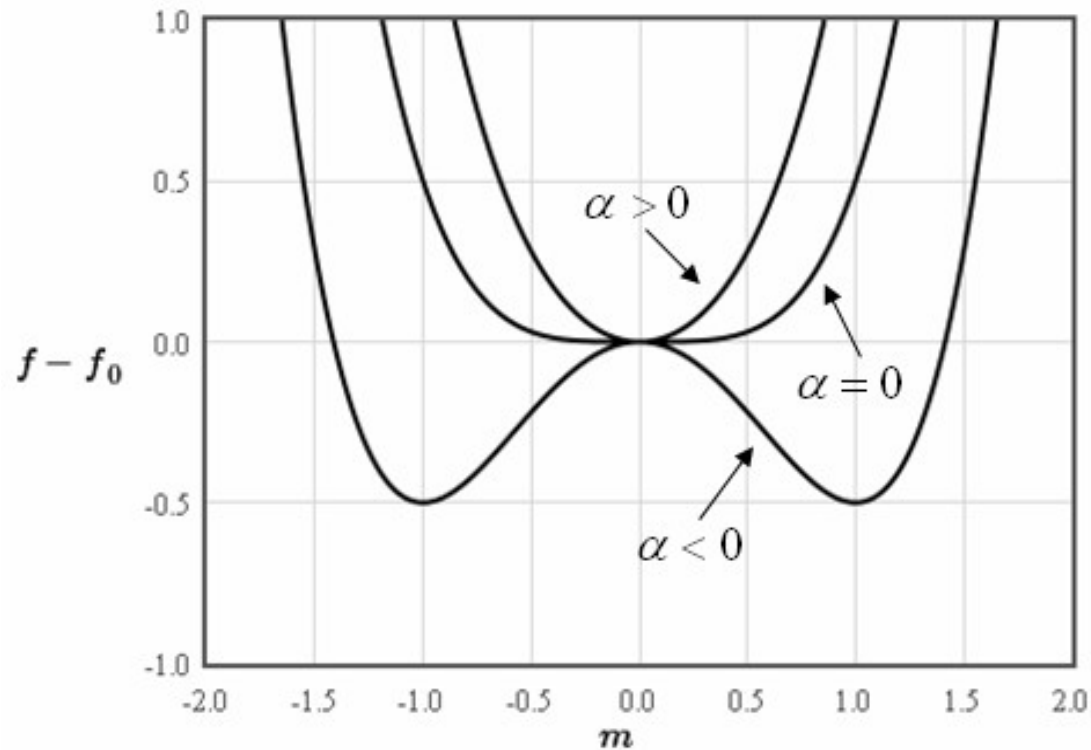


# Landau Theory of Phase Transitions

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Expand the free energy in terms of the order parameter

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$$f = f_0 + \alpha m^2 + \frac{1}{2} \beta m^4 + \dots$$

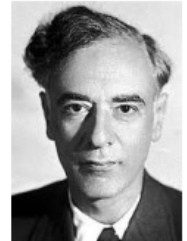
The odd terms are not physical.

## Landau theory of second order phase transitions

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Normally, to calculate thermodynamic properties like the free energy, the entropy, or the specific heat, it is necessary to determine the microscopic states of system by solving the Schrödinger equation. For crystals, the microscopic states are labeled by  $k$  and the solutions of the Schrödinger equation are typically expressed as a dispersion relation where the energy is given for each  $k$ . The dispersion relation can be used to calculate the density of states and the density of states can be used to calculate the thermodynamic properties. This is typically a long and numerically intensive calculation.

Landau realized that near a phase transition an approximate form for the free energy can be constructed without first calculating the microscopic states. He recognized it is always possible to identify an order parameter that is zero on the high temperature side of the phase transition and nonzero on the low temperature side of the phase transition. For instance, the magnetization can be considered the order parameter at a ferromagnetic - paramagnetic phase transition. For a structural phase transition from a cubic phase to a tetragonal phase, the order parameter can be taken to be  $c/a - 1$  where  $c$  is the length of the long side of the tetragonal unit cell and  $a$  is the length of the short side of the tetragonal unit cell.

