Dielectric properties of metals

http://lamp.tu-graz.ac.at/~hadley/ss2/linearresponse/dmetal.php

Advanced Solid State Physics

Optical properties of a diffusive metal

It is assumed that electrons in a diffusive metal scatter so often that we can average over the scatering events. The differential equation that describes the motion of the electrons is,

$$m\frac{d\vec{v}}{dt}+\frac{e\vec{v}}{\mu}=-e\vec{E}.$$

Here m is the mass of an electron, $\vec{v}$ is the velocity of the electron, $-e$ is the charge of an electron, and $\vec{E}$ is the electric field. When a constant electric field is applied, the solution is,

$$\vec{v}=-\mu\vec{E}.$$

Thus the (negatively charged) electrons move in the opposite direction as the electric field.

If the electric field is pulsed on, the reponse of the electrons is described by the impulse response function $g(t)$. The impulse response function satisfies the equation,

$$m\frac{dg}{dt}+\frac{eg}{\mu}=-e\delta(t).$$

When the electric field is pulsed on, the electrons suddenly start moving and then their velocity decays exponentially to zero in a time $\tau=m\mu/e$.

$$g(t)=-\frac{e}{m}\exp(-t/\tau).$$

The scattering time τ and the electron density n are the only two parameters that are needed to describe many of the optical properties of a diffusive metal. The form below can be used to input τ and n and then a script calculates and plots the impulse response function, the Fourier transform of the impulse response function, the mobility, the dc conductivity, the frequency dependent complex conductivity, the electric susceptibility, the dielectric function, the plasma frequency, the index of refraction, the extinction coefficient, and the reflectance.

τ = 1E-11 [s] n = 1E28 [m^{-3}] submit

Mobility $\mu = 1.76$ m² V^{-1} s^{-1}

DC conductivity $\sigma_0 = 2.82e+9\ \Omega^{-1}$ m^{-1}

Plasma frequency $\omega_p = 5.64e+15$ rad/s, $\omega_p\tau = 5.64e+4$.

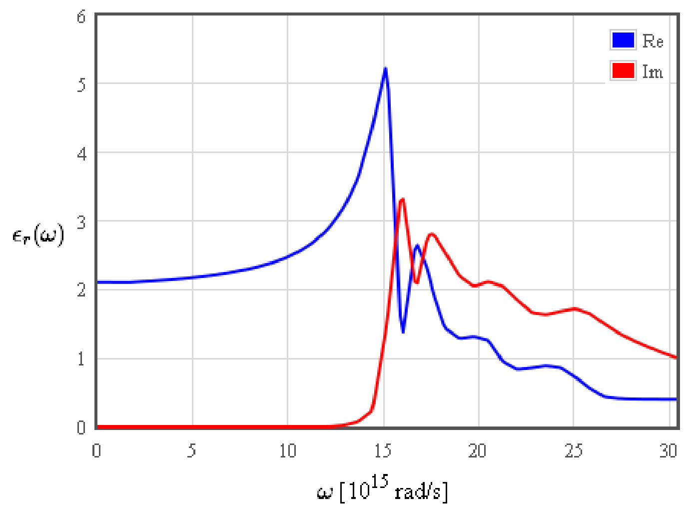
Impulse response function

The optical properties of SiO_2 (glass)

nanophotonics.csic.es

Dielectric function

The relative dielectric constant describes the relationship between the electric displacement $\vec{D}$ and the electric field $\vec{E}$, $\vec{D} = \epsilon_r \epsilon_0 \vec{E} = \vec{P} + \epsilon_0 \vec{E}$.

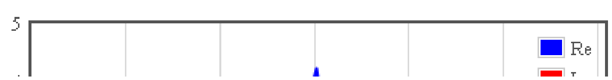


There are two conventions for dielectric function. Either it is assumed that the time dependence of $\vec{D}$, $\vec{P}$, and $\vec{E}$ is $\exp(-i\omega t)$ and the plot of the dielectric function looks as it is shown above, or it is assumed that the time dependence of $\vec{D}$, $\vec{P}$, and $\vec{E}$ is $\exp(i\omega t)$ and the imaginary part of the has the opposite sign as in the plot above. Here we will assume a time dependence of $\exp(-i\omega t)$.

