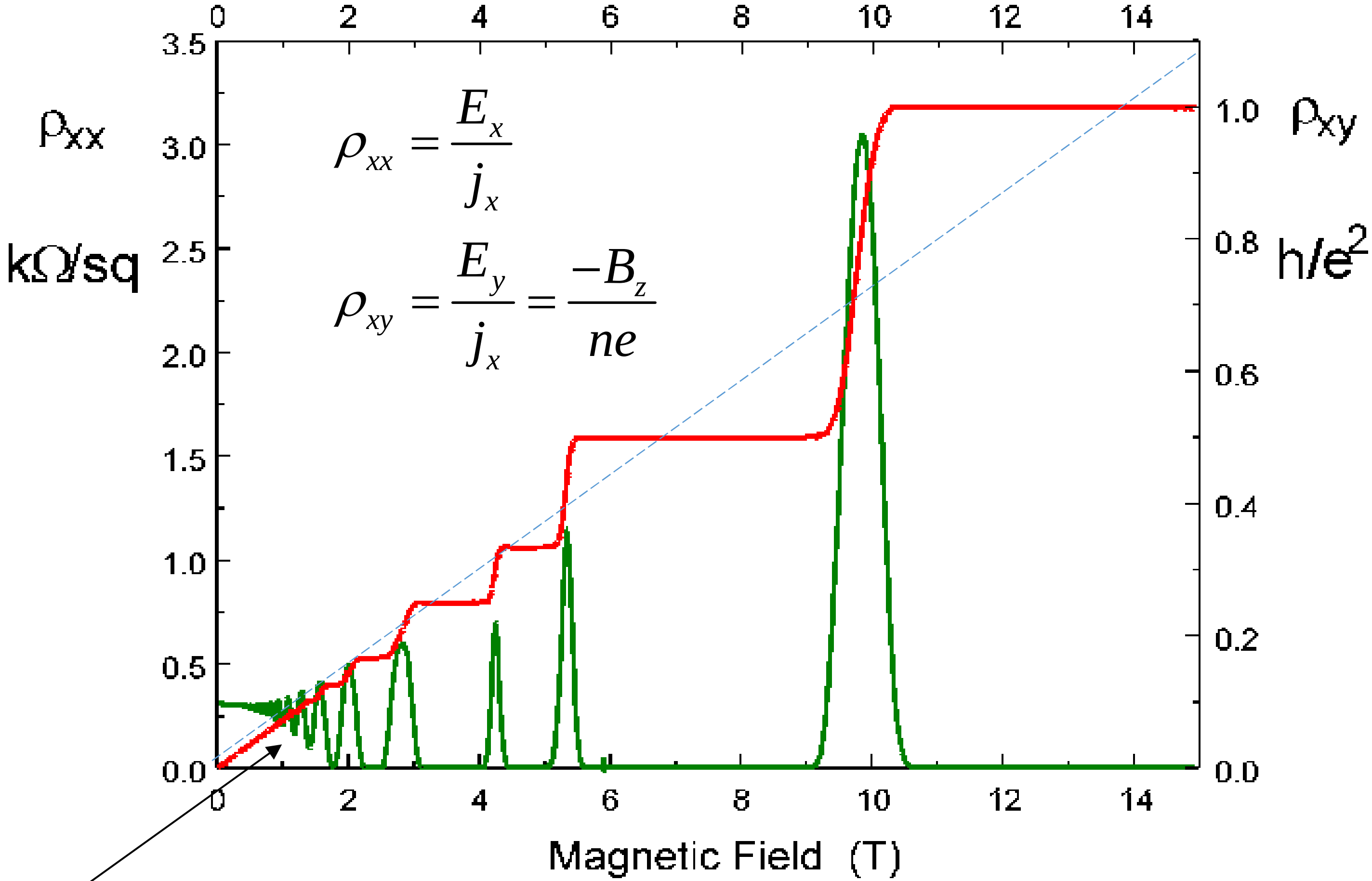
Quantum Hall Effect
Fermi Surfaces
Magnetism

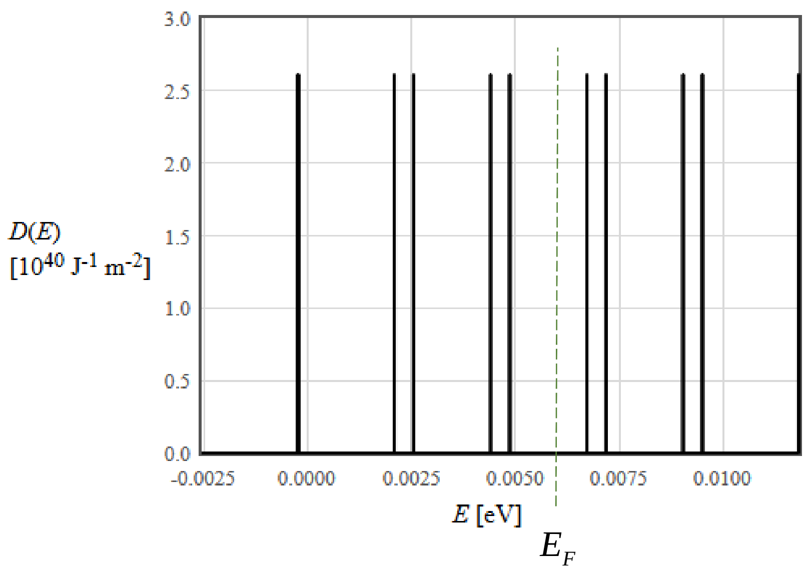
Quantum Hall Effect



Shubnikov-De Haas oscillations

Resistance standard
25812.807557(18) Ω

Quantum hall effect



If the Fermi energy is between Landau levels, the electron density *n* is an integer *v* times the degeneracy of the Landau level $n = D_0 v$

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$

Each Landau level can hold the same number of electrons.

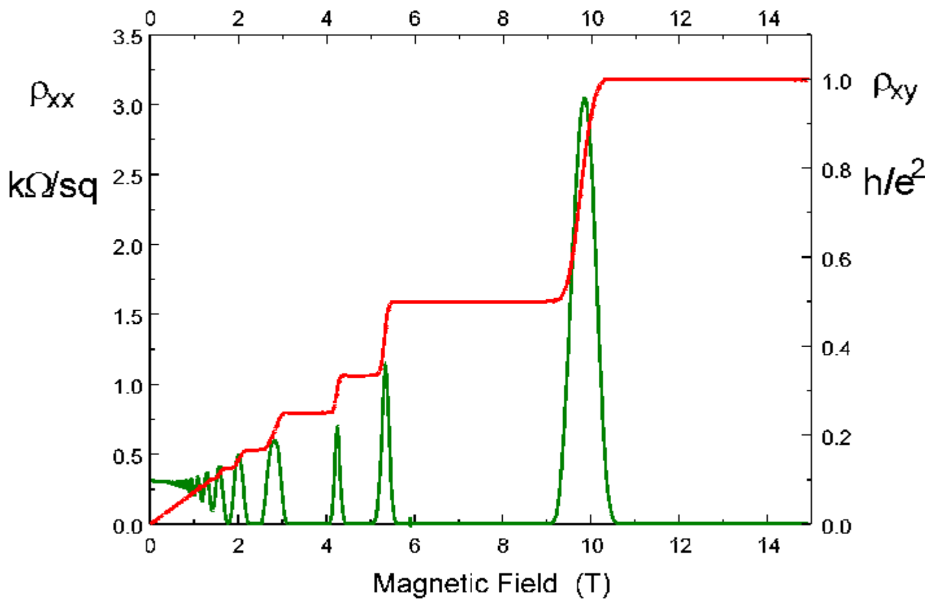
$$\rho_{xy} = \frac{-B_z}{ne} = \frac{-hD_0}{ve^2 D_0} = \frac{-h}{ve^2}$$

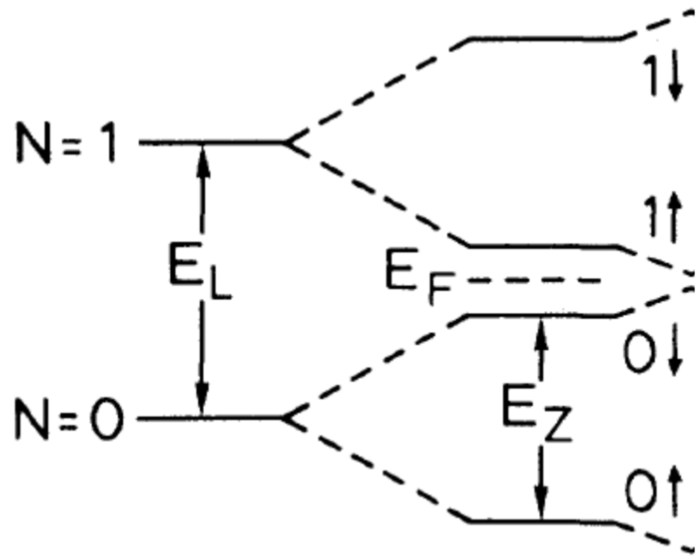
$$D_0 = \frac{m\omega_c}{2\pi\hbar} = \frac{eB_z}{h}$$

