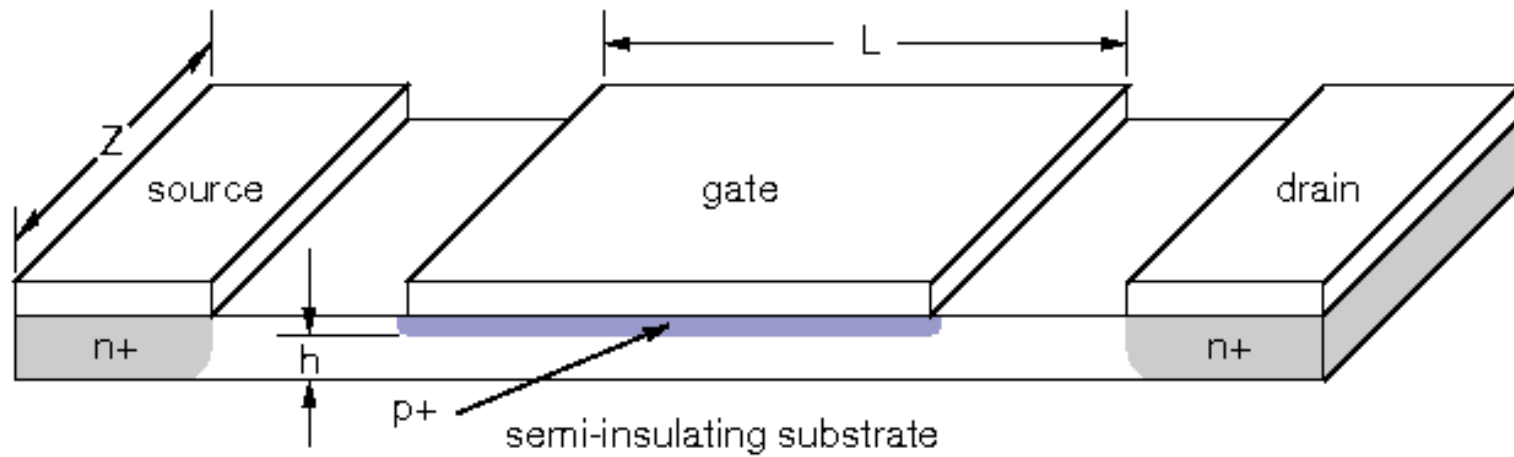


Semiconductor Devices

Electrons in a
magnetic field

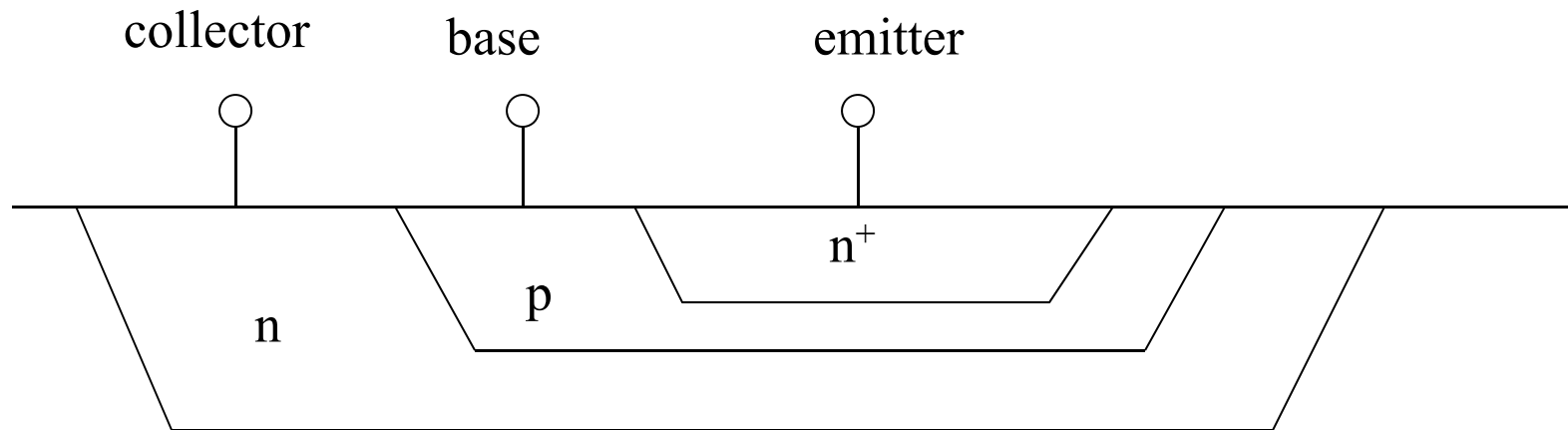
JFETs