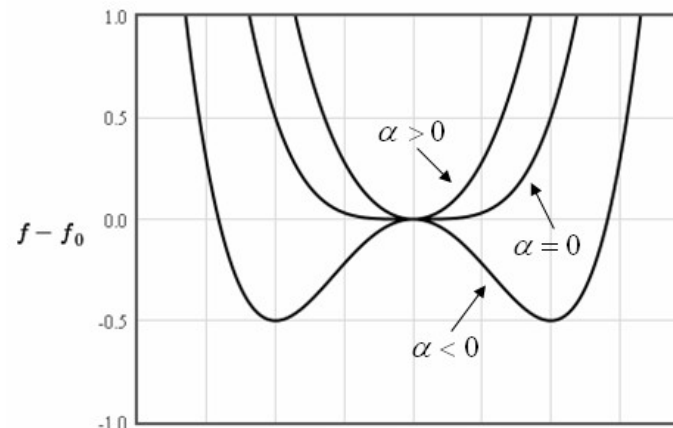


Lev Landau

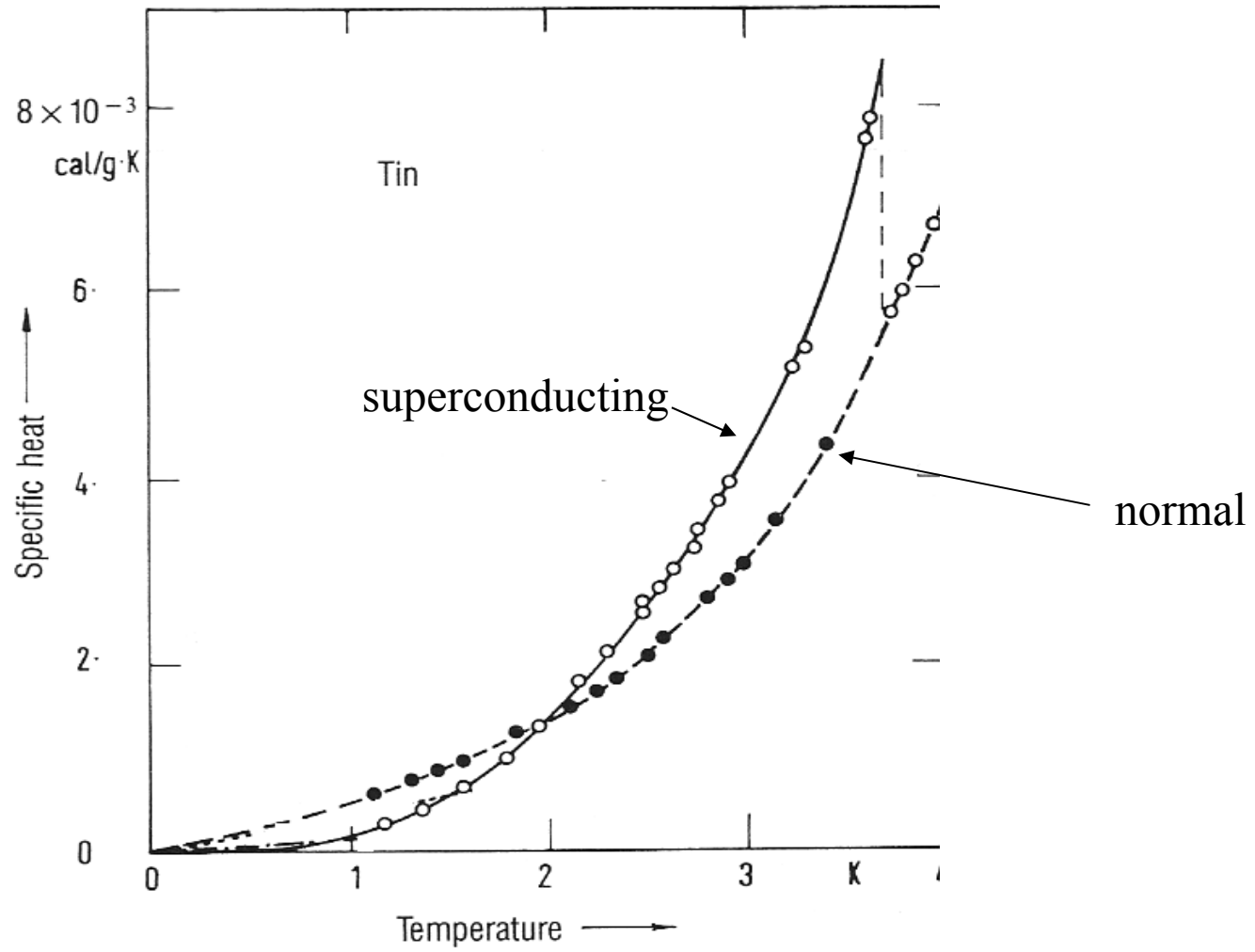
At a second order phase transition, the order parameter increases continuously from zero starting at the critical temperature of the phase transition. An example of this is the continuous increase of the magnetization at a ferromagnetic - paramagnetic phase transition. Since the order parameter is small near the phase transition, to a good approximation the free energy of the system can be approximated by the first few terms of a Taylor expansion of the free energy in the order parameter.

$$f(T) = f_0(T) + \alpha m^2 + \frac{1}{2} \beta m^4 \quad \alpha_0 > 0, \quad \beta > 0.$$

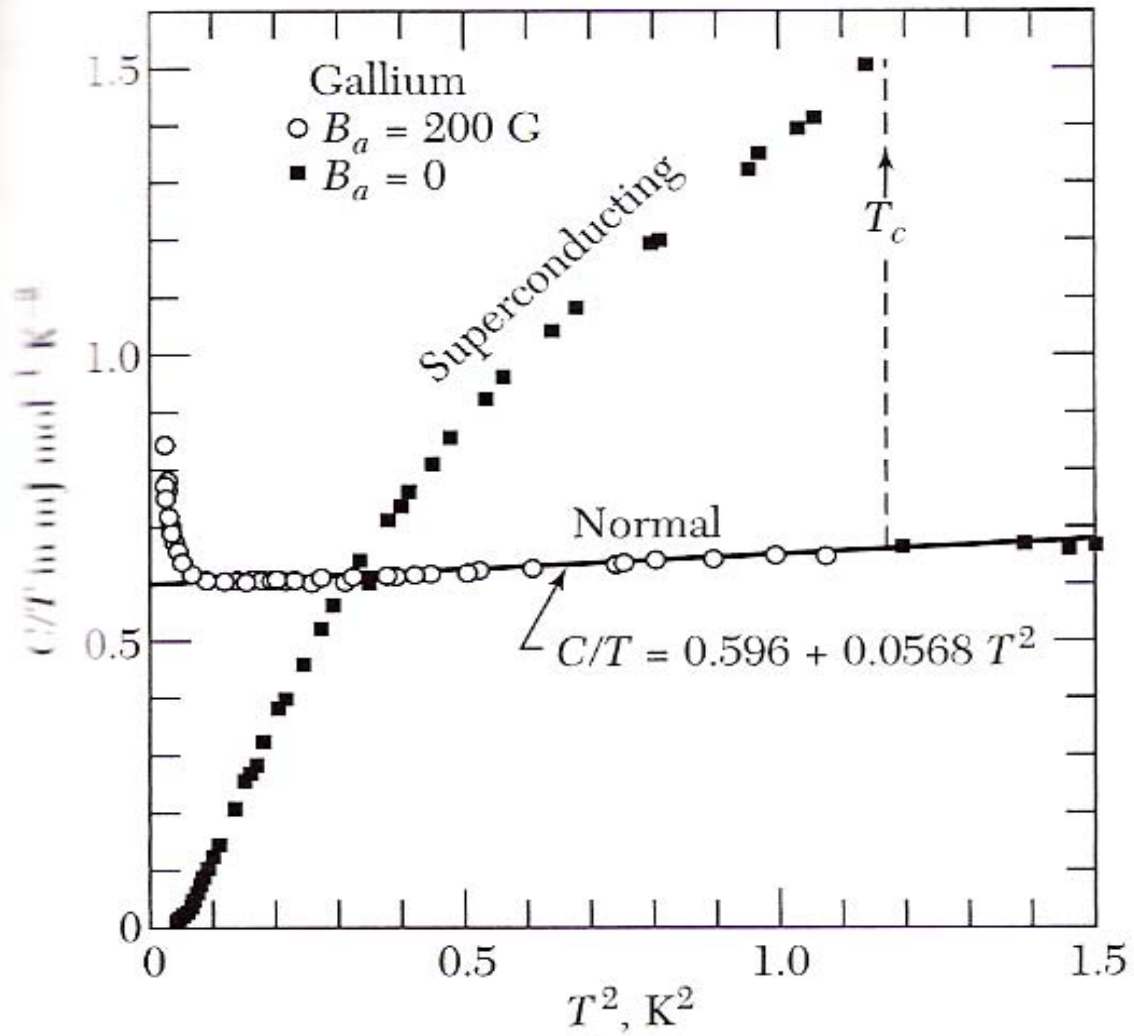
Here  $m$  is the order parameter,  $\alpha$  and  $\beta$  are parameters, and  $f_0(T)$  describes the temperature dependence of the high temperature phase near the phase transition. It is assumed that  $\beta > 0$  so that the free energy has a minimum for finite values of the order parameter. When  $\alpha > 0$ , there is only one minimum at  $m = 0$ . When  $\alpha < 0$  there are two minima with  $m \neq 0$ .