$$\omega_c = \frac{eB_z}{m} \qquad B_z = \frac{hD_0}{e}$$

Quantum hall effect

$$\rho_{xy} = \frac{h}{\nu e^2}$$



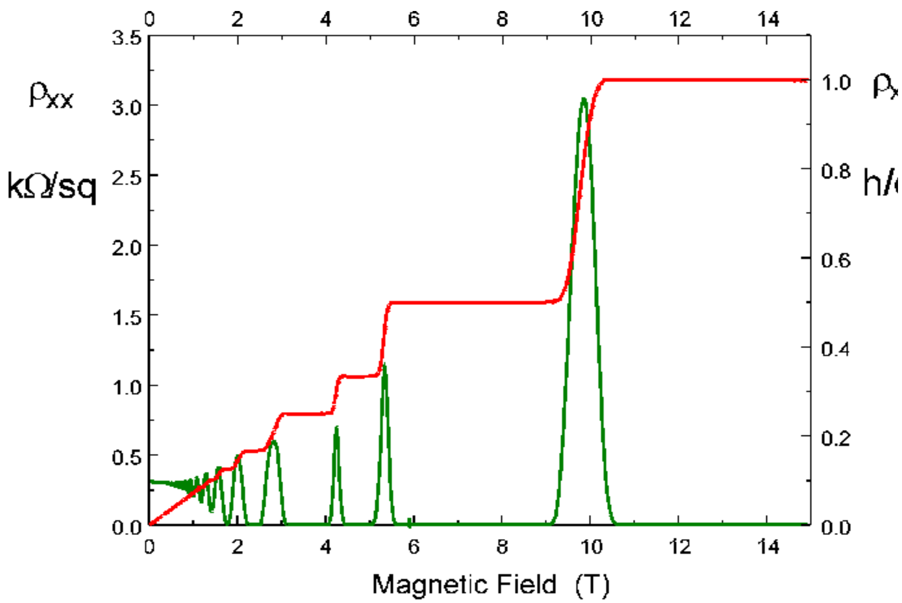


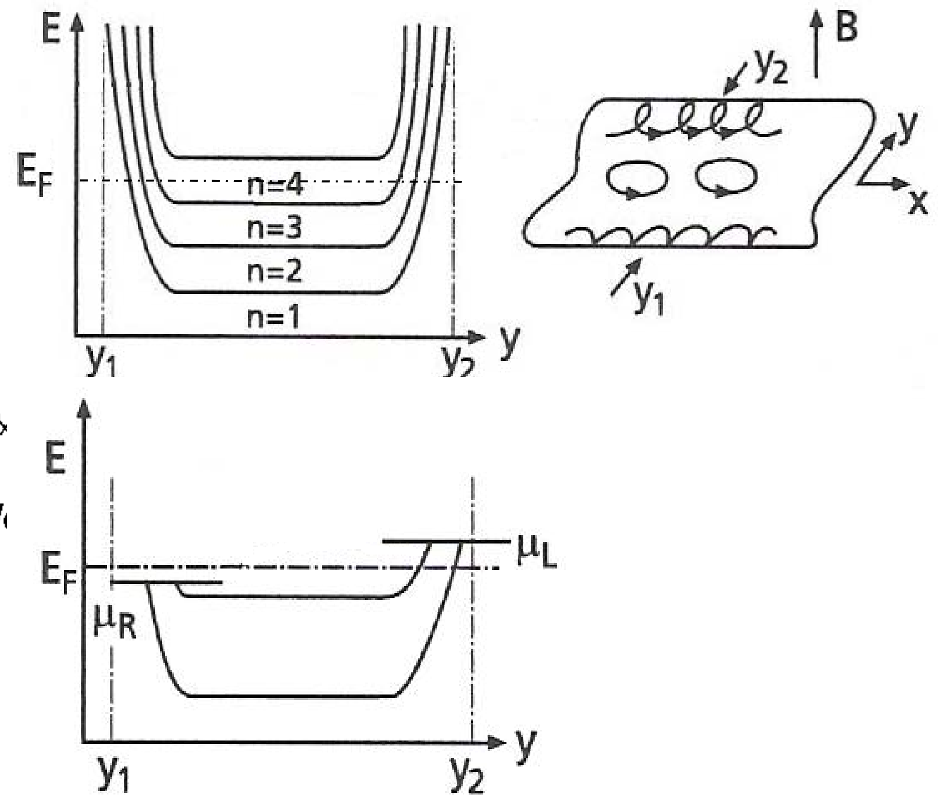
S. Koch, R. J. Haug, and K. v. Klitzing, Phys. Rev. B 47, 4048–4051 (1993)

Quantum Hall effect

Edge states are responsible for the zero resistance in ρ_{xx}

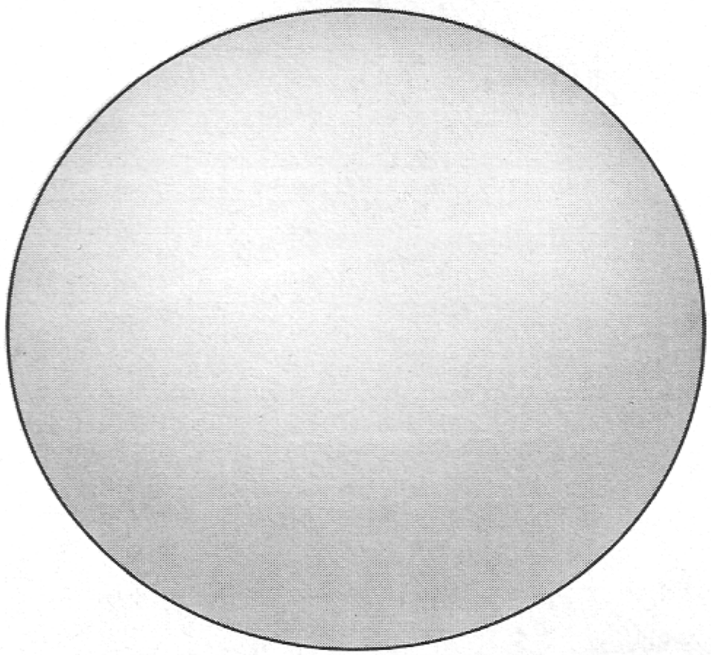
On the plateaus, resistance goes to zero because there are no states to scatter into.



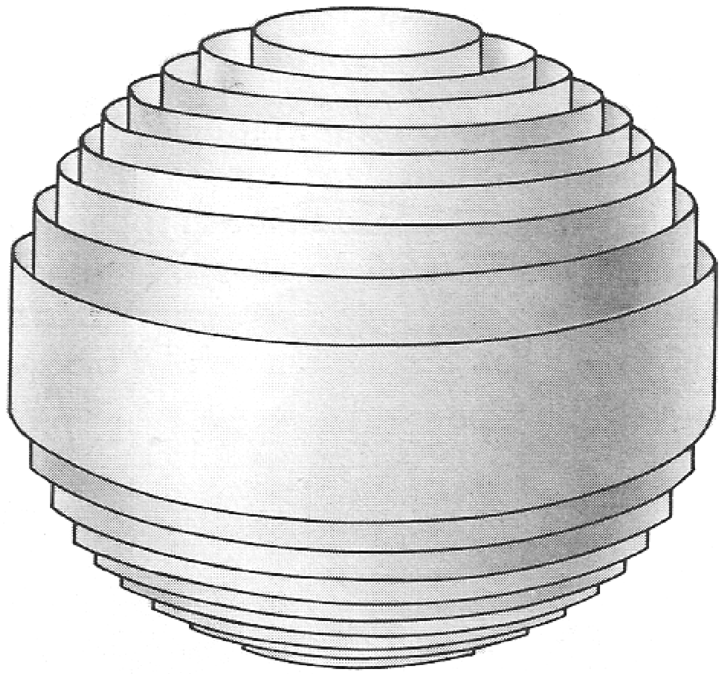


Ibach & Lueth (modified)