Junction Field Effect Transistors



low noise

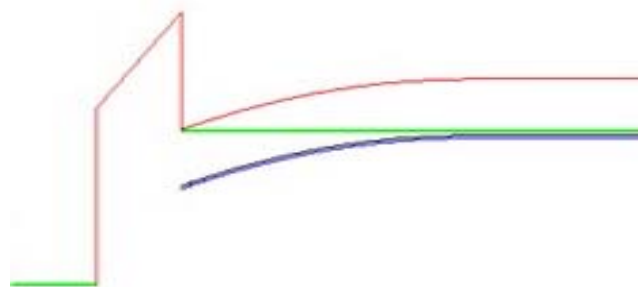
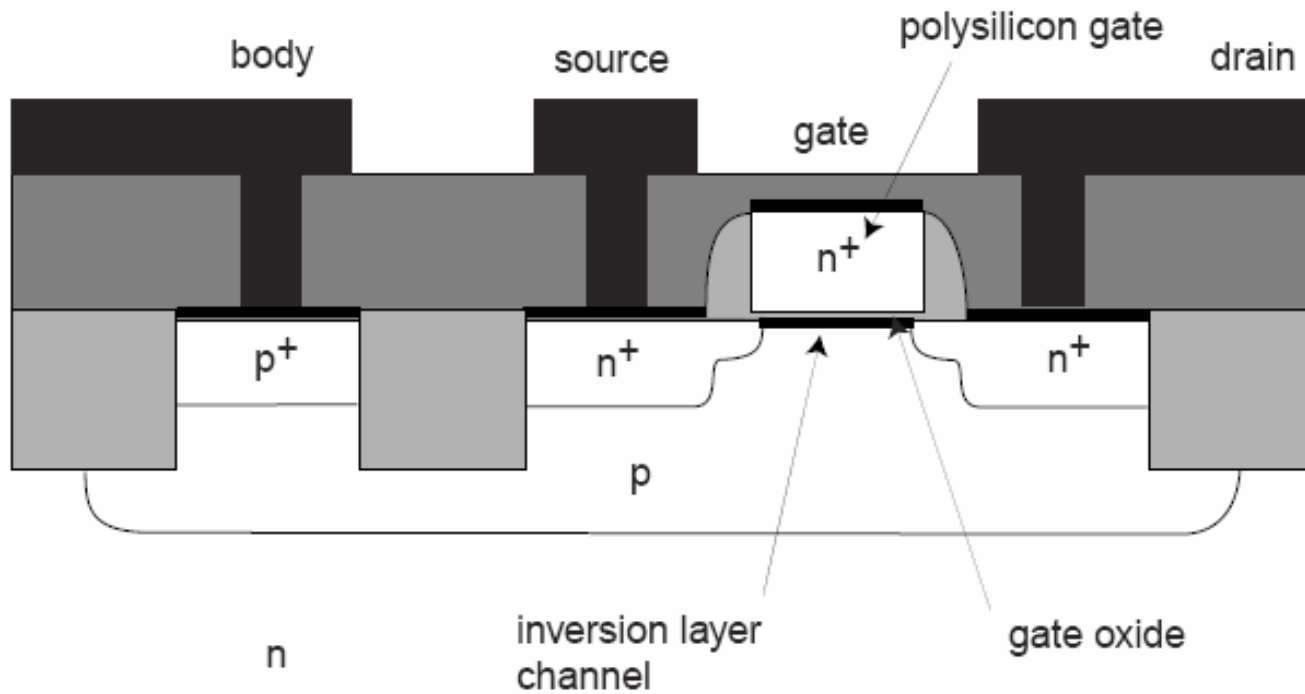
Bipolar transistor