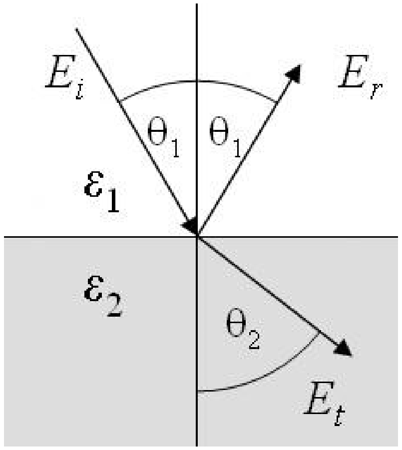
Electric susceptibility

The electric susceptibility χ_E describes the relationship between the polarization $\vec{P}$ and the electric field $\vec{E}$, $\vec{P} = \epsilon_0 \chi_E \vec{E}$.

$$\chi_E = \epsilon_r - 1$$



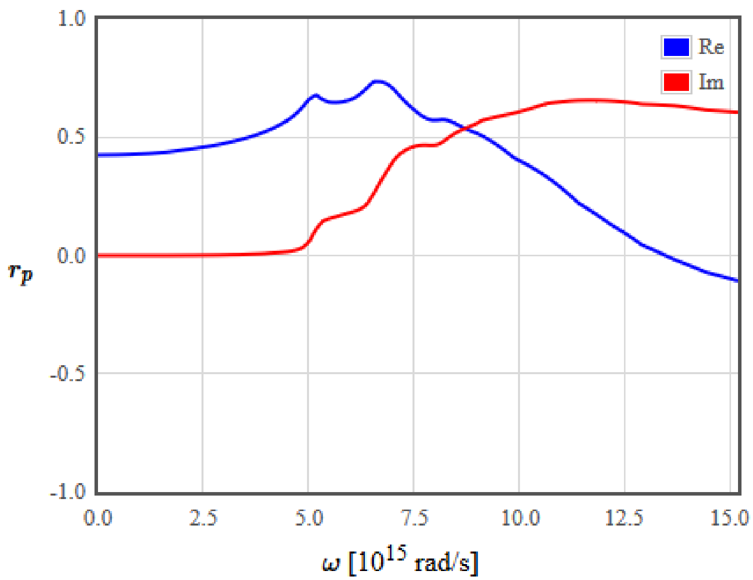
Ellipsometry

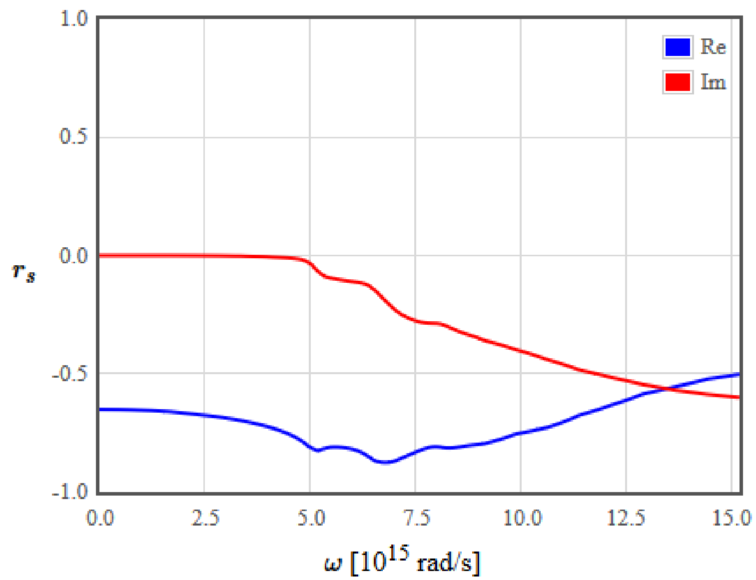


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

