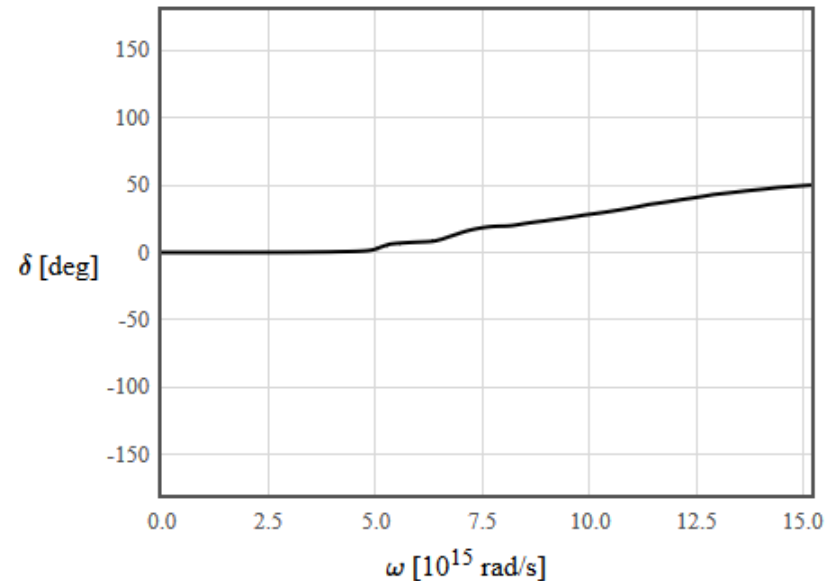
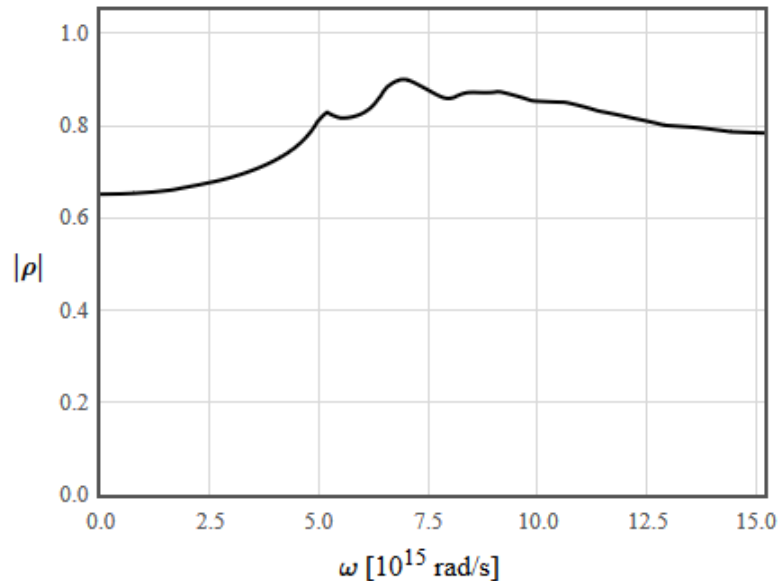


Ellipsometry measures the change of polarization upon reflection. The measured signal depends on the thickness and the dielectric constant.