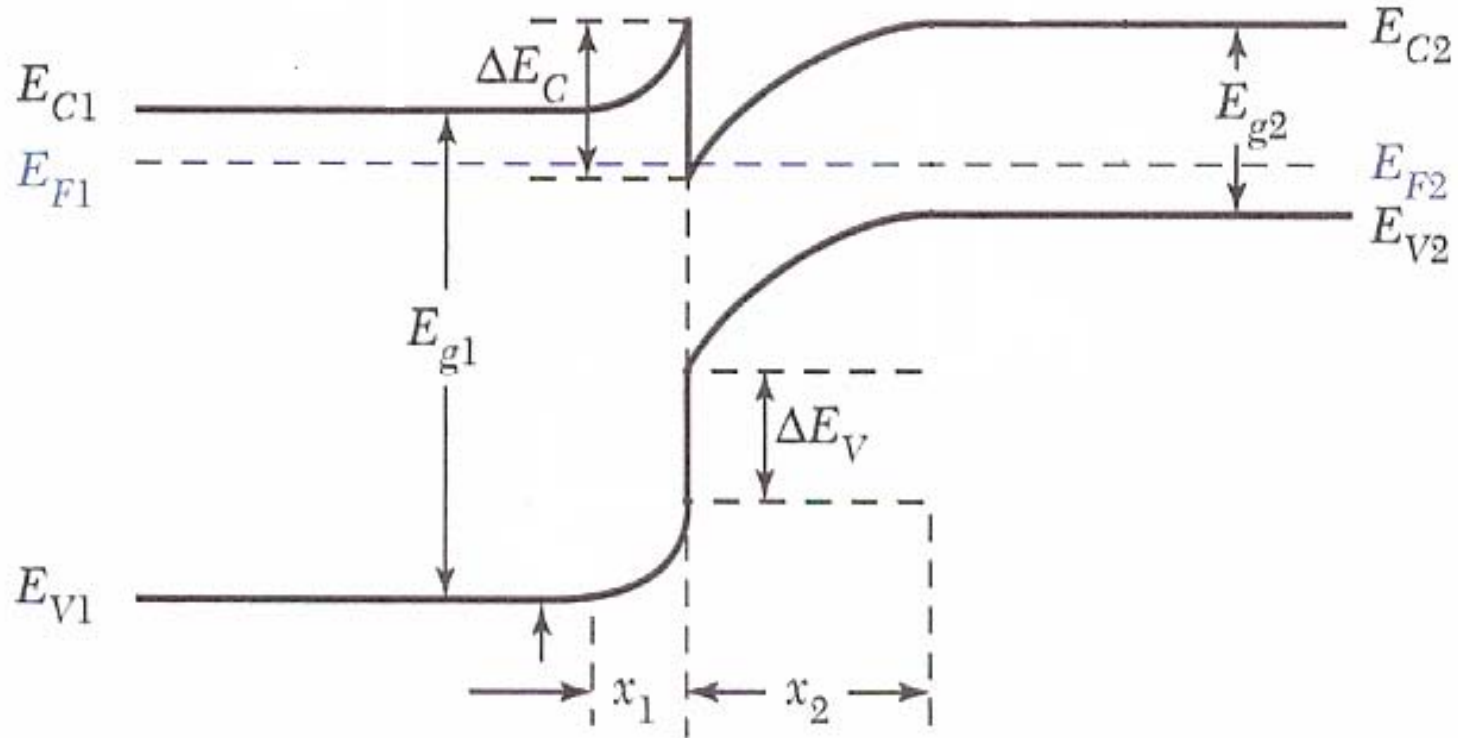
lightly doped p substrate

MOSFETs

Metal-oxide semiconductor field effect transistors



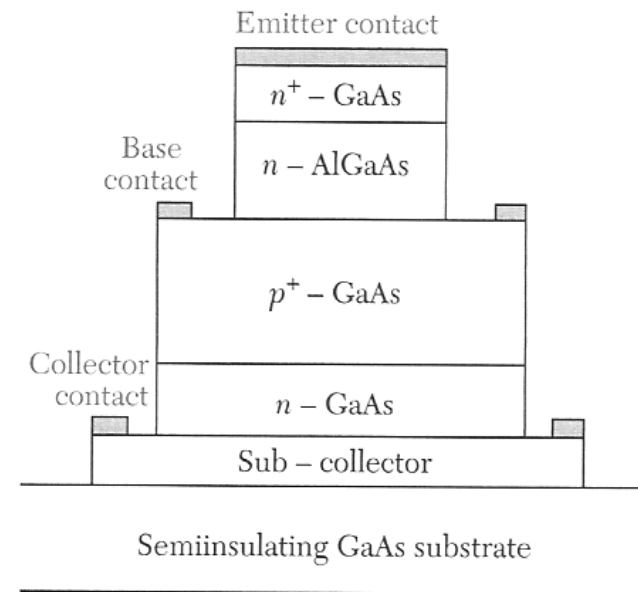
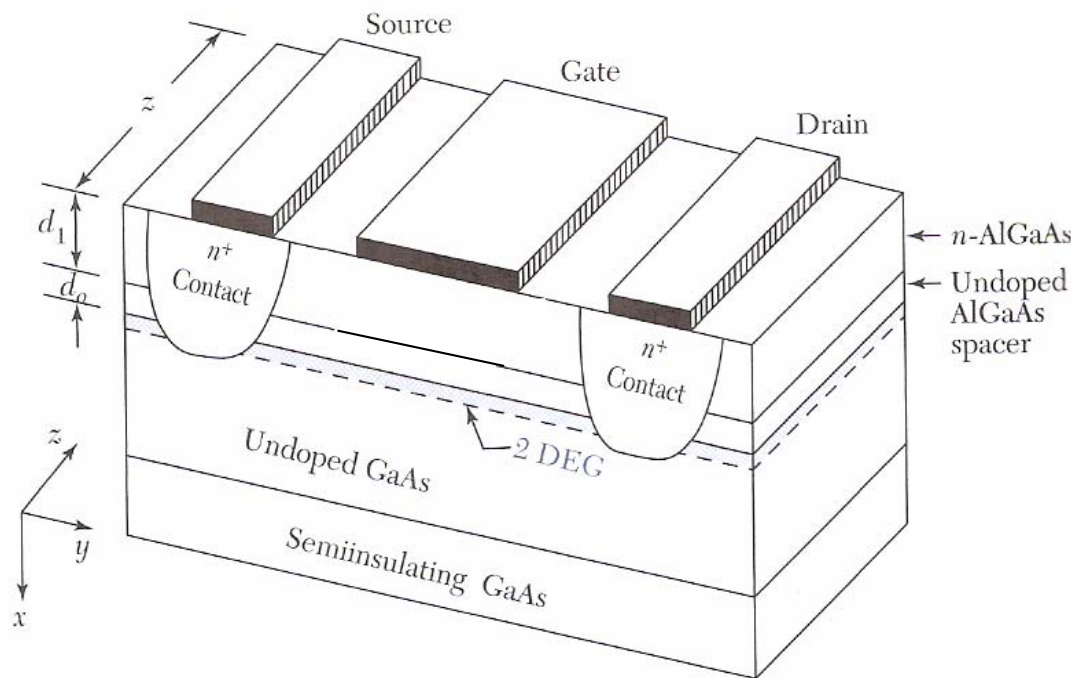
Heterojunctions