Fermi sphere in a magnetic field



$B = 0$

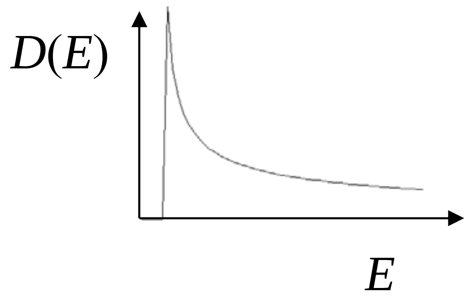


$B \neq 0$

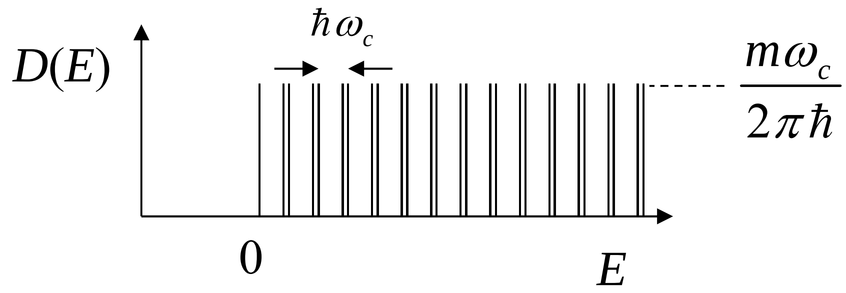
Landau cylinders

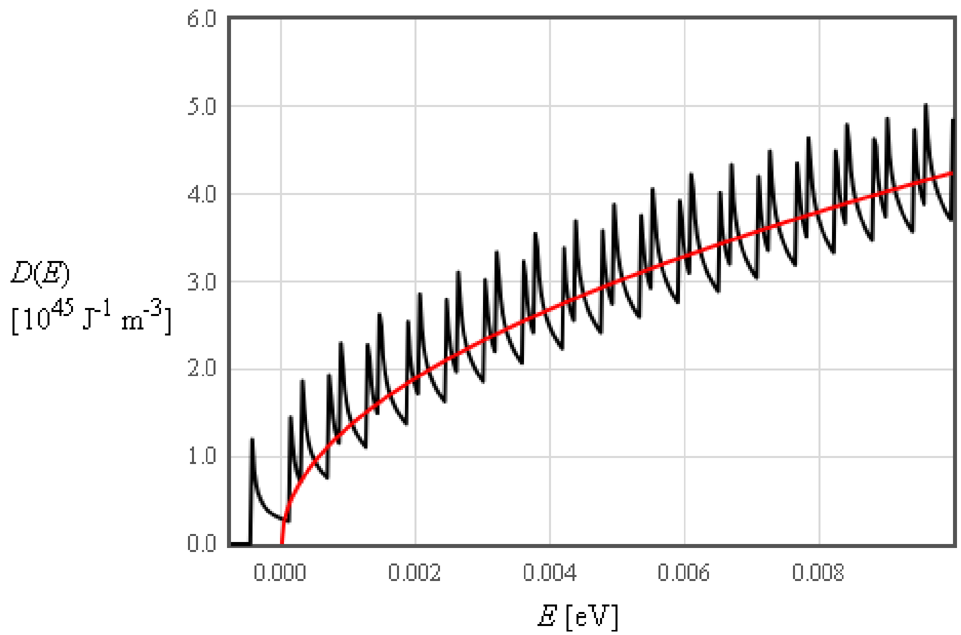
Density of states 3d

convolution of



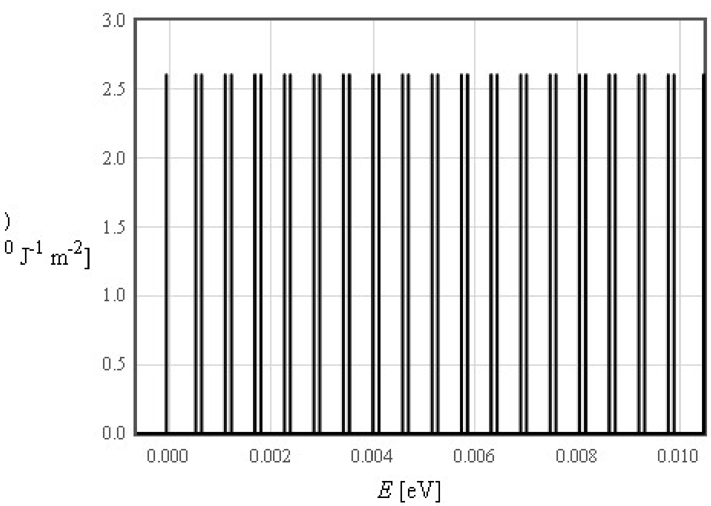
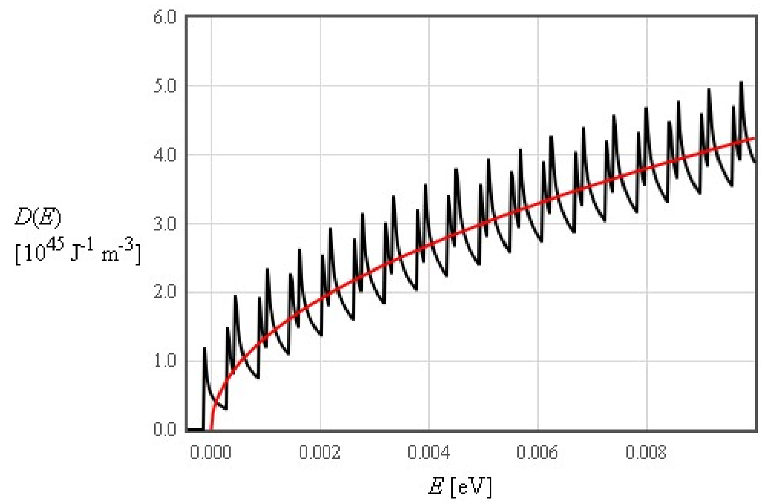
and





$$D(E)=\frac{(2m)^{3/2}\omega_c}{8\pi^2\hbar^2}\left(\sum_{\nu=0}^{\infty}\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)}}+\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)}}\right) \quad \mathrm{J^{-1}m^{-3}}$$

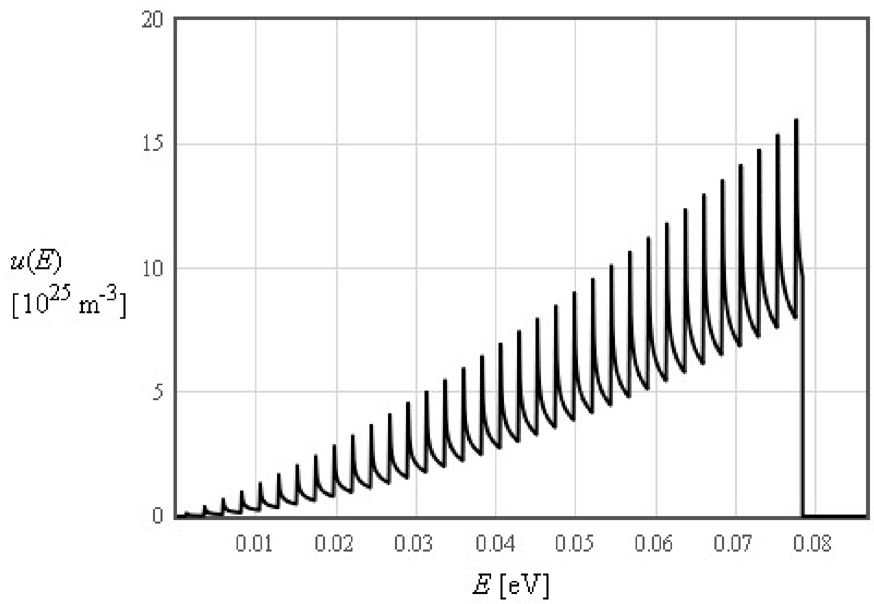
quation for free electrons a magnetic field in 2 and 3 dimensions.

2-D Schrödinger equation	3-D Schrödinger equation
$i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$	$i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$
$\psi=g_v(x)\exp\left(ik_y y\right)$ $g_v(x)$ is a harmonic oscillator wavefunction	$\psi=g_v(x)\exp\left(ik_y y\right)\exp\left(ik_z z\right)$ $g_v(x)$ is a harmonic oscillator wavefunction
$E=\hbar\omega_c\left(v+\frac{1}{2}\right)$ J $v=0,1,2,\cdots$ $\quad\omega_c=\frac{\lvert eB_z\rvert}{m}$	$E=\frac{\hbar^2k_z^2}{2m}+\hbar\omega_c\left(v+\frac{1}{2}\right)$ J $v=0,1,2,\cdots$ $\quad\omega_c=\frac{\lvert eB_z\rvert}{m}$
$\sum_{v=0}^{\infty}\delta\left(E-\hbar\omega_c\left(v+\frac{1}{2}\right)-\frac{g\mu_B}{2}B\right)+\delta\left(E-\hbar\omega_c\left(v+\frac{1}{2}\right)+\frac{g\mu_B}{2}B\right)$ $\text{J}^{-1}\text{m}^{-2}$  Calculate DoS	$D(E)=\frac{(2m)^{3/2}\omega_c}{8\pi^2\hbar^2}\left(\sum_{v=0}^{\infty}\frac{H\left(E-\hbar\omega_c\left(v+\frac{1}{2}-g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(v+\frac{1}{2}-g/4\right)}}+\frac{H\left(E-\hbar\omega_c\left(v+\frac{1}{2}+g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(v+\frac{1}{2}+g/4\right)}}\right)$ $\text{J}^{-1}\text{m}^{-3}$  Calculate DoS
$E_F=\hbar\omega\left(\text{Int}\left(\frac{\pi\hbar n}{\cdots}\right)+\frac{1}{\cdots}\right)$	