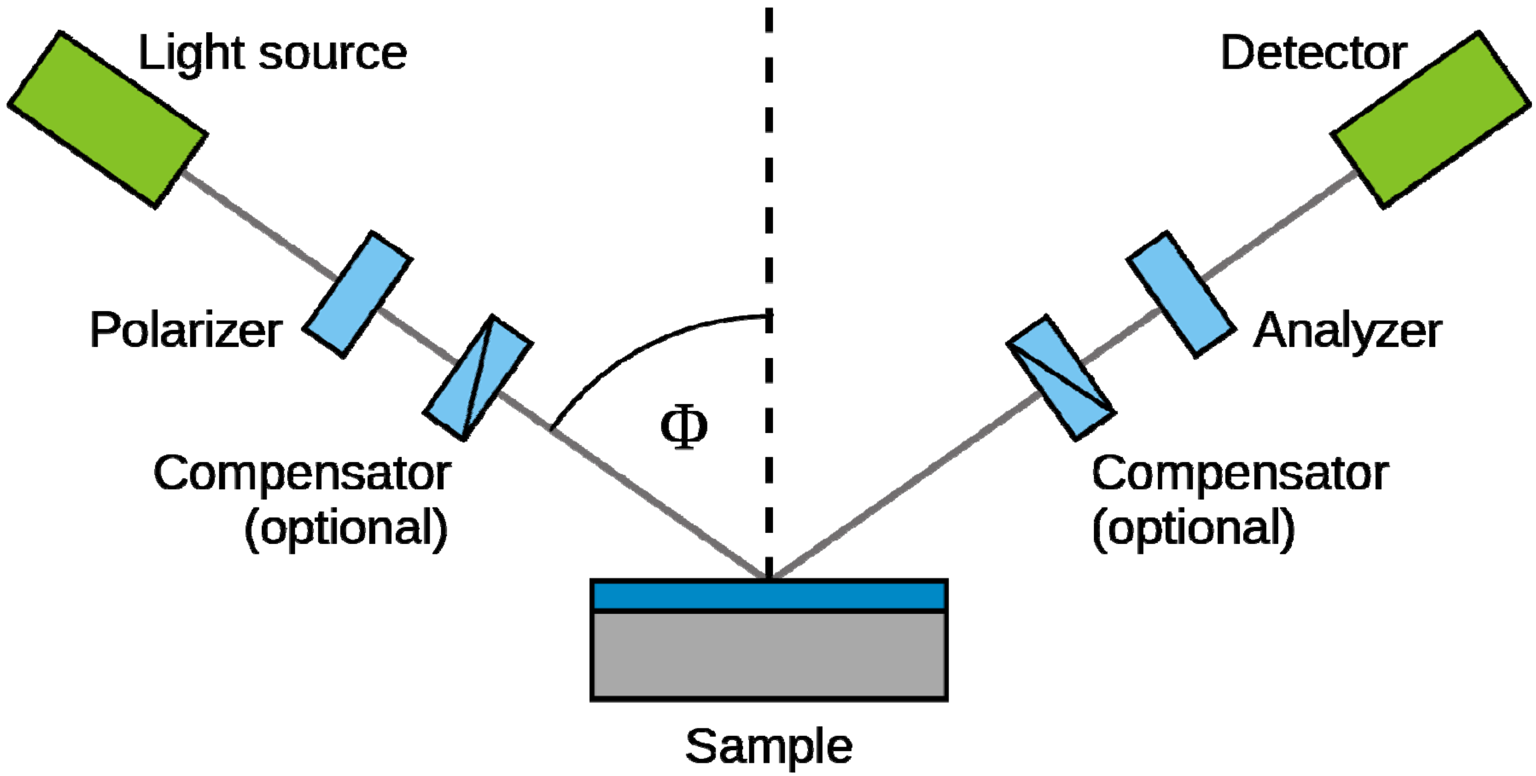
$$r_p = \frac{E_{rp}}{E_{ip}} = \frac{\sqrt{\epsilon_2}\cos\theta_1 - \sqrt{\epsilon_1}\cos\theta_2}{\sqrt{\epsilon_2}\cos\theta_1 + \sqrt{\epsilon_1}\cos\theta_2}$$

$$r_s = \frac{E_{sr}}{E_{si}} = \frac{\sqrt{\epsilon_2}\cos\theta_2 - \sqrt{\epsilon_1}\cos\theta_1}{\sqrt{\epsilon_1}\cos\theta_1 + \sqrt{\epsilon_2}\cos\theta_2}$$