Quantum hall effect
Quantized conductance
HBTs
HEMTs

HEMT High electron mobility transistor

HBT Hetero junction bipolar transistor



Charged particle in a magnetic field

$$\vec{F} = -e\vec{v} \times \vec{B} = ma$$

$$evB_z = \frac{mv^2}{R}$$

solve for velocity

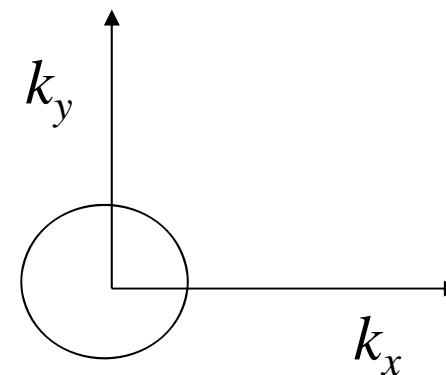
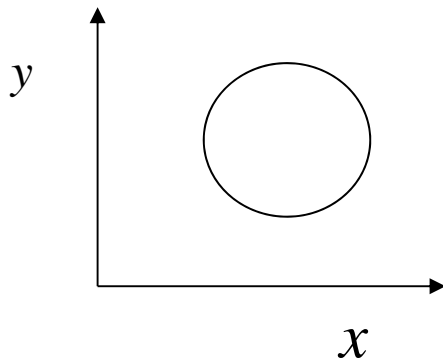
$$v = \frac{eB_z R}{m}$$

$$v = \omega_c R$$

$$\omega_c = \frac{eB_z}{m}$$



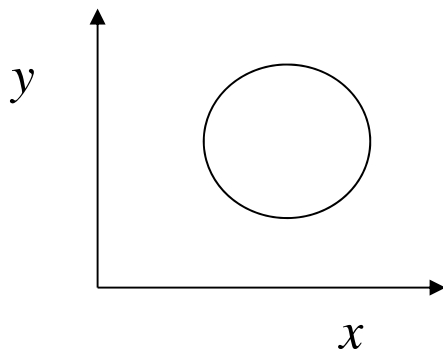
Magnetron



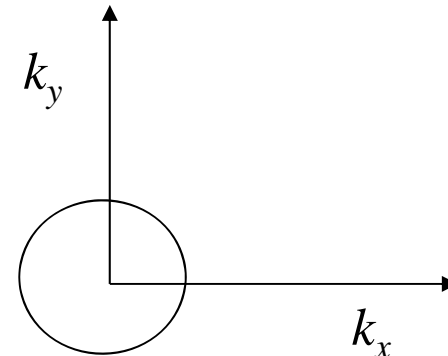
Why don't metals in a B field radiate?



Magnetron



$$\omega_c = \frac{eB_z}{m}$$



There are no lower lying states to fall into. We need a quantum description.

$$\frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

Magnetic force depends on the velocity, not on the position.