Energy spectral density 3d

At $T = 0$

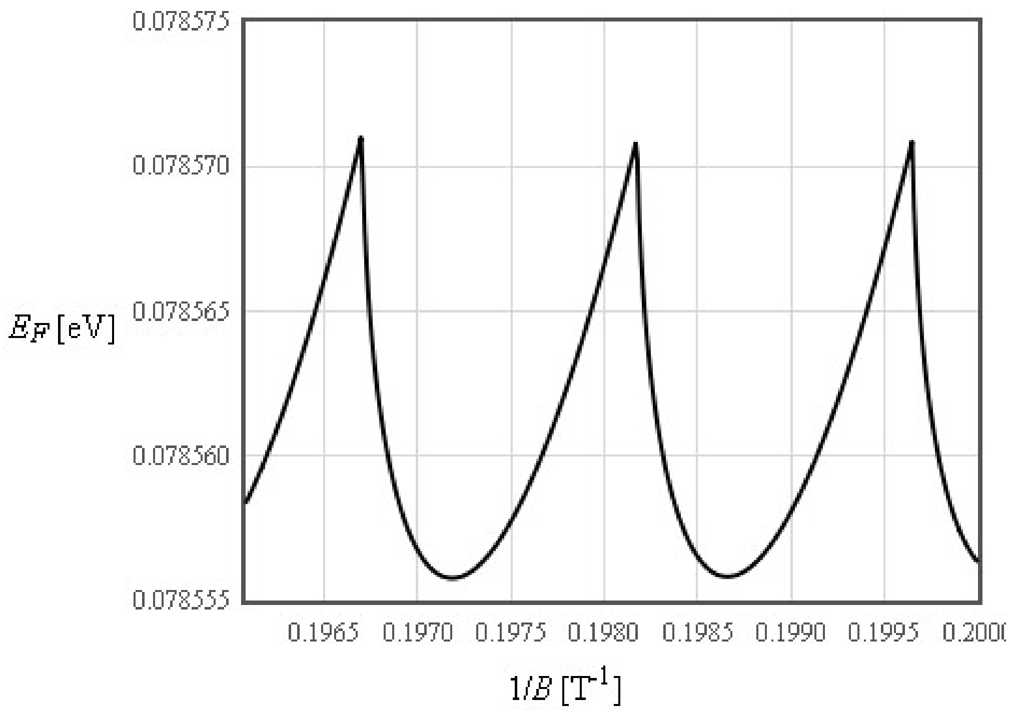
$$u(E) = ED(E)\,f(E)$$



$$u(T=0) = \int_{-\infty}^{E_F} ED(E)\,dE$$

Fermi energy 3d

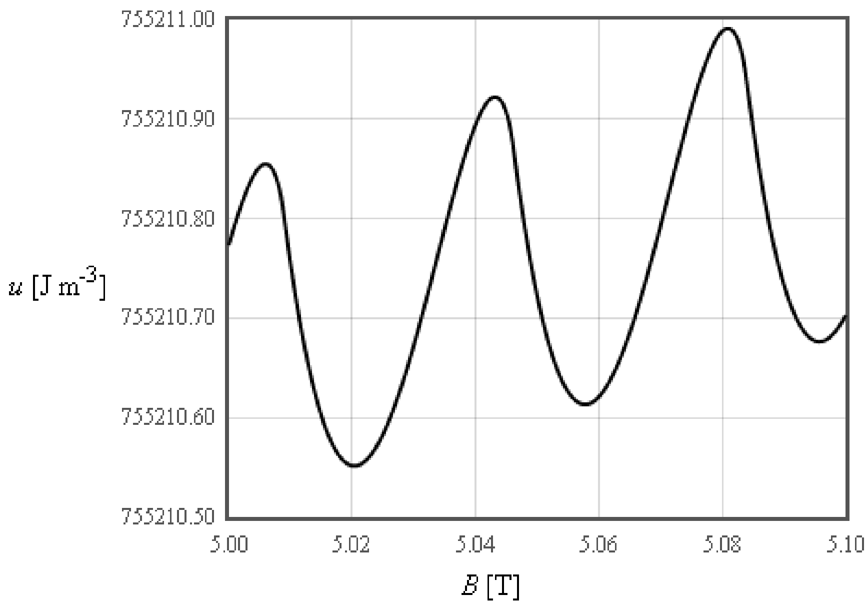
$$n = \int_{-\infty}^{E_F} D(E)\,dE$$



Periodic in $1/B$

Internal energy 3d

$$u = \int_{-\infty}^{E_F} E D(E) dE$$



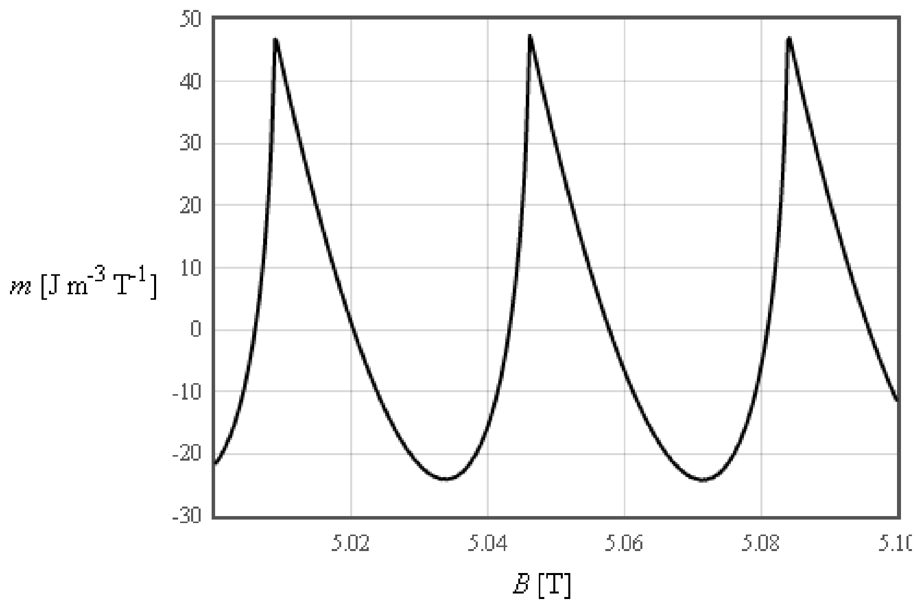
At $T = 0$

$$u = \frac{(2m)^{3/2}\omega_c}{4\pi^2\hbar^2} \sum_{\nu=0}^{\nu<\frac{E_F}{\hbar\omega_c}-\frac{1}{2}} \int_{\hbar\omega_c(\nu+\frac{1}{2})}^{E_F} \frac{E dE}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}\right)}} \quad \text{J m}^{-3}$$

$$u = \frac{(2m)^{3/2}\omega_c}{6\pi^2\hbar^2} \sum_{\nu=0}^{\nu<\frac{E_F}{\hbar\omega_c}-\frac{1}{2}} \left(2\hbar\omega_c\left(\nu+\frac{1}{2}\right)+E_F\right)\sqrt{E_F-\hbar\omega_c\left(\nu+\frac{1}{2}\right)} \quad \text{J m}^{-3}$$

Magnetization 3d

$$m = -\frac{du}{dB}$$



Periodic in $1/B$

At finite temperatures this function would be smoother

de Haas - van Alphen oscillations

Practically all properties are periodic in 1/*B*

Internal energy

$$u = \int_{-\infty}^{\infty} E D(E) f(E) dE$$

Specific heat

$$c_v = \left(\frac{\partial u}{\partial T}\right)_{V=const}$$

Entropy

$$s = \int \frac{c_v}{T} dT$$

Helmholtz free energy

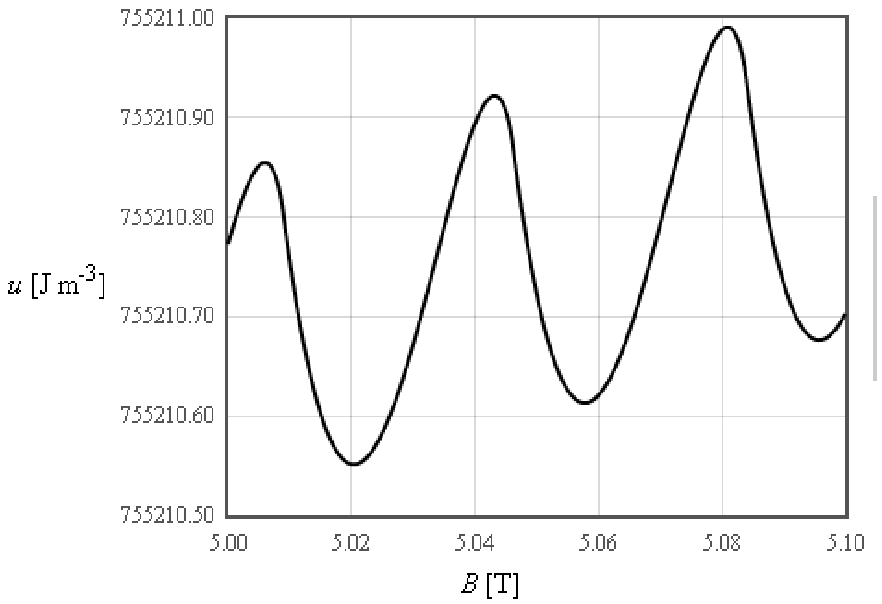
$$f = u - Ts$$

Pressure

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T=const}$$

Bulk modulus

$$B = -V \frac{\partial P}{\partial V}$$

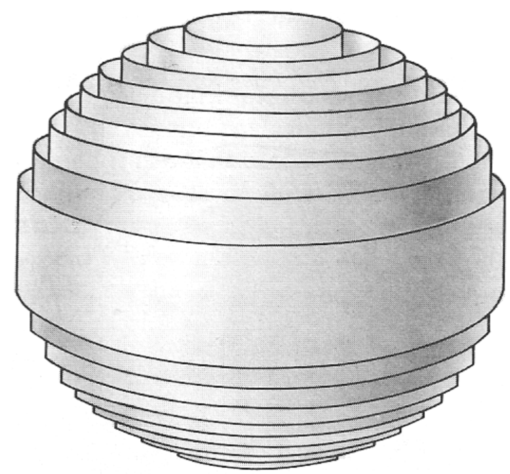


Magnetization

$$M = -\frac{dU}{dH}$$

Fermi sphere in a magnetic field

Cross sectional area $S = \pi k_F^2$



$$\hbar\omega_c\left(v + \tfrac{1}{2}\right) = \frac{\hbar^2 k_F^2}{2m}$$

$$\hbar\frac{eB_v}{m}\left(v + \tfrac{1}{2}\right) = \frac{\hbar^2 k_F^2}{2m}$$

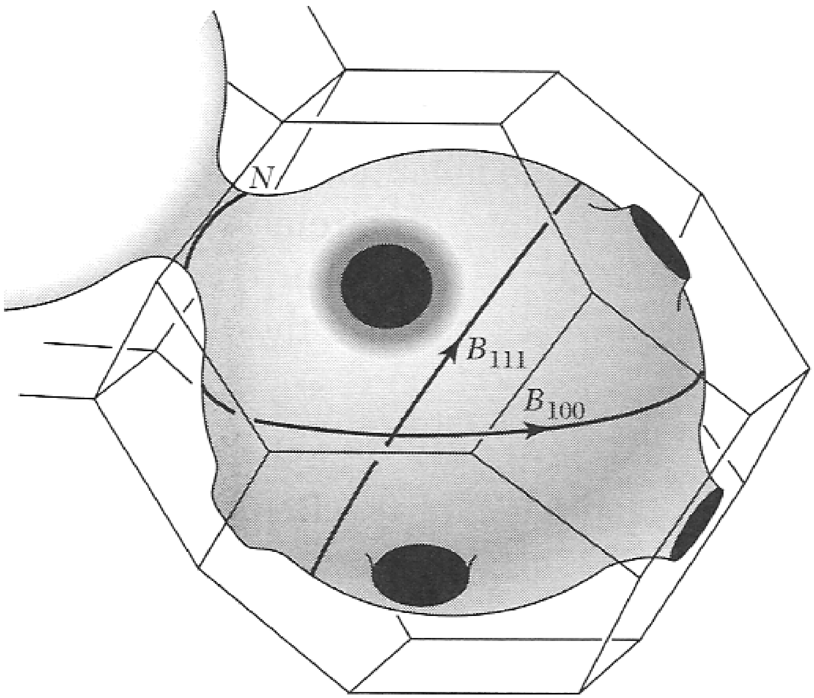
$$\frac{2\pi e}{\hbar}\left(v + 1 + \tfrac{1}{2}\right) = \frac{S}{B_{v+1}} \qquad \frac{2\pi e}{\hbar}\left(v + \tfrac{1}{2}\right) = \frac{S}{B_v}$$

Subtract right from left

$$S\left(\frac{1}{B_{v+1}} - \frac{1}{B_v}\right) = \frac{2\pi e}{\hbar}$$

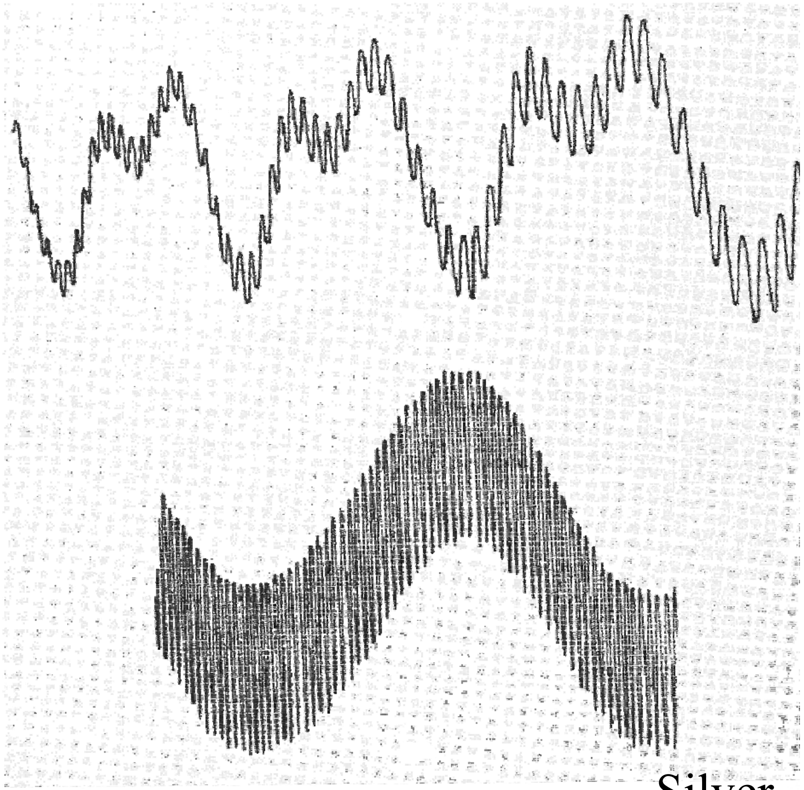
From the periodic of the oscillations, you can determine the cross sectional area *S*.