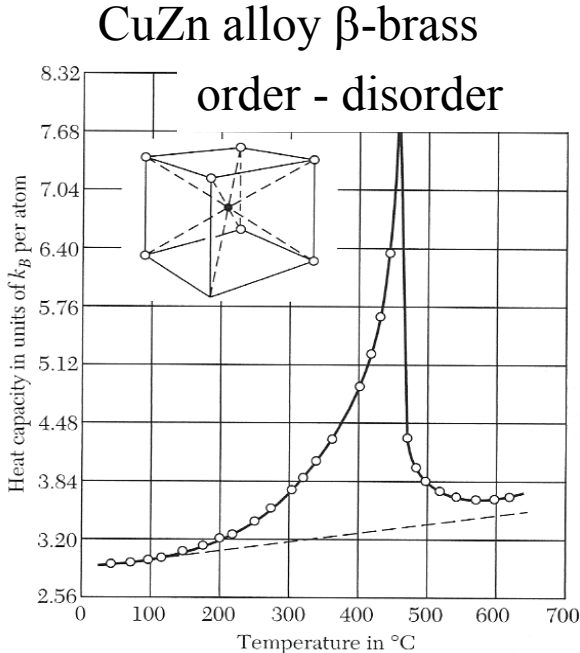
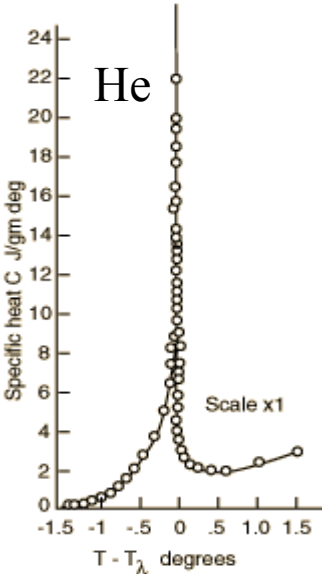
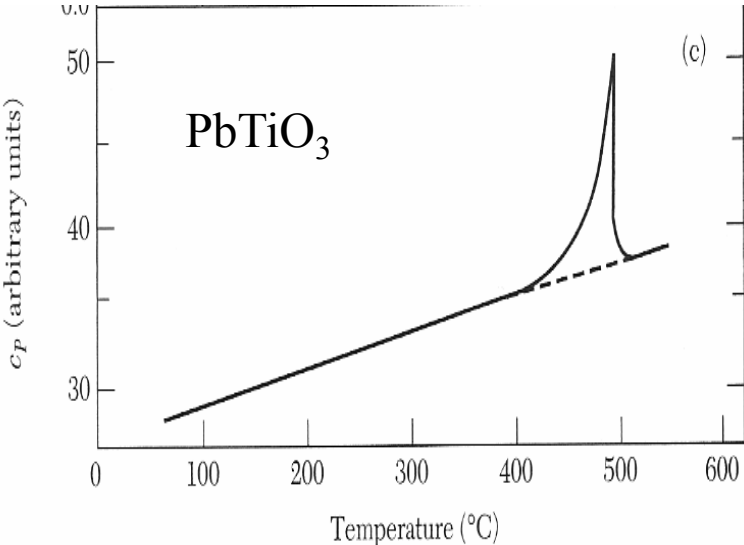
# Specific heat



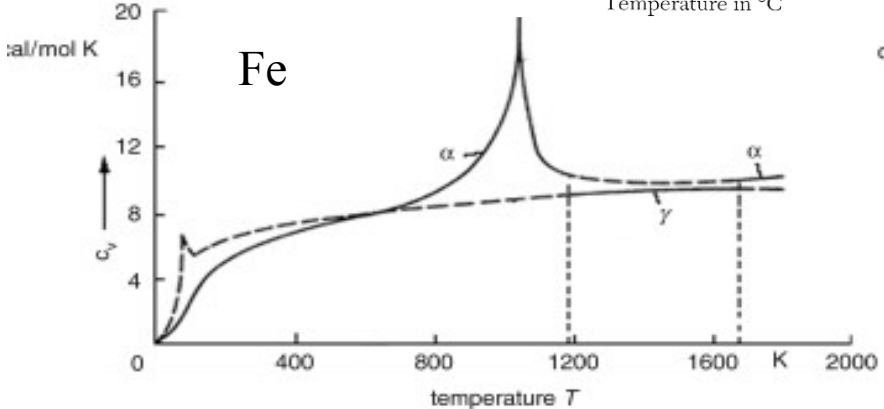
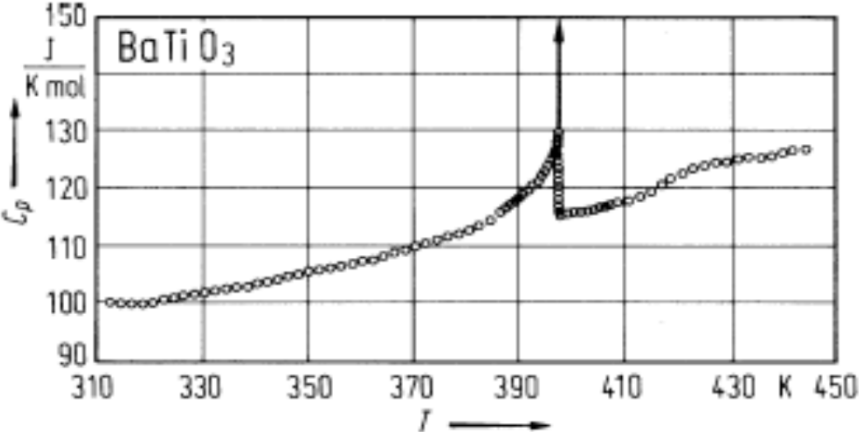
# Specific heat