Ellipsometry

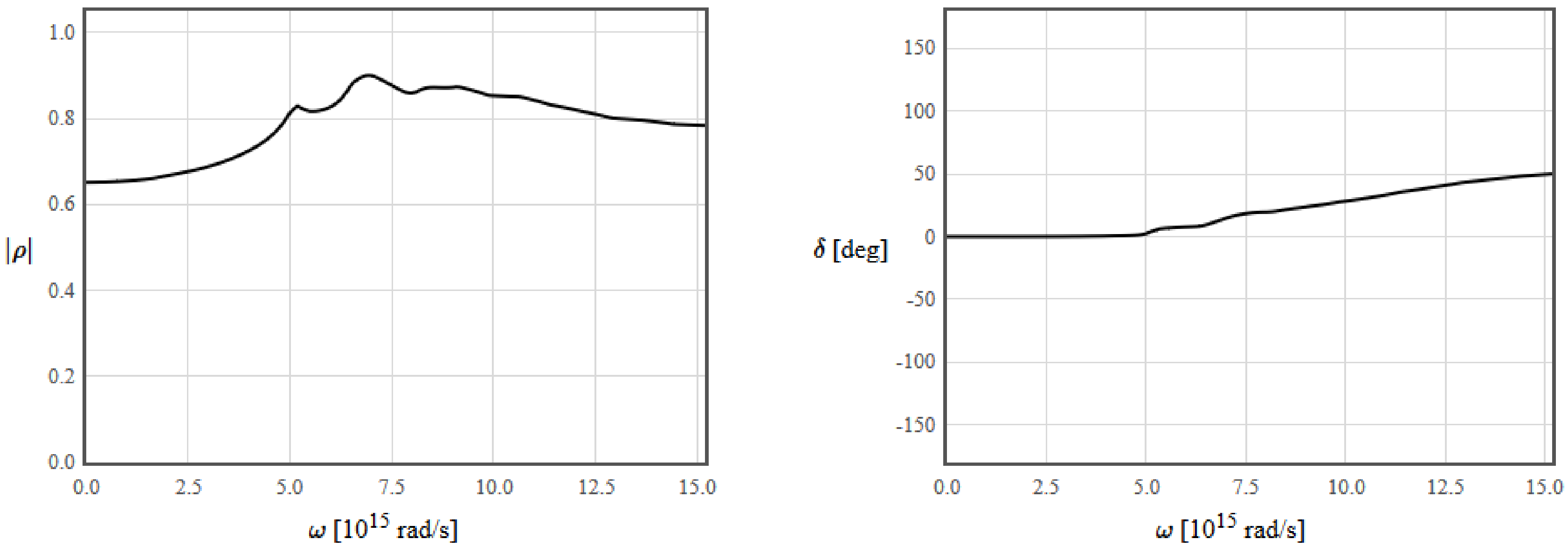


Ellipsometry measures the change of polarization upon reflection. The measured signal depends on the thickness and the dielectric constant.

http://en.wikipedia.org/wiki/Ellipsometry

Ellipsometry

$$\rho = \frac{r_p}{r_s} = |\rho| e^{i\delta}$$



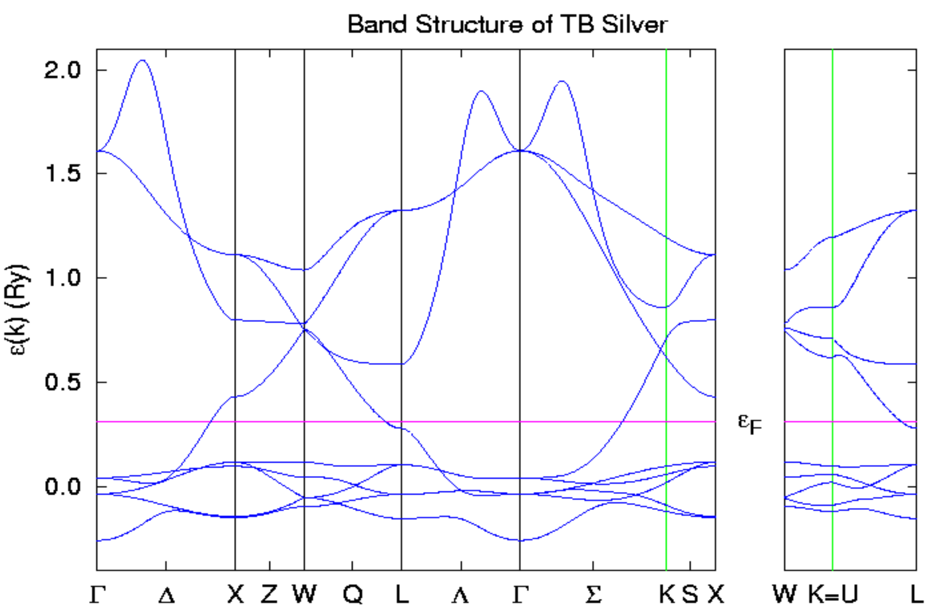
The ratio of the two reflected polarizations is insensitive to instabilities of light source or atmospheric absorption.