Experimental determination of the Fermi surface



Kittel

Rhenium

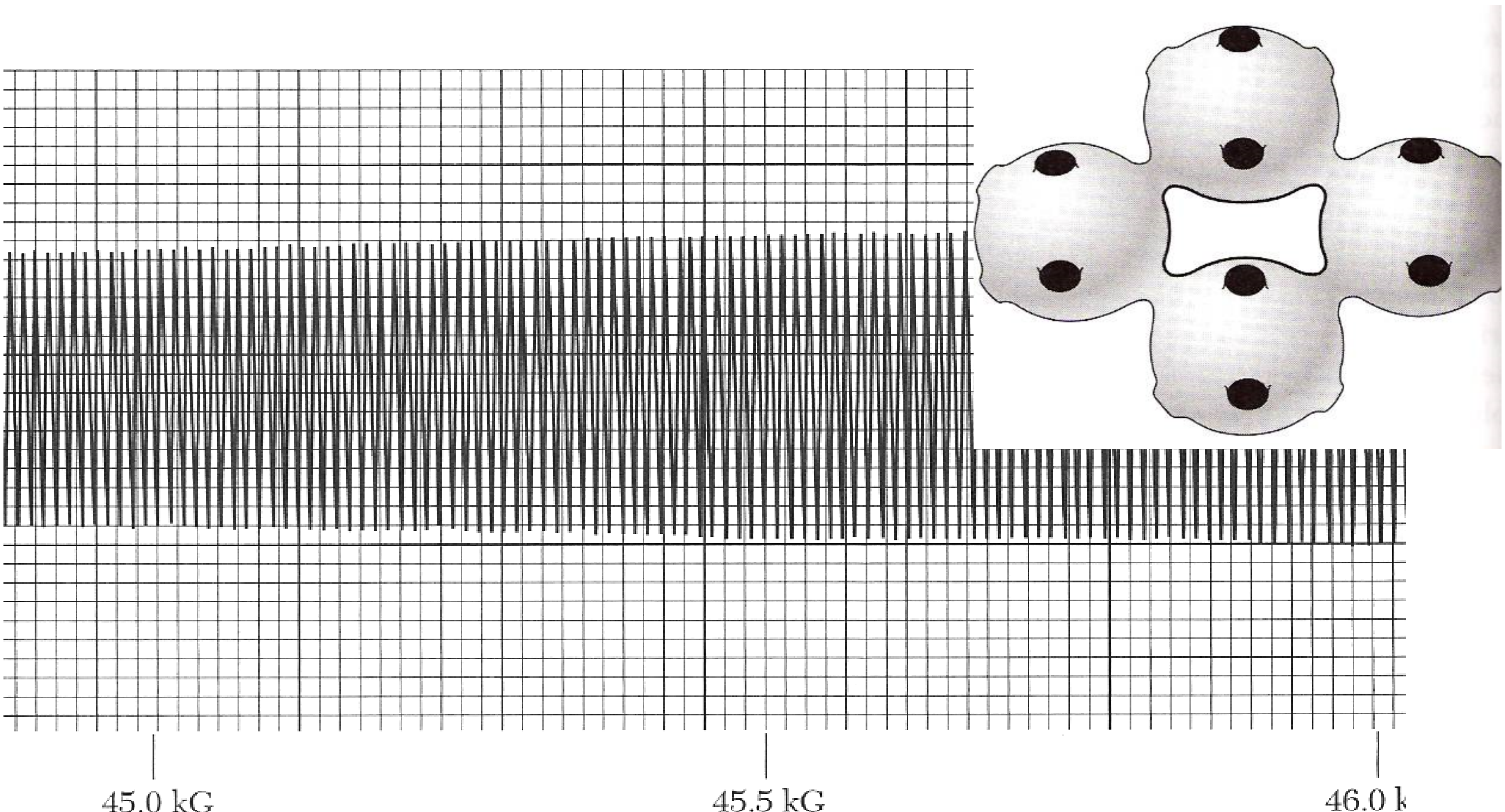


Silver

de Haas - van Alphen

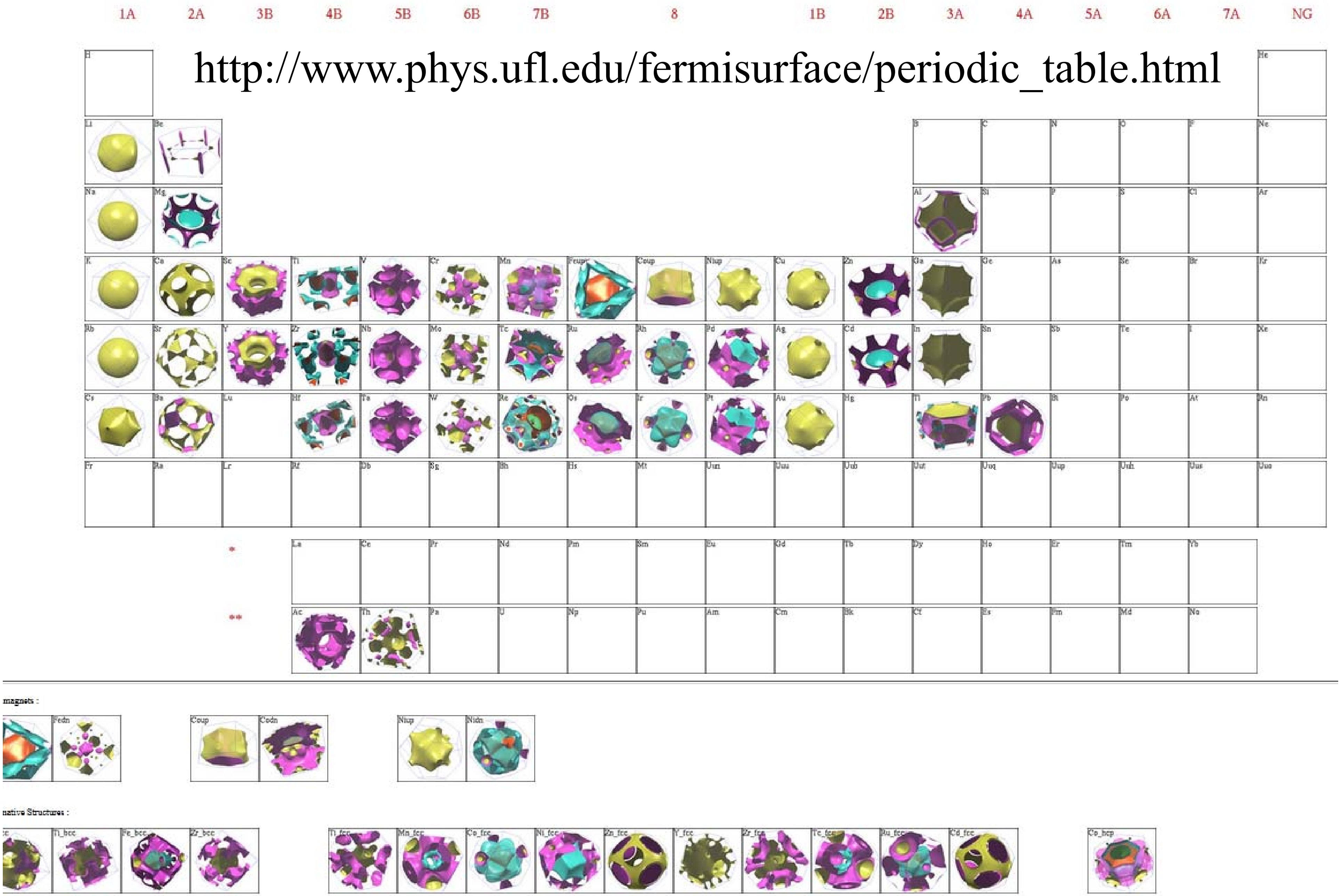
De Haas - van Alphen effect

The magnetic moment of gold oscillates periodically with $1/B$



Kittel

http://www.phys.ufl.edu/fermisurface/periodic_table.html



Magnetism

diamagnetism
paramagnetism
ferromagnetism (Fe, Ni, Co)
ferrimagnetism (Magneteisenstein)
antiferromagnetism

$$H = -\sum_i \frac{\hbar^2}{2m_e}\nabla_i^2 - \sum_A \frac{\hbar^2}{2m_A}\nabla_A^2 - \sum_{i,A}\frac{Z_A e^2}{4\pi\varepsilon_0 r_{iA}} + \sum_{i<j}\frac{e^2}{4\pi\varepsilon_0 r_{ij}} + \sum_{A<B}\frac{Z_A Z_B e^2}{4\pi\varepsilon_0 r_{AB}}$$

Coulomb interactions cause ferromagnetism not magnetic interactions.

Magnetism

magnetic intensity

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right)$$

magnetic induction field

magnetization

$$\vec{M} = \chi \vec{H}$$

χ is the magnetic susceptibility

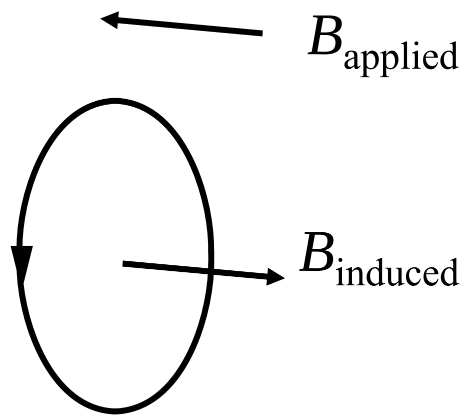
$\chi < 0$ diamagnetic

$\chi > 0$ paramagnetic

χ is typically small (10^{-5}) so $B \approx \mu_0 H$

Diamagnetism

A free electron in a magnetic field will travel in a circle



The magnetic created by the current loop is opposite the applied field.

Diamagnetism

Dissipationless currents are induced in a diamagnet that generate a field that opposes an applied magnetic field.