<http://en.wikipedia.org/wiki/Ellipsometry>

Ellipsometry

$$\rho = \frac{r_p}{r_s} = |\rho|e^{i\delta}$$



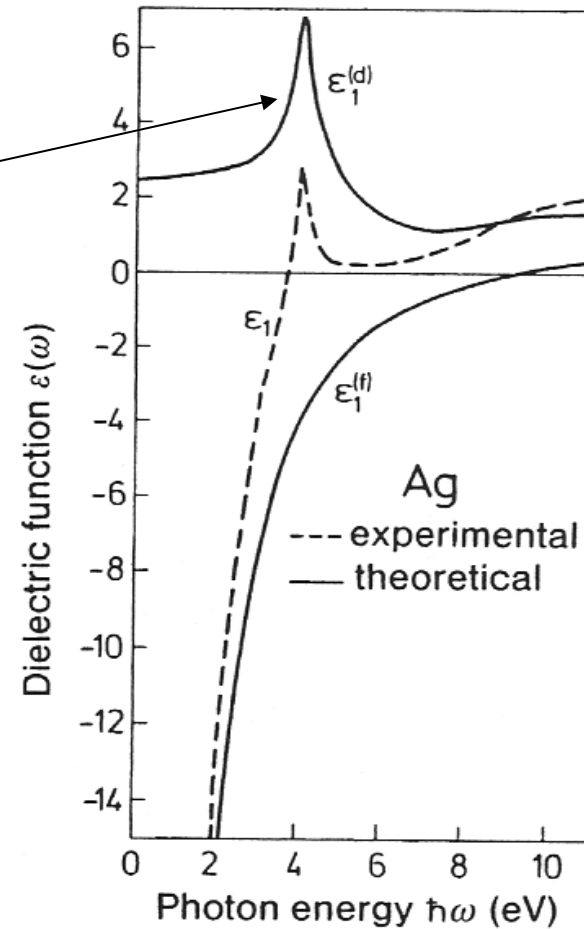
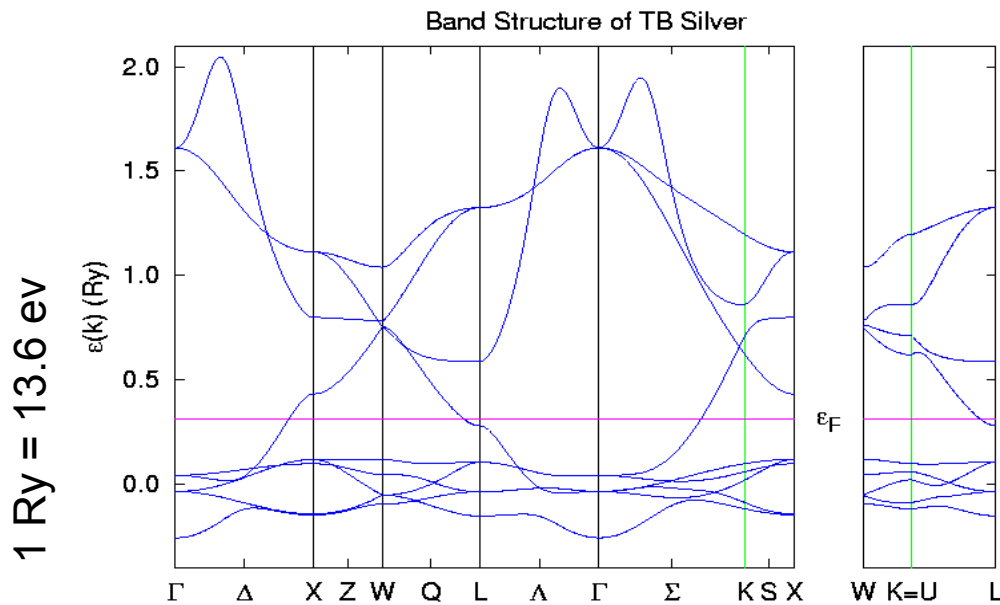
The ratio of the two reflected polarizations is insensitive to instabilities of light source or atmospheric absorption.

Intraband transitions

When the bands are parallel, there is a peak in the absorption (ϵ'')

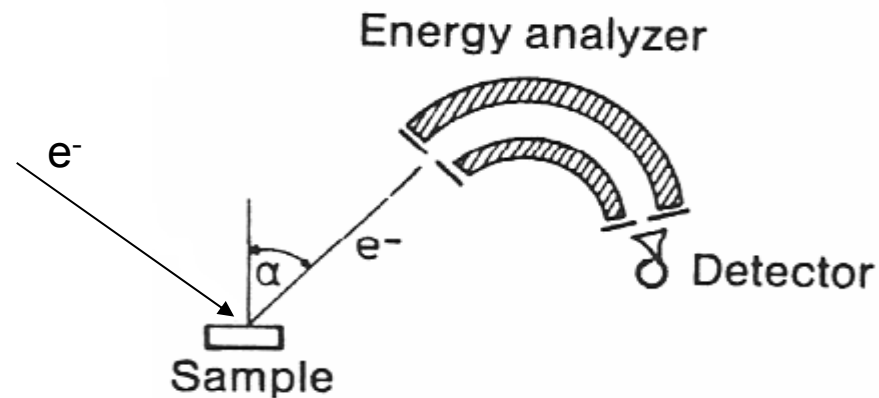
$$\hbar\omega = E_c(\vec{k}) - E_v(\vec{k})$$