Intraband transitions

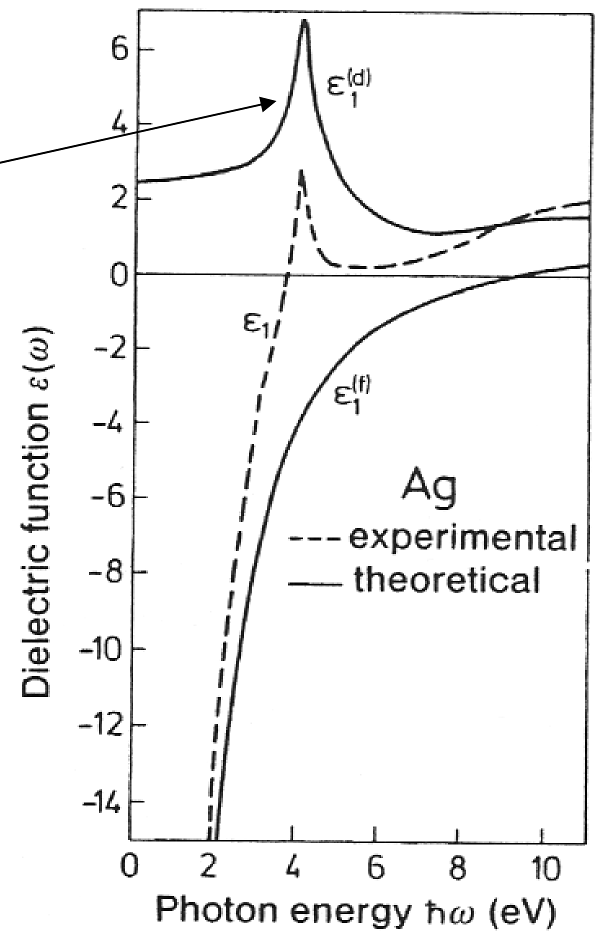
When the bands are parallel, there is a peak in the absorption (ε")