Electrons in a magnetic field

Lorentz force law $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Euler Lagrange equations $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0$

Lagrangian: $L = \frac{1}{2} m v^2 - qV(\vec{r}, t) + q\vec{v} \cdot \vec{A}(\vec{r}, t)$

Kittel: Appendix G

<http://lamp.tu-graz.ac.at/~hadley/ss2/IQHE/cpimf.php>

Lagrangian

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0$$

$$L = \frac{1}{2} m v^2 - qV(\vec{r}, t) + q\vec{v} \cdot \vec{A}(\vec{r}, t)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial v_x} \right) = \frac{d}{dt} (m v_x + q A_x) = m \frac{d v_x}{dt} + q \frac{d A_x}{dt}$$

$$= m \frac{d v_x}{dt} + q \left(\frac{d x}{d t} \frac{\partial A_x}{\partial x} + \frac{d y}{d t} \frac{\partial A_x}{\partial y} + \frac{d z}{d t} \frac{\partial A_x}{\partial z} + \frac{\partial A_x}{\partial t} \right)$$

$$= m \frac{d v_x}{dt} + q \left(v_x \frac{\partial A_x}{\partial x} + v_y \frac{\partial A_x}{\partial y} + v_z \frac{\partial A_x}{\partial z} + \frac{\partial A_x}{\partial t} \right),$$

$$\frac{\partial L}{\partial x} = -q \frac{\partial V}{\partial x} + q \left(v_x \frac{\partial A_x}{\partial x} + v_y \frac{\partial A_y}{\partial x} + v_z \frac{\partial A_z}{\partial x} \right)$$

$$m \frac{d v_x}{dt} = -q \left(\frac{\partial V}{\partial x} + \frac{\partial A_x}{\partial t} \right) + q \left(v_y \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) + v_z \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) \right)$$

Lagrangian

$$m \frac{dv_x}{dt} = -q \left(\frac{\partial V}{\partial x} + \frac{\partial A_x}{\partial t} \right) + q \left(v_y \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) + v_z \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) \right)$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \quad \text{and} \quad \vec{B} = \nabla \times \vec{A}$$

$$m \frac{dv_x}{dt} = q \left(E_x + (\vec{v} \times \vec{B})_x \right)$$

$$L = \frac{1}{2} m v^2 - qV(\vec{r}, t) + q\vec{v} \cdot \vec{A}(\vec{r}, t)$$

conjugate variable: $p_x = \frac{\partial L}{\partial v_x} = m v_x + q A_x$

kinetic momentum $v_x = \frac{1}{m} (p_x - q A_x)$ field momentum (inductance)

Hamiltonian

$$v_x = \frac{1}{m}(p_x - qA_x)$$

$$L = \frac{1}{2}mv^2 - qV(\vec{r}, t) + q\vec{v} \cdot \vec{A}(\vec{r}, t)$$

Legendre transformation $H = \vec{v} \cdot \vec{p} - L$

Classical result $\rightarrow H = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + qV$

$$\vec{p} \rightarrow -i\hbar\nabla$$

Schrödinger equation $i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}(-i\hbar\nabla - q\vec{A})^2 \psi + qV\psi$

Landau Levels

free particles in a magnetic field

$$\frac{1}{2m}(-i\hbar\nabla - q\vec{A})^2 \psi(\vec{r}) = E\psi(\vec{r}). \quad V=0$$

Landau gauge $\vec{A} = B_z x \hat{y}.$

$$\vec{B} = \nabla \times \vec{A} = B_z x \hat{y} = \left(\frac{dA_z}{dy} - \frac{dA_y}{dz} \right) \hat{x} + \left(\frac{dA_x}{dz} - \frac{dA_z}{dx} \right) \hat{y} + \left(\frac{dA_y}{dx} - \frac{dA_x}{dy} \right) \hat{z}.$$

$$\vec{B} = B_z \hat{z}.$$

Landau Levels

free particles in a magnetic field

$$\frac{1}{2m} \left(-i\hbar\nabla - q\vec{A} \right)^2 \psi(\vec{r}) = E\psi(\vec{r}). \quad V=0$$

Landau gauge $\vec{A} = B_z x \hat{y}$.

$$\left(-i\hbar\nabla - q\vec{A} \right)^2 = \left(-i\hbar\nabla - qB_z x \hat{y} \right) \cdot \left(-i\hbar\nabla - qB_z x \hat{y} \right)$$

$$-i\hbar\nabla \cdot (-qB_z x \hat{y}) = -i\hbar \left(\frac{d}{dx} \hat{x} + \frac{d}{dy} \hat{y} + \frac{d}{dz} \hat{z} \right) \cdot (-qB_z x \hat{y}) = i\hbar q B_z x \frac{d}{dy}$$

$$\frac{1}{2m} \left(-\hbar^2 \nabla^2 + i2\hbar q B_z x \frac{d}{dy} + q^2 B_z^2 x^2 \right) \psi = E\psi.$$