Intraband (d-band) absorption



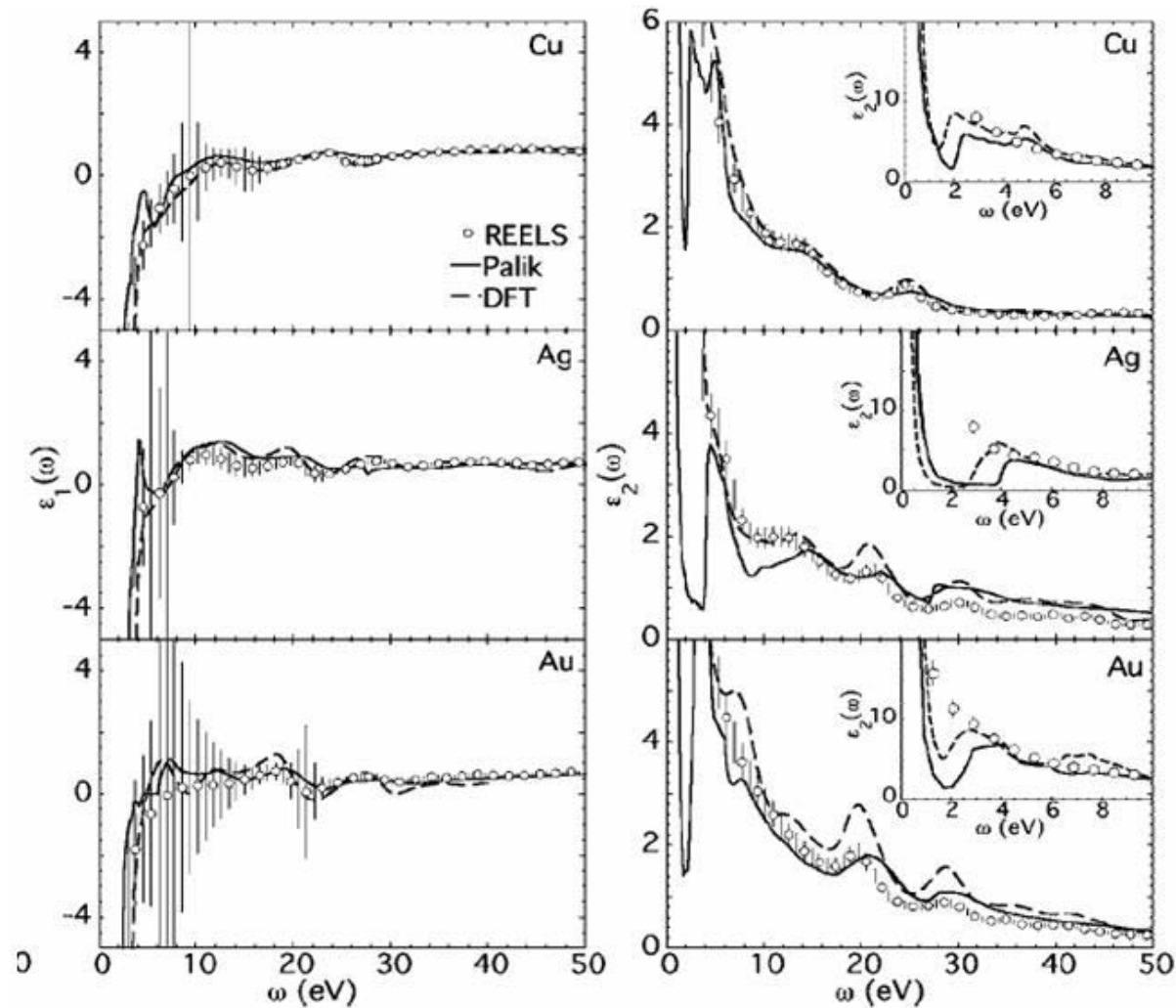
Ibach & Lueeth

Reflection Electron Energy Loss Spectroscopy



Fast electrons moving through the solid generate a time dependent electric field. If the polarization moves out of phase with this field, energy will be lost. This is detected in the reflected electrons.

Dielectric function of Cu, Ag, and Au obtained from reflection electron energy loss spectra, optical measurements, and density functional theory



Werner (TU Vienna) APL 89 213106 (2006)

Microwave engineering

Microwave frequencies are a few GHz

The wavelength is smaller than the circuit

Losses in metals increase with increasing frequency

Losses in dielectrics increase with increasing frequency

There is a characteristic length scale called the skin depth which tells us how far into a metal fields penetrate before they are reflected out.