# Specific heat



**BaTiO<sub>3</sub>. Heat capacity vs. temperature [76H].**



# Landau theory, susceptibility

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Add a magnetic field

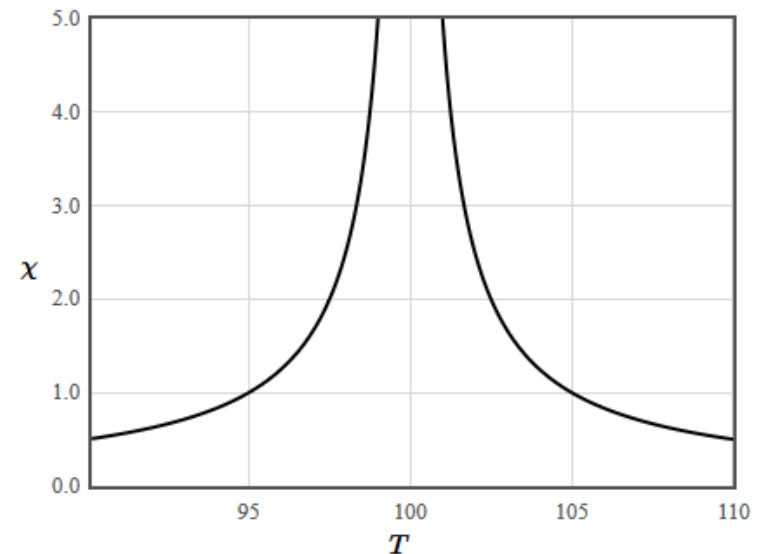
$$f = f_0 + \alpha_0 (T - T_c) m^2 + \frac{1}{2} \beta m^4 - mB$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 - B = 0$$

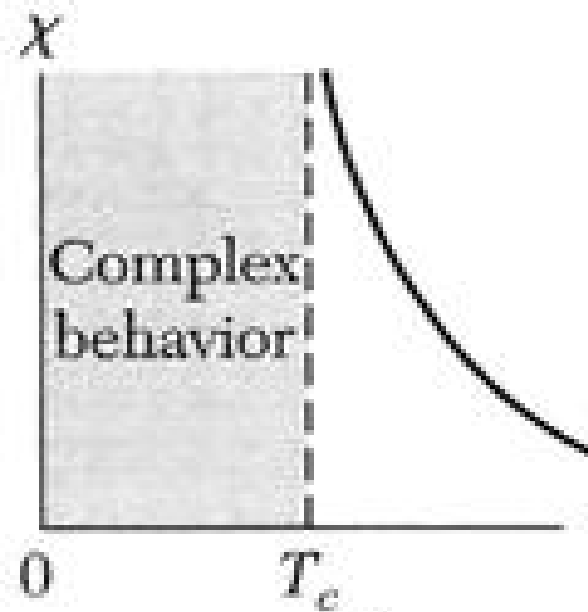
Above  $T_c$ ,  $m$  is finite for finite  $B$ . For small  $m$ ,

$$m = \frac{B}{2\alpha_0 (T - T_c)} \quad T > T_c$$

$$\chi = \frac{1}{2\alpha_0 (T - T_c)} \quad \text{Curie-Weiss}$$



## Ferromagnetism

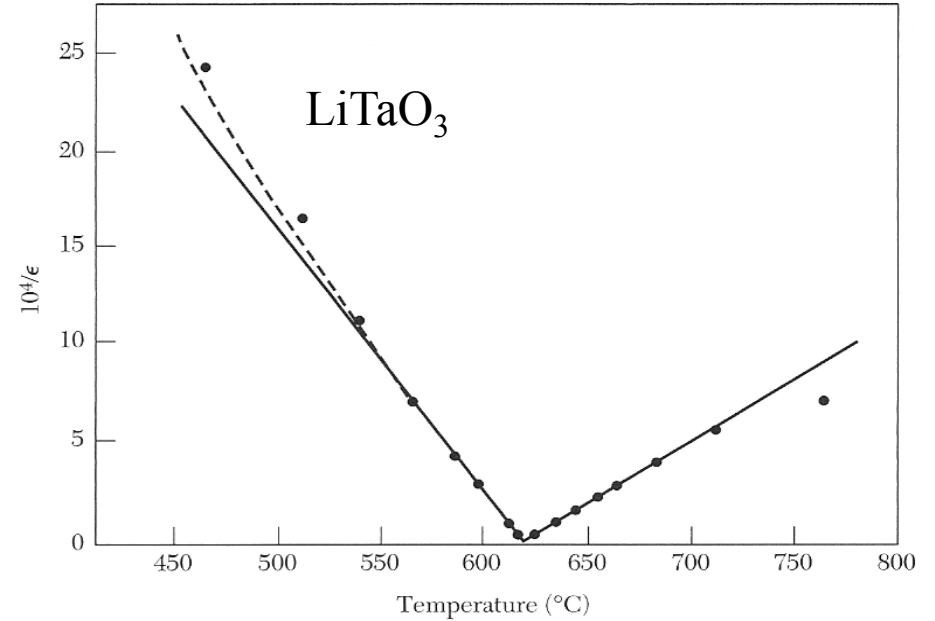
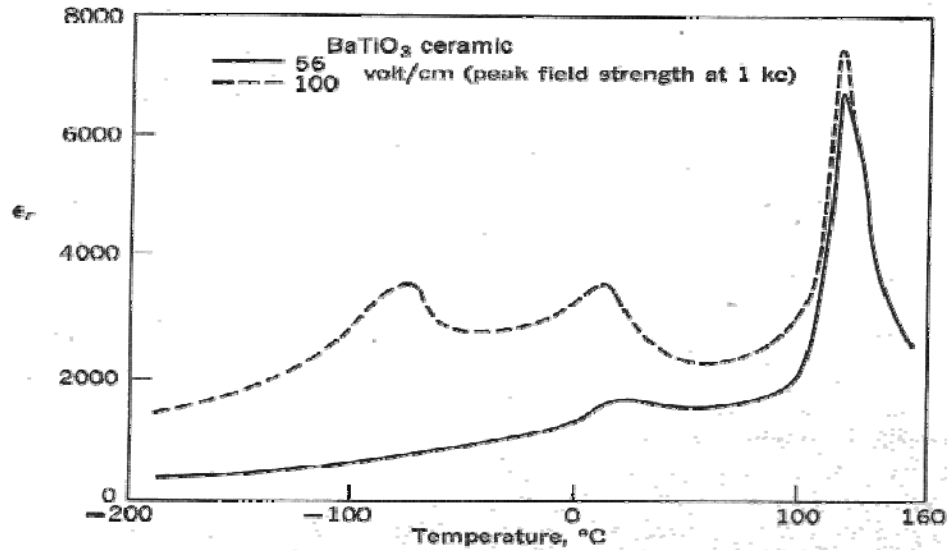