The solution has the form

$$\psi = e^{ik_y y} e^{ik_z z} \phi(x).$$

Landau Levels

$$\frac{1}{2m} \left(-\hbar^2 \nabla^2 + i2\hbar q B_z x \frac{d}{dy} + q^2 B_z^2 x^2 \right) \psi = E\psi.$$

The solution has the form

$$\psi = e^{ik_y y} e^{ik_z z} \phi(x).$$

Substitute this into the equation

$$\frac{1}{2m} \left(-\hbar^2 \frac{\partial^2}{\partial x^2} + \hbar^2 k_z^2 + \hbar^2 k_y^2 - 2\hbar q B_z k_y x + q^2 B_z^2 x^2 \right) \phi(x) = E\phi(x).$$

Landau Levels

The equation for $\phi(x)$ is

$$\frac{1}{2m} \left(-\hbar^2 \frac{\partial^2}{\partial x^2} + \hbar^2 k_z^2 + \underbrace{\hbar^2 k_y^2 - 2\hbar q B_z k_y x + q^2 B_z^2 x^2}_{(\hbar k_y - q B_z x)^2} \right) \phi(x) = E \phi(x).$$

$$\frac{1}{2m} \left(-\hbar^2 \frac{\partial^2}{\partial x^2} + q^2 B_z^2 \left(x - \frac{\hbar k_y}{q B_z} \right)^2 \right) \phi(x) = \left(E - \frac{\hbar^2 k_z^2}{2m} \right) \phi(x).$$

This is the equation for a harmonic oscillator

$$\left(\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{K}{2} (x - x_0)^2 \right) \phi(x) = E' \phi(x).$$

Landau Levels

$$\frac{1}{2m} \left(-\hbar^2 \frac{\partial^2}{\partial x^2} + q^2 B_z^2 (x - x_0)^2 \right) \phi(x) = \left(E - \frac{\hbar^2 k_z^2}{2m} \right) \phi(x). \quad x_0 = \frac{\hbar k_y}{q B_z}$$
$$\left(\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{K}{2} (x - x_0)^2 \right) \phi(x) = E' \phi(x).$$

This is the equation for a harmonic oscillator

$$\frac{K}{2} \Leftrightarrow \frac{q^2 B_z^2}{2m}$$

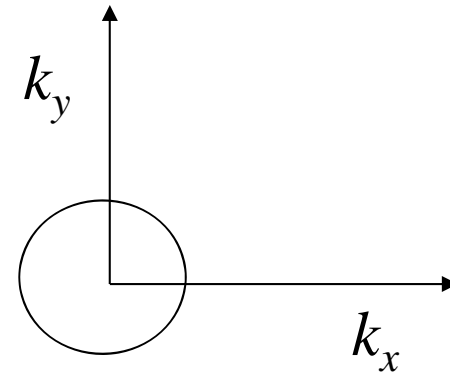
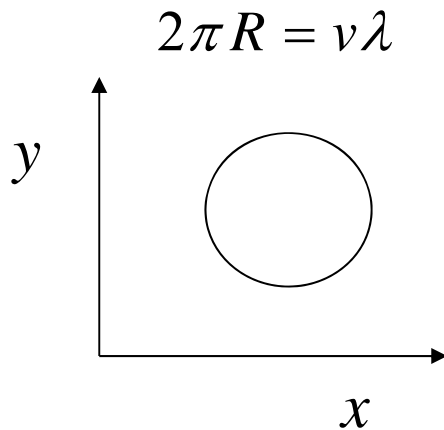
$$\omega_c = \sqrt{\frac{K}{m}} \Leftrightarrow \frac{q B_z}{m}$$

$$E_{k_z, \nu} = \frac{\hbar^2 k_z^2}{2m} + \hbar \omega_c \left(\nu + \frac{1}{2} \right)$$