Skin depth $\omega\tau \ll 1$

$$\sigma(\omega) = ne\mu \left(\frac{1 - i\omega\tau}{1 + \omega^2\tau^2} \right) \approx ne\mu = \sigma_0 \quad \omega\tau \ll 1$$

Ohm's law $\vec{J} = \sigma_0 \vec{E}$

Take the curl $\frac{1}{\sigma_0} \nabla \times \vec{J} = \nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$ Faraday's law

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \text{Ampere's law}$$

$$\frac{1}{\sigma_0 \mu_0} \nabla \times \nabla \times \vec{B} = -\frac{d\vec{B}}{dt}$$

Vector identity $\nabla \times \nabla \times \vec{B} = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$

$$\frac{1}{\sigma_0 \mu_0} \nabla^2 \vec{B} = \frac{d\vec{B}}{dt}$$

Skin depth

$$\frac{1}{\sigma_0 \mu_0} \nabla^2 \vec{B} = \frac{d\vec{B}}{dt}$$

Assume harmonic dependence $B_0 e^{i(kx - \omega t)}$

$$\frac{k^2}{\sigma_0 \mu_0} = i\omega$$

$$k = \sqrt{i\omega\sigma_0\mu_0} = \sqrt{\frac{\omega\sigma_0\mu_0}{2}} + i\sqrt{\frac{\omega\sigma_0\mu_0}{2}}$$

Skin depth $\delta = \sqrt{\frac{2}{\mu_0\sigma_0\omega}}$

Exponential decay



Light $\omega < \omega_p$ is reflected out of a metal. The waves penetrate a length δ .

Skin depth

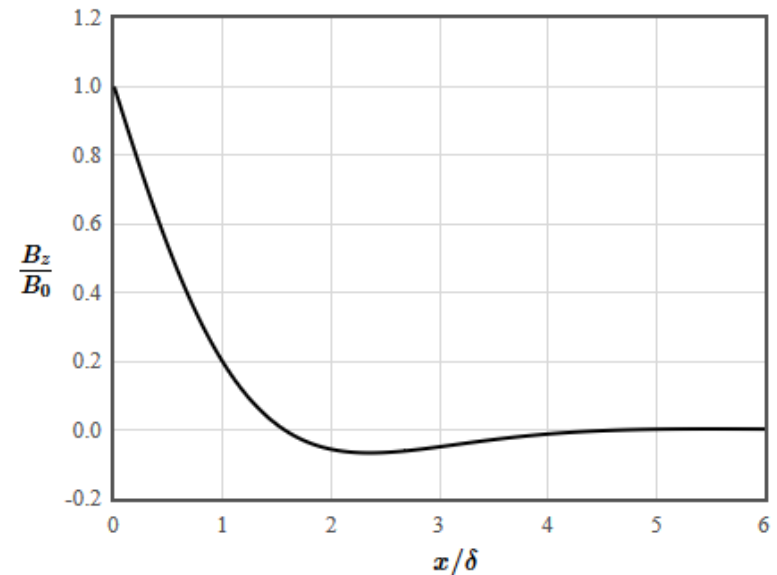
$$\vec{B} = B_0 e^{-x/\delta} e^{i(x/\delta - \omega t)} \hat{z}$$

$$\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B} = \vec{B} = \frac{B_0(1-i)}{\mu_0 \delta} e^{-x/\delta} e^{i(x/\delta - \omega t)} \hat{y}$$

$$1-i = \sqrt{2} e^{-i\pi/4}$$

$$\vec{J} = \frac{\sqrt{2} B_0}{\mu_0 \delta} e^{-x/\delta} e^{i(x/\delta - \omega t - \pi/4)} \hat{y}$$

$$\vec{E} = \frac{\vec{J}}{\sigma_0} = \frac{\sqrt{2} B_0}{\mu_0 \delta \sigma_0} e^{-x/\delta} e^{i(x/\delta - \omega t - \pi/4)} \hat{y}$$



The electric field lags behind the magnetic field by 45 degrees.