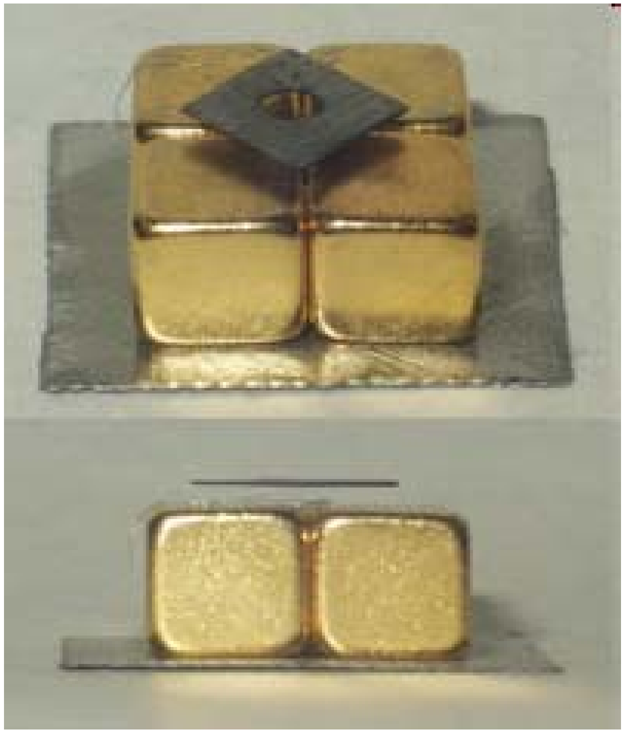
Current flow without dissipation is a quantum effect. There are no lower lying states to scatter into. This creates a current that generates a field that opposes the applied field.

$\chi = -1$ superconductor (perfect diamagnet)

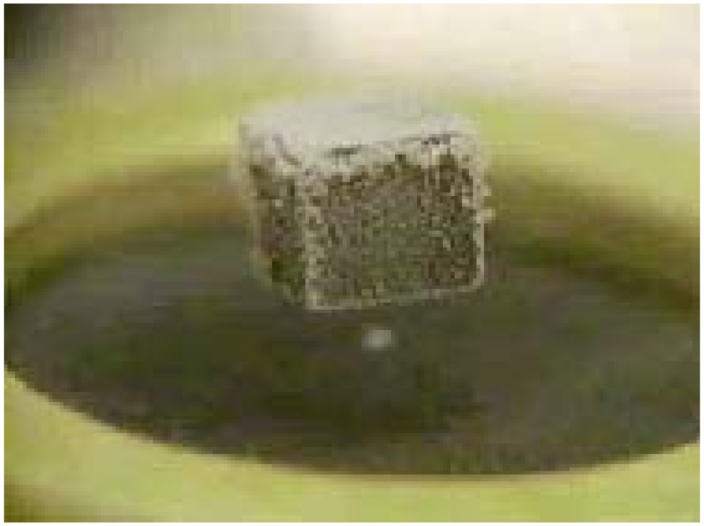
$\chi \sim -10^{-6} - 10^{-5}$ normal materials

Diamagnetism is always present but is often overshadowed by some other magnetic effect.

Levitating diamagnets



Levitating pyrolytic carbon



NOT: Lenz's law

$$V = -\frac{d\Phi}{dt}$$

Levitating frogs

χ for water is -9.05×10^{-6}

16 Tesla magnet at the Nijmegen High Field Magnet Laboratory

http://www.hfml.ru.nl/froglev.html

Andre Geim

2000 Ig Nobel Prize for levitating a frog with a magnet

The Nobel Prize in Physics 2010
Andre Geim, Konstantin Novoselov

The Nobel Prize in Physics 2010

Nobel Prize Award Ceremony

Andre Geim

Biographical
Nobel Lecture
Banquet Speech
Interview
Nobel Diploma
Photo Gallery
Other Resources

Konstantin Novoselov

Andre Geim

Born: 1958, Sochi, Russia

Affiliation at the time of the award: University of Manchester, Manchester, United Kingdom

Prize motivation: "for groundbreaking experiments regarding the two-dimensional material graphene"

Diamagnetism

A dissipationless current is induced by a magnetic field that opposes the applied field.

$$\vec{M} = \chi \vec{H}$$

Diamagnetic susceptibility	
Copper	-9.8×10^{-6}
Diamond	-2.2×10^{-5}
Gold	-3.6×10^{-5}
Lead	-1.7×10^{-5}
Nitrogen	-5.0×10^{-9}
Silicon	-4.2×10^{-6}
water	-9.0×10^{-6}
bismuth	-1.6×10^{-4}

Most stable molecules have a closed shell configuration and are diamagnetic.

Paramagnetism

Materials that have a magnetic moment are paramagnetic.

An applied field aligns the magnetic moments in the material making the field in the material larger than the applied field.

The internal field is zero at zero applied field (random magnetic moments).

$$\vec{M} = \chi \vec{H}$$

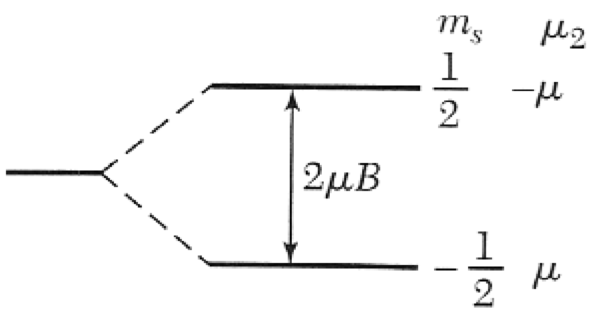
Paramagnetic susceptibility	
Aluminum	2.3×10^{-5}
Calcium	1.9×10^{-5}
Magnesium	1.2×10^{-5}
Oxygen	2.1×10^{-6}
Platinum	2.9×10^{-4}
Tungsten	6.8×10^{-5}

Boltzmann factors

To take the average value of quantity A

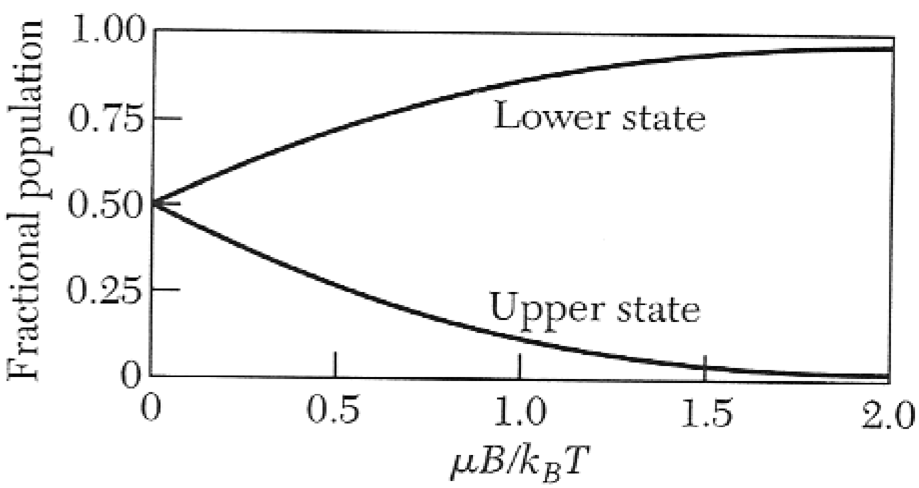
$$\langle A \rangle = \frac{\sum_i A_i e^{-E_i / k_B T}}{\sum_i e^{-E_i / k_B T}}$$

Spin populations



$$\frac{N_1}{N} = \frac{\exp(\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$\frac{N_2}{N} = \frac{\exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

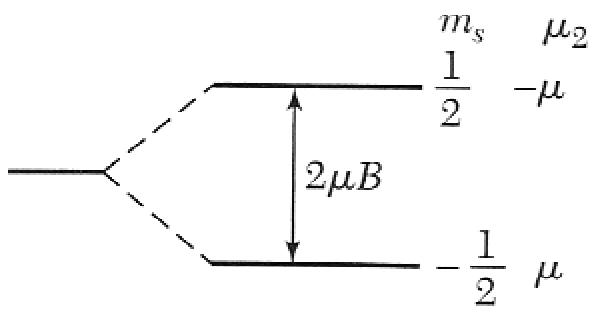


$$M = (N_1 - N_2)\mu$$

$$= N\mu \frac{\exp(\mu B / k_B T) - \exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$= N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$$

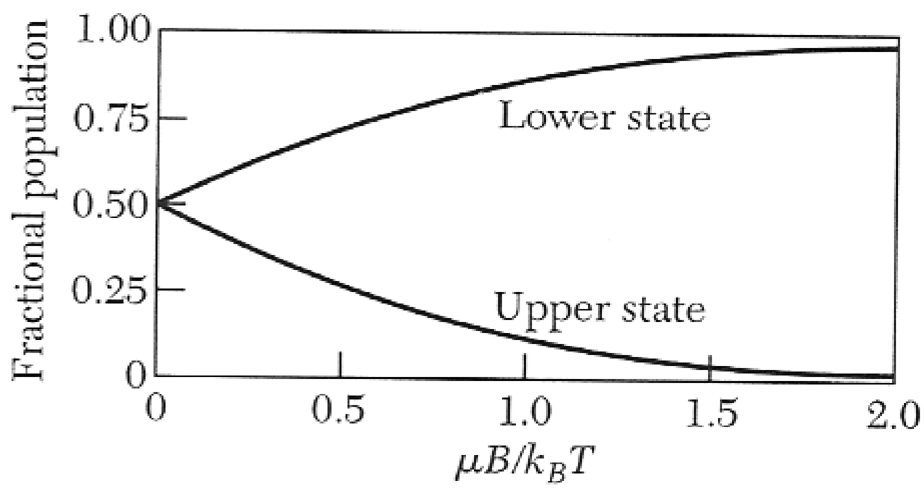
Paramagnetism, spin 1/2

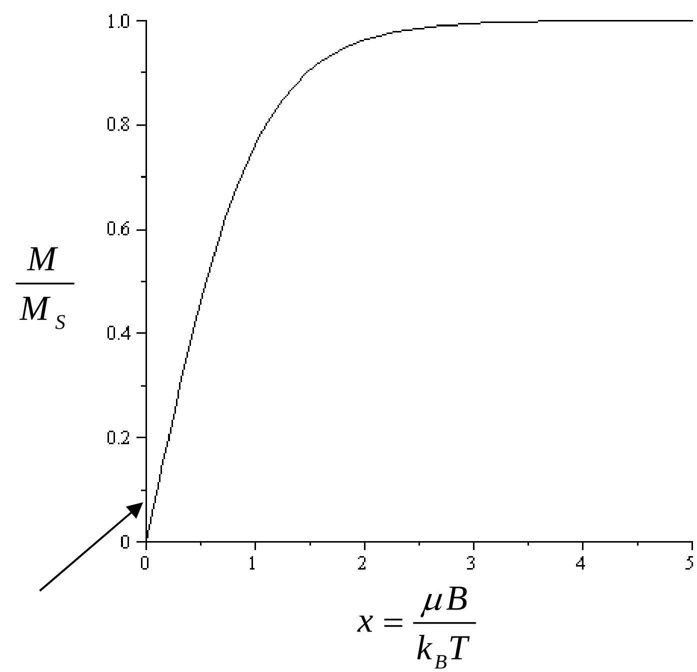


$$M = N\mu \tanh\left(\frac{\mu B}{k_B T}\right) \approx \frac{N\mu^2 B}{k_B T} = \frac{CB}{T}$$