$$\omega_c = \frac{q B_z}{m}$$

Charged particle in a magnetic field

Bohr - Sommerfeld quantization

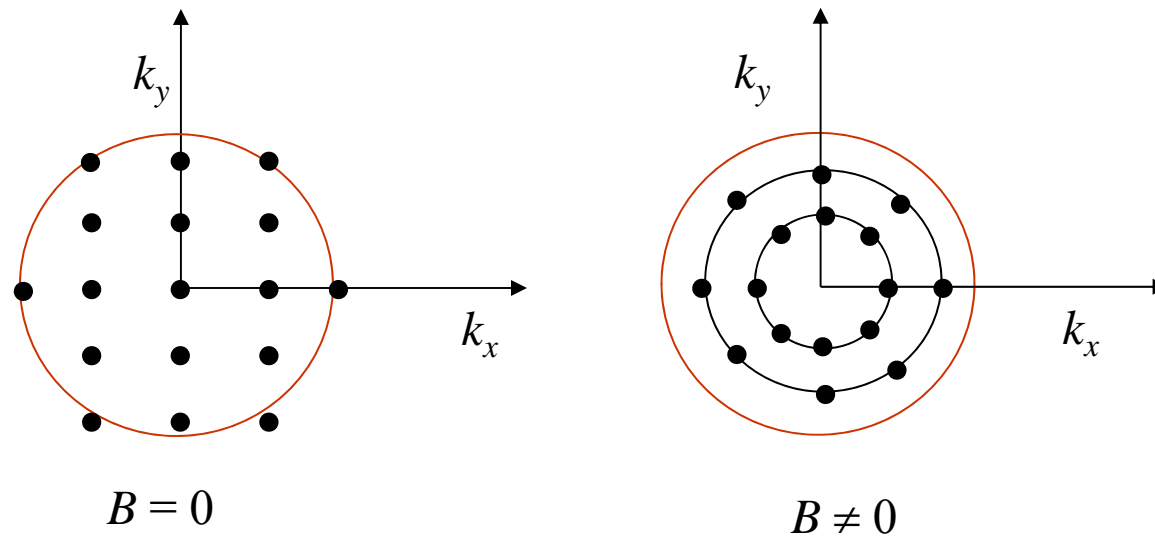


Circular motion is harmonic motion. Harmonic motion is quantized.

$$E_\nu = \hbar\omega_c \left(\nu + \frac{1}{2}\right) = \frac{\hbar^2}{2m} (k_x^2 + k_y^2)$$

ν labels the Landau level

Landau levels



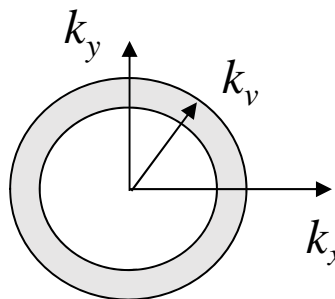
The global density of states in k -space does not change

In 2-D, the k -volume per k state is: $\left(\frac{2\pi}{L}\right)^2$

Density of states 2D

$$E_\nu = \hbar \omega_c \left(\nu + \frac{1}{2} \right)$$

The number of states between ring $\nu-1$ and ring ν is


$$\frac{\pi (k_\nu^2 - k_{\nu-1}^2)}{\left(\frac{2\pi}{L} \right)^2} \quad \frac{\hbar^2 k_\nu^2}{2m} = \hbar \omega_c \left(\nu + \frac{1}{2} \right)$$

$$k_\nu^2 - k_{\nu-1}^2 = \frac{2m\omega_c}{\hbar} \left[\left(\nu + \frac{1}{2} \right) - \left(\nu - 1 + \frac{1}{2} \right) \right] = \frac{2m\omega_c}{\hbar}$$

The number of states between ring $\nu-1$ and ring ν is $\frac{m\omega_c}{2\pi\hbar} L^2$

The density of states per spin is $\frac{m\omega_c}{2\pi\hbar}$

Spin

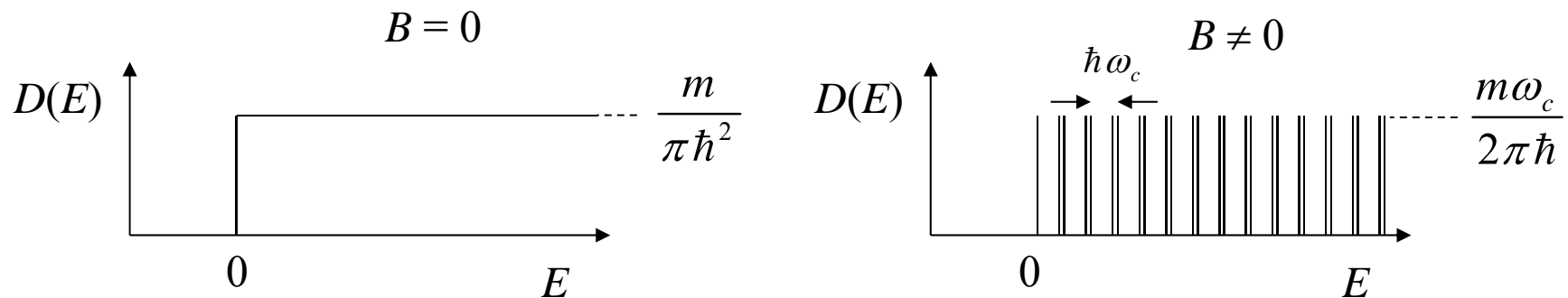
In a magnetic field, there is a shift of the energy of the electrons because of their spin.