$$\chi = \frac{C}{T - T_c}$$

Curie-Weiss law  
( $T > T_c$ )



# Landau theory of phase transitions

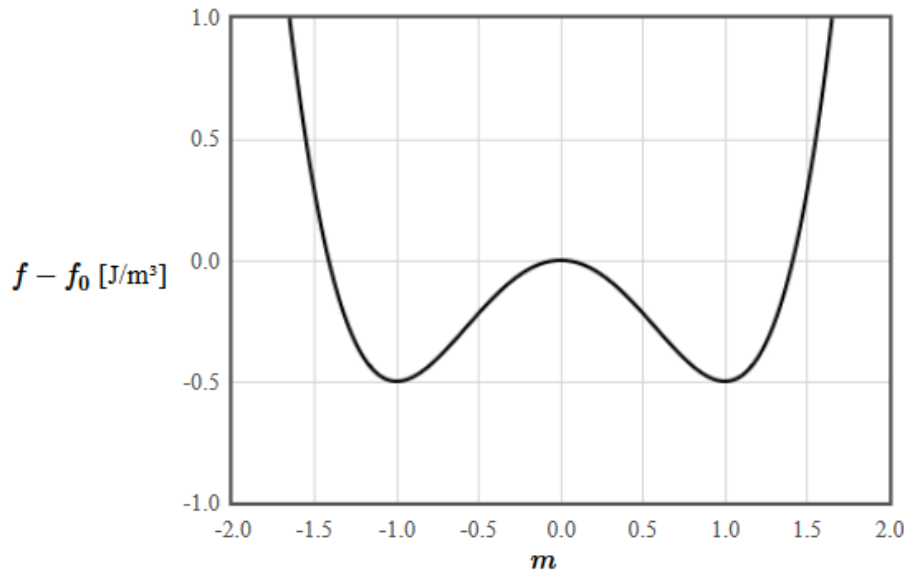


$$\epsilon_r = 1 + \chi$$

$$\chi = \frac{1}{2\alpha_0 (T - T_c)}$$

Curie-Weiss law

# Fitting the $\alpha_0$ and $\beta$ parameters

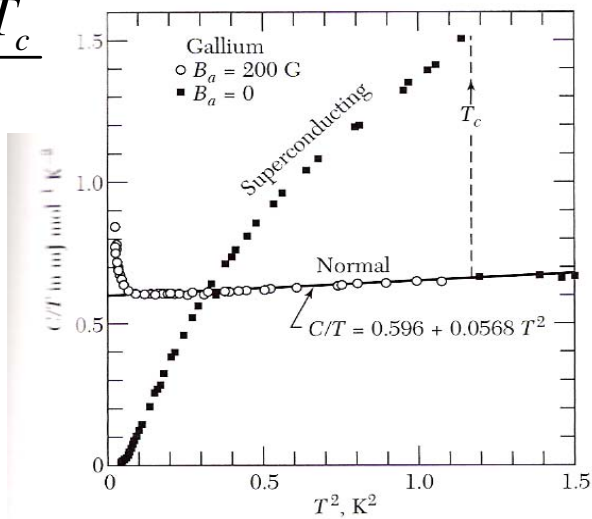


$\alpha_0 =$    
 $\beta =$    
 $T =$    
 $T_c =$    
 $f_0(T) =$

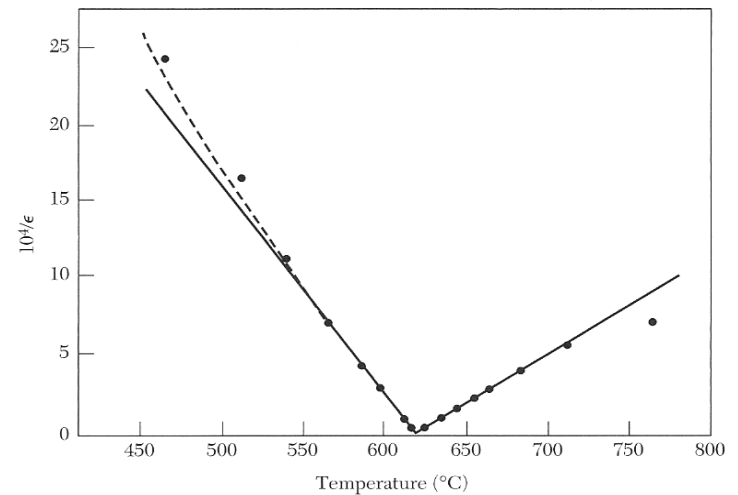
**Superconductivity**

**Ferromagnetism**

$$\Delta c_v = \frac{2\alpha_0^2 T_c}{\beta}$$

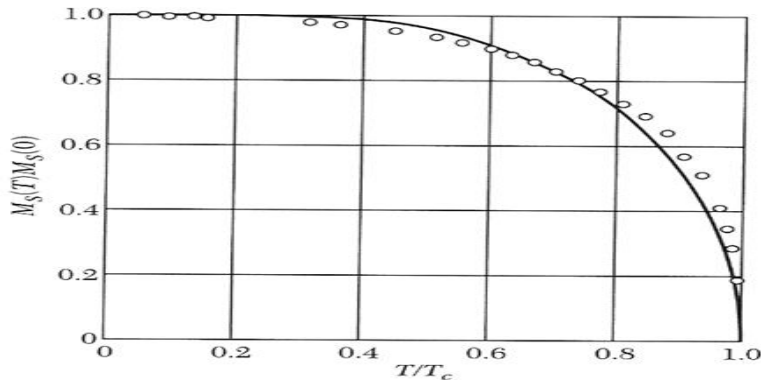


$$\chi = \frac{1}{2\alpha_0 (T - T_c)}$$

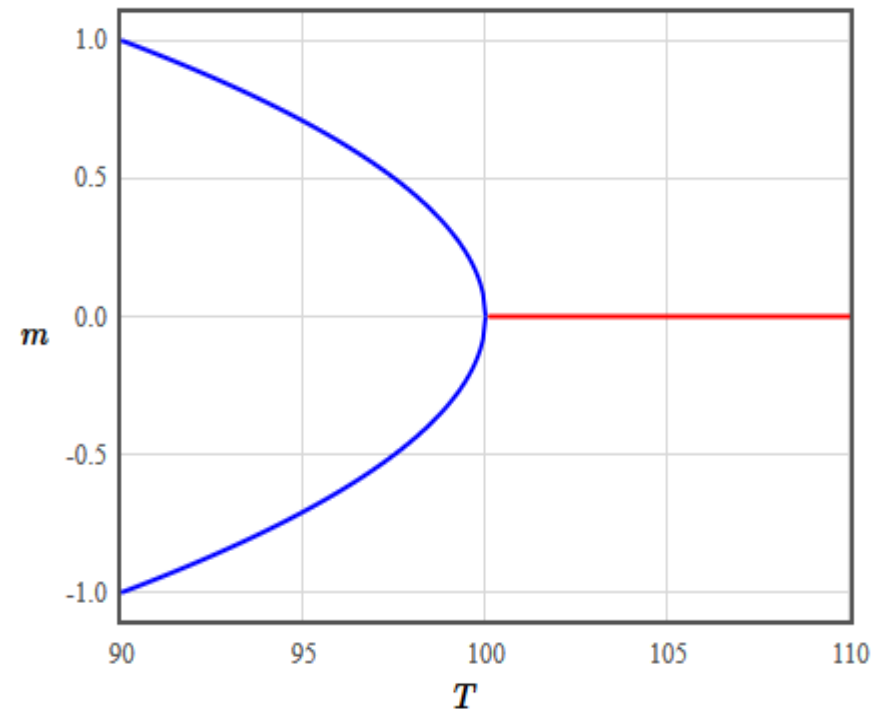