$$\hbar\omega = E_c(\vec{k}) - E_v(\vec{k})$$

Intraband (d-band) absorption

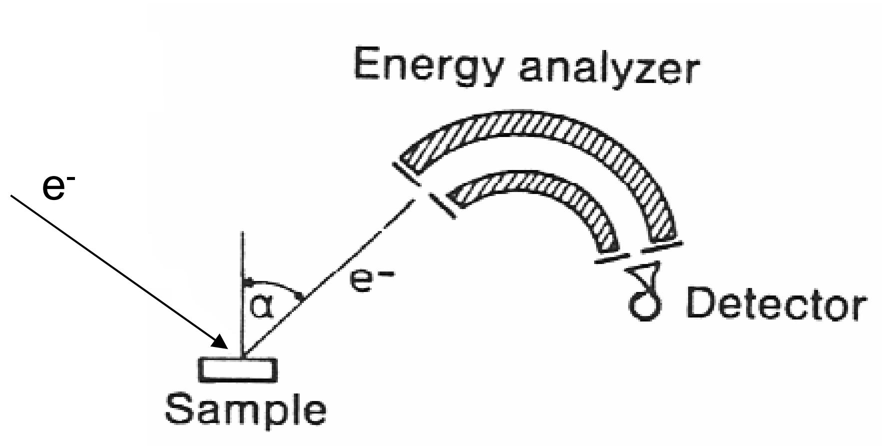


1 Ry = 13.6 ev



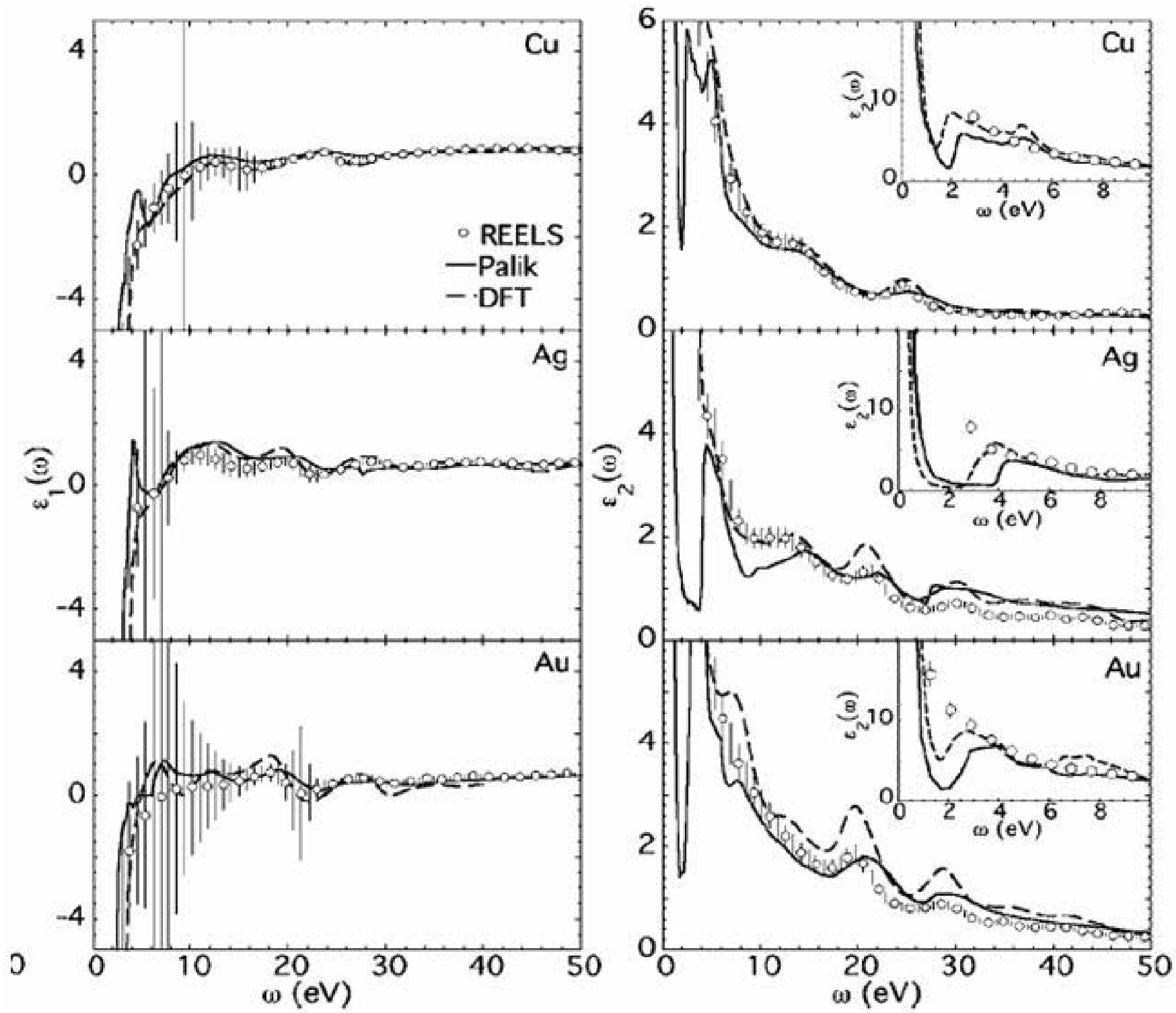
Ibach &Lueth

Reflection Electron Energy Loss Spectroscopy



Fast electrons moving through the solid generate and a time dependent electric field. If the polarization moves out of phase with this field, energy will be lost. This is detected in the reflected electrons.

Dielectric function of Cu, Ag, and Au obtained from reflection electron energy loss spectra, optical measurements, and density functional theory



Werner (TU Vienna) APL 89 213106 (2006)

Microwave engineering

Microwave frequencies are a few GHz

The wavelength is smaller than the circuit

Losses in metals increase with increasing frequency

Losses in dielectrics increase with increasing frequency

There is a characteristic length scale called the skin depth which tells us how far into a metal fields penetrate before they are reflected out.

Skin depth $\omega\tau << 1$

$$\sigma(\omega) = ne\mu\left(\frac{1 - i\omega\tau}{1 + \omega^2\tau^2}\right) \approx ne\mu = \sigma_0 \qquad \omega\tau << 1$$

Ohm's law $\qquad \vec{J} = \sigma_0 \vec{E}$

Take the curl $\qquad \dfrac{1}{\sigma_0}\nabla \times \vec{J} = \nabla \times \vec{E} = -\dfrac{d\vec{B}}{dt} \qquad$ Faraday's law

$$\nabla \times \vec{B} = \mu_0 \vec{J} \qquad \text{Ampere's law}$$

$$\frac{1}{\sigma_0 \mu_0}\nabla \times \nabla \times \vec{B} = -\frac{d\vec{B}}{dt}$$