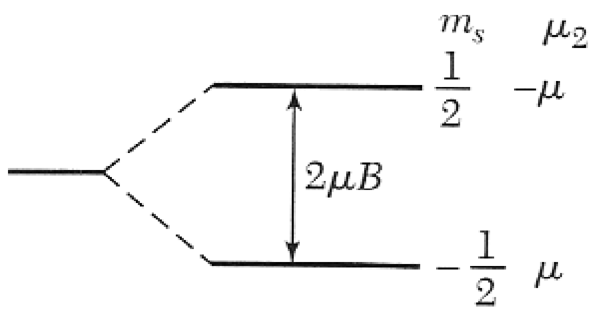
for $\mu B << k_B T$

Curie law





Paramagnetism, spin 1/2



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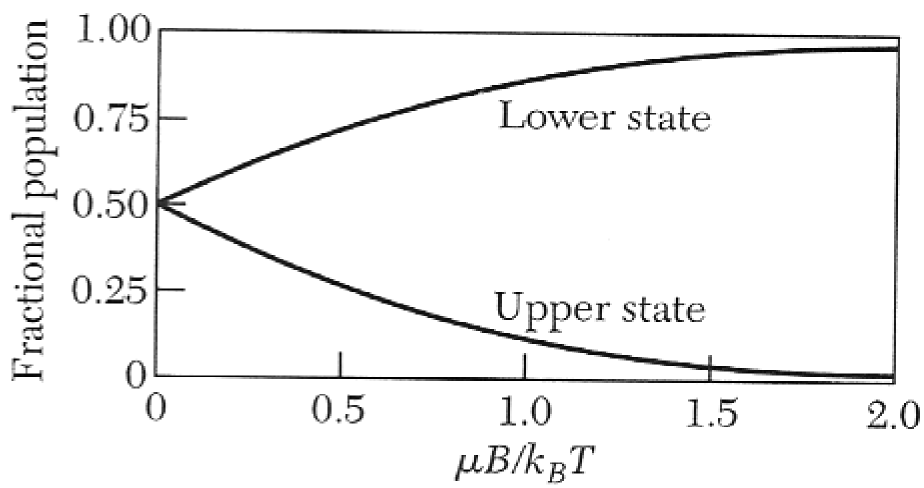
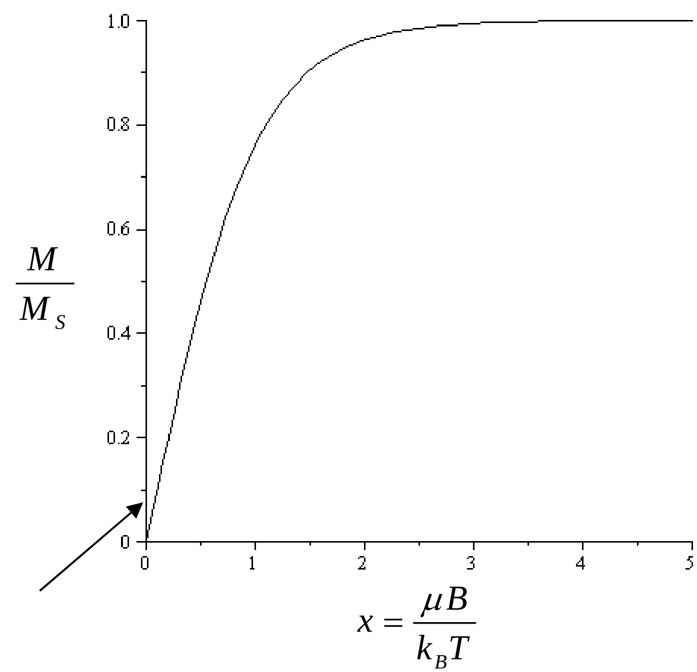
for $\mu B << k_B T$

Curie law





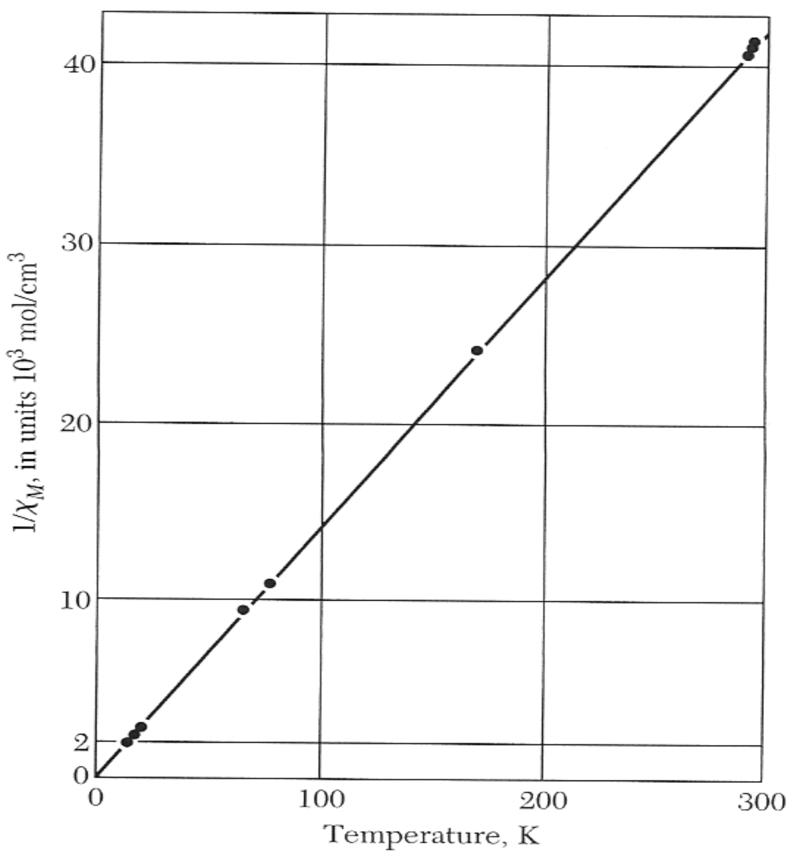
Curie law

for $\mu B << k_B T \quad M = CB/T$

$$\chi \propto \left.\frac{dM}{dB}\right|_{B=0} = \frac{C}{T}$$

C is the Curie constant

$Gd(C_2H_3SO_4)_3 9H_2O$



Atomic physics

In atomic physics, the possible values of the magnetic moment of an atom in the direction of the applied field can only take on certain values.

Total angular momentum

$J = L + S$ Orbital L + spin S angular momentum

Magnetic quantum number

$$m_J = -J, -J+1, \cdots J-1, J$$

Allowed values of the magnetic moment in the z direction

$$\mu_Z = m_j g_J \mu_B$$

Lande g factor → g_J

Bohr magneton → μ_B

$$g_J \approx \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

Brillouin functions

Average value of the magnetic quantum number

$$\langle m_J \rangle = \frac{\sum_{-J}^{J} m_J e^{-E(m_J)/k_B T}}{\sum_{-J}^{J} e^{-E(m_J)/k_B T}} = \frac{\sum_{-J}^{J} m_J e^{-m_J g_J \mu_B B/k_B T}}{\sum_{-J}^{J} e^{-m_J g_J \mu_B B/k_B T}} = -\frac{1}{Z}\frac{dZ}{dx}$$

Lande g factor

Bohr magneton

$$x = g_J \mu_B B / k_B T$$

$$Z = \sum_{-J}^{J} e^{-m_J x} = \frac{\sinh\left(\left(2J+1\right)\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)}$$

check by synthetic division

Brillouin functions

$$Z = \sum_{-J}^{J} e^{-m_J x} = \frac{\sinh\left(\left(2J+1\right)\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)}$$

$J = 1/2$

$$\begin{array}{r} e^{\frac{x}{2}} + e^{-\frac{x}{2}} \\ e^{\frac{x}{2}} - e^{-\frac{x}{2}} \overline{\big)\, e^{x} - e^{-x}} \\ e^{x} - 1 \\ \hline 1 - e^{-x} \\ 1 - e^{-x} \\ \hline 0 \end{array}$$

$$M = N g_J \mu_B \left\langle m_J \right\rangle = -N g_J \mu_B \frac{1}{Z} \frac{dZ}{dx}$$

Brillouin function

$$M = N g \mu_B J \left(\frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} \frac{g \mu_B J B}{k_B T} \right) - \frac{1}{2J} \coth\left(\frac{1}{2J} \frac{g \mu_B J B}{k_B T} \right) \right)$$

Hund's rules from atomic physics

Hund calculated the energies of atomic states:

$$\frac{\langle \psi_{Ne3s} | H | \psi_{Ne3s} \rangle}{\langle \psi_{Ne3s} | \psi_{Ne3s} \rangle} < \frac{\langle \psi_{Ne3p} | H | \psi_{Ne3p} \rangle}{\langle \psi_{Ne3p} | \psi_{Ne3p} \rangle} < \frac{\langle \psi_{Ar4s} | H | \psi_{Ar4s} \rangle}{\langle \psi_{Ar4s} | \psi_{Ar4s} \rangle} < \frac{\langle \psi_{Ne3d} | H | \psi_{Ne3d} \rangle}{\langle \psi_{Ne3d} | \psi_{Ne3d} \rangle}$$

H includes *e*-*e* interactions

He formulated the following rules:

Electrons fill atomic orbitals following these rules:

1. Maximize the total spin *S* allowed by the exclusion principle

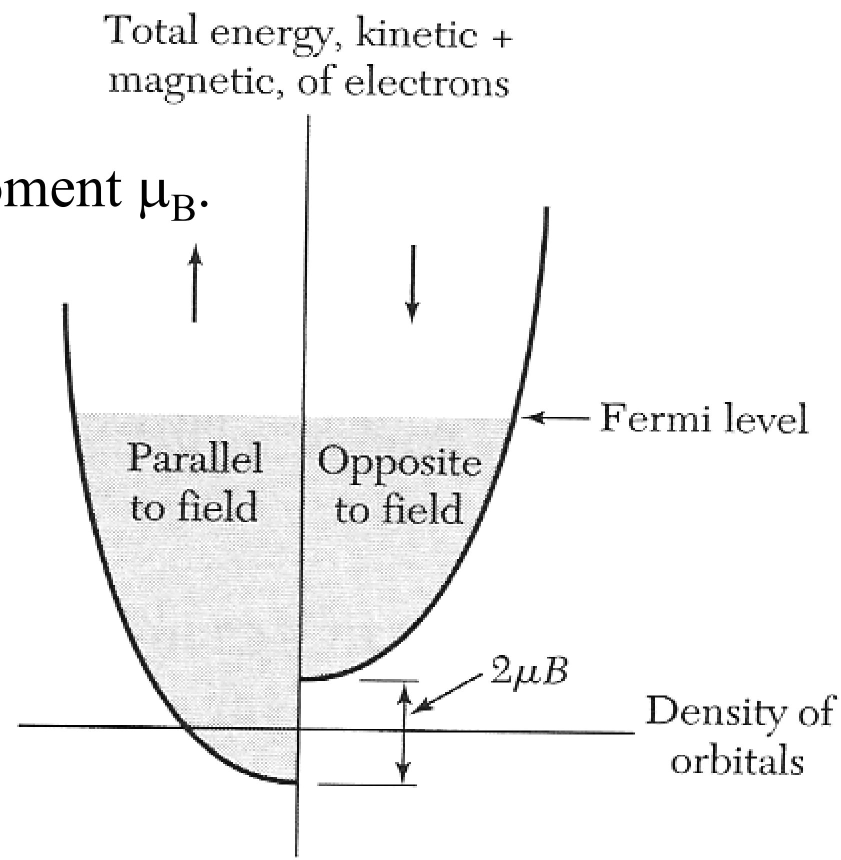
2. Maximize the orbital angular momentum *L*

3. $J=|L-S|$ when the shell is less than half full, $J=|L+S|$ when the shell is more than half full.

Pauli paramagnetism

Paramagnetic contribution due to free electrons.

Electrons have an intrinsic magnetic moment μ_B.



$$n_+ \approx \frac{1}{2}n + \frac{1}{2}\mu_B B D(E_F)$$

$$n_- \approx \frac{1}{2}n - \frac{1}{2}\mu_B B D(E_F)$$

$$M = \mu_B(n_+ - n_-)$$

$$M = \mu_B^2 D(E_F) B = \mu_0 \mu_B^2 D(E_F) H$$

$$\chi = \frac{dM}{dH} = \mu_0 \mu_B^2 D(E_F)$$

If E_F is 1 eV, a field of B = 17000 T is needed to align all of the spins.

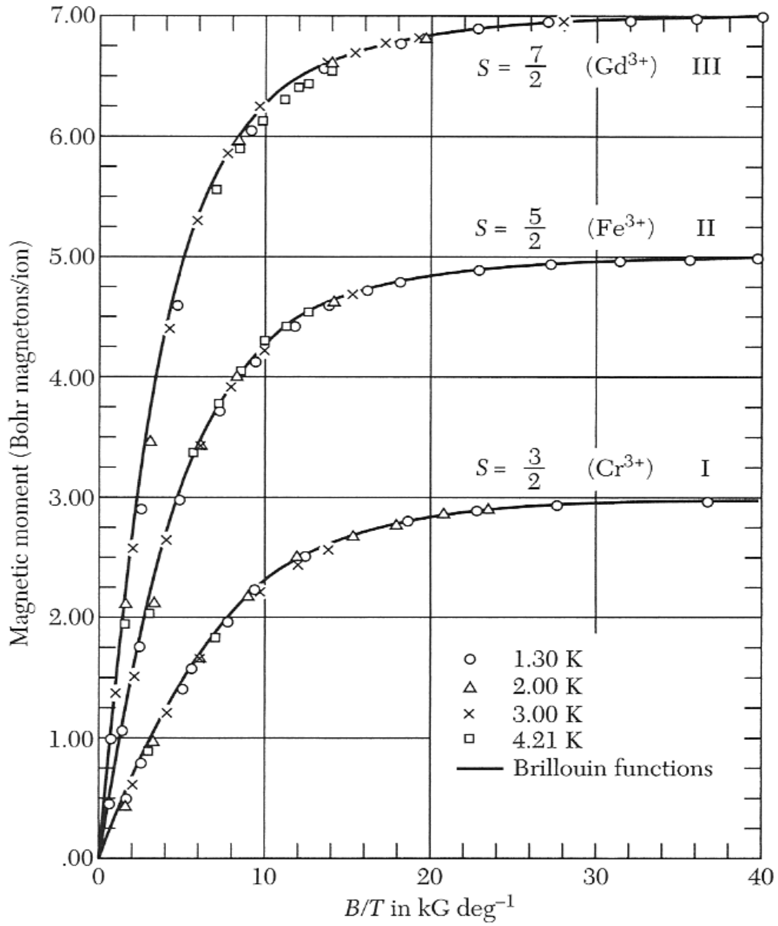
Pauli paramagnetism is much smaller than the paramagnetism due to atomic moments and almost temperature independent because $D(E_F)$ doesn't change very much with temperature.

Hund's rules (f - shell)

n	$l_z = 3,$	$2,$	$1,$	$0,$	$-1,$	$-2,$	-3	S	$L = \lvert\Sigma l_z\rvert$	J	
1	↓							1/2	3	5/2	$J = \lvert L - S\rvert$
2	↓	↓						1	5	4	
3	↓	↓	↓					3/2	6	9/2	
4	↓	↓	↓	↓				2	6	4	
5	↓	↓	↓	↓	↓			5/2	5	5/2	
6	↓	↓	↓	↓	↓	↓		3	3	0	
7	↓	↓	↓	↓	↓	↓	↓	7/2	0	7/2	
8	↓↑	↑	↑	↑	↑	↑	↑	3	3	6	$J = L + S$
9	↓↑	↓↑	↑	↑	↑	↑	↑	5/2	5	15/2	
10	↓↑	↓↑	↓↑	↑	↑	↑	↑	2	6	8	
11	↓↑	↓↑	↓↑	↓↑	↑	↑	↑	3/2	6	15/2	
12	↓↑	↓↑	↓↑	↓↑	↓↑	↑	↑	1	5	6	
13	↓↑	↓↑	↓↑	↓↑	↓↑	↓↑	↑	1/2	3	7/2	
14	↓↑	↓↑	↓↑	↓↑	↓↑	↓↑	↓↑	0	0	0	

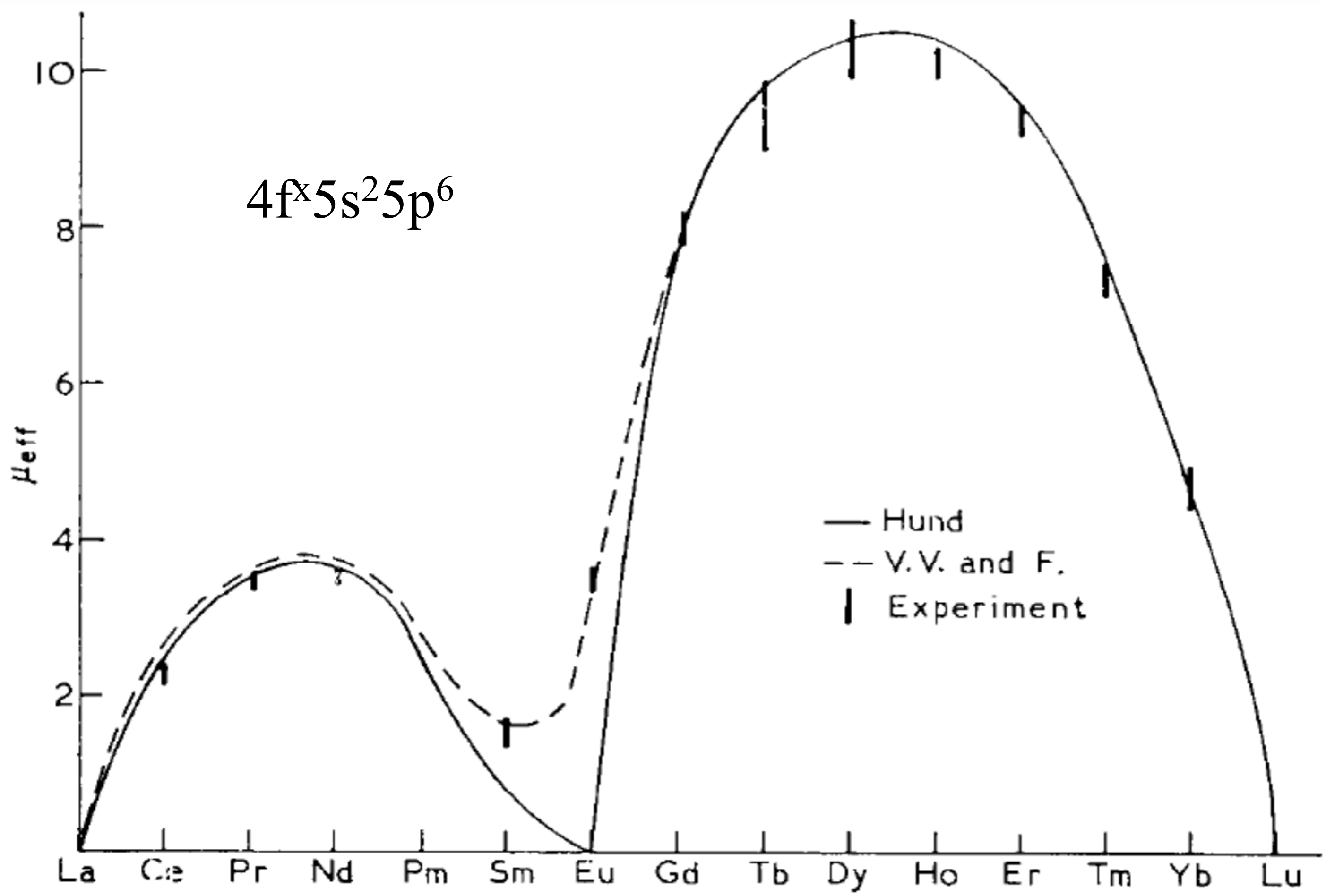
The half filled shell and completely filled shell have zero total angular mo

Paramagnetism



$$M = Ng\mu_B J\left(\frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}\frac{g\mu_B JB}{k_B T}\right)-\frac{1}{2J}\coth\left(\frac{1}{2J}\frac{g\mu_B JB}{k_B T}\right)\right)$$

Quantum Mechanics: The Key to Understanding Magnetism John H. van Vleck



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