Surface resistance

At low frequencies:

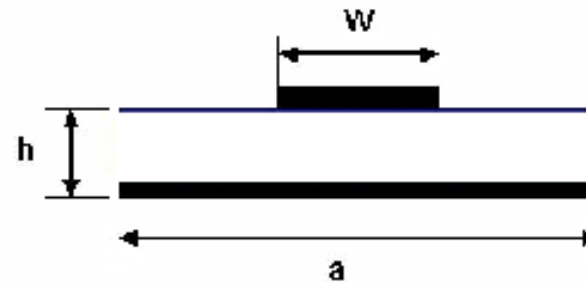
$$R = \frac{\rho \ell}{wt} = \frac{\ell}{\sigma_0 wt}$$

When $\delta < t$:

$$R = \frac{\ell}{\sigma_0 w \delta}$$

for $\ell = w$

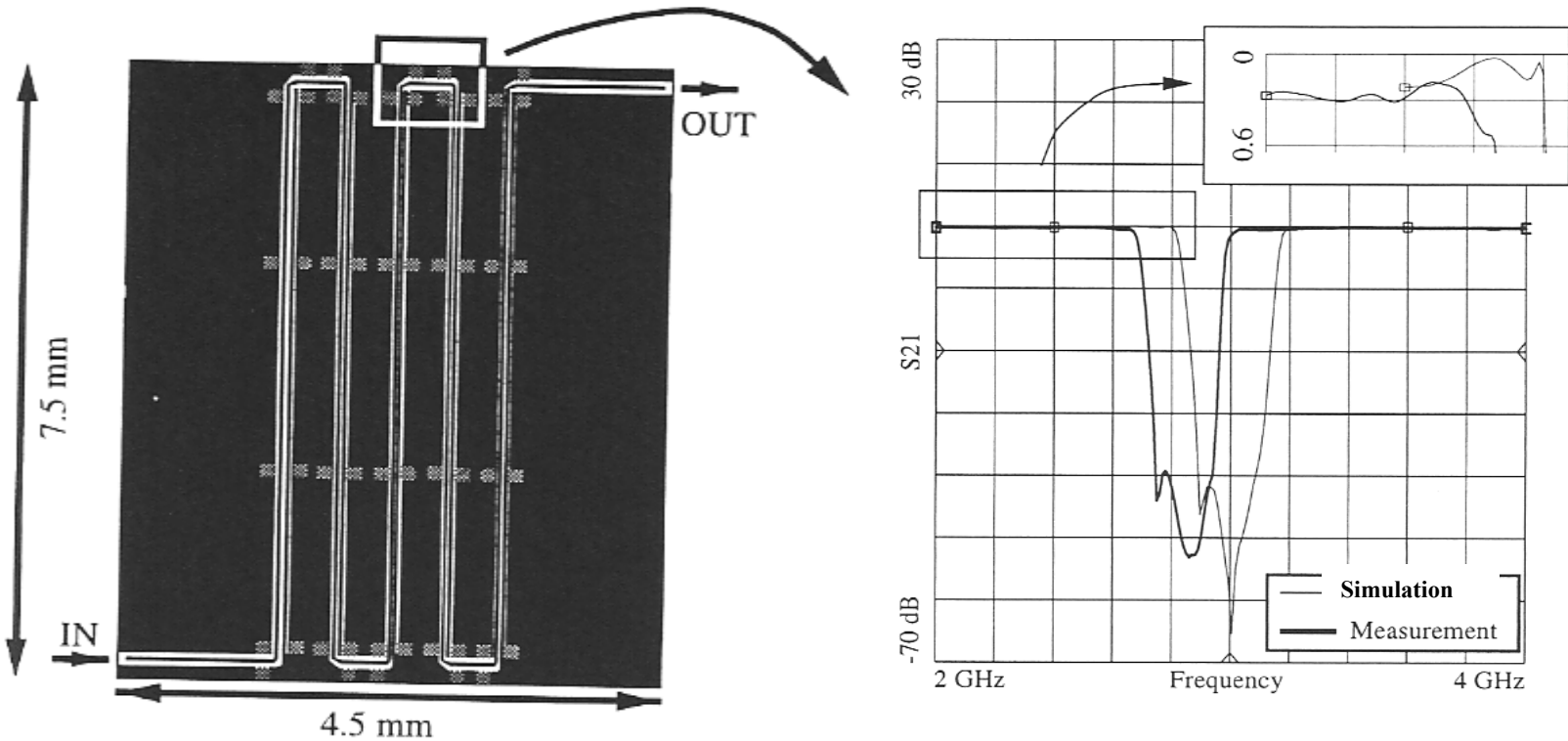
$$R_s = \frac{1}{\sigma_0 \delta} \propto \sqrt{\omega}$$



Complex signal processing at high frequencies > 1 GHz is difficult because the losses increase with frequency.

Usually you mix down to a lower frequency as soon as possible.

Superconducting filter



Complex signal processing at high frequencies > 1 GHz is difficult because the losses increase with frequency.