Vector identity $\qquad \nabla \times \nabla \times \vec{B} = \nabla\left(\cancelto{0}{\nabla \cdot \vec{B}}\right) - \nabla^2 \vec{B}$

$$\frac{1}{\sigma_0 \mu_0}\nabla^2 \vec{B} = \frac{d\vec{B}}{dt}$$

Skin depth

$$\frac{1}{\sigma_0 \mu_0} \nabla^2 \vec{B} = \frac{d\vec{B}}{dt}$$

Assume harmonic dependence $B_0 e^{i(kx - \omega t)}$

$$\frac{k^2}{\sigma_0 \mu_0} = i\omega$$

$$k = \sqrt{i\omega\sigma_0\mu_0} = \sqrt{\frac{\omega\sigma_0\mu_0}{2}} + i\sqrt{\frac{\omega\sigma_0\mu_0}{2}}$$

Exponential decay

Skin depth $\delta = \sqrt{\frac{2}{\mu_0 \sigma_0 \omega}}$

Light $\omega < \omega_p$ is reflected out of a metal. The waves penetrate a length δ.

Skin depth

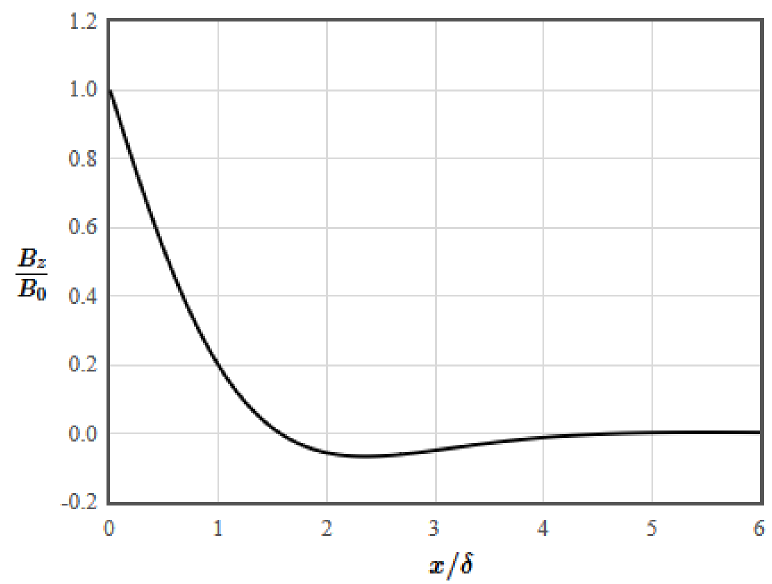
$$\vec{B} = B_0 e^{-x/\delta} e^{i(x/\delta - \omega t)} \hat{z}$$

$$\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B} = \vec{B} = \frac{B_0(1-i)}{\mu_0 \delta} e^{-x/\delta} e^{i(x/\delta - \omega t)} \hat{y}$$

$$1 - i = \sqrt{2} e^{-i\pi/4}$$

$$\vec{J} = \frac{\sqrt{2} B_0}{\mu_0 \delta} e^{-x/\delta} e^{i(x/\delta - \omega t - \pi/4)} \hat{y}$$

$$\vec{E} = \frac{\vec{J}}{\sigma_0} = \frac{\sqrt{2} B_0}{\mu_0 \delta \sigma_0} e^{-x/\delta} e^{i(x/\delta - \omega t - \pi/4)} \hat{y}$$



The electric field lags behind the magnetic field by 45 degrees.