# Landau theory of phase transitions

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$$m = \sqrt{\frac{\alpha_0 (T_c - T)}{\beta}} \quad T < T_c$$

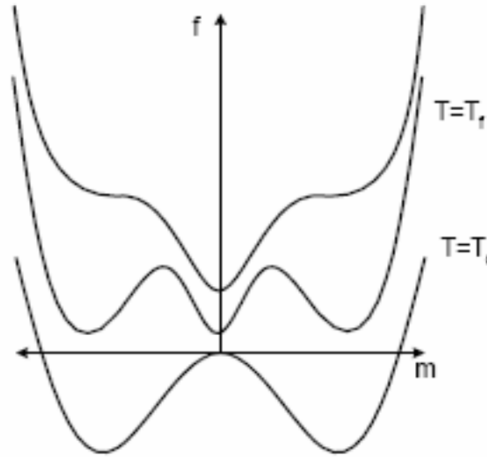


$\frac{\alpha_0}{\beta}$  can be determined from the temperature dependence of the order parameter

# First order transitions

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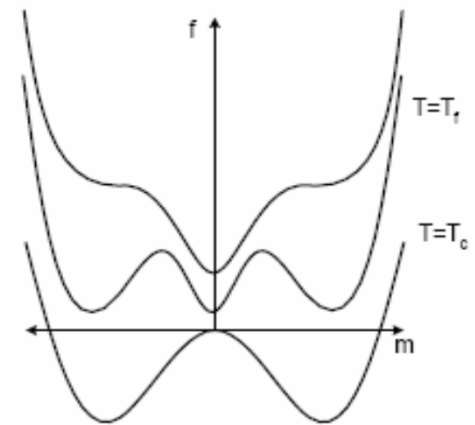
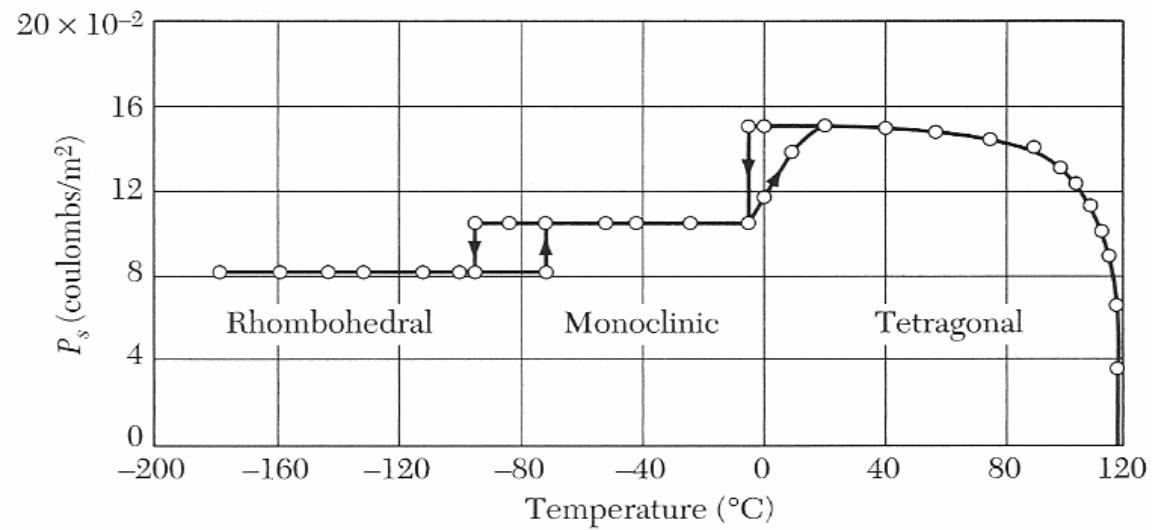
$$f = f_0 + \alpha_0 (T - T_c) m^2 + \frac{1}{2} \beta m^4 + \frac{1}{3} \gamma m^6 + \dots \quad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0$$



There is a jump in the order parameter at the phase transition.

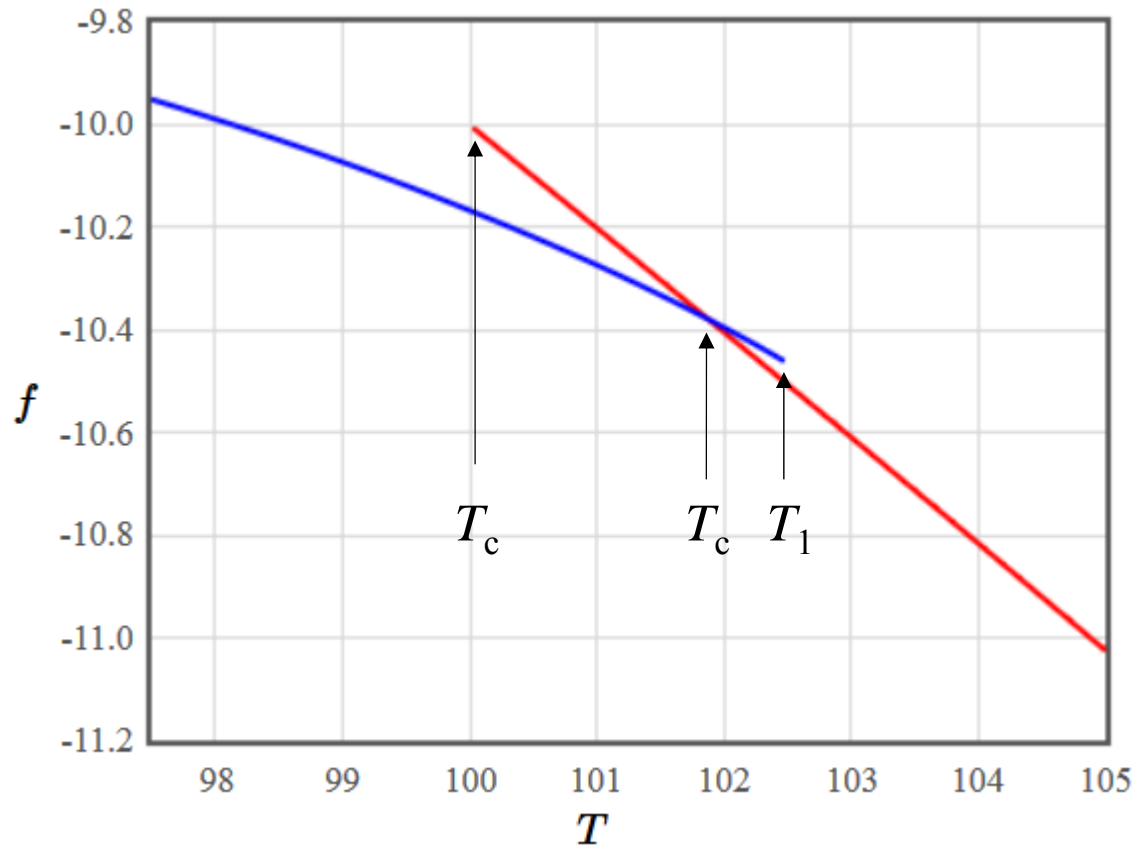
# First order transitions

BaTiO<sub>3</sub>



$T_c?$

---



# First order transitions

---

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \frac{1}{2} \beta m^4 + \frac{1}{3} \gamma m^6 + \dots \quad \beta < 0 \quad \gamma > 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 = 0$$

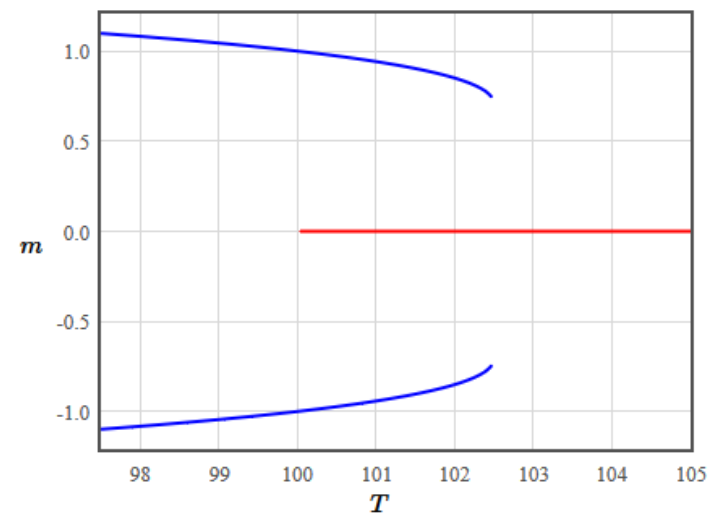
One solution for  $m = 0$ .

$$\alpha_0 (T - T_c) + \beta m^2 + \gamma m^4 = 0$$

$$m^2 = 0, \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c) \gamma}}{2\gamma}$$

There will be a minimum at finite  $m$  as long as  $m^2$  is real

$$T_1 = \frac{\beta^2}{4\alpha_0 \gamma} + T_c$$



# Jump in the order parameter

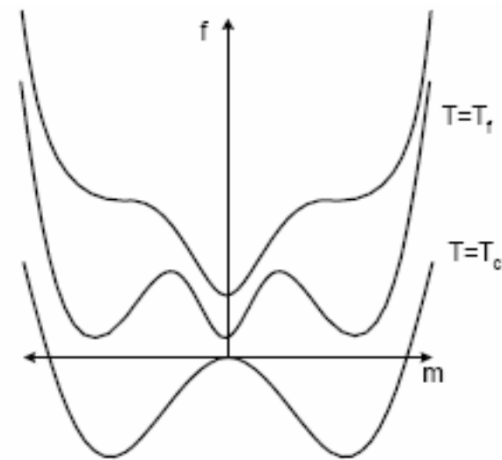
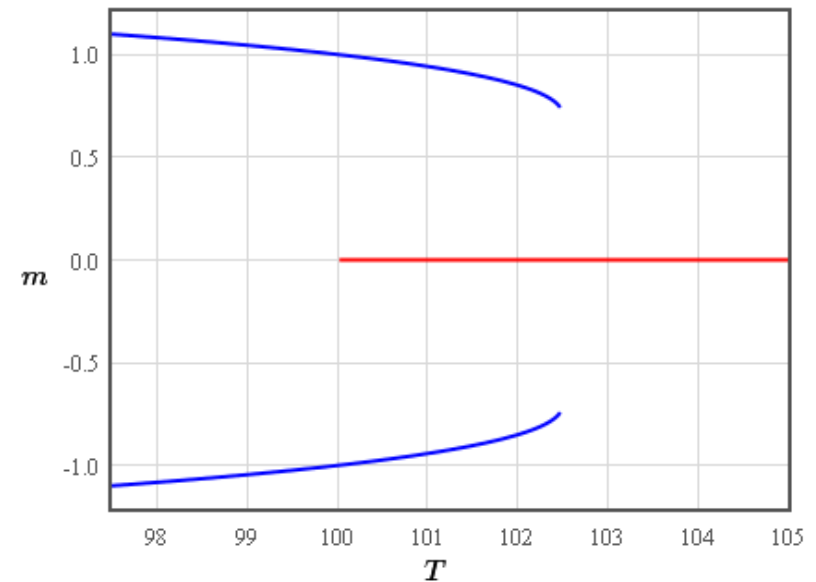
$$m^2 = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0(T - T_c)\gamma}}{2\gamma}$$

At  $T_c$

$$\Delta m = \sqrt{\frac{-\beta}{\gamma}}$$

At  $T_1$

$$\Delta m = \sqrt{\frac{-\beta}{2\gamma}}$$





# First order transitions, entropy, $c_v$

---

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \frac{1}{2} \beta m^4 + \frac{1}{3} \gamma m^6 + \dots \quad \beta < 0$$

$$m = 0, \pm \sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c) \gamma}}{2\gamma}}$$

$$s = -\frac{\partial f}{\partial T} = \frac{\alpha_0}{2\gamma} \left( \beta - \sqrt{\beta^2 - 4\alpha_0 \gamma (T - T_c)} \right)$$

$$c_v = T \frac{\partial s}{\partial T} = \frac{\alpha_0^2 T}{\sqrt{\beta^2 - 4\alpha_0 \gamma (T - T_c)}}$$

branch where the order parameter is nonzero

# First order transitions, susceptibility

---

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \frac{1}{2} \beta m^4 + \frac{1}{3} \gamma m^6 - mB \quad \beta < 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 - B = 0$$

At the minima  $B = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5$

For small  $m$ ,

$$\chi = \left. \frac{dm}{dB} \right|_{m=0} = \frac{1}{2\alpha_0 (T - T_c)} \quad \text{Curie - Weiss}$$

$$\chi = \left. \frac{dm}{dB} \right|_{m=\sqrt{\frac{-\beta}{2\gamma}}} = \frac{1}{2\alpha_0 (T - T_1)}$$

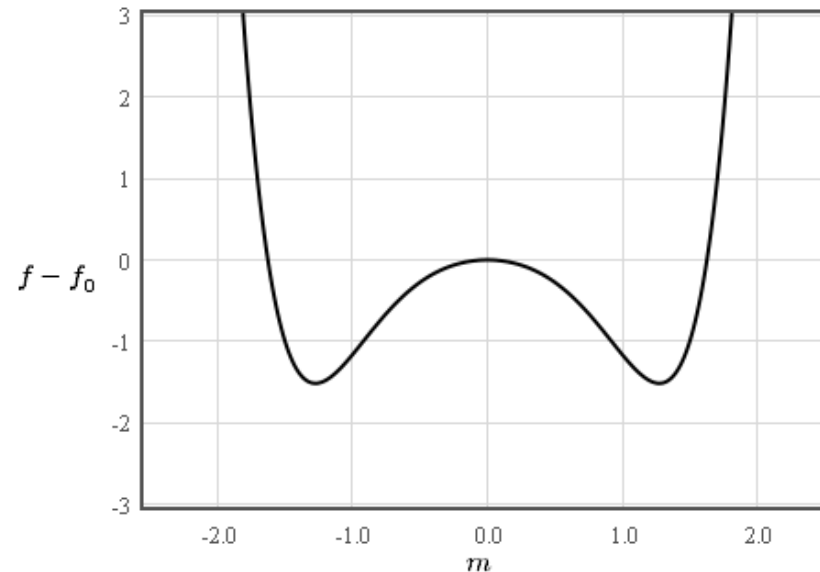
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## Landau theory of a first order phase transition

The free energy for a first order transition in Landau theory is,

$$f(T) = f_0(T) + \alpha_0(T - T_c)m^2 + \frac{1}{2}\beta m^4 + \frac{1}{3}\gamma m^6 \quad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0.$$

Here  $f_0(T)$  describes the temperature dependence of the high temperature phase near the phase transition.



$\alpha_0 =$

$\beta =$

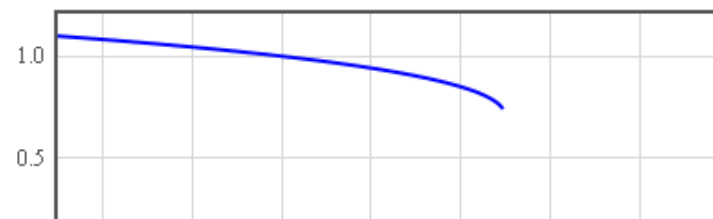
$\gamma =$

$T =$

$T_c =$

$f_0(T) =$

Order parameter



# Quasiparticles

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