$$E = -\vec{\mu} \cdot \vec{B} = \pm \frac{g}{2} \mu_B B$$

Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$

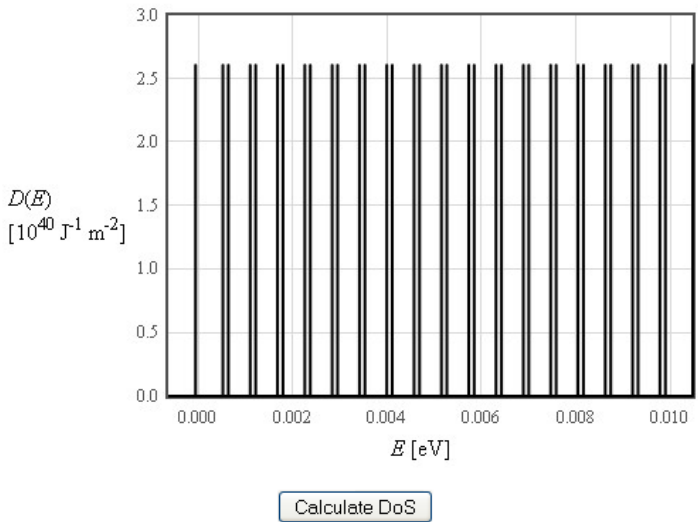
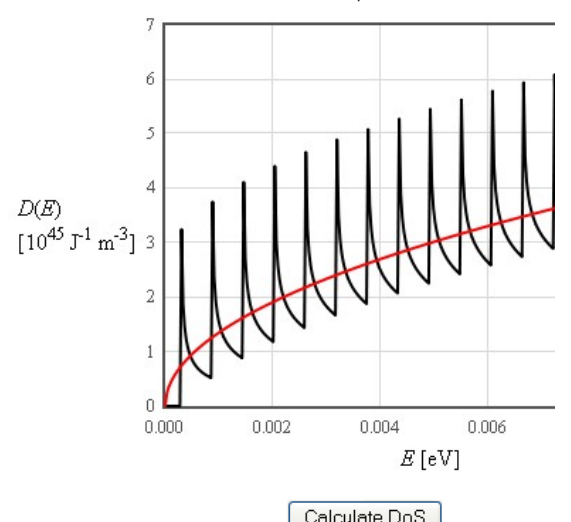
g-factor $g \approx 2$

$$\hbar\omega_c = \frac{\hbar e B}{m} = 2\mu_B B$$



$$D(E) = \frac{m\omega_c}{2\pi\hbar} \sum_{\nu=0}^{\infty} \delta\left(E - \hbar\omega_c \left(\nu + \frac{1}{2} - \frac{g}{4}\right)\right) + \delta\left(E - \hbar\omega_c \left(\nu + \frac{1}{2} + \frac{g}{4}\right)\right)$$

ization of the Schrödinger equation for free electrons a magnetic field in 2 and 3 dimensions.

	2-D Schrödinger equation	3-D Schrödinger equation
Eigenfunction solutions	$i\hbar \frac{d\psi}{dt} = \frac{1}{2m} (-i\hbar\nabla - e \vec{A})^2 \psi$ $\psi = g_v(x) \exp(ik_y y)$ $g_v(x) \text{ is a harmonic oscillator wavefunction}$	$i\hbar \frac{d\psi}{dt} = \frac{1}{2m} (-i\hbar\nabla - e \vec{A})^2 \psi$ $\psi = g_v(x) \exp(ik_y y) \exp(ik_z z)$ $g_v(x) \text{ is a harmonic oscillator wavefun}$
Energy eigenvalues	$E = \hbar\omega_c \left(v + \frac{1}{2}\right) \text{ J}$ $v = 0, 1, 2, \dots \quad \omega_c = \frac{ eB_z }{m}$	$E = \frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c \left(v + \frac{1}{2}\right) \text{ J}$ $v = 0, 1, 2, \dots \quad \omega_c = \frac{ eB_z }{m}$
Density of states	$D(E) = \frac{m\omega_c}{2\pi\hbar} \sum_{v=0}^{\infty} \delta\left(E - \hbar\omega_c \left(v + \frac{1}{2}\right) - \frac{g\mu_B}{2} B\right) + \delta\left(E - \hbar\omega_c \left(v + \frac{1}{2}\right) + \frac{g\mu_B}{2} B\right) \text{ J}^{-1}\text{m}^{-2}$ 	$D(E) = \frac{(2m)^{3/2} \omega_c}{4\pi^2 \hbar^2} \sum_{v=0}^{\infty} \frac{H\left(E - \hbar\omega_c \left(v + \frac{1}{2}\right)\right)}{\sqrt{E - \hbar\omega_c \left(v + \frac{1}{2}\right)}}$ 
	$E_F = \hbar\omega_c \left(\text{Int} \left(\frac{\pi \hbar n}{m\omega_c} \right) + \frac{1}{2} \right)$ 