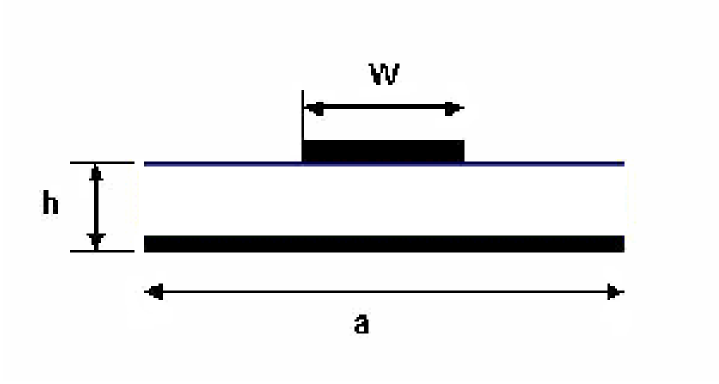
Surface resistance

At low frequencies: $$R = \frac{\rho \ell}{wt} = \frac{\ell}{\sigma_0 wt}$$

When $\delta < t$: $$R = \frac{\ell}{\sigma_0 w \delta}$$

for $$\ell = w$$

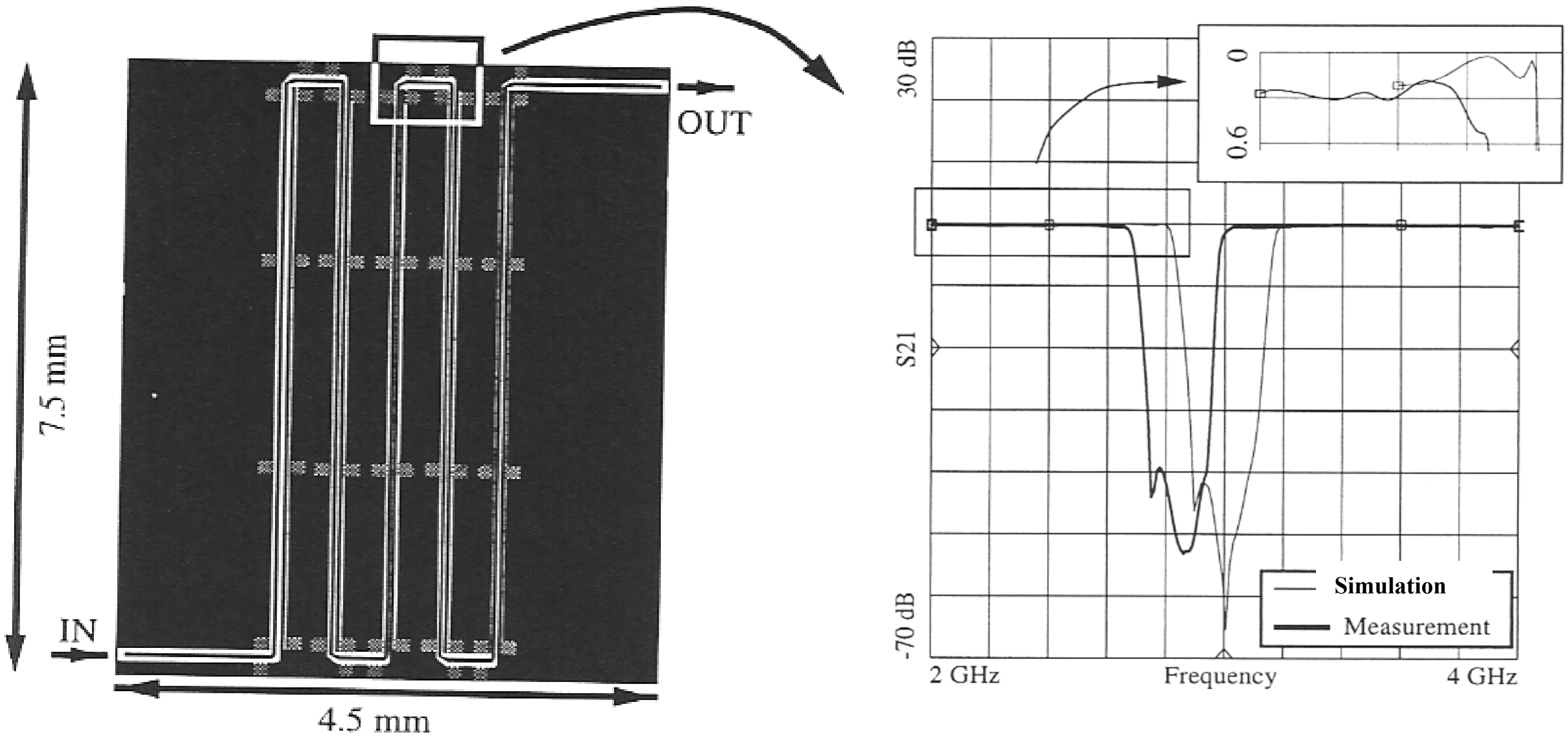
$$R_s = \frac{1}{\sigma_0 \delta} \propto \sqrt{\omega}$$



Complex signal processing at high frequencies > 1 GHz is difficult because the losses increase with frequency.

Usually you mix down to a lower frequency as soon as possible.

Superconducting